Resonance Y-shape Solitons and Mixed Solutions for a (2+1)-dimensional Generalized Caudrey-Dodd-Gibbon-Kotera-Sawada Equation in Fluid Mechanics

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Resonance Y-shape solitons and mixed solutions for a (2+1)-dimensional generalized Caudrey-Dodd-Gibbon-Kotera-Sawada equation in fluid mechanics

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Abstract

Under the well-known bilinear method of Hirota, the specific expression for N-soliton solutions of (2+1)-dimensional generalized Caudrey-Dodd-Gibbon-Kotera-Sawada(gCDGKS) equation in fluid mechanics is given. By defining a noval restrictive condition on N-soliton solutions, resonant Y-type and X-type soliton solutions are generated. Under the previous new constraints, combined with the velocity resonance method and module resonant method, the mixed solutions of resonant Y-type solitons and line waves, breather solutions are found. Finally, with the support of long wave limit method, the interaction between resonant Y-type solitons and higher-order lumps is shown, and the motion trajectory equation before and after the interaction between lumps and resonant Y-type solitons is derived.

Keywords: (2+1)-dimensional gCDGKS equation; a new constrained condition; resonance Y-type soliton; hybrid solutions

1 Introduction

As a local nonlinear wave, solitons have many interesting properties [1]. By adding some constraints on the N-soliton solution obtained by Hirota bilinear method, one can obtain many types of bound molecules. Through the velocity resonance mechanism, the soliton molecular solution first observed in physical experiments is obtained [2–6]. By using the module resonant method, the breather solution which changes periodically with time can be gained [7, 8]. Through long wave limit approach, one can gain lump solution [9, 10]. By combining the above methods, one can also get a hybrid solution consisting of soliton molecules, breather solutions and lump solutions [11–16].

Resonant soliton is a special kind of soliton existing in integrable system, which has been widely studied. As we all know, If phase shift of the colliding solitons becomes infinite or converges to infinity, solitons will resonate [17]. The well-known soliton fusion and fission is a resonance phenomenon [18, 19]. When it comes to the resonant Y-type soliton solutions, people

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usually directly restrict N-soliton solution derived from Hirota bilinear method. At first, most people set the form of the solution [20, 21] to

$$u = aln(1 + \sum_{j=1}^{N} e^{k_j x + r_j y + w_j t + \phi_j})_{xx},$$

however, due to many formal constraints, the form of the solution is a little simple, so it is very laborious to obtain interactions between resonant Y-shaped solitons and other solutions. Later, Chen and others constructed another constraint, which obtained the mixed solutions of resonant Y-type soliton and 1-order lump [22]. The resonant multi soliton solution can be obtained by the linear superposition principle [23]. Recently, in order to obtain more interaction solutions between resonant Y-type solitons and other solutions, Li et al proposed a new constraint [24–27]. By this method, we can obtain the mixed solutions of resonant Y-shaped solitons and line waves, higher-order lumps, breather solutions and resonant Y-shaped solitons respectively.

A (2+1)-dimensional gCDGKS equation [28] is derived

$$\begin{aligned} &36u_t + (u_{4x} + 15uu_{xx} + 15u^3)_x - \alpha v_{yy} - \beta(u_{xxy} + 3uu_y + 3u_x v_y) = 0, \\ &u = v_x, \end{aligned} \tag{1}$$

where α and β are two arbitrary constants. For equation (1), Deng et al have obtained Nth-order Pfaffian and periodic wave solutions [29], Peng et al have also derived solitary and lump waves and their interaction phenomena [28].

In fluid mechanics, if $\alpha = 5, \beta = 5$, equation (1) will be reduced to (2+1)-dimensional CDGKS equation,

$$\begin{aligned} &36u_t + (u_{4x} + 15uu_{xx} + 15u^3)_x - 5v_{yy} - 5(u_{xxy} + 3uu_y + 3u_x v_y) = 0, \\ &u = v_x, \end{aligned}$$

which was first introduced in [30]. In addition, quasi-periodic solutions [31], diverse solitons, interaction solutions [32], rogue waves, generalized and classical lump solutions [33] of (2+1)-dimensional CDGKS equation have been obtained.

If $\alpha = 5, \beta = 5$ and $u_y = 0$, equation (1) will be reduced to (1+1)-dimensional CDGKS equation.

If $\alpha = 5, t = 36T, \beta = 5$ and $u_y = 0$, equation (1) will be reduced to Sawada–Kotera equation for the long waves in shallow water under the gravity, which have gained many interesting results [34–36]. And also be used in lattice dynamics, quantum mechanics and nonlinear optics.

This article is divided into five sections to describe our content. In the second section, N-soliton solution for equation (1) is given directly with Hirota bilinear method. The third section is divided into two parts. In the first part, based on N-soliton solution and taking advantage of new constraints, we obtain the resonant Y-shaped solitons and some noval interaction solutions, especially the resonant X-type solution; In the second part, mixed solution of resonant Y-shaped soliton and line wave, breather wave are obtained under new constraints by velocity resonance and mode resonance methods. In section 4, with the support of long wave limit approach, the interaction between resonant Y-type solitons and higher-order lumps is shown, and the motion trajectory equation before and after the interaction between lumps and resonant Y-type solitons is dissussed detailly. The fifth section is a short conclusion.

2 N-soliton solution

Using logarithmic transformation

$$u = 2(lnf)_{xx}, \tag{2}$$

the equation (1) can be transformed into the following bilinear equation:

$$(36D_xD_t + D_x^6 - \alpha D_y^2 - \beta D_x^3 D_y)(f \cdot f) = 0,$$

where D_x, D_y, D_t are defined as follows [37]:

$$D_x^{m_1} D_y^{m_2} D_t^{m_3}(f \cdot f) = (\partial x - \partial x')^{m_1}(\partial y - \partial y')^{m_2}(\partial t - \partial t')^{m_3}(f \cdot f)|_{x'=x, y'=y, t'=t}.$$

For the sake of obtaining the N-soliton solution, based on famous bilinear method, f in (2) is directly constructed as

$$f = \sum_{\mu=0,1} exp(\sum_{j=1}^{N} \mu_j \eta_j + \sum_{j<s}^{N} \mu_j \mu_s A_{js}), \tag{3}$$

with

$$\begin{aligned} \eta_j &= k_j x + r_j y + w_j t + \phi_j, \\ w_j &= \frac{-k_j^6 + \beta k_j^3 r_j + \alpha r_j^2}{36k_j}, \end{aligned} \tag{4}$$

and

$$e^{A_{js}} = -\frac{36(k_j - k_s)(w_j - w_s) + (k_j - k_s)^6 - \alpha(r_j - r_s)^2 - \beta(k_j - k_s)^3(r_j - r_s)}{36(k_j + k_s)(w_j + w_s) + (k_j + k_s)^6 - \alpha(r_j + r_s)^2 - \beta(k_j + k_s)^3(r_j + r_s)}. \tag{5}$$

Here, k_j, r_j and ϕ_j are arbitrary constants, $\sum_{\mu=0,1}$ is all possible combinations of $u_i = 0, 1(i = 1, ..., N)$.

3 Resonant Y-shape soliton and interactions

3.1 Resonance Y-shape soliton

On the basis of $exp(x + ln(0)) = 0exp(x) = 0$, we can eliminate some terms of equation (3). Therefore, in order to obtain mixed solutions of L-resonance Y-type soliton and P-resonance Y-type soliton, we need to make the following constraints on certain parameters for the above N-soliton solution:

$$e^{A_{js}} = 0(N = L + P, 1 \leq j < s \leq L, L < j < s \leq N), \tag{6}$$

where

$$\begin{aligned} r_s =& \frac{\beta k_j^3 - 3\beta k_j^2 k_s + 2\beta k_j k_s^2 + 2\alpha r_j}{2\alpha k_j} k_s \\ &\pm \frac{\sqrt{(k_j - k_s)^2 k_j [(\beta^2 - 20\alpha)k_j^3 - 4k_j k_s (k_j - k_s)(\beta^2 - 5\alpha) + 12\beta\alpha r_j]}}{2\alpha k_j} k_s. \end{aligned} \tag{7}$$

Obviously, it can be seen from the expression of r_s that there are positive and negative cases in equation (7), which explains that there will be two opposite cases for resonant Y-type soliton.

When $N=2$, we take the parameters in (2) as $\alpha=1, \beta=5, k_1=\dfrac{1}{2}, k_2=\dfrac{1}{3}, r_1=1, r_2=\dfrac{139}{216}\pm\dfrac{29\sqrt{5}}{216}, \phi_1=0, \phi_2=0$ when $t=0$, and its density and three-dimensional plot are in figure a, b, d, e of Fig. 1.

When $N=3$, we take the parameters in (2) as $\alpha=-1, \beta=-2, k_1=\dfrac{1}{2}, k_2=\dfrac{1}{4}, k_3=\dfrac{1}{5}, r_1=\dfrac{1}{2}, r_2=\dfrac{1}{4}-\dfrac{\sqrt{111}}{64}, r_3=\dfrac{1}{3}, \phi_1=0, \phi_2=0, \phi_3=0$ when $t=0$, and its density and three-dimensional plot are in figure c, f of Fig. 1.

As shown in Fig. 1 (a,d) and Fig. 1 (b,e), two 2-resonant Y-type soliton openings have different directions, and only r_2 of their parameters are different. In addition, Fig. 1 (c,f) gives a 3-resonant Y-type soliton.

If $N=4$, i.e. $L=2, P=2$, one can get the expression describing the interaction phenomenon of two 2-resonant Y-type soliton is

$$u_1 = 2(lnf_1)_{xx}, \tag{8}$$

with

$$f_1 = 1+e^{\eta_1}+e^{\eta_1+\eta_3+A_{13}}+e^{\eta_2}+e^{\eta_1+\eta_4+A_{14}}+e^{\eta_3}+e^{\eta_2+\eta_3+A_{23}}+e^{\eta_4}+e^{\eta_2+\eta_4+A_{24}}.$$

Here parameters e^{η_j} and $e^{A_{js}}$ are limited by (4),(5),(6), and (7).

Via selecting appropriate constants, there are three main situations for interaction between two 2-resonant Y-shape soliton. Fig. 2, Fig. 3 and Fig. 4 show detailly the interaction phenomena of resonance Y-shape solutions with two fission, two fusion, and one fission with one fusion. Because the interaction between line waves is elastic and resonant Y-type solitons are simplified by N-soliton solutions, interaction between resonant Y-shape soliton is also elastic.

If we take

$$u_2 = 2(lnf_2)_{xx}, \tag{9}$$

where

$$f_2 = 1+e^{\eta_1}+e^{\eta_2}+e^{\eta_3}+e^{\eta_2+\eta_3+A_{23}},$$

and r_2, r_3 and $e^{A_{23}}$ are determined respectively by conditions (7) and (5), one can get a resonant X-type soliton, which is formed by two resonant Y-shape soliton with opposite opening directions. It is a new type of double open resonant Y-type soliton. As shown in Fig. 5, the double open resonant Y-type solitons are one in fission and one in fusion.

3.2 Interaction between line wave molecule, breather and resonant Y-type soliton

Based on the local velocity resonance method and constraints condition (7), a method to directly obtain the interaction between resonant Y-type solitons and line solitons is proposed. The following conditions are set on the N-soliton solution (3):

$$\begin{aligned} &e^{A_{js}}=0, \frac{k_{L+2p-1}}{k_{L+2p}}=\frac{r_{L+2p-1}}{r_{L+2p}}=\frac{w_{L+2p-1}}{w_{L+2p}}\neq\pm1, \\ &(1\leq j<s\leq L, 1\leq p\leq P, N=L+2P). \end{aligned} \tag{10}$$

If $L=2, P=1$, we will gain a hybrid solution of a line soliton and 2-resonant Y-type soliton of equation (1), which is expressed as

$$u_3 = 2(lnf_3)_{xx}, \tag{11}$$

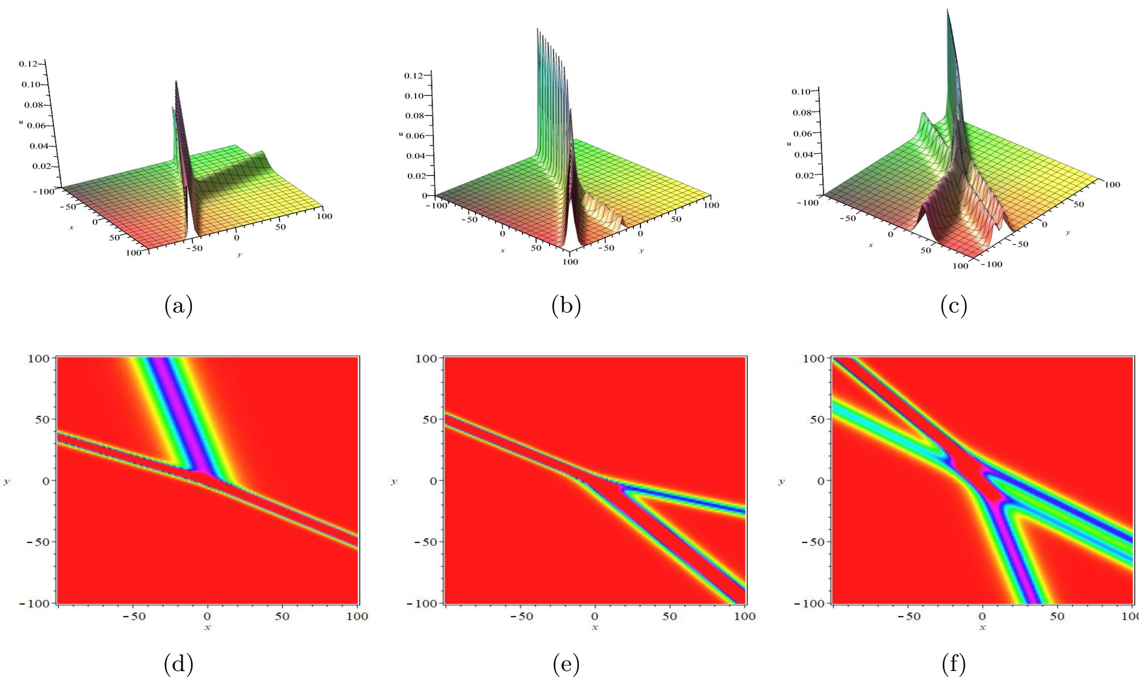


Figure 1: A 2-resonance Y-shape soliton of (2) with $\alpha = 1, \beta = 5, k_1 = \frac{1}{2}, k_2 = \frac{1}{3}, r_1 = 1, \phi_1 = 0, \phi_2 = 0$ when $t = 0$, and:(a,d)$r_2 = \frac{139}{216} + \frac{29\sqrt{5}}{216}$;(b,e)$r_2 = \frac{139}{216} - \frac{29\sqrt{5}}{216}$. 3-resonance Y-shape soliton of (2) with $\alpha = -1, \beta = -2, k_1 = \frac{1}{2}, k_2 = \frac{1}{4}, k_3 = \frac{1}{5}, r_1 = \frac{1}{2}, r_2 = \frac{1}{4} - \frac{\sqrt{111}}{64}, r_3 = \frac{1}{3}, \phi_1 = 0, \phi_2 = 0, \phi_3 = 0$ when $t = 0$ in (c,f).

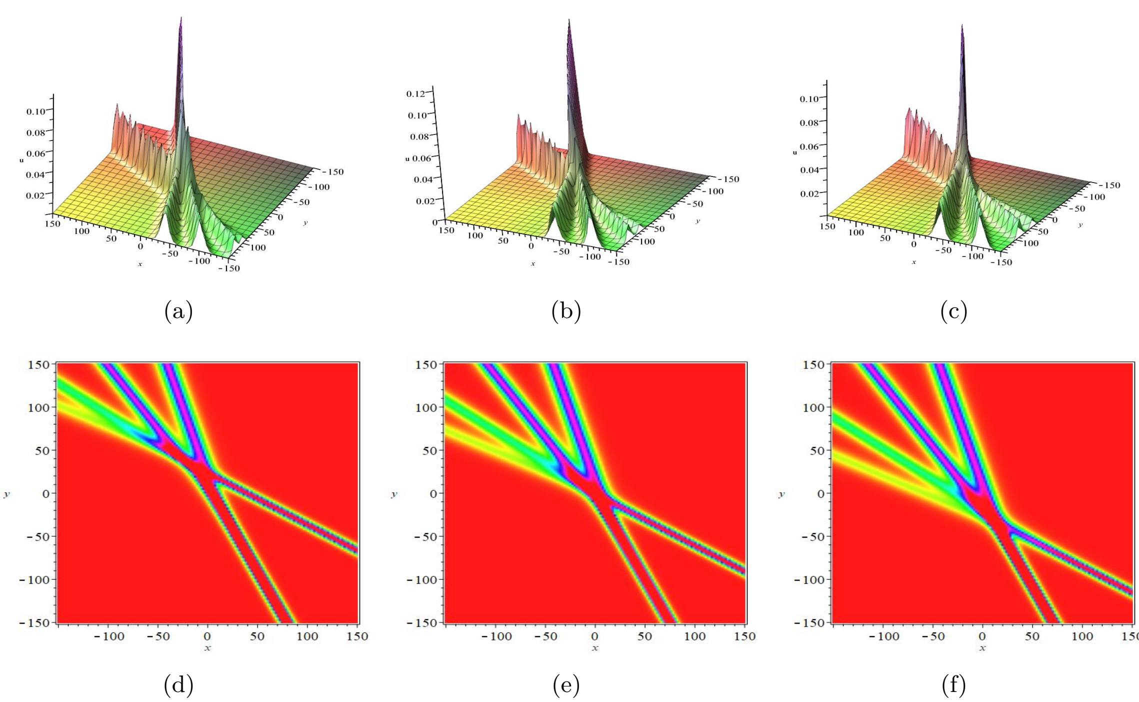


Figure 2: The interaction phenomena of two fission resonance Y-type solitons of (8) with $\alpha = 1, \beta = 1, k_1 = \frac{3}{10}, k_2 = \frac{1}{8}, k_3 = \frac{1}{2}, k_4 = \frac{1}{4}, r_1 = \frac{1}{2}, r_2 = \frac{8021}{38400} + \frac{7\sqrt{466}}{3200}, r_3 = \frac{1}{4}, r_4 = \frac{11}{64}, \phi_1 = 0, \phi_2 = 0, \phi_3 = 0, \phi_4 = 0$ when $t = -500$ in (a) and (d), $t = 0$ in (b) and (e), $t = 500$ in (c) and (f), respectively.

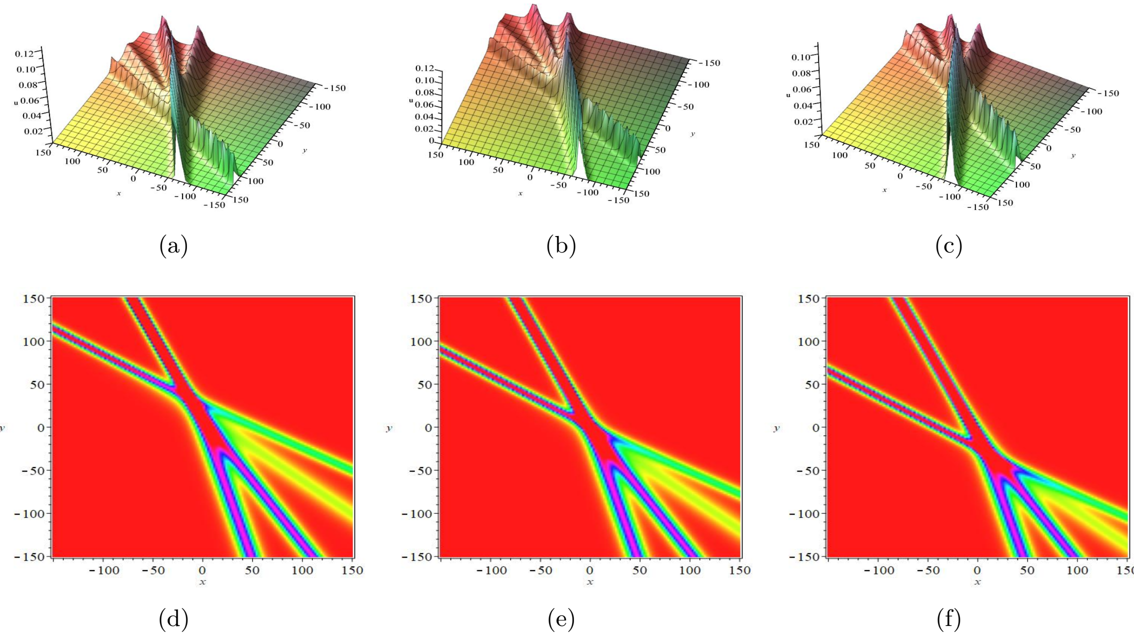


Figure 3: The interaction phenomena of two fusion resonance Y-type solitons of (8) with $\alpha = 1, \beta = 1, k_1 = \frac{3}{10}, k_2 = \frac{1}{8}, k_3 = \frac{1}{2}, k_4 = \frac{1}{4}, r_1 = \frac{1}{2}, r_2 = \frac{8021}{38400} - \frac{7\sqrt{466}}{3200}, r_3 = \frac{1}{4}, r_4 = \frac{5}{64}, \phi_1 = 0, \phi_2 = 0, \phi_3 = 0, \phi_4 = 0$ when $t = -500$ in (a) and (d), $t = 0$ in (b) and (e), $t = 500$ in (c) and (f), respectively.

$$\begin{aligned} f_3 =& 1 + e^{\eta_1} + e^{\eta_1+\eta_3+A_{13}} + e^{\eta_2} + e^{\eta_1+\eta_4+A_{14}} + e^{\eta_1+\eta_3+A_{13}+\eta_4+A_{14}+A_{34}} + e^{\eta_3} + e^{\eta_2+\eta_3+A_{23}} \\ &+ e^{\eta_4} + e^{\eta_2+\eta_4+A_{24}} + e^{\eta_2+\eta_3+A_{23}+\eta_4+A_{24}+A_{34}} + e^{\eta_3+\eta_4+A_{34}}, \end{aligned} \tag{12}$$

where

$$\begin{aligned} r_2 =& \frac{\beta k_1^3 - 3\beta k_1^2 k_2 + 2\beta k_1 k_2^2 + 2\alpha r_1}{2\alpha k_1} k_2 \\ &\pm \frac{\sqrt{(k_1 - k_2)^2 k_1[(\beta^2 - 20\alpha)k_1^3 - 4k_1 k_2(k_1 - k_2)(\beta^2 - 5\alpha) + 12\beta\alpha r_1]}}{2\alpha k_1} k_2, \end{aligned} \tag{13}$$

and

$$\frac{k_3}{k_4} = \frac{r_3}{r_4} = \frac{w_3}{w_4} \neq \pm 1.$$

The interaction between 2-resonant Y-type soliton and a line soliton is shown in Fig. 6.

Based on module resonant and constraints condition (7), a method to directly obtain solutions on resonant Y-shape soliton and breather wave interactions is proposed. The following conditions are set on the N-soliton solution (3):

$$\begin{aligned} &e^{A_{js}} = 0, \eta_{L+2p-1} = \eta_{L+2p}^*, \\ &(1 \leq j < s \leq L, 1 \leq p \leq P, N = L + 2P). \end{aligned}$$

If $L = 2, P = 1$, we will get a hybrid solution of a breather wave and 2-resonant Y-type soliton of equation (1), which is expressed as

$$u_4 = 2(lnf_3)_{xx}, \tag{14}$$

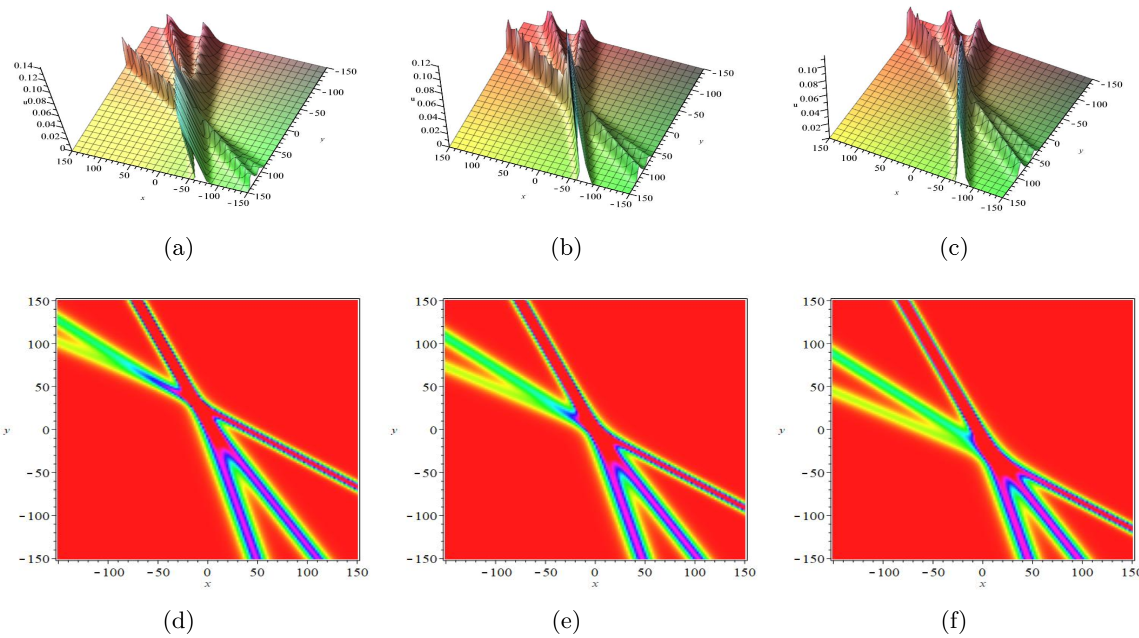


Figure 4: The interaction phenomena of one fission with one fusion resonance Y-type soliton of (8) with $\alpha = 1, \beta = 1, k_1 = \frac{3}{10}, k_2 = \frac{1}{8}, k_3 = \frac{1}{2}, k_4 = \frac{1}{4}, r_1 = \frac{1}{2}, r_2 = \frac{8021}{38400} + \frac{7\sqrt{466}}{3200}, r_3 = \frac{1}{4}, r_4 = \frac{5}{64}, \phi_1 = 0, \phi_2 = 0, \phi_3 = 0, \phi_4 = 0$ when $t = -500$ in (a) and (d), $t = 0$ in (b) and (e), $t = 500$ in (c) and (f), respectively.

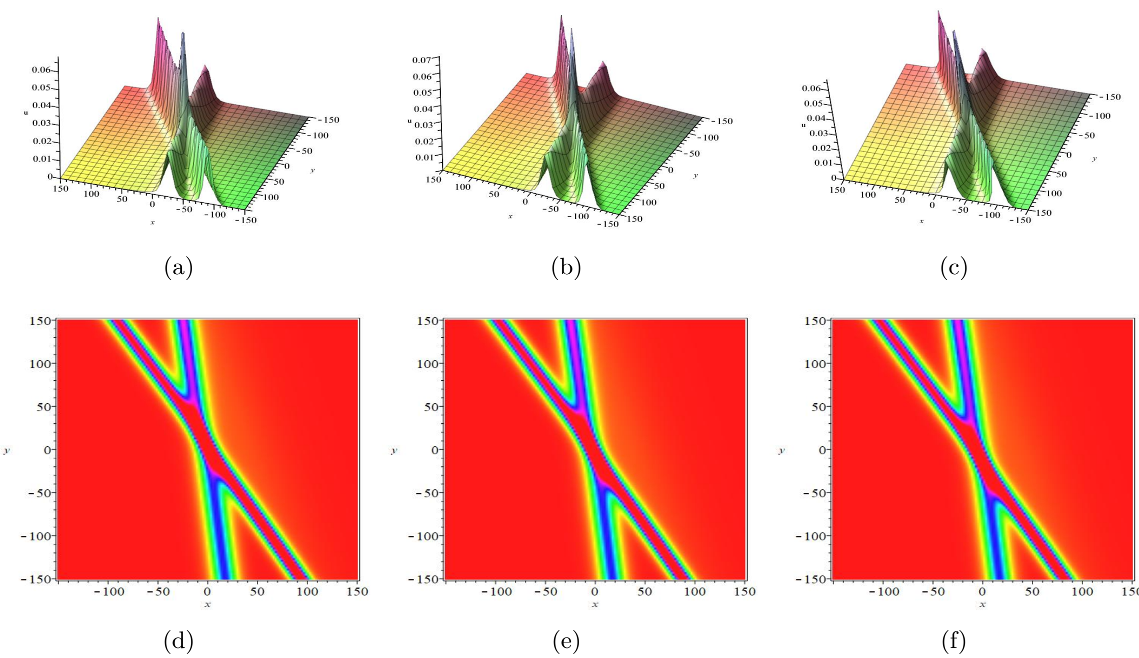


Figure 5: Resonant X-type soliton of (9) with $\alpha = 5, \beta = 5, k_1 = \frac{1}{2}, k_2 = \frac{1}{4}, k_3 = \frac{1}{5}, r_1 = \frac{1}{5}, r_2 = \frac{1}{10} + \frac{9\sqrt{5}}{320}, r_3 = \frac{83}{1000} - \frac{27\sqrt{5}}{1000}, \phi_1 = 0, \phi_2 = 0, \phi_3 = 0$ when $t = -100$ in (a) and (d), $t = 0$ in (b) and (e), $t = 100$ in (c) and (f), respectively.

where

$$k_3 = k_4^*, r_3 = r_4^*, \phi_3 = \phi_4^*$$

and f_3 and r_2 are constrained by (12) and (13).

It can be observed from Fig. 7 that the interaction between 2-resonant Y-type soliton and a breather wave is not affected by time, and the 2-resonant Y-type soliton moves only in the horizontal direction.

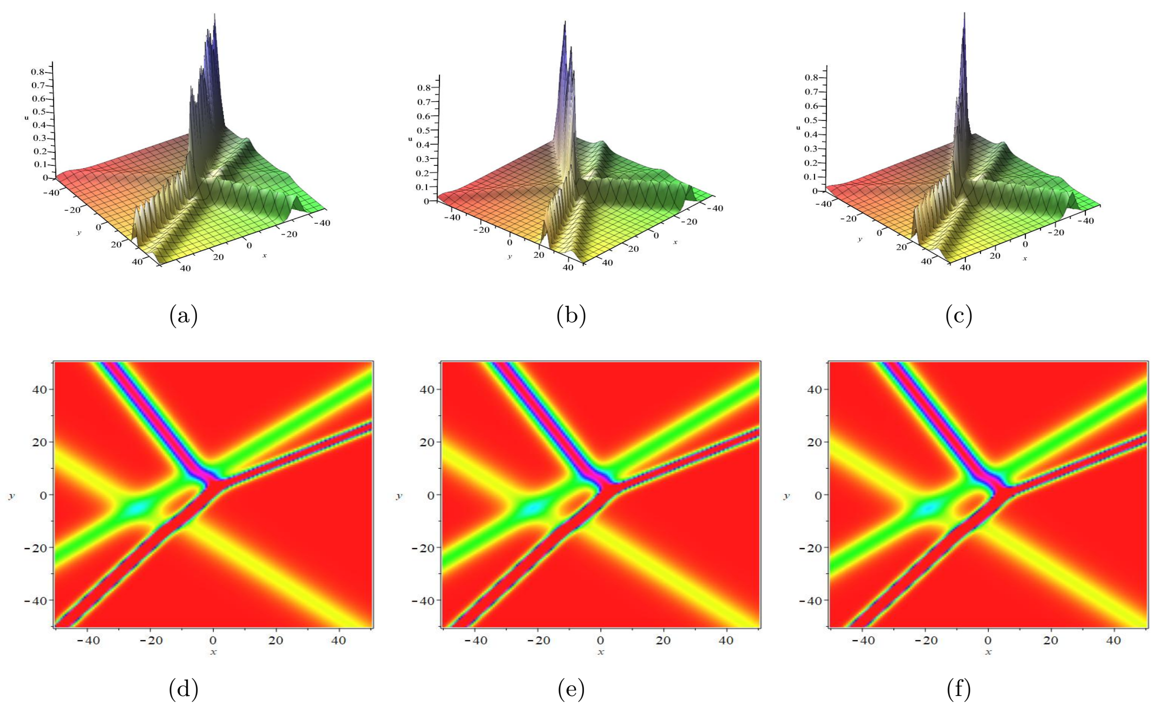


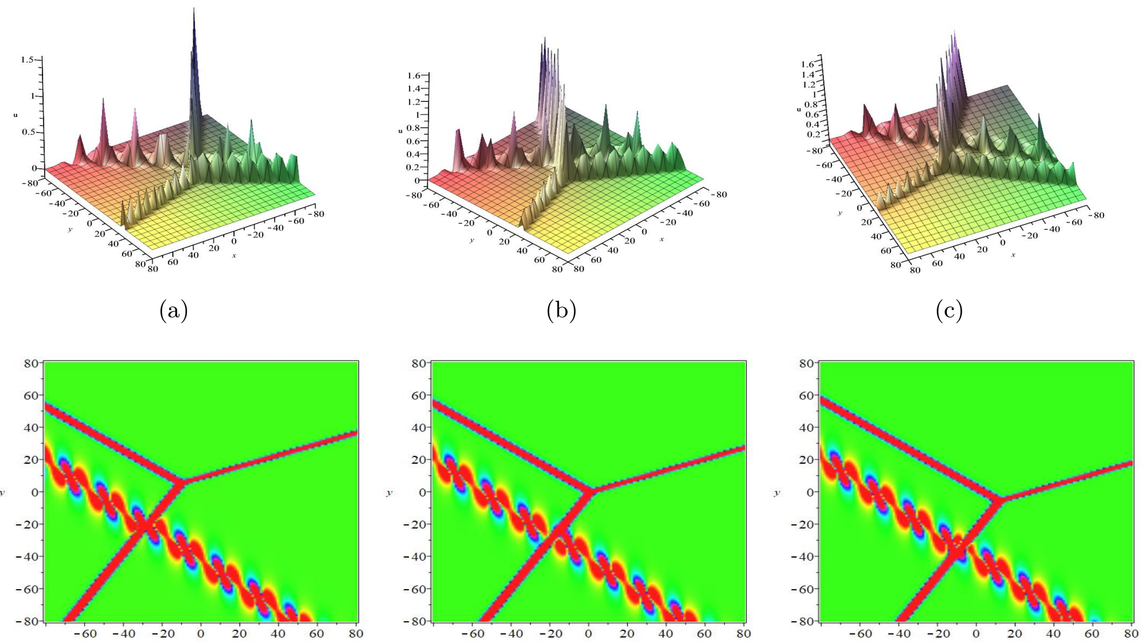
Figure 6: The interaction between a line soliton and 2-resonant Y-type soliton of (11) with $\alpha = -1, \beta = -1, k_1 = -\frac{4}{3}, k_2 = -\frac{3}{4}, k_3 = -\frac{1}{3}, k_4 = \frac{1}{4}, r_1 = \frac{6}{5}, r_2 = \frac{683}{960} + \frac{7\sqrt{14430}}{960}, r_3 = \frac{1}{54} + \frac{\sqrt{38481}}{432}, r_4 = \frac{1}{3}, \phi_1 = 0, \phi_2 = 0, \phi_3 = -3, \phi_4 = 5$ when $t = -50$ in (a) and (d), $t = 0$ in (b) and (e), $t = 50$ in (c) and (f), respectively.

4 Interaction between resonant Y-shape soliton solution and lump solution

Based on constraint condition (7) and usual long-wave limit method, an approach to obtain interaction between high-order lump waves and resonant Y-type solitons is proposed directly. The following limiting conditions are set on the N-soliton solution(3):

$$\begin{aligned} &k_{2m-1} = k_{2m}^* = K_{2m-1}\delta, r_{2m-1} = r_{2m}^* = R_{2m-1}\delta, \phi_{2m-1} = \phi_{2m} = \pi i, \\ &\delta \to 0, e^{A_{js}} = 0, N = 2L + P, (1 \leq m \leq L, 2L < j < s \leq N). \end{aligned} \tag{15}$$

Referring to an interesting work [38], we can know that the coordinates of lump's centre before and after interacting with resonant Y-shape soliton are given by parameters $\{K_{2m-1}, R_{2m-1}, K_{2m}, R_{2m}\}$. Next, we use $\{x_b, y_b\}$ to represent before the interaction, $\{x_a, y_a\}$ to represent after the interac-



(a) (b) (c)

(d) (e) (f)

Figure 7: The interaction between a breather wave and 2-resonant Y-type soliton of (14) with $\alpha = -3, \beta = 3, k_1 = -\frac{16}{9}, k_2 = -\frac{7}{8}, k_3 = \frac{1}{5} - \frac{i}{3}, k_4 = \frac{1}{5} + \frac{i}{3}, r_1 = \frac{6}{5}, r_2 = \frac{124747}{207360} + \frac{91\sqrt{108915}}{15552}, r_3 = \frac{1}{4}, r_4 = \frac{1}{4}, \phi_1 = 0, \phi_2 = 0, \phi_3 = 10, \phi_4 = 10$ when $t = -30$ in (a) and (d), $t = 0$ in (b) and (e), $t = 30$ in (c) and (f), respectively.

tion.

$$
\begin{aligned}
x_b &= \frac{\alpha R_{2m-1} R_{2m}}{36 K_{2m-1} K_{2m}} t + \sum_{s=2L+1}^{N} h_b(\lambda_s) \Phi_s, \\
y_b &= -\frac{\alpha (K_{2m-1} R_{2m} + K_{2m} R_{2m-1})}{36 K_{2m-1} K_{2m}} t + \sum_{s=2L+1}^{N} h_b(\lambda_s) \Theta_s, \\
x_a &= \frac{\alpha R_{2m-1} R_{2m}}{36 K_{2m-1} K_{2m}} t + \sum_{s=2L+1}^{N} h_a(\lambda_s) \Phi_s, \\
y_a &= -\frac{\alpha (K_{2m-1} R_{2m} + K_{2m} R_{2m-1})}{36 K_{2m-1} K_{2m}} t + \sum_{s=2L+1}^{N} h_a(\lambda_s) \Theta_s,
\end{aligned}
$$

where

$$
\begin{aligned}
\lambda_s &= \frac{\alpha k_s R_{2m-1} R_{2m}}{36 K_{2m-1} K_{2m}} - \frac{\alpha r_s (K_{2m-1} R_{2m} + K_{2m} R_{2m-1})}{36 K_{2m-1} K_{2m}} + \frac{-k_s^6 + \beta k_s^3 r_s + \alpha r_s^2}{36 k_s}, \\
\Phi_s &= \frac{-b_{2m-1,s} R_{2m} + b_{2m,s} R_{2m-1}}{K_{2m-1} R_{2m} - R_{2m-1} K_{2m}}, \Theta_s = \frac{b_{2m-1,s} K_{2m} - b_{2m,s} K_{2m-1}}{K_{2m-1} R_{2m} - R_{2m-1} K_{2m}},
\end{aligned}
$$

$$
h_b(x) = \begin{cases} 1, x < 0, \\ 0, x \geq 0, \end{cases} \quad h_a(x) = \begin{cases} 0, x \leq 0, \\ 1, x > 0, \end{cases}
$$

and

$$b_{js}=\begin{cases}\dfrac{6\beta K_j^2K_s^2(K_jR_s+K_sR_j)}{\alpha(K_jR_s-K_sR_j)^2}, & 1\leq j<s<2L,\\ \dfrac{-6k_s^2K_j^2[(-5k_s^3+\beta r_s)K_j+\beta k_sR_j]}{(-5k_s^6+2\beta r_sk_s^3-\alpha r_s^2)K_j^2+k_sR_jK_j(\beta k_s^3+2\alpha r_s)-\alpha k_s^2R_j^2}, & 1\leq j\leq 2L, s>2L.\end{cases}$$

What's more, the height of the lump solution will not increase or decrease before and after its interaction with one resonant Y-shape soliton. It is a fixed value, recorded as h_u.

$$h_u=\frac{2\alpha(K_{2m-1}R_{2m}-K_{2m}R_{2m-1})^2}{3\beta K_{2m-1}K_{2m}(K_{2m-1}R_{2m}+K_{2m}R_{2m-1})}.$$

If $L=1, P=2$, we will get a hybrid solution of one lump solution and one 2-resonant Y-type soliton. Under condition (15), the function f in (3) becomes

$$f_4=\theta_1\theta_2+b_{12}+(\theta_1\theta_2+b_{23}\theta_1+b_{13}\theta_2+b_{13}b_{23}+b_{12})e^{\eta_3}+(\theta_1\theta_2+b_{24}\theta_1+b_{14}\theta_2+b_{14}b_{24}+b_{12})e^{\eta_4}, \quad (16)$$

where $\theta_j=K_jx+R_jy+\dfrac{\alpha R_j^2}{36K_j}t$.

Then, bring a function (16) into the logarithmic transformation (2), we can gain interaction between one lump and one 2-resonant Y-shape soliton of u.

Taking the parameters $\alpha=36, \beta=36, K_1=-\frac{6}{5}+\frac{6}{5}i, K_2=-\frac{6}{5}-\frac{6}{5}i, R_1=\frac{4}{3}, R_2=\frac{4}{3}, k_3=\dfrac{5}{4}, r_3=\frac{1}{2}, k_4=\frac{3}{10}, r_4=\frac{1701}{8000}+\dfrac{19\sqrt{16246}}{8000}, \phi_1=\pi i, \phi_2=\pi i, \phi_3=0, \phi_4=0$, Fig. 8 shows dynamic process of the interaction between a single lump solution and a resonance Y-type soliton. Before and after interaction, motion path of the single lump solution is $y=\frac{9}{5}x$ and $y=\frac{9}{5}x-\dfrac{74378116249560}{17368865139001}$, respectively.

From Fig. 8, one can see that after lump wave collides with the resonant Y-type soliton, the velocity, shape and amplitude do not change and continue to propagate, but the trajectory is offset. Next, we can study more complex resonant mixed solutions in the same way.

If $L=2, P=2$, we will get a hybrid solution of two lumps and one 2-resonant Y-type soliton. Under condition (15), the function f in (3) becomes

$$\begin{aligned}f_5=&\theta_1\theta_2\theta_3\theta_4+b_{12}\theta_3\theta_4+b_{13}\theta_2\theta_4+b_{14}\theta_2\theta_3+b_{23}\theta_1\theta_4+b_{24}\theta_1\theta_3+b_{34}\theta_1\theta_2+b_{12}b_{34}+b_{13}b_{24}+\\&b_{14}b_{23}+(b_{15}b_{25}b_{35}b_{45}+b_{15}b_{25}b_{35}\theta_4+b_{15}b_{25}b_{45}\theta_3+b_{15}b_{25}\theta_3\theta_4+b_{15}b_{35}b_{45}\theta_2+b_{15}b_{35}\theta_2\theta_4\\&+b_{15}b_{45}\theta_2\theta_3+b_{15}\theta_2\theta_3\theta_4+b_{25}b_{35}b_{45}\theta_1+b_{25}b_{35}\theta_1\theta_4+b_{25}b_{45}\theta_1\theta_3+b_{25}\theta_1\theta_3\theta_4+b_{35}b_{45}\theta_1\theta_2\\&+b_{35}\theta_1\theta_2\theta_4+b_{45}\theta_1\theta_2\theta_3+\theta_1\theta_2\theta_3\theta_4+b_{12}b_{35}b_{45}+b_{12}b_{35}\theta_4+b_{12}b_{45}\theta_3+b_{12}\theta_3\theta_4+b_{13}b_{25}b_{45}\\&+b_{13}b_{25}\theta_4+b_{13}b_{45}\theta_2+b_{13}\theta_2\theta_4+b_{14}b_{25}b_{35}+b_{14}b_{25}\theta_3+b_{14}b_{35}\theta_2+b_{14}\theta_2\theta_3+b_{15}b_{23}b_{45}\\&+b_{15}b_{23}\theta_4+b_{15}b_{24}b_{35}+b_{15}b_{24}\theta_3+b_{15}b_{25}b_{34}+b_{15}b_{34}\theta_2+b_{23}b_{45}\theta_1+b_{23}\theta_1\theta_4+b_{24}b_{35}\theta_1\\&+b_{24}\theta_1\theta_3+b_{25}b_{34}\theta_1+b_{34}\theta_1\theta_2+b_{12}b_{34}+b_{13}b_{24}+b_{14}b_{23})e^{\eta_5}\\&+(b_{16}b_{26}b_{36}b_{46}+b_{16}b_{26}b_{36}\theta_4+b_{16}b_{26}b_{46}\theta_3+b_{16}b_{26}\theta_3\theta_4+b_{16}b_{36}b_{46}\theta_2+b_{16}b_{36}\theta_2\theta_4+\\&b_{16}b_{46}\theta_2\theta_3+b_{16}\theta_2\theta_3\theta_4+b_{26}b_{36}b_{46}\theta_1+b_{26}b_{36}\theta_1\theta_4+b_{26}b_{46}\theta_1\theta_3+b_{26}\theta_1\theta_3\theta_4+b_{36}b_{46}\theta_1\theta_2\\&+b_{36}\theta_1\theta_2\theta_4+b_{46}\theta_1\theta_2\theta_3+\theta_1\theta_2\theta_3\theta_4+b_{12}b_{36}b_{46}+b_{12}b_{36}\theta_4+b_{12}b_{46}\theta_3+b_{12}\theta_3\theta_4+\\&b_{13}b_{26}b_{46}+b_{13}b_{26}\theta_4+b_{13}b_{46}\theta_2+b_{13}\theta_2\theta_4+b_{14}b_{26}b_{36}+b_{14}b_{26}\theta_3+b_{14}b_{36}\theta_2+b_{14}\theta_2\theta_3+\\&b_{16}b_{23}b_{46}+b_{16}b_{23}\theta_4+b_{16}b_{24}b_{36}+b_{16}b_{24}\theta_3+b_{16}b_{26}b_{34}+b_{16}b_{34}\theta_2+b_{23}b_{46}\theta_1+b_{23}\theta_1\theta_4+\\&b_{24}b_{36}\theta_1+b_{24}\theta_1\theta_3+b_{26}b_{34}\theta_1+b_{34}\theta_1\theta_2+b_{12}b_{34}+b_{13}b_{24}+b_{14}b_{23})e^{\eta_6},\end{aligned} \quad (17)$$

where $\theta_j = K_j x + R_j y + \dfrac{\alpha R_j^2}{36K_j} t$.

Then, bring a function (17) into the logarithmic transformation (2), we can gain interaction between two lumps and one 2-resonant Y-shape soliton of u.

Taking parameters $\alpha = 5, \beta = 5, K_1 = \dfrac{1}{2} + i, K_2 = \dfrac{1}{2} - i, K_3 = \dfrac{1}{4} + \frac{3}{5}i, K_4 = \dfrac{1}{4} - \frac{3}{5}i, R_1 = -\dfrac{11}{5}, R_2 = -\dfrac{11}{5}, R_3 = -\dfrac{11}{10}, R_4 = -\dfrac{11}{10}, k_5 = \dfrac{11}{10}, k_6 = -\dfrac{11}{10}, r_5 = \dfrac{1}{2}, r_6 = -\dfrac{4493}{1000} - \dfrac{33\sqrt{2453}}{1000}, \phi_1 = \pi i, \phi_2 = \pi i, \phi_3 = \pi i, \phi_4 = \pi i, \phi_5 = 0, \phi_6 = 0$, Fig. 9 shows the interaction between the 2-order lump solution and a resonance Y-type soliton.

5 Conclusions

In this paper, subject to N-soliton solutions of (2+1)-dimensional gCDGKS equation, we successfully construct resonant Y-shape soliton and their interactions with a number of other solutions. Firstly, by proposing a noval restrictive condition for the N-soliton solution, resonant Y-shape soliton solution of (2) is discussed, especially the three interactions of two 2-resonant Y-type soliton solutions: two fusion, two fission and one fusion with one fission. By selecting appropriate parameters, a new double opening resonant Y-type soliton can be obtained. As illustrated in Fig. 5, it can be referred to as a resonant X-shape soliton because its shape resembles the letter "X". Secondly, combined with velocity resonance method and mode resonance method, interaction solutions of these resonant Y-shape solitons with line waves and breather solutions are gained. Finally, with the support of long wave limit method, the hybrid solution composed of resonant Y-shape soliton and higher order lump solutions is given, and the coordinates of lump's centre before and after interacting with resonant Y-shape soliton are derived.

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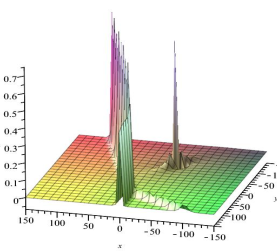
Compliance with ethical standards Conflict

Conflict of interest

There are no conflicts of interest to this work.

Data Availability Statements

Data sharing not applicable to this article as no datasets were generated.



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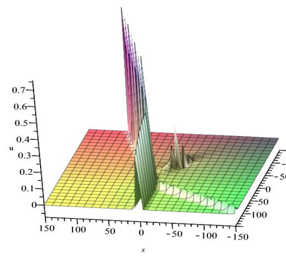

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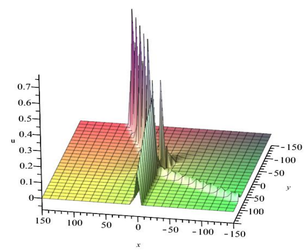

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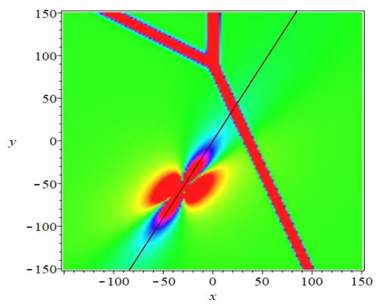

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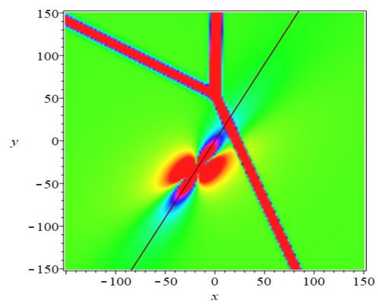

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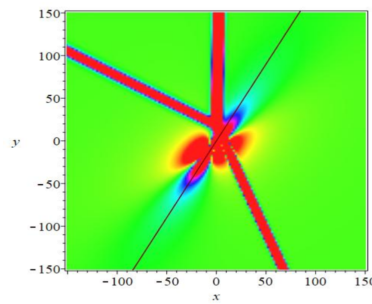

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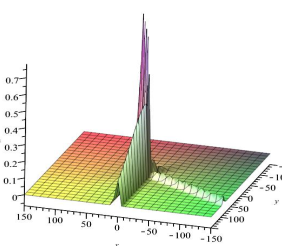

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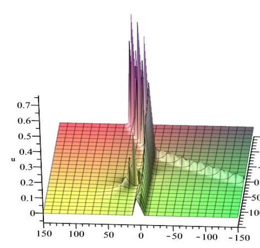

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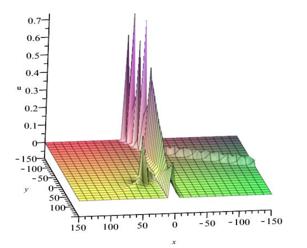

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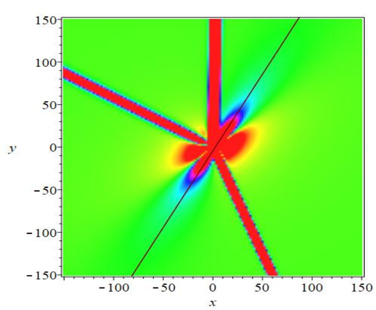

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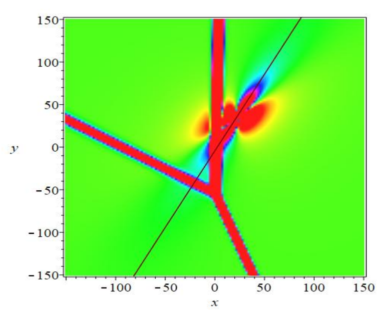

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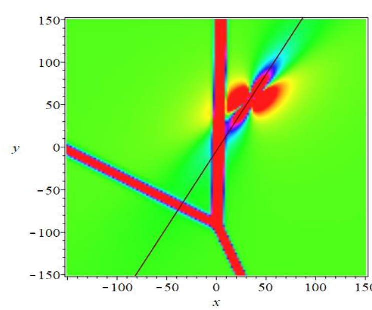
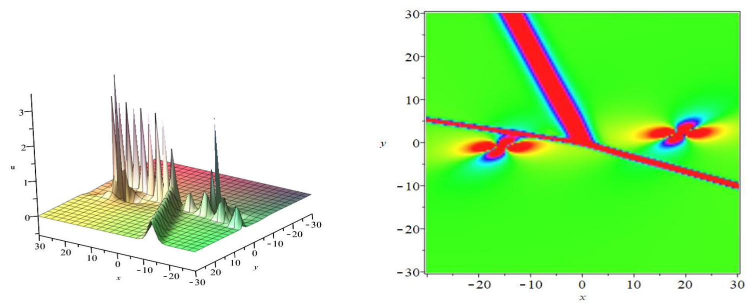


(l)

Figure 8: Three-dimensional plots and density plots of interaction between a single lump solution and a resonance Y-type soliton with $\alpha = 36, \beta = 36, K_1 = -\frac{6}{5} + \frac{6}{5}i, K_2 = -\frac{6}{5} - \frac{6}{5}i, R_1 = \frac{4}{3}, R_2 = \frac{4}{3}, k_3 = \dfrac{5}{4}, r_3 = \frac{1}{2}, k_4 = \frac{3}{10}, r_4 = \frac{1701}{8000} + \dfrac{19\sqrt{16246}}{8000}, \phi_1 = \pi i, \phi_2 = \pi i, \phi_3 = 0, \phi_4 = 0$ when $t = -50$ in (a) and (g), $t = -30$ in (b) and (h), $t = -10$ in (c) and (i), $t = 0$ in (d) and (j), $t = 30$ in (e) and (k), $t = 50$ in (e) and (l), respectively.



(a) (b)

Figure 9: Three-dimensional plots and density plots of the interaction between two lumps and one resonance Y-type soliton with $\alpha = 5, \beta = 5, K_1 = \frac{1}{2} + i, K_2 = \frac{1}{2} - i, K_3 = \frac{1}{4} + \frac{3}{5}i, K_4 = \frac{1}{4} - \frac{3}{5}i, R_1 = -\frac{11}{5}, R_2 = -\frac{11}{5}, R_3 = -\frac{11}{10}, R_4 = -\frac{11}{10}, k_5 = \frac{11}{10}, k_6 = -\frac{11}{10}, r_5 = \frac{1}{2}, r_6 = -\frac{4493}{1000} - \frac{33\sqrt{2453}}{1000}, \phi_1 = \pi i, \phi_2 = \pi i, \phi_3 = \pi i, \phi_4 = \pi i, \phi_5 = 0, \phi_6 = 0$ when $t = 0$.

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