

Resonances and Unitarity in Weak Boson Scattering at the LHC

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Alboreanu/Kilian/JR, arXiv:0806.4145 (**JHEP**); M. Mertens, 2005;

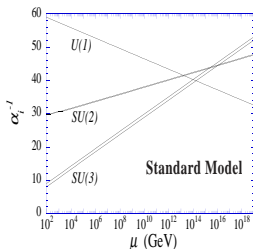
Kilian/Kobel/JR/Schumacher, work in progress;

Beyer/Kilian/Krstonošić/Mönig/JR/Schmitt/Schröder, **EPJC** **48** (2006), 353 [ILC version]

Seminar, Wien, 24.01.2013

Open Questions

- Unification of all interactions (?)
- Baryon asymmetrie $\Delta N_B - \Delta N_{\bar{B}} \sim 10^{-9}$
missing CP violation
- Flavour: three generations
- Tiny neutrino masses: $m_\nu \sim \frac{v^2}{M}$
- Dark Matter:
 - ▶ stable
 - ▶ only weakly interacting
 - ▶ $m_{DM} \sim 100 \text{ GeV}$
- Quantum theory of gravity
- Cosmic inflation
- Cosmological constant



Model-Independent Description of the EW sector

- ▶ **Higgs boson observed** (?) ... what is the EW sector?
- ▶ Aim: describe any physics beyond the SM as generically as possible
- ▶ Implement what we know about the SM
- ▶ Implements $SU(2)_L \times U(1)_Y$ gauge invariance
- ▶ Building blocks (including longitudinal modes):

$$\psi \text{ (SM fermions)}, \quad W_\mu^a \text{ (} a = 1, 2, 3\text{)}, \quad B_\mu, \quad \Sigma = \exp \left[\frac{-i}{v} w^a \tau^a \right]$$

- ▶ Minimal Lagrangian including gauge interactions

$$\mathcal{L}_{\min} = \sum_{\psi} \bar{\psi}(i\not{D})\psi - \frac{1}{2g^2} \text{tr} [\mathbf{W}_{\mu\nu} \mathbf{W}^{\mu\nu}] - \frac{1}{2g'^2} \text{tr} [\mathbf{B}_{\mu\nu} \mathbf{B}^{\mu\nu}] + \frac{v^2}{4} \text{tr} [(\mathbf{D}_\mu \Sigma)(\mathbf{D}^\mu \Sigma)]$$

The Fundamental Building Blocks

- ▶ $\mathbf{V} = \Sigma(\mathbf{D}\Sigma)^\dagger$ (longitudinal vectors), $\mathbf{T} = \Sigma\tau^3\Sigma^\dagger$ (neutral component)
- ▶ **Unitary gauge** (no Goldstones): $\mathbf{w} \equiv 0$, i.e., $\Sigma \equiv 1$.

$$\begin{aligned}\mathbf{V} &\longrightarrow -\frac{ig}{2} \left[\sqrt{2}(W^+\tau^+ + W^-\tau^-) + \frac{1}{c_w} Z\tau^3 \right] \\ \mathbf{T} &\longrightarrow \tau^3\end{aligned}$$

- ▶ **Gaugeless limit** (only Goldstones) ($g, g' \rightarrow 0$):

$$\begin{aligned}\mathbf{V} &\longrightarrow \frac{i}{v} \left\{ \sqrt{2}\partial w^+\tau^+ + \sqrt{2}\partial w^-\tau^- + \partial z\tau^3 \right\} + O(v^{-2}) \\ \mathbf{T} &\longrightarrow \tau^3 + 2\sqrt{2}\frac{i}{v} (w^+\tau^+ - w^-\tau^-) + O(v^{-2})\end{aligned}$$

So \mathbf{T} projects out the neutral part:

$$\text{tr}[\mathbf{T}\mathbf{V}] = \frac{2i}{v} \left[\partial z + \frac{i}{v} (w^+\partial w^- - w^-\partial w^+) \right] + O(v^{-3})$$

Electroweak Chiral Lagrangian

Complete Lagrangian contains infinitely many parameters

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{min}} - \sum_{\psi} \bar{\psi}_L \Sigma M \psi_R + \beta_1 \mathcal{L}'_0 + \sum_i \alpha_i \mathcal{L}_i + \frac{1}{v} \sum_i \alpha_i^{(5)} \mathcal{L}^{(5)} + \frac{1}{v^2} \sum_i \alpha_i^{(6)} \mathcal{L}^{(6)} + \dots$$

$$\mathcal{L}'_0 = \frac{v^2}{4} \text{tr} [\mathbf{T} \mathbf{V}_{\mu}] \text{tr} [\mathbf{T} \mathbf{V}^{\mu}]$$

$$\mathcal{L}_1 = \text{tr} [\mathbf{B}_{\mu\nu} \mathbf{W}^{\mu\nu}]$$

$$\mathcal{L}_2 = \text{itr} [\mathbf{B}_{\mu\nu} [\mathbf{V}^{\mu}, \mathbf{V}^{\nu}]]$$

$$\mathcal{L}_3 = \text{itr} [\mathbf{W}_{\mu\nu} [\mathbf{V}^{\mu}, \mathbf{V}^{\nu}]]$$

$$\mathcal{L}_4 = \text{tr} [\mathbf{V}_{\mu} \mathbf{V}_{\nu}] \text{tr} [\mathbf{V}^{\mu} \mathbf{V}^{\nu}]$$

$$\mathcal{L}_5 = \text{tr} [\mathbf{V}_{\mu} \mathbf{V}^{\mu}] \text{tr} [\mathbf{V}_{\nu} \mathbf{V}^{\nu}]$$

$$\mathcal{L}_6 = \text{tr} [\mathbf{V}_{\mu} \mathbf{V}_{\nu}] \text{tr} [\mathbf{T} \mathbf{V}^{\mu}] \text{tr} [\mathbf{T} \mathbf{V}^{\nu}]$$

$$\mathcal{L}_7 = \text{tr} [\mathbf{V}_{\mu} \mathbf{V}^{\mu}] \text{tr} [\mathbf{T} \mathbf{V}_{\nu}] \text{tr} [\mathbf{T} \mathbf{V}^{\nu}]$$

$$\mathcal{L}_8 = \frac{1}{4} \text{tr} [\mathbf{T} \mathbf{W}_{\mu\nu}] \text{tr} [\mathbf{T} \mathbf{W}^{\mu\nu}]$$

$$\mathcal{L}_9 = \frac{1}{2} \text{tr} [\mathbf{T} \mathbf{W}_{\mu\nu}] \text{tr} [\mathbf{T} [\mathbf{V}^{\mu}, \mathbf{V}^{\nu}]]$$

$$\mathcal{L}_{10} = \frac{1}{2} (\text{tr} [\mathbf{T} \mathbf{V}_{\mu}] \text{tr} [\mathbf{T} \mathbf{V}^{\mu}])^2$$

Indirect info on new physics in β_1, α_i, \dots (Flavor physics only in M)

Electroweak precision observables (LEP I/II, SLC):

$$\Delta S = -16\pi\alpha_1$$

$$\Delta T = 2\beta_1/\alpha_{\text{QED}}$$

$$\Delta U = -16\pi\alpha_8$$

$$\alpha_1 = 0.0026 \pm 0.0020$$

$$\beta_1 = -0.00062 \pm 0.00043$$

$$\alpha_8 = -0.0044 \pm 0.0026$$

Anomalous triple and quartic gauge couplings

$$\mathcal{L}_{TGC} = ie \left[g_1^\gamma A_\mu \left(W_\nu^- W^{+\mu\nu} - W_\nu^+ W^{-\mu\nu} \right) + \kappa^\gamma W_\mu^- W_\nu^+ A^{\mu\nu} + \frac{\lambda^\gamma}{M_W^2} W_\mu^{-\nu} W_{\nu\rho}^+ A^{\rho\mu} \right] \\ + ie \frac{c_w}{s_w} \left[g_1^Z Z_\mu \left(W_\nu^- W^{+\mu\nu} - W_\nu^+ W^{-\mu\nu} \right) + \kappa^Z W_\mu^- W_\nu^+ Z^{\mu\nu} + \frac{\lambda^Z}{M_W^2} W_\mu^{-\nu} W_{\nu\rho}^+ Z^{\rho\mu} \right]$$

SM values: $g_1^{\gamma,Z} = \kappa^{\gamma,Z} = 1$, $\lambda^{\gamma,Z} = 0$ and $\delta_Z = \frac{\beta_1 + g'^2 \alpha_1}{c_w^2 - s_w^2}$, $g_{1/2}^{VV'} = 1$, $h^{ZZ} = 0$

$$\Delta g_1^\gamma = 0$$

$$\Delta \kappa^\gamma = g^2(\alpha_2 - \alpha_1) + g^2 \alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^Z = \delta_Z + \frac{g^2}{c_w^2} \alpha_3$$

$$\Delta \kappa^Z = \delta_Z - g'^2(\alpha_2 - \alpha_1) + g^2 \alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^{\gamma\gamma} = \Delta g_2^{\gamma\gamma} = 0$$

$$\Delta g_2^{ZZ} = 2\Delta g_1^{\gamma Z} - \frac{g^2}{c_w^4}(\alpha_5 + \alpha_7)$$

$$\Delta g_1^{\gamma Z} = \Delta g_2^{\gamma Z} = \delta_Z + \frac{g^2}{c_w^2} \alpha_3$$

$$\Delta g_1^{WW} = 2c_w^2 \Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) + g^2 \alpha_4$$

$$\Delta g_1^{ZZ} = 2\Delta g_1^{\gamma Z} + \frac{g^2}{c_w^4}(\alpha_4 + \alpha_6)$$

$$\Delta g_2^{WW} = 2c_w^2 \Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) - g^2(\alpha_4 + 2\alpha_5)$$

$$h^{ZZ} = g^2[\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})]$$

Anomalous triple and quartic gauge couplings

$$\begin{aligned}
 \mathcal{L}_{QGC} = & e^2 \left[g_1^{\gamma\gamma} A^\mu A^\nu W_\mu^- W_\nu^+ - g_2^{\gamma\gamma} A^\mu A_\mu W^{-\nu} W_\nu^+ \right] \\
 & + e^2 \frac{c_w}{s_w} \left[g_1^{\gamma Z} A^\mu Z^\nu \left(W_\mu^- W_\nu^+ + W_\mu^+ W_\nu^- \right) - 2g_2^{\gamma Z} A^\mu Z_\mu W^{-\nu} W_\nu^+ \right] \\
 & + e^2 \frac{c_w^2}{s_w^2} \left[g_1^{ZZ} Z^\mu Z^\nu W_\mu^- W_\nu^+ - g_2^{ZZ} Z^\mu Z_\mu W^{-\nu} W_\nu^+ \right] \\
 & + \frac{e^2}{2s_w^2} \left[g_1^{WW} W^{-\mu} W^{+\nu} W_\mu^- W_\nu^+ - g_2^{WW} \left(W^{-\mu} W_\mu^+ \right)^2 \right] + \frac{e^2}{4s_w^2 c_w^4} h^{ZZ} (Z^\mu Z_\mu)^2
 \end{aligned}$$

SM values: $g_1^{\gamma,Z} = \kappa^{\gamma,Z} = 1$, $\lambda^{\gamma,Z} = 0$ and $\delta_Z = \frac{\beta_1 + g'^2 \alpha_1}{c_w^2 - s_w^2}$, $g_{1/2}^{VV'} = 1$, $h^{ZZ} = 0$

$$\Delta g_1^\gamma = 0$$

$$\Delta \kappa^\gamma = g^2(\alpha_2 - \alpha_1) + g^2 \alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^Z = \delta_Z + \frac{g^2}{c_w^2} \alpha_3$$

$$\Delta \kappa^Z = \delta_Z - g'^2(\alpha_2 - \alpha_1) + g^2 \alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^{\gamma\gamma} = \Delta g_2^{\gamma\gamma} = 0$$

$$\Delta g_2^{ZZ} = 2\Delta g_1^{\gamma Z} - \frac{g^2}{c_w^4}(\alpha_5 + \alpha_7)$$

$$\Delta g_1^{\gamma Z} = \Delta g_2^{\gamma Z} = \delta_Z + \frac{g^2}{c_w^2} \alpha_3$$

$$\Delta g_1^{WW} = 2c_w^2 \Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) + g^2 \alpha_4$$

$$\Delta g_1^{ZZ} = 2\Delta g_1^{\gamma Z} + \frac{g^2}{c_w^4}(\alpha_4 + \alpha_6)$$

$$\Delta g_2^{WW} = 2c_w^2 \Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) - g^2(\alpha_4 + 2\alpha_5)$$

$$h^{ZZ} = g^2[\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})]$$

Parameters and Scales, Resonances

α_i measurable at ILC

- ▶ $\alpha_i \ll 1$ (LEP)
- ▶ $\alpha_i \gtrsim 1/16\pi^2 \approx 0.006$ (renormalize divergencies, $16\pi^2\alpha_i \gtrsim 1$)

Translation of parameters into new physics scale Λ : $\alpha_i = v^2/\Lambda^2$

- ▶ Operator normalization is arbitrary
- ▶ Power counting can be intricate

To be specific: consider resonances that couple to EWSB sector

Resonance mass gives detectable shift in the α_i

- ▶ Narrow resonances \Rightarrow particles
- ▶ Wide resonances \Rightarrow continuum

$\beta_1 \ll 1 \Rightarrow SU(2)_c$ custodial symmetry (weak isospin, broken by hypercharge $g' \neq 0$ and fermion masses)

	$J = 0$	$J = 1$	$J = 2$
$I = 0$	σ^0 (Higgs ?)	ω^0 (γ'/Z' ?)	f^0 (Graviton ?)
$I = 1$	π^\pm, π^0 (2HDM ?)	ρ^\pm, ρ^0 (W'/Z' ?)	a^\pm, a^0
$I = 2$	$\phi^{\pm\pm}, \phi^\pm, \phi^0$ (Higgs triplet ?)	—	$t^{\pm\pm}, t^\pm, t^0$

accounts for **weakly and strongly interacting models**

Model-Independent Way – Effective Field Theories



How to clearly separate effects of **heavy degrees of freedom**?

Toy model: Two interacting scalar fields φ, Φ

$$\mathcal{Z}[j, J] = \int \mathcal{D}[\Phi] \mathcal{D}[\varphi] \exp \left[i \int dx \left(\frac{1}{2} (\partial\varphi)^2 - \frac{1}{2} \Phi (\square + M^2) \Phi - \lambda \varphi^2 \Phi - \dots + J \Phi + j \varphi \right) \right]$$

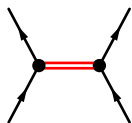
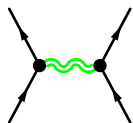
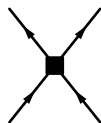
Low-energy effective theory \Rightarrow integrating out **heavy degrees of freedom (DOF)** in path integrals, set up **Power Counting**

Completing the square:

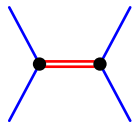
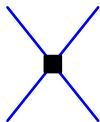
$$\Phi' = \Phi + \frac{\lambda}{M^2} \left(1 + \frac{\partial^2}{M^2} \right)^{-1} \varphi^2 \Rightarrow$$

$$\frac{1}{2} (\partial\Phi)^2 - \frac{1}{2} M^2 \Phi^2 - \lambda \varphi^2 \Phi = -\frac{1}{2} \Phi' (M^2 + \partial^2) \Phi' + \frac{\lambda^2}{2M^2} \varphi^2 \left(1 + \frac{\partial^2}{M^2} \right)^{-1} \varphi^2.$$

Effective Dim. 6 Operators

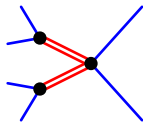
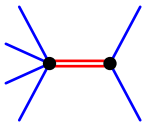
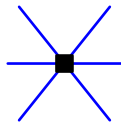

 \longrightarrow


$$\mathcal{O}_{JJ}^{(I)} = \frac{1}{F^2} \text{tr} [J^{(I)} \cdot J^{(I)}]$$

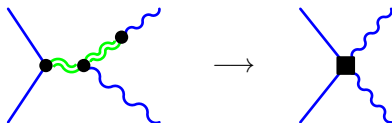

 \longrightarrow


$$\mathcal{O}'_{h,1} = \frac{1}{F^2} ((Dh)^\dagger h) \cdot (h^\dagger (Dh)) - \frac{v^2}{2} |Dh|^2$$

$$\mathcal{O}'_{hh} = \frac{1}{F^2} (h^\dagger h - v^2/2) (Dh)^\dagger \cdot (Dh)$$


 \longrightarrow


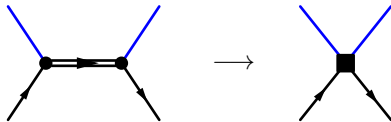
$$\mathcal{O}'_{h,3} = \frac{1}{F^2} \frac{1}{3} (h^\dagger h - v^2/2)^3$$



$$\mathcal{O}'_{WW} = -\frac{1}{F^2} \frac{1}{2} (h^\dagger h - v^2/2) \text{tr} [W_{\mu\nu} W^{\mu\nu}]$$

$$\mathcal{O}_B = \frac{1}{F^2} \frac{i}{2} (D_\mu h)^\dagger (D_\nu h) B^{\mu\nu}$$

$$\mathcal{O}'_{BB} = -\frac{1}{F^2} \frac{1}{4} (h^\dagger h - v^2/2) B_{\mu\nu} B^{\mu\nu}$$



$$\mathcal{O}_{Vq} = \frac{1}{F^2} \bar{q} h (\not{D} h) q$$

Oblique Corrections: S , T , U

$$\Delta T \sim \Delta \rho \sim \Delta M_Z^2 \mathbf{Z} \cdot \mathbf{Z}$$

$$\Delta S \sim W^0_{\mu\nu} B^{\mu\nu}, \Delta U \sim W^0_{\mu\nu} W^{0\mu\nu}$$

- ◇ All low-energy effects order v^2/F^2 (Wilson coefficients)
- ◇ Low-energy observables with low-energy input G_F , α , M_Z affected by **non-oblique** contributions:

$$G_F = \frac{1}{v} \longrightarrow \frac{1}{v} (1 - \alpha \Delta T + \delta), \quad \boxed{\delta \equiv -\frac{v^2}{4} f_{JJ}^{(3)}}$$

$$S_{\text{eff}} = \Delta S$$

$$T_{\text{eff}} = \Delta T - \frac{1}{\alpha} \delta$$

$$U_{\text{eff}} = [\Delta U = 0] + \frac{4s_w^2}{\alpha} \delta$$

- ▶ non-oblique flavour-dependent corrections \Rightarrow enforce **flavour-dependent EW fit**

Integrating out resonances

Consider leading order effects of resonances on EW sector:

$$\mathcal{L}_\Phi = z [\Phi (M_\Phi^2 + DD) \Phi + 2\Phi J] \quad \Rightarrow \quad \mathcal{L}_\Phi^{\text{eff}} = -\frac{z}{M^2} J J + \frac{z}{M^4} J (DD) J + \mathcal{O}(M^{-6})$$

- ▶ Simplest example: scalar singlet σ :

$$\mathcal{L}_\sigma = -\frac{1}{2} [\sigma (M_\sigma^2 + \partial^2) \sigma - g_\sigma v \sigma \text{tr} [\mathbf{V}_\mu \mathbf{V}^\mu] - h_\sigma \text{tr} [\mathbf{T} \mathbf{V}_\mu] \text{tr} [\mathbf{T} \mathbf{V}^\mu]]$$

- ▶ Effective Lagrangian

$$\mathcal{L}_\sigma^{\text{eff}} = \frac{v^2}{8M_\sigma^2} [g_\sigma \text{tr} [\mathbf{V}_\mu \mathbf{V}^\mu] + h_\sigma \text{tr} [\mathbf{T} \mathbf{V}_\mu] \text{tr} [\mathbf{T} \mathbf{V}^\mu]]^2$$

- ▶ leads to **anomalous quartic couplings**

$$\alpha_5 = g_\sigma^2 \left(\frac{v^2}{8M_\sigma^2} \right) \quad \alpha_7 = 2g_\sigma h_\sigma \left(\frac{v^2}{8M_\sigma^2} \right) \quad \alpha_{10} = 2h_\sigma^2 \left(\frac{v^2}{8M_\sigma^2} \right)$$

- ▶ Special case: SM Higgs with $g_\sigma = 1$ and $h_\sigma = 0$

Coupl. strengths, Anomal. Couplings, Power Counting

Scalar resonance width ($M_\sigma \gg M_W, M_Z$):

$$\Gamma_\sigma = \frac{g_\sigma^2 + \frac{1}{2}(g_\sigma^2 + 2h_\sigma^2)^2}{16\pi} \left(\frac{M_\sigma^3}{v^2} \right) + \Gamma(\text{non} - WW, ZZ)$$

Largest allowed coupling for a broad continuum: $\Gamma \sim M \gg \Gamma(\text{non} - WW, ZZ) \sim 0$
translates to bounds for effective Lagrangian (e.g. scalar with no isospin violation):

$$\alpha_5 \leq \frac{4\pi}{3} \left(\frac{v^4}{M_\sigma^4} \right) \approx \frac{0.015}{(M_\sigma \text{ in TeV})^4} \Rightarrow 16\pi^2 \alpha_5 \leq \frac{2.42}{(M_\sigma \text{ in TeV})^4}$$

Scalar: $\Gamma \sim g^2 M^3, \alpha \sim g^2/M^2 \Rightarrow \alpha_{\max} \sim 1/M^4$

Vector: $\Gamma \sim g^2 M, \alpha \sim g^2/M^2 \Rightarrow \alpha_{\max} \sim 1/M^2$

Tensor: $\Gamma \sim g^2 M^3, \alpha \sim g^2/M^2 \Rightarrow \alpha_{\max} \sim 1/M^4$

Vector triplet (simplified)

$$\mathcal{L}_\rho = -\frac{1}{8} \text{tr} [\boldsymbol{\rho}_{\mu\nu} \boldsymbol{\rho}^{\mu\nu}] + \frac{M_\rho^2}{4} \text{tr} [\boldsymbol{\rho}_\mu \boldsymbol{\rho}^\mu] + \frac{ig_\rho v^2}{2} \text{tr} [\boldsymbol{\rho}_\mu \mathbf{V}^\mu]$$

$1/M^2$ term renormalizes kinetic energy (i.e. v), hence unobservable:

$$\mathcal{L}_\rho^{\text{eff}} = \frac{g_\rho^2 v^4}{4M_\rho^2} \text{tr} [(\mathbf{D}_\mu \Sigma)(\mathbf{D}^\mu \Sigma)] + \mathcal{O}(1/M_\rho^4)$$

Vector Resonances

$$\begin{aligned}
 \mathcal{L}_\rho = & -\frac{1}{8} \text{tr} [\rho_{\mu\nu} \rho^{\mu\nu}] + \frac{M_\rho^2}{4} \text{tr} [\rho_\mu \rho^\mu] + \frac{\Delta M_\rho^2}{8} (\text{tr} [\mathbf{T} \rho_\mu])^2 + i \frac{\mu_\rho}{2} g \text{tr} [\rho_\mu \mathbf{W}^{\mu\nu} \rho_\nu] \\
 & + i \frac{\mu'_\rho}{2} g' \text{tr} [\rho_\mu \mathbf{B}^{\mu\nu} \rho_\nu] + i \frac{g_\rho v^2}{2} \text{tr} [\rho_\mu \mathbf{V}^\mu] + i \frac{h_\rho v^2}{2} \text{tr} [\rho_\mu \mathbf{T}] \text{tr} [\mathbf{T} \mathbf{V}^\mu] \\
 & + \frac{g' v^2 k_\rho}{2 M_\rho^2} \text{tr} [\rho_\mu [\mathbf{B}^{\nu\mu}, \mathbf{V}_\nu]] + \frac{g v^2 k'_\rho}{4 M_\rho^2} \text{tr} [\rho_\mu [\mathbf{T}, \mathbf{V}_\nu]] \text{tr} [\mathbf{T} \mathbf{W}^{\nu\mu}] \\
 & + \frac{g v^2 k''_\rho}{4 M_\rho^2} \text{tr} [\mathbf{T} \rho_\mu] \text{tr} [[\mathbf{T}, \mathbf{V}_\nu] \mathbf{W}^{\nu\mu}] + i \frac{\ell_\rho}{M_\rho^2} \text{tr} [\rho_{\mu\nu} \mathbf{W}^\nu{}_\rho \mathbf{W}^{\rho\mu}] \\
 & + i \frac{\ell'_\rho}{M_\rho^2} \text{tr} [\rho_{\mu\nu} \mathbf{B}^\nu{}_\rho \mathbf{W}^{\rho\mu}] + i \frac{\ell''_\rho}{M_\rho^2} \text{tr} [\rho_{\mu\nu} \mathbf{T}] \text{tr} [\mathbf{T} \mathbf{W}^\nu{}_\rho \mathbf{W}^{\rho\mu}]
 \end{aligned}$$

all $\alpha_i \sim 1/M_\rho^4$, except for $\beta_1 \sim \Delta\rho \sim T \sim h_\rho^2/M_\rho^2$

4-fermion contact interaction $j_\mu j^\mu \sim 1/M_\rho^2$ (eff. T and U parameter)

vector coupling $j_\mu V^\mu \sim 1/M_\rho^2$ (eff. S parameter)

Mismatch: measured fermionic vs. bosonic coupling g

Nyffeler/Schenk, 2000; Kilian/JR, 2003

Effects on Triple Gauge Couplings

- ▶ $\mathcal{O}(1/M^2)$: Renormalization of ZWW coupling
- ▶ $\mathcal{O}(1/M^4)$: shifts in Δg_1^Z , $\Delta \kappa^\gamma$, $\Delta \kappa^Z$, λ^γ , λ^Z

Effects on Quartic Gauge Couplings

- ▶ $\mathcal{O}(1/M^4)$, orthogonal (in α_4 - α_5 space) to scalar case

WHIZARD

Kilian/Ohl/JRR: DESY/Freiburg/Siegen/Würzburg, hep-ph/0102195, 0708.4233



- ▶ Multi-Purpose event generator for collider and astroparticle physics
- ▶ Acronym: **W**, **H**iggs, **Z**, **A**nd **R**espective **D**ecays (deprecated)
 - ▶ Fast adaptive multi-channel Monte-Carlo integration
 - ▶ Very efficient phase space and event generation
 - ▶ Optimized/-al matrix elements
 - ▶ Recent version: 2.1.1 (18.09.2012)
 - `http://projects.hepforge.org/whizard` und
 - `http://whizard.event-generator.org`
 - ▶ Parton shower (k^\perp -ordered and analytical) Kilian/JRR/Schmidt/Wiesler, 1112.1039
 - ▶ Underlying Event: preliminary (for 2.1)
 - ▶ Arbitrary processes: matrix element generator (O'Mega)
 - ▶ 2.0 Features: ME/PS matching, cascades, versatile new steering syntax, WHIZARD as shared library
- ▶ Interface to `FeynRules` Christensen/Duhr/Fuks/JRR/Speckner, 1010.3215
- ▶ Versatile input language: SINDARIN

WHIZARD

Kilian/Ohl/JRR: DESY/Freiburg/Siegen/Würzburg, hep-ph/0102195, 0708.4233



- ▶ Multi-Purpose event generator for collider and astroparticle physics
- ▶ Focus: LHC, ILC, CLIC, SM, QCD, **BSM**

MODEL TYPE	with CKM matrix	trivial CKM
QED with e, μ, τ, γ	—	QED
QCD with d, u, s, c, b, t, g	—	QCD
Standard model	SM_CKM	SM
SM with anomalous couplings	SM_ac_CKM	SM_ac
SM with anomalous top couplings	—	SM_top
SM with K matrix	—	SM_KM
MSSM	MSSM_CKM	MSSM
MSSM with Gravitinos	—	MSSM_Grav
NMSSM	NMSSM_CKM	NMSSM
extended SUSY models	—	PSSSM
Littlest Higgs	—	Littlest
Littlest Higgs with ungauged $U(1)$	—	Littlest_Eta
Littlest Higgs with T parity	—	Littlest_Tpar
Simplest Little Higgs (anomaly free)	—	Simplest
Simplest Little Higgs (universal)	—	Simplest_univ
UED	—	UED
3-Site Higgsless Model	—	Thresh1
Noncommutative SM (inoff.)	—	NCSM
SM with Z'	—	Zprime
SM with Gravitino and Photino	—	GravTest
Augmentable SM template	—	Template

easy to
implement new models

- ▶ Interface to FeynRules
- ▶ Versatile input language: SINDARIN

Christensen/Duhr/Fuks/JRR/Speckner, 1010.3215

Anomalous Gauge Couplings at LHC

ILC:

Beyer/Kilian/Krstonošić/Mönig/JR/Schröder/Schmidt, 2006

LHC:

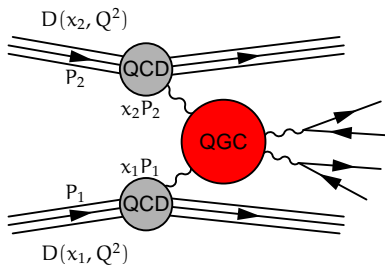
Mertens, 2006; Kilian/Kobel/Mader/JR/Schumacher

Anomalous quartic gauge couplings, by chiral EW Lagrangian:

$$\mathcal{L}_4 = \alpha_4 \frac{g^2}{2} \left\{ [(W^+ W^+)(W^- W^-) + (W^+ W^-)^2] + \frac{2}{c_W^2} (W^+ Z)(W^- Z) + \frac{1}{2c_W^4} (ZZ)^2 \right\}$$

$$\mathcal{L}_5 = \alpha_5 \frac{g^2}{2} \left\{ (W^+ W^-)^2 + \frac{2}{c_W^2} (W^+ W^-)(ZZ) + \frac{1}{2c_W^4} (ZZ)^2 \right\}$$

(all leptons, incl. τ):



$$pp \rightarrow jj(ZZ/WW) \rightarrow jj\ell^-\ell^+\nu_e\bar{\nu}_e$$

$$\sigma \approx 40 \text{ fb}$$

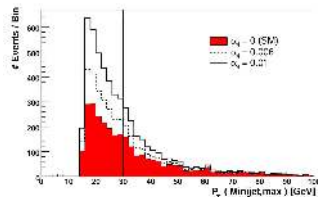
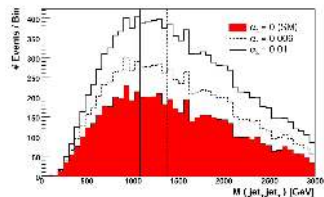
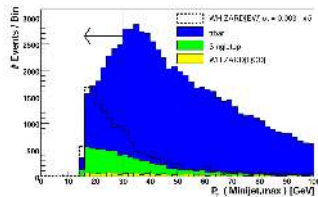
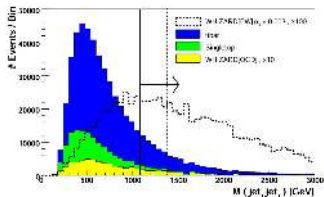
Background:

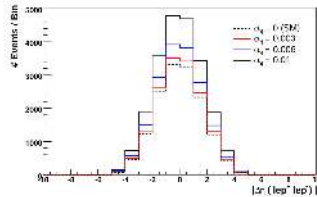
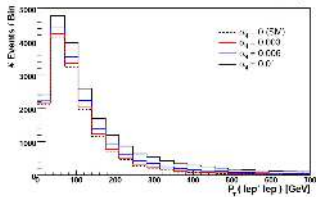
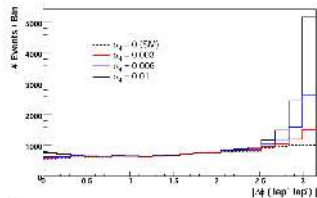
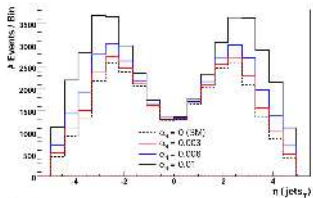
- ▶ $t\bar{t} \rightarrow WbWb$, $\sigma \approx 52 \text{ pb}$
- ▶ Single t , misrec. jet: $\sigma \approx 4.8 \text{ pb}$
- ▶ QCD: $\sigma \approx 0.21 \text{ pb}$

Tagging and Cuts:

- ▶ $\ell\ell jj$ -Tag, $\eta_{tag}^{min} < \eta_\ell < \eta_{tag}^{max}$, b -Veto
- ▶ $|\Delta\eta_{jj}| > 4.4$, $M_{jj} > 1080$ GeV
- ▶ Minijet-Veto: $p_{T,j} < 30$ GeV
- ▶ $E_j > 600, 400$ GeV, $p_{T,j}^1 > 60, 24$ GeV

Improves S/\sqrt{B} from 3.3 to 29.7





Results: (1σ Sensitivity to α_s)

Coupl.	ILC (1 ab^{-1})	LHC (100 fb^{-1})
α_4	0.0088	0.00160
α_5	0.0071	0.00098

Limits for Λ [TeV]:

Spin	$I = 0$	$I = 1$	$I = 2$
0	1.39	1.55	1.95
1	1.74	2.67	—
2	3.00	3.01	5.84

Isospin decomposition

- ▶ Lowest order chiral Lagrangian (incl. anomalous couplings)

$$\mathcal{L} = -\frac{v^2}{4} \text{tr} [\mathbf{V}_\mu \mathbf{V}^\mu] + \alpha_4 \text{tr} [\mathbf{V}_\mu \mathbf{V}_\nu] \text{tr} [\mathbf{V}^\mu \mathbf{V}^\nu] + \alpha_5 (\text{tr} [\mathbf{V}_\mu \mathbf{V}^\mu])^2$$

- ▶ Leads to the following amplitudes: $s = (p_1 + p_2)^2$ $t = (p_1 - p_3)^2$ $u = (p_1 - p_4)^2$

$$\boxed{\mathcal{A}(s, t, u) =:}$$

$$\begin{aligned} \mathcal{A}(w^+ w^- \rightarrow zz) &= \frac{s}{v^2} + 8\alpha_5 \frac{s^2}{v^4} + 4\alpha_4 \frac{t^2 + u^2}{v^4} \\ \mathcal{A}(w^+ z \rightarrow w^+ z) &= \frac{t}{v^2} + 8\alpha_5 \frac{t^2}{v^4} + 4\alpha_4 \frac{s^2 + u^2}{v^4} \\ \mathcal{A}(w^+ w^- \rightarrow w^+ w^-) &= -\frac{u}{v^2} + (4\alpha_4 + 2\alpha_5) \frac{s^2 + t^2}{v^4} + 8\alpha_4 \frac{u^2}{v^4} \\ \mathcal{A}(w^+ w^+ \rightarrow w^+ w^+) &= -\frac{s}{v^2} + 8\alpha_4 \frac{s^2}{v^4} + 4(\alpha_4 + 2\alpha_5) \frac{t^2 + u^2}{v^4} \\ \mathcal{A}(zz \rightarrow zz) &= 8(\alpha_4 + \alpha_5) \frac{s^2 + t^2 + u^2}{v^4} \end{aligned}$$

- ▶ (Clebsch-Gordan) Decomposition into isospin eigenamplitudes

$$\mathcal{A}(I=0) = 3\mathcal{A}(s, t, u) + \mathcal{A}(t, s, u) + \mathcal{A}(u, s, t)$$

$$\mathcal{A}(I=1) = \mathcal{A}(t, s, u) - \mathcal{A}(u, s, t)$$

$$\mathcal{A}(I=2) = \mathcal{A}(t, s, u) + \mathcal{A}(u, s, t)$$

Unitarity of Amplitudes

UV-incomplete theories could violate unitarity

Cross section:
$$\sigma = \int d\Omega \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |\mathcal{M}|^2$$

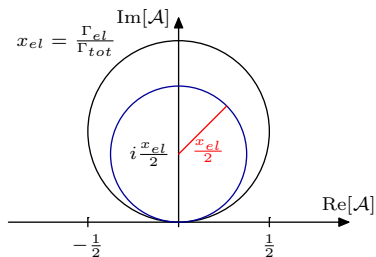
Optical Theorem (Unitarity of the S(cattering) Matrix):

$$\sigma_{\text{tot}} = \text{Im} [\mathcal{M}_{ii}(t=0)] / s \quad t = -s(1 - \cos\theta)/2$$

Partial wave amplitudes:
$$\mathcal{M}(s, t, u) = 32\pi \sum_{\ell} (2\ell + 1) \mathcal{A}_{\ell}(s) P_{\ell}(\cos\theta)$$

Assuming only elastic scattering:

$$\sigma_{\text{tot}} = \sum_{\ell} \frac{32\pi(2\ell+1)}{s} |\mathcal{A}_{\ell}|^2 \stackrel{!}{=} \sum_{\ell} \frac{32\pi(2\ell+1)}{s} \text{Im} [\mathcal{A}_{\ell}] \Rightarrow \boxed{|\mathcal{A}_{\ell}|^2 = \text{Im} [\mathcal{A}_{\ell}]}$$



Argand circle

$$\boxed{|\mathcal{A}(s) - \frac{i}{2}| = \frac{1}{2}}$$

Resonance:
$$\mathcal{A}(s) = \frac{-M\Gamma_{el}}{s - M^2 + iM\Gamma_{tot}}$$

Counterclockwise circle, **radius** $\frac{x_{el}}{2}$

Pole at $s = M^2 - iM\Gamma_{tot}$

Unitarity in the EW sector: SM

► Project out isospin eigenamplitudes

Lee, Quigg, Thacker, 1973

$$\mathcal{A}_\ell(s) = \frac{1}{32\pi} \int_{-s}^0 \frac{dt}{s} \mathcal{A}(s, t, u) P_\ell(1 + 2t/s) \quad \cos \theta = 1 + 2t/s$$

Remember Legendre polynomials: $P_0(s) = 1$ $P_1(s) = \cos \theta$ $P_2(s) = (3 \cos^2 \theta - 1)/2$

► SM longitudinal isospin eigenamplitudes ($\mathcal{A}_{I, \text{spin}=J}$):

$$\mathcal{A}_{I=0} = 2 \frac{s}{v^2} P_0(s) \quad \mathcal{A}_{I=1} = \frac{t-u}{v^2} = \frac{s}{v^2} P_1(s) \quad \mathcal{A}_{I=2} = -\frac{s}{v^2} P_2(s)$$

$$\mathcal{A}_{0,0} = \frac{s}{16\pi v^2}$$

$$\mathcal{A}_{1,1} = \frac{s}{96\pi v^2}$$

$$\mathcal{A}_{2,0} = -\frac{s}{32\pi v^2}$$

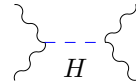
exceeds unitarity bound $|\mathcal{A}_{IJ}| \lesssim \frac{1}{2}$ at:

$$I = 0 : \quad E \sim \sqrt{8\pi} v = 1.2 \text{ TeV}$$

$$I = 1 : \quad E \sim \sqrt{48\pi} v = 3.5 \text{ TeV}$$

$$I = 2 : \quad E \sim \sqrt{16\pi} v = 1.7 \text{ TeV}$$

Higgs exchange:



$$\mathcal{A}(s, t, u) = -\frac{M_H^2}{v^2} \frac{s}{s - M_H^2}$$

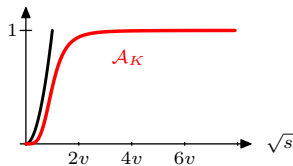
Unitarity: $M_H \lesssim \sqrt{8\pi} v \sim 1.2 \text{ TeV}$

K-Matrix Unitarization and friends

K-Matrix unitarization

$$\mathcal{A}_K(s) = \frac{\mathcal{A}(s)}{1 - i\mathcal{A}(s)} = \mathcal{A}(s) \frac{1 + i\mathcal{A}(s)}{1 + \mathcal{A}(s)^2}$$

Unitarization by infinitely heavy and wide resonance

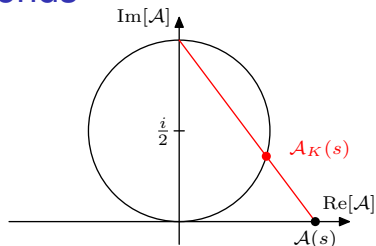


Padé unitarization

separates higher chiral orders

$$\mathcal{A}_P(s) = \frac{\mathcal{A}^{(0)}(s)^2}{\mathcal{A}^{(0)}(s) - \mathcal{A}^{(1)}(s) - i\mathcal{A}^{(0)}(s)^2}$$

each partial wave dominated by single resonance



- ▶ Low-energy theorem (LET): $\frac{s}{v^2}$
- ▶ K-Matrix amplitude: $|\mathcal{A}(s)|^2 = \frac{s^2}{s^2 + v^4} \xrightarrow{s \rightarrow \infty} 1$
- ▶ Poles $\pm iv$: M_0, Γ large

“Naive” Unitarization

Extreme case:

$$\mathcal{A}_N(s) = e^{i\mathcal{A}(s)} \sin \mathcal{A}(s)$$

Infinitely many resonances becoming denser for $s \rightarrow \infty$

BSM Unitarized Resonances: e.g. Scalar Singlet

Assumptions:

- ▶ LHC is able to detect a resonance in the EW sector
- ▶ Further resonances might exist, but out of reach or not detectable
- ▶ Describe 1st resonance by correct amplitude
- ▶ Use K-matrix unitarization to define a consistent model

Example: Scalar Singlet

- ▶ $\mathcal{L}_\sigma = -\frac{1}{2}\sigma(M_\sigma^2 + \partial^2)\sigma + \frac{g_\sigma v}{2}\sigma \text{tr}[\mathbf{V}_\mu \mathbf{V}^\mu]$
- ▶ Feynman rules: $\sigma w^+ w^- : -\frac{2ig_\sigma}{v}(k_+ \cdot k_-)$ $\sigma zz : -\frac{2ig_\sigma}{v}(k_1 \cdot k_2)$

- ▶ Amplitude (s-channel exchange):

$$\mathcal{A}^\sigma(s, t, u) = \frac{g_\sigma^2}{v^2} \frac{s^2}{s - M^2}$$

- ▶ Isospin eigenamplitudes:

$$\mathcal{A}_0^\sigma(s, t, u) = \frac{g_\sigma^2}{v^2} \left(3 \frac{s^2}{s - M^2} + \frac{t^2}{t - M^2} + \frac{u^2}{u - M^2} \right)$$

$$\mathcal{A}_1^\sigma(s, t, u) = \frac{g_\sigma^2}{v^2} \left(\frac{t^2}{t - M^2} - \frac{u^2}{u - M^2} \right)$$

$$\mathcal{A}_2^\sigma(s, t, u) = \frac{g_\sigma^2}{v^2} \left(\frac{t^2}{t - M^2} + \frac{u^2}{u - M^2} \right)$$

Unitarizing the scalar singlet

Alboteanu/Kilian/JR, 2008

$$\mathcal{A}_{00}^\sigma(s) = 3 \frac{g_\sigma^2}{v^2} \frac{s^2}{s-M^2} + 2 \frac{g_\sigma^2}{v^2} \mathcal{S}_0(s)$$

$$\mathcal{A}_{02}^\sigma(s) = 2 \frac{g_\sigma^2}{v^2} \mathcal{S}_2(s) = A_{22}^\sigma(s)$$

$$\mathcal{A}_{11}^\sigma(s) = 2 \frac{g_\sigma^2}{v^2} \mathcal{S}_1(s)$$

$$\mathcal{A}_{13}^\sigma(s) = 2 \frac{g_\sigma^2}{v^2} \mathcal{S}_3(s)$$

$$\mathcal{A}_{20}^\sigma(s) = 2 \frac{g_\sigma^2}{v^2} \mathcal{S}_0(s)$$

- ▶ S -wave coefficients no longer polynomial, e.g.:

$$\mathcal{S}_0(s) = M^2 - \frac{s}{2} + \frac{M^4}{s} \log \frac{s}{s+M^2}$$

- ▶ s -channel pole must be explicitly subtracted:

$$A_{IJ}(s) = A_{IJ}^{(0)}(s) + F_{IJ}(s) + \frac{G_{IJ}(s)}{s-M^2},$$

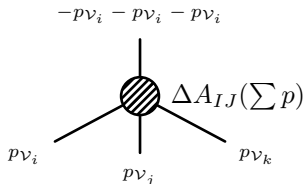
- $F_{IJ}(s)$ is finite
- $G_{IJ}(s) \propto s$ (vector), $\propto s^2$ (scalar, tensor)

$$\hat{A}_{IJ}(s) = \frac{A_{IJ}(s)}{1 - \frac{i}{32\pi} A_{IJ}(s)} = A_{IJ}^{(0)}(s) + 32\pi i \Delta A_{IJ}(s),$$

$$\Delta A_{IJ}(s) = 32\pi i \left(1 + \frac{i}{32\pi} A_{IJ}^{(0)}(s) + \frac{s-M^2}{\frac{i}{32\pi} G_{IJ}(s) - (s-M^2) \left[1 - \frac{i}{32\pi} (A_{IJ}^{(0)}(s) + F_{IJ}(s)) \right]} \right)$$

Implementation and Taxonomy of Resonances

- ▶ Explicit “time arrow” in WHIZARD



- trace back pairs of momenta at quartic vertices to external legs
- guarantee for only s -channel insertions

- ▶ Consider the following resonances:

$$\mathcal{L}_\sigma = -\frac{1}{2}\sigma(M_\sigma^2 + \partial^2)\sigma + \sigma j_\sigma$$

$$\mathcal{L}_\phi = -\frac{1}{2}\left[\frac{1}{2}\text{tr}[\phi(M_\sigma^2 + \partial^2)\phi] + \text{tr}[\phi j_\phi]\right]$$

$$\mathcal{L}_\rho = \frac{1}{2}\left[\frac{M_\rho^2}{2}\text{tr}[\rho_\mu\rho^\mu] - \frac{1}{4}\text{tr}[\rho_{\mu\nu}\rho^{\mu\nu}] + \text{tr}[j_\rho^\mu\rho_\mu]\right]$$

$$\mathcal{L}_f = \mathcal{L}_{\text{kin}} - \frac{M_f^2}{2}f_{\mu\nu}f^{\mu\nu} + f_{\mu\nu}j_f^{\mu\nu}$$

$$\mathcal{L}_a = \mathcal{L}_{\text{kin}} - \frac{M_a^2}{4}\text{tr}[\mathbf{t}_{\mu\nu}\mathbf{t}^{\mu\nu}] + \frac{1}{2}\text{tr}[\mathbf{t}_{\mu\nu}\mathbf{j}_a^{\mu\nu}]$$

$$j_\sigma = \frac{g\sigma v}{2}\text{tr}[\mathbf{V}_\mu\mathbf{V}^\mu]$$

$$\mathbf{j}_\phi = -\frac{g\phi v}{2}\left(\mathbf{V}_\mu \otimes \mathbf{V}^\mu - \frac{\tau^{aa}}{6}\text{tr}[\mathbf{V}_\mu\mathbf{V}^\mu]\right)$$

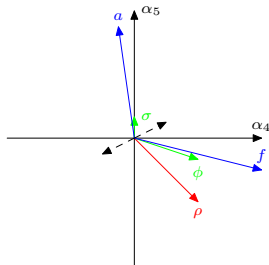
$$\mathbf{j}_\rho^\mu = ig_\rho v^2\mathbf{V}^\mu$$

$$j_f^{\mu\nu} = -\frac{gf v}{2}\left(\text{tr}[\mathbf{V}^\mu\mathbf{V}^\nu] - \frac{g^{\mu\nu}}{4}\text{tr}[\mathbf{V}_\rho\mathbf{V}^\rho]\right)$$

$$\mathbf{j}_a^{\mu\nu} = -\frac{g_a v}{2}\left[\frac{1}{2}(\mathbf{V}^\mu \otimes \mathbf{V}^\nu + \mathbf{V}^\nu \otimes \mathbf{V}^\mu) - \frac{g^{\mu\nu}}{4}\mathbf{V}_\rho \otimes \mathbf{V}^\rho - \frac{\tau^{aa}}{6}\text{tr}[\mathbf{V}^\mu\mathbf{V}^\nu] + \frac{g^{\mu\nu}\tau^{aa}}{24}\text{tr}[\mathbf{V}_\rho\mathbf{V}^\rho]\right]$$

Taxonomy of resonances/Loops

Resonance	σ	ϕ	ρ	f	a
$\Gamma[g^2 M^2/(64\pi v^2)]$	6	1	$\frac{4}{3}(\frac{v^2}{M^2})$	$\frac{1}{5}$	$\frac{1}{30}$
$\Delta\alpha_4[(16\pi\Gamma/M)(v^4/M^4)]$	0	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{5}{2}$	$-\frac{5}{8}$
$\Delta\alpha_5[(16\pi\Gamma/M)(v^4/M^4)]$	$\frac{1}{12}$	$-\frac{1}{12}$	$-\frac{3}{4}$	$-\frac{5}{8}$	$\frac{35}{8}$



- ▶ Loop corrections to LET can be switched on/off:
(μ renormalization scale)

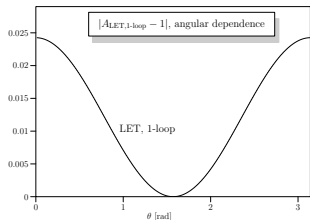
$$A_C^{1\text{-loop}}(s, t, u) = \frac{1}{16\pi^2} \left[\left(\frac{1}{2} \ln \frac{\mu^2}{|s|} + 8C_5 \right) \frac{s^2}{v^4} + \left(\frac{t(s+2t)}{6v^4} \ln \frac{\mu^2}{|t|} + 4C_4 \frac{t^2}{v^4} \right) + (t \leftrightarrow u) \right],$$

- ▶ Finite scheme-dep. matching coefficients/NLO counterterms
(e.g. heavy Higgs regulator $\mu = M_H$ [Dawson/Willenbrock, 1989](#))

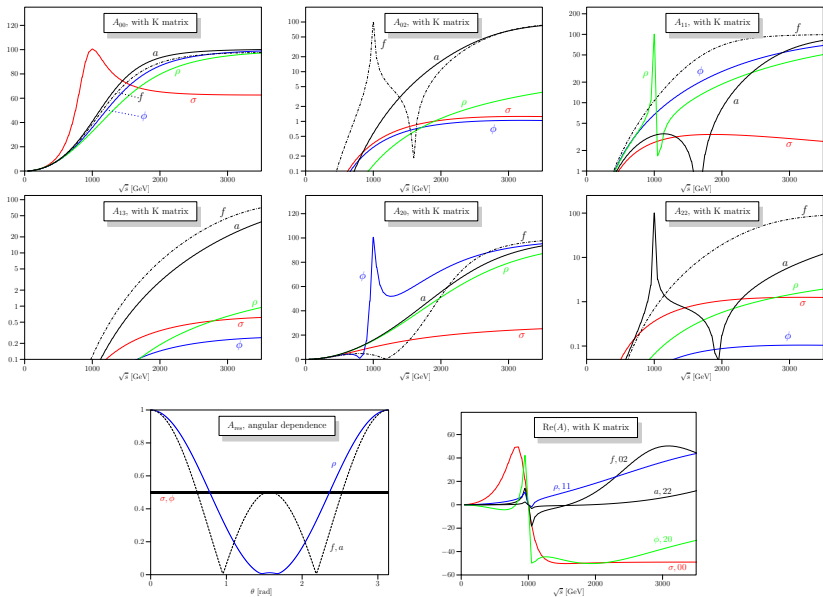
$$C_4 = -\frac{1}{18} \approx -0.056, \quad C_5 = \frac{9\pi}{16\sqrt{3}} - \frac{37}{36} \approx -0.0075.$$

$$\alpha_4^{(1)} = \frac{1}{16\pi^2} \left(C_4 - \frac{1}{12} \ln \frac{\mu^2}{\mu_0^2} \right)$$

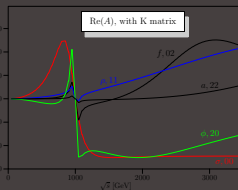
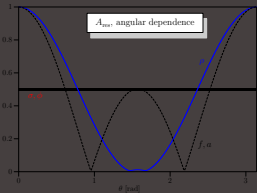
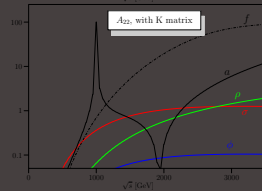
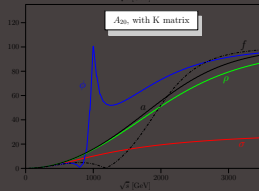
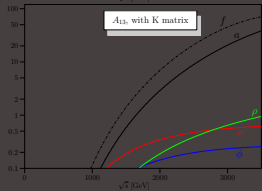
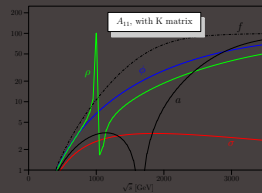
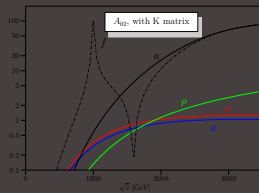
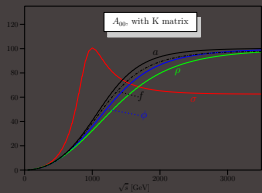
$$\alpha_5^{(1)} = \frac{1}{16\pi^2} \left(C_5 - \frac{1}{24} \ln \frac{\mu^2}{\mu_0^2} \right)$$



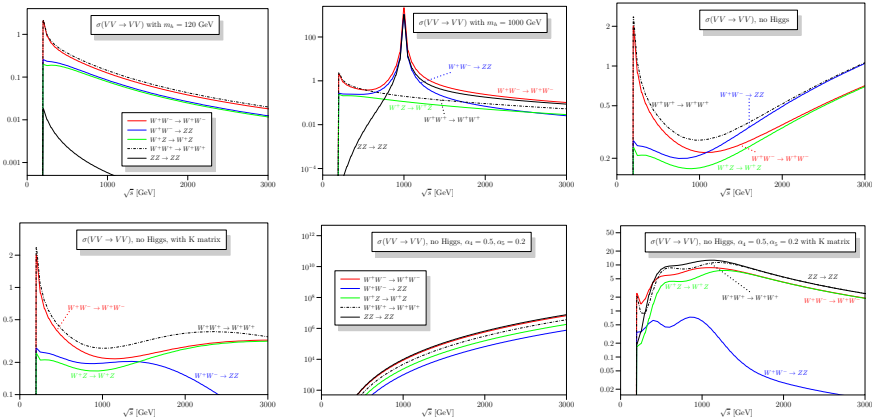
Eigenamplitudes



Eigenamplitudes

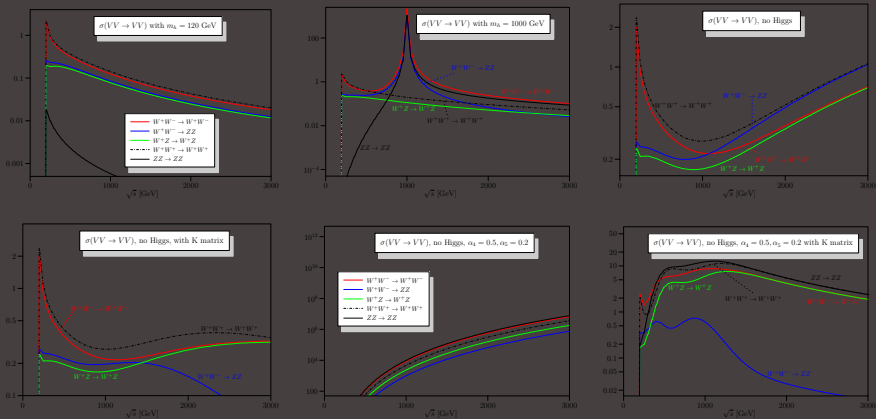


"Partonic" cross sections (I)



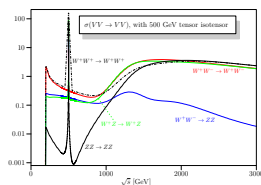
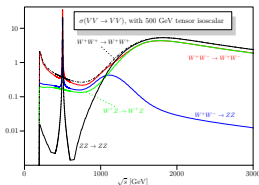
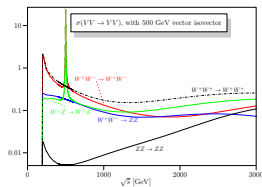
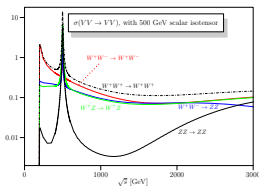
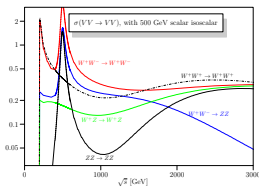
► Cross sections (in nb)

“Partonic” cross sections (I)



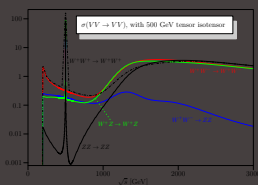
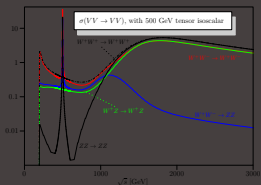
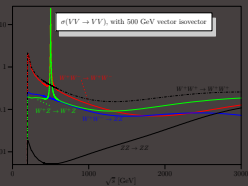
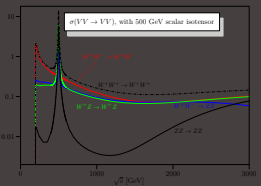
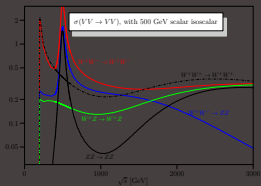
► Cross sections (in nb)

“Partonic” cross sections (II)



- ▶ $\sigma(VV \rightarrow VV)$ in nb $M_R = 500$ GeV
- ▶ all amplitudes K-matrix unitarized
- ▶ Cut of 15° around the beam axis

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- ▶ $\sigma(VV \rightarrow VV)$ in nb $M_R = 500$ GeV
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The Effective W approximation

- ▶ $M_{\mathcal{V}}, \hat{t}_i$ small corrections, \mathcal{V} nearly onshell:

$$\sigma(q_1 q_2 \rightarrow q'_1 q'_2 \mathcal{V}'_1 \mathcal{V}'_2) \approx \sum_{\lambda_1, \lambda_2} \int dx_1 dx_2 F_{q_1 \rightarrow q'_1 \mathcal{V}_1}^{\lambda_1}(x_1) F_{q_2 \rightarrow q'_2 \mathcal{V}_2}^{\lambda_2}(x_2) \sigma_{\mathcal{V}_1 \mathcal{V}_2 \rightarrow \mathcal{V}'_1 \mathcal{V}'_2}(x_1 x_2 s)$$

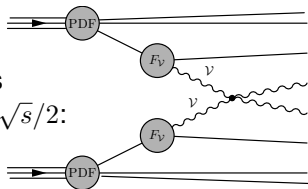
- ▶ In addition to Weizsäcker-Williams: longitudinal polarisation

$$F_{q \rightarrow q' \mathcal{V}}^+(x) = \frac{(V-A)^2 + (V+A)^2(1-x)^2}{16\pi^2 x} \left[\ln \left(\frac{p_{\perp, \max}^2 + (1-x)m_{\mathcal{V}}^2}{(1-x)m_{\mathcal{V}}^2} \right) - \frac{p_{\perp, \max}^2}{p_{\perp, \max}^2 + (1-x)m_{\mathcal{V}}^2} \right]$$

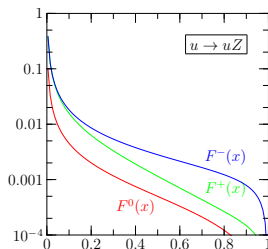
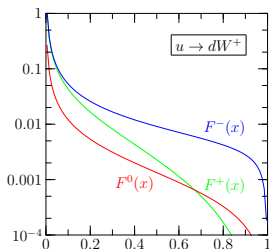
$$F_{q \rightarrow q' \mathcal{V}}^-(x) = \frac{(V+A)^2 + (V-A)^2(1-x)^2}{16\pi^2 x} \left[\ln \left(\frac{p_{\perp, \max}^2 + (1-x)m_{\mathcal{V}}^2}{(1-x)m_{\mathcal{V}}^2} \right) - \frac{p_{\perp, \max}^2}{p_{\perp, \max}^2 + (1-x)m_{\mathcal{V}}^2} \right]$$

$$F_{q \rightarrow q' \mathcal{V}}^0(x) = \frac{V^2 + A^2}{8\pi^2} \frac{2(1-x)}{x} \frac{p_{\perp, \max}^2}{p_{\perp, \max}^2 + (1-x)m_{\mathcal{V}}^2}$$

- ▶ Dominant contribution from small \mathcal{V} virtualities
- ▶ Transverse momentum cutoff $p_{\perp, \max} \leq (1-x)\sqrt{s}/2$:
 - ▶ longitudinal pol.: finite for $p_{\perp, \max} \rightarrow \infty$
 - ▶ Transversal pol.: logarithmic singularity



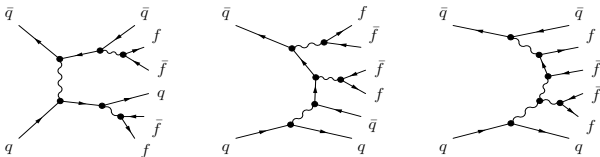
► EWA structure functions: W (left) and Z (right)



– Emission from u , $\sqrt{s} = 2$ TeV
emission

– preferred at high energy: transversal emission

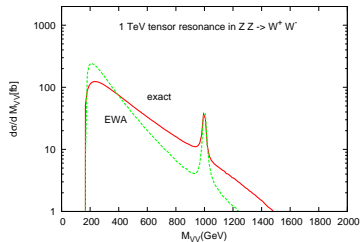
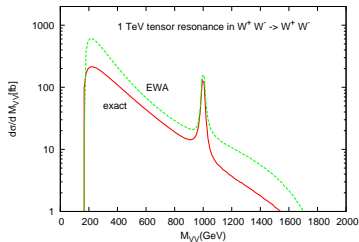
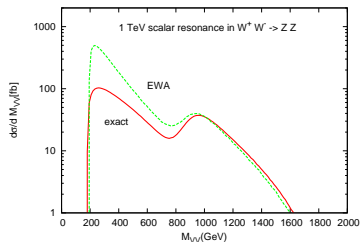
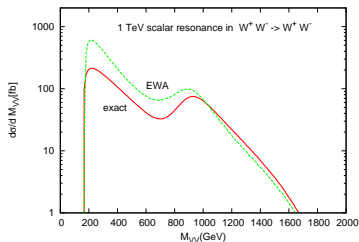
► Problem: Irreducible background to weak-boson scattering



– Double ISR/FSR

– t -channel like diagrams

► Coulomb-singularity (peak): cut on $p_{T,V} \gtrsim 30$ GeV



- ▶ **Effective W approx.** vs. **WHIZARD full matrix elements**
- ▶ Shapes/normalization of distributions heavily affected
- ▶ **EWA: Sideband subtraction completely screwed up!**

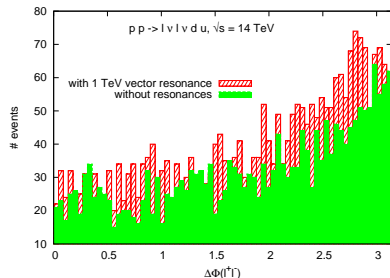
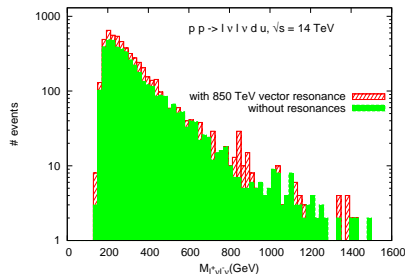
LHC Example: Vector Isovector

Alboteanu/Kilian/JR, 2008

- ▶ Example: 850 GeV vector resonance, coupling $g_\rho = 1$
- ▶ (Theory) Cuts:
 - $p_\perp(\ell\nu) > 30$ GeV
 - $|\delta R(\ell\nu)| < 1.5$
 - $\theta(u/d) > 0.5^\circ$
- ▶ Integrated luminosity: 225 fb^{-1}
- ▶ Discriminator: angular correlations $\Delta\phi(\ell\ell)$
- ▶ Ongoing ATLAS study

Kobel/JR/Schumacher

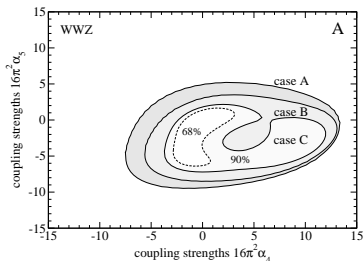
 - Cut analysis/NN
 - More kinematic observables
 - Geant4 FullSim (special points)
 - all resonances, parameter scans



ILC Results: Triboson production

$e^+e^- \rightarrow WWZ/ZZZ$, dep. on $(\alpha_4 + \alpha_6)$, $(\alpha_5 + \alpha_7)$, $\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})$

Polarization populates longitudinal modes, suppresses SM bkgd.



Simulation with WHIZARD Kilian/Ohl/JR

1 TeV, 1 ab^{-1} , full 6-fermion final states, SIMDET fast simulation

Observables: M_{WW}^2 , M_{WZ}^2 , $\sphericalangle(e^-, Z)$

A) unpol., B) 80% e_R^- , C) 80% e_R^- , 60% e_L^+

$16\pi^2 \times$	WWZ			ZZZ no pol.	best
	no pol.	e^- pol.	both pol.		
$\Delta\alpha_4^+$	9.79	4.21	1.90	3.94	1.78
$\Delta\alpha_4^-$	-4.40	-3.34	-1.71	-3.53	-1.48
$\Delta\alpha_5^+$	3.05	2.69	1.17	3.94	1.14
$\Delta\alpha_5^-$	-7.10	-6.40	-2.19	-3.53	-1.64

32 % hadronic decays

Durham jet algorithm

Bkgd. $t\bar{t} \rightarrow 6$ jets

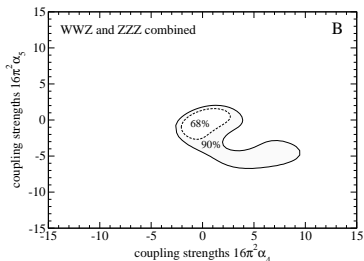
Veto against $E_{\text{mis}}^2 + p_{\perp, \text{mis}}^2$

No angular correlations yet

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Veto against $E_{\text{mis}}^2 + p_{\perp, \text{mis}}^2$

No angular correlations yet

Vector Boson Scattering

1 TeV, 1 ab^{-1} , full $6f$ final states, 80 % e_R^- , 60 % e_L^+ polarization, binned likelihood

Contributing channels: $WW \rightarrow WW, WW \rightarrow ZZ, WZ \rightarrow WZ, ZZ \rightarrow ZZ$

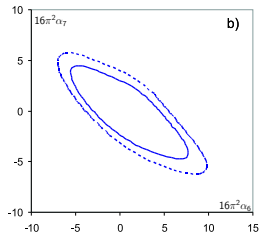
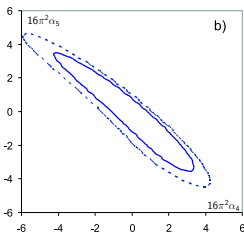
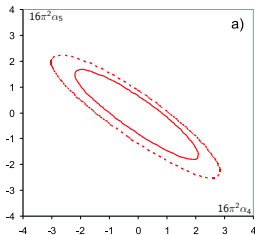
Process	Subprocess	σ [fb]
$e^+e^- \rightarrow \nu_e \bar{\nu}_e q\bar{q}q\bar{q}$	$WW \rightarrow WW$	23.19
$e^+e^- \rightarrow \nu_e \bar{\nu}_e q\bar{q}q\bar{q}$	$WW \rightarrow ZZ$	7.624
$e^+e^- \rightarrow \nu \bar{\nu} q\bar{q}q\bar{q}$	$V \rightarrow VVV$	9.344
$e^+e^- \rightarrow \nu_e q\bar{q}q\bar{q}$	$WZ \rightarrow WZ$	132.3
$e^+e^- \rightarrow e^+e^- q\bar{q}q\bar{q}$	$ZZ \rightarrow ZZ$	2.09
$e^+e^- \rightarrow e^+e^- q\bar{q}q\bar{q}$	$ZZ \rightarrow W^+W^-$	414.
$e^+e^- \rightarrow b\bar{b}X$	$e^+e^- \rightarrow t\bar{t}$	331.768
$e^+e^- \rightarrow q\bar{q}q\bar{q}$	$e^+e^- \rightarrow W^+W^-$	3560.108
$e^+e^- \rightarrow q\bar{q}q\bar{q}$	$e^+e^- \rightarrow ZZ$	173.221
$e^+e^- \rightarrow e\nu q\bar{q}$	$e^+e^- \rightarrow e\nu W$	279.588
$e^+e^- \rightarrow e^+e^- q\bar{q}$	$e^+e^- \rightarrow e^+e^- Z$	134.935
$e^+e^- \rightarrow X$	$e^+e^- \rightarrow q\bar{q}$	1637.405

$SU(2)_c$ conserved case, all channels

coupling	σ^-	σ^+
$16\pi^2\alpha_4$	-1.41	1.38
$16\pi^2\alpha_5$	-1.16	1.09

$SU(2)_c$ broken case, all channels

coupling	σ^-	σ^+
$16\pi^2\alpha_4$	-2.72	2.37
$16\pi^2\alpha_5$	-2.46	2.35
$16\pi^2\alpha_6$	-3.93	5.53
$16\pi^2\alpha_7$	-3.22	3.31
$16\pi^2\alpha_{10}$	-5.55	4.55

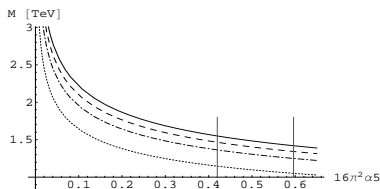


Interpretation as limits on resonances

Consider the width to mass ratio, $f_\sigma = \Gamma_\sigma/M_\sigma$

$SU(2)$ conserving scalar singlet

$$M_\sigma = v \left(\frac{4\pi f_\sigma}{3\alpha_5} \right)^{\frac{1}{4}}$$

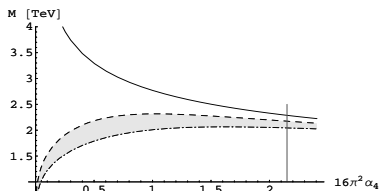


$f = 1.0$ (full), 0.8 (dash), 0.6 (dot-dash), 0.3 (dot)

$SU(2)$ broken vector triplet

needs input from TGC covariance matrix

$$M_{\rho^\pm} = v \left(\frac{12\pi\alpha_4 f_{\rho^\pm}}{\alpha_4^2 + 2(\alpha_2^\lambda)^2 + s_w^2 (\alpha_4^\lambda)^2 / (2c_w^2)} \right)^{\frac{1}{4}}$$



upper/lower limit from λ_Z , grey area: magnetic moments

**Final
result:**

Spin	$I = 0$	$I = 1$	$I = 2$
0	1.55	—	1.95
1	—	2.49	—
2	3.29	—	4.30

Spin	$I = 0$	$I = 1$	$I = 2$
0	1.39	1.55	1.95
1	1.74	2.67	—
2	3.00	3.01	5.84

Summary/Conclusions

- ▶ New Physics generically encoded in EW Chiral Lagrangian
- ▶ Triple/Quartic gauge couplings measured either
 - via triple boson production
 - via vector boson scattering
- ▶ interpreted as resonances coupled to EW bosons
- ▶ “Correct” description for first resonance (also [very] broad)
- ▶ Beyond that: assure unitarity (K matrix)
- ▶ Sensitivity rises with number of intermediate states:
 - LHC sensitivity limited in pure EW sector: $0.6 - 2 \text{ TeV}$
 - ILC : $1.5 - 6 \text{ TeV}$
- ▶ Full analysis including all channels/backgrounds with WHIZARD
- ▶ Complete ATLAS study is under way

One Ring to Find them ... One Ring to Rule them Out

One Ring to Find them ... One Ring to Rule them Out

