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by

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RESONANCES IN PHOTON-PHOTON SCATTERING

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ABSTRACT

A quantity called "stickiness" is introduced which should be largest for $J \neq 0$ glueballs and can be measured in two photon scattering and radiative J/ψ decay. An argument is reviewed suggesting that light $J = 0$ glueballs may have large couplings to two photons. The analysis of radiative decays of η and η' is reviewed and a plea made to desist from false claims that they are related to $\Gamma(\pi^0 \rightarrow \gamma\gamma)$ by SU(3) symmetry. It is shown that two photon studies can refute the difficult-to-refute hypothesis that $\xi(2220)$ or $\zeta(8320)$ are Higgs bosons. A gallery of rogue resonances and resonance candidates is presented which could usefully be studied in $\gamma\gamma$ scattering, including especially the low mass dipion.

1. Introduction

It is gratifying to see the many new experimental results on two photon widths of resonances obtained in the last year. Clearly the strength of the $\gamma\gamma$ coupling is a fundamental parameter of any resonance. We can use this information to test our understanding of the $\bar{q}q$ meson spectrum and to aid in the identification of new states, for instance, states with gluonic constituents. Naively we expect unmixed glueball states to have small $\gamma\gamma$ couplings, but I will discuss a remarkable low energy theorem, due to Novikov, Shifman, Vainshtein, and Zakharov, which implies that light $J = 0$ glueballs may have large $\gamma\gamma$ couplings, at least as large as $\bar{q}q$ mesons.[1)]

My talk is in five parts. In Section 2 I will discuss the $\gamma\gamma$ coupling of glueballs, beginning with the naive expectation that pure glueballs have small $\gamma\gamma$ widths. I propose a quantitative measure, stickiness, which for a given state X is the ratio $\Gamma(\psi \to \gamma X)/\Gamma(X \to \gamma\gamma)$ with phase space factors removed. Stickiness is a measure of the color charge of the constituents relative to their electric charge, so that glueballs should be much stickier than $\bar{q}q$ states. The available experimental data in the pseudoscalar and tensor channels indicates that $\iota(1440)$ and $\Theta(1700)$ are both rather sticky objects.

In Section 3 I will present the low energy theorem of Novikov et al., which is a consequence of the trace[2] and chiral anomalies[3] in the flavor singlet channel. Here my discussion largely follows Sharpe's talk at the Spring, 1984 Vanderbilt conference.[4] The conclusion is that the lightest particle in the scalar or pseudoscalar channel with large couplings to two gluons must also have large couplings to two photons. In the pseudoscalar channel this is probably $\eta'(958)$ (whose large coupling to two gluons can be understood without invoking a large glueball admixture).[5] We might also anticipate some enhancement for heavier states such as $\iota(1440)$ but it is very difficult to make even a semi-quantitative estimate. A light glueball might still be the lightest particle with large gluonic couplings in the scalar channel, in which case it could be copiously produced in $\gamma\gamma$ collisions. In his talk at this conference[6] Kück has reported on the PLUTO measurement of $\gamma\gamma \to \pi^+\pi^-$ which shows an apparent dip at ~ 600 MeV. This might be the manifestation of an enhancement at $\leq$ 500 MeV (where the experimental efficiency dies) or it could be the effect of interference effects, as discussed by Sharpe, Jaffe, and Pennington.[7] Despite the apparent lack of structure in the I=0 s-wave $\pi\pi$ phase shift, it is still important to look carefully at the low mass dipion in $\gamma\gamma \to \pi\pi$ and $\psi \to \gamma\pi\pi$.

Section 4 concerns the $\gamma\gamma$ decays of η and η'. Frequently, at this conference and elsewhere, we are shown so-called SU(3) predictions for these decays, which are used to determine the η-η' mixing angle. In fact these predictions do <u>not</u> follow from SU(3) but are based on a dynamical assumption, tantamount to ideal mixing, which is very badly violated in the η-η' system (this violation of ideal mixing is the essence of the U(1) problem). In Section 4 I present an analysis which does not require this problematical assumption.[8] In this analysis the decays $\eta \to \gamma\gamma$, $\eta' \to \gamma\gamma$ and $\eta \to \pi^+\pi^-\gamma$ are all deduced from the chiral anomaly. This allows comparison with the data without the need for an ideal-mixing or so-called "nonet symmetry" assumption. A second analysis applies vector dominance to $\eta \to \gamma\gamma$, $\rho \to \eta\gamma$, $\eta' \to \gamma\gamma$ and $\eta' \to \rho\gamma$ to test the quark charge assignments of QCD.[9] While this second analysis depends largely on the well known ratio $\Gamma(\eta' \to \gamma\gamma)/\Gamma(\eta' \to \rho\gamma)$, the first analysis requires clarification of the presently confused experimental results for $\eta \to \gamma\gamma$ and $\eta' \to \gamma\gamma$.

In Section 5 I describe an important test that can be made in $\gamma\gamma$ physics of the hypotheses that $\xi(2220)$ or $\zeta(8320)$ (but not both) are Higgs bosons in two doublet models with enhanced couplings to $Q = +2/3$ or $Q = -1/3$ charge quarks respectively. These are difficult hypotheses to test experimentally. If either particle can be detected in $\gamma\gamma$ scattering it would imply that it is <u>not</u> such a Higgs boson.

In Section 6 I conclude with a Rogues Gallery of particles and particle-candidates which could profitably be studied in $\gamma\gamma$ collisions. We would like to either measure or bound the $\gamma\gamma$ widths of all of these rogues.

2. Stickiness

Naively, using perturbation theory as a guide, the decay of a hypothetical pure glueball G to two photons proceeds by a qq loop. Therefore comparing its $\gamma\gamma$ width to the $\gamma\gamma$ width of a meson M of the same spin-parity and comparable mass, we expect a suppression

$$\frac{\Gamma(G \to \gamma\gamma)}{\Gamma(M \to \gamma\gamma)} \sim \left(\frac{\alpha_s}{\pi}\right)^2 \tag{1}$$

Similarly in perturbation theory the ratio of radiative decay widths from the ψ is enhanced by

$$\frac{\Gamma(\psi \to \gamma G)}{\Gamma(\psi \to \gamma M)} \sim \left(\frac{1}{\alpha_s}\right)^2 \tag{2}$$

This suggests that we consider the stickines S_X of the state X, defined as the ratio

$$S_X = \frac{\Gamma(\psi \to \gamma X)}{\Gamma(X \to \gamma\gamma)} \cdot \frac{LIPS(X \to \gamma\gamma)}{LIPS(\psi \to \gamma X)} \tag{3}$$

where LIPS means Lorentz Invariant Phase Space. We do not attribute any significance to the absolute normalization of S_X. Now from Eqs. (1) and (2) we expect a glueball G to be much more sticky than a meson M of the same J^{PC},

$$S_G \gg S_M \tag{4}$$

Since S_X is proportional to the ratio $|\langle X|gg\rangle|^2/|\langle X|\gamma\gamma\rangle|^2$ it probes the relative color and electric charges of the constituents of X.

Stickiness has two advantages, one experimental, the other theoretical. The experimental advantage is that we do not need to know the branching ratio of the state X, B(X $\to$ final state), in order to measure S_X. For example, we can determine S_θ from measurements of $\Gamma(\psi \to \gamma\theta \to \gamma\bar{K}K)$ and $\sigma(\gamma\gamma \to \theta \to \bar{K}K)$ without knowing the branching ratio $B(\theta \to \bar{K}K)$. The theoretical advantage is that dynamical factors tend to cancel in the stickiness ratio, though the cancellation is not perfect since in $\psi \to \gamma X$ the two gluons which couple to X may be off-mass-shell whereas the two photons in $\gamma\gamma \to X$ are essentially on-shell. The imaginary part of $\mathcal{M}(\psi \to \gamma X)$ corresponds to on-shell gluons by the Cutkosky-Landau rules, and the off-shell effects are only in the real part.

Consider the pseudoscalar and tensor channels, which are of interest in the glueball hunt. For $J^{PC}(X) = 0^{-+}$ both $\psi \to \gamma X$ and $X \to \gamma\gamma$ have p-wave phase space, so with an arbitrary normalization N I define

$$S_X(0^{-+}) = N\left(\frac{m_X}{k_{\psi\gamma X}}\right)^3 \frac{\Gamma(\psi \to \gamma X)}{\Gamma(X \to \gamma\gamma)} \tag{5}$$

where $k_{\psi\gamma X}$ is the energy of the energy of the photon in $\psi \to \gamma X$ in the ψ rest frame. I choose N so that $S_\eta = 1$. Then using the experimental data I find

$$S_\iota : S_{\eta'} : S_\eta : S_{\pi^0} \cong (>10) : 2\tfrac{1}{2} : 1 : .02 \tag{6}$$

with experimental errors omitted. The small value of S_{π^0} is the expected consequence of the pion isospin. The lower limit for S_ι is based on the experimental upper limit $\Gamma(\iota \to \gamma\gamma)\cdot B(\iota \to \bar{K}K\pi) \leq$ 7 KeV. Notice that $\iota(1440)$ is apppreciably more sticky than $\eta'(958)$:

$$S_\iota / S_{\eta'} \gtrsim 4 \tag{7}$$

For the tensors, $J^{PC}(X) = 2^{++}$, $X \to \gamma\gamma$ and $\psi \to \gamma X$ can proceed by s-wave or d-wave. I will assume the s-wave dominates and define

$$S_X(2^{++}) = N \frac{m_X}{k_{\psi\gamma X}} \cdot \frac{\Gamma(\psi \to \gamma X)}{\Gamma(X \to \gamma\gamma)} \tag{8}$$

with N chosen to give $S_f = 1$. Then the experimental data implies

$$S_\xi : S_\theta : S_{f'} : S_f \cong (>1) : (>20) : 14 : 1 \tag{9}$$

where again I have omitted the experimental errors, which are in some cases considerable. For the sake of comparison I have assumed $J(\xi) = 2$. The lower limits for S_ξ and S_θ are based on new preliminary 95% CL upper limits from TASSO first presented at this conference,[10) $\Gamma(\xi \to \gamma\gamma)\cdot B(\xi \to \bar{K}K) < 0.5$ KeV and $\Gamma(\theta \to \gamma\gamma)\cdot B(\theta \to \bar{K}K) < 0.14$ KeV. We see that θ is very sticky compared to f, though f′ is also rather sticky.

The large value of $S_{f'}/S_f$ is not unexpected. If we assume ideal mixing, $f = 1/\sqrt{2}(\bar{u}u + \bar{d}d)$ and $f' = \bar{s}s$, then a naive calculation gives

$$S_{f'}/S_f = \frac{1}{2}\Big/\left[\frac{\frac{1}{9}}{\frac{1}{\sqrt{2}}\left(\frac{4}{9}+\frac{1}{9}\right)}\right]^2 \tag{10}$$

The factor 1/2 is the expected ratio $\Gamma(\psi \to \gamma f')/\Gamma(\psi \to \gamma f)$ with phase space removed. The remaining factor 25/2 is the square of the weighted sum of the square of the constituent quark charges, the naive expectation for $\Gamma(f \to \gamma\gamma)/\Gamma(f' \to \gamma\gamma)$ with phase space removed. Including the experimental uncertainties which were omitted in (9), I find combining errors in quadrature that

$$S_{f'}/S_f = 14 \pm 8 \tag{11}$$

This is consistent with the naive estimate (10) at the 1σ level.

As the upper limit on $\Gamma(\theta \to \gamma\gamma)$ improves, the stickiness of θ becomes increasingly evident. It suggests that θ is not a $\bar{q}q$ meson, since of the $\bar{q}q$ mesons only $\bar{s}s$ would be very sticky, while θ is much too light to be the $\bar{s}s$ radial excitation of f'.

3. Low Energy Theorems for J = 0 States.

Contrary to the previous section, light J = 0 glueballs may have $\gamma\gamma$ couplings as large as $\bar{q}q$ mesons. This conclusion applies to the lightest scalar and pseudoscalar particle which couples appreciably to two gluons. For pseudoscalars this is probably $\eta'(958)$, whose large coupling to two gluons can be understood without assuming a large glueball admixture (see below and Ref. 5). The implications for a heavier pseudoscalar glueball - such as $\iota(1440)$ may be - are less clear, though some enhancement above the naive expectation of Section 2 seems likely. In the scalar channel, many theoretical estimates have suggested a very light glueball, $\leq$ 1 GeV, and there are amusing hints in $\psi \to \omega\pi\pi$[11)] and $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$[12)] for structure around ~ 500-600 MeV. The low energy theorem suggests that $\gamma\gamma \to \pi\pi$ is well worth studying down to the lowest possible dipion masses. The report[6] of a rise toward threshold below 600 MeV certainly deserves further investigation. We need to measure the spectrum as close to threshold as possible, in both $\gamma\gamma \to \pi\pi$ and $\psi \to \gamma\pi\pi$.

The remarkable low energy theorems[1] follow from the trace[2] and chiral anomalies.[3] Let $\theta^{\mu\nu}$ be the QCD stress tensor for just the light matter fields u, d, s, and let $A_1{}^\mu$ be the SU(3) flavor singlet axial current. We then consider the matrix elements between the vacuum and a two photon state of the trace of the stress tensor $\theta \equiv \theta_\mu{}^\mu$ and the divergence of the axial current, ∂A_1. We neglect the light quark masses, $m_{u,d,s} \to 0$, but include anomalies of both QCD and QED. The result is

$$\langle 0|\,\theta\,|\gamma_1\gamma_2\rangle = \langle 0|\,\frac{\beta}{2g}\,G\cdot G + \frac{\alpha R}{6\pi}\,F\cdot F\,|\gamma_1\gamma_2\rangle \tag{12a}$$

$$\langle 0|\,\partial A_1\,|\gamma_1\gamma_2\rangle = \langle 0|\,\frac{\sqrt{3}\,\alpha_s}{2\sqrt{2}\,\pi}\,G\cdot\tilde{G} + \frac{\alpha}{\sqrt{6}\,\pi}\,F\cdot\tilde{F}\,|\gamma_1\gamma_2\rangle \tag{12b}$$

Here $\beta = \beta(g)$ is the QCD β function for three flavors, $R = 2$ is the R of e^+e^- annihilation for the three light flavors, $G\cdot G = G_{\mu\nu}G^{\mu\nu}$ is the square of the gluon field strength tensor, $G\cdot\tilde{G} = \frac{1}{2}\,\epsilon_{\mu\nu\alpha\beta}\,G^{\mu\nu}G^{\alpha\beta}$, and $F^{\mu\nu}$ is the photon field strength tensor. Because of the Adler-Bardeen theorem, Eq. (12b) is exact to any finite order in α and α_S. Equation (12a) does have $O(\alpha)$ corrections but the important and remarkable feature is that there are no $O(\alpha_S)$ QCD corrections to the $O(\alpha)$ QED anomaly.[13] This means that the low energy theorem derived from Eq. (12a) does not have uncalculable QCD corrections.

The crucial observation of Ref. (1) is that the left sides of Eqs. (12) are $O(k^4)$, where k_i are the photon momenta, so that the $O(k^2)$ terms on the right side must cancel. If there are no singularities, then Lorentz invariance, gauge invariance, and Bose statistics imply that the left sides are $O(k^4)$. In the massless quark limit the triangle diagrams do contribute $1/k^2$ singularities to the left sides,[14] but these are not really present in a confining theory. Goldstone bosons could also contribute $1/k^2$ poles to the left side, but these are not expected in QCD because of nonperturbative effects - the U(1) effects for the pseudoscalars[15] and analogous effects for the scalars.[16]

The cancellation of the QED and QCD anomaly terms then gives the following remarkable results:

$$\langle 0 | \frac{\beta}{2g} G \cdot G | \gamma_1 \gamma_2 \rangle = \frac{\alpha R}{6\pi} F_1 \cdot F_2 + O(k^4) \tag{13a}$$

$$\langle 0 | \frac{\sqrt{3}\,\alpha_s}{2\sqrt{2}\,\pi} G \cdot \tilde{G} | \gamma_1 \gamma_2 \rangle = \frac{\alpha}{\sqrt{6}\,\pi} F_1 \cdot \tilde{F}_2 + O(k^4) \tag{13b}$$

where $F_i^{\mu\nu} = k_i^{\mu}\varepsilon_i^{\nu} - k_i^{\nu}\varepsilon_i^{\mu}$, k_i and ε_i being the momentum and polarization tensor of the photon γ_i. Equation (13) is remarkable because there are no powers of α_s on the right side!

We can easily use Eq. (13) to obtain low energy theorems for the $\gamma\gamma$ widths of the lightest J = 0 particles which dominate the two gluon channel. Consider first the scalar, $J^{PC} = 0^{++}$. Suppose there is a very light scalar glueball S, which couples with strength F_S (analogous to F_π) to the G·G operator:

$$\langle 0 | \frac{\beta}{2g} G \cdot G | S \rangle = F_S m_S^2 \tag{14}$$

We assume that the left side of Eq. (13a) is dominated by the S pole,

$$\langle 0 | \frac{\beta}{2g} G \cdot G | \gamma_1 \gamma_2 \rangle = \frac{F_S m_S^2 \cdot \langle S | \gamma_1 \gamma_2 \rangle}{m_S^2 - (k_1 + k_2)^2} \tag{15}$$

Then from Eq. (13)

$$\langle S|\gamma_1\gamma_2\rangle = \frac{1}{F_S}\,\frac{\alpha R}{6\pi}\,F_1\cdot F_2 + O(k^4) \tag{16}$$

If S is light enough we can neglect the $O(k^4)$ terms in (16) and compute the two photon width

$$\Gamma(S\to\gamma\gamma) \cong \frac{\alpha^2 R^2 m_S^3}{144\,\pi^3 F_S^2} \tag{17}$$

With $S \to \sigma$ this is just the old POT/PCDC low energy theorem that Crewther, Ellis and I found for the would-be "dilaton", - the would-be Goldstone bosons of scale invariance.[2] Under the stated assumptions, Eqs. (16) and (17) should be approximately valid even though there is no Goldstone limit of scale invariance in QCD.

If F_S is of the order of the PCAC constant F_π, then $\Gamma(S \to \gamma\gamma)$ would be of the same order as the $\gamma\gamma$ widths of $\bar{q}q$ mesons. The actual value of F_S depends on dynamics which is difficult to estimate reliably. The ITEP sum rules imply[17] $F_S \simeq 300$ MeV which for $m_S = 500$ MeV gives $\Gamma(S \to \gamma\gamma) \simeq 70$ eV. Another estimate,[18] based on a different approach to the ITEP sum rules, finds a value of F_S smaller by one order of magnitude, implying a value for $\Gamma(S \to \gamma\gamma)$ larger by two orders of magnitude.

The result in the pseudoscalar channel is analogous. We assume that the left side of Eq. (13b) is dominated by the pole of a single light pseudoscalar P. Defining F_P by

$$\langle 0|\frac{\sqrt{3}\alpha_s}{2\sqrt{2}\pi}\,G\cdot\tilde{G}\,|P\rangle = F_P\,m_P^2 \tag{18}$$

we find the low energy theorem

$$\langle P|\gamma_1\gamma_2\rangle = \frac{1}{F_P}\,\frac{\alpha}{\sqrt{6}\pi}\,F_1\cdot\tilde{F}_2 + O(k^4) \tag{19}$$

and the width

$$\Gamma(P\rightarrow\gamma\gamma) \cong \frac{\alpha^2 m_P^3}{24\pi^3 F_P^2} \tag{20}$$

If the lightest pseudoscalar P happens to be the flavor singlet meson

$$\eta_1 = \frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d + \bar{s}s) \tag{21}$$

then Eq. (19) is just the familiar low energy theorem for $\eta_1 \rightarrow \gamma\gamma$. We define the singlet PCAC constant F_1 by

$$\langle 0|A_1^\mu|\eta_1(k)\rangle = i k^\mu F_1 \tag{22}$$

F_1 is analogous to F_π but they are not related by any symmetry. Now in the chiral limit and neglecting electroweak corrections, ∂A_1 is just given by the QCD anomaly, so that

$$\langle 0|\partial A_1|\eta_1\rangle = \langle 0|\frac{\sqrt{3}\,\alpha_s}{2\sqrt{2}\,\pi} G\cdot\tilde{G}|\eta_1\rangle \tag{23}$$

or, comparing Eqs. (18) and (22),

$$F_1 = F_P \tag{24}$$

With $F_P = F_1$ and $P = \eta_1$, Eq. (19) is the usual low energy theorem, which can be obtained more directly using broken chiral symmetry and η_1 pole dominance.

Incidentally, Eq. (24) shows that the large coupling of $\eta'(958)$ to gluons, which is indicated by the large value of $\Gamma(\psi \rightarrow \gamma\eta')$, can be understood as a consequence of approximate chiral SU(3) and the chiral anomaly, with no need to assume a large glueball admixtude in η'. That is, we see from Eqs. (23) and (24) that as $m_{u,d,s} \rightarrow 0$, the QCD chiral anomaly requires the $\bar{q}q$ state η_1 to have the same coupling to the gluon bilinear operator $(\sqrt{3}\,\alpha_s/2\sqrt{2}\,\pi)\, G\cdot G$ as it has to the $\bar{q}q$ bilinear ∂A_1! The relevant masses $m_{u,d,s}$ are the <u>current</u> quark masses, which are indeed small. The qualitative conclusion which follows from approximate chiral symmetry is that the large coupling of η' to gluons is expected in the conventional picture, which identifies η' predominantly with η_1.

It is straightforward to include the effect of $\eta_1-\eta_8$ mixing on the low energy theorems for $\eta \to \gamma\gamma$ and $\eta' \to \gamma\gamma$. We consider broken chiral symmetry, $m_{u,d,s} \neq 0$, and allow an arbitrary $\eta-\eta'$ mixing angle θ, defined conventionally as

$$\begin{aligned} \eta &= \cos\theta\ \eta_8 - \sin\theta\ \eta_1 \\ \eta' &= \sin\theta\ \eta_8 + \cos\theta\ \eta_1 \end{aligned} \qquad (25)$$

The PCAC constants are defined by

$$\begin{aligned} \langle 0 |\, A_8^\mu \,| \eta_8 \rangle &= i k^\mu F_8 \\ \langle 0 |\, A_1^\mu \,| \eta_1 \rangle &= i k^\mu F_1 \end{aligned} \qquad (26)$$

We assume that SU(3) symmetry holds for the current matrix elements,

$$\begin{aligned} \langle 0 |\, A_8^\mu \,| \eta_1 \rangle &= 0 \\ \langle 0 |\, A_1^\mu \,| \eta_8 \rangle &= 0 \end{aligned} \qquad (27)$$

so that for instance

$$\langle 0 |\, A_1^\mu \,| \eta' \rangle = i k^\mu \cos\theta\ F_1 \qquad (28)$$

The constant F_8 is related to F_π by SU(3) symmetry

$$F_8 \cong F_\pi \tag{29}$$

but there is no symmetry relating F_1 to F_8 or F_π. Because of the presence of the anomaly and nonperturbative gluonic effects in the flavor singlet channel, there is no simple dynamical reason to expect F_1 to equal F_π. They are equal to lending order in the $1/N_{color}$ expansion, but in the same order the η' would be lighter than $\sqrt{3}\, m_\pi$ - a rather unsuccessful prediction. The value of F_1/F_π depends on complicated dynamics. The experimental data should be analyzed without assuming $F_1 = F_\pi$ or any other value. Instead we should try to determine F_1 from the data, as discussed in the next Section.

4. Radiative Decays of η and η'

At this conference and others, the widths for $\eta \to \gamma\gamma$ and $\eta' \to \gamma\gamma$ are used to determine the $\eta - \eta'$ mixng angle defined by means of a so-called "SU(3)" comparison with $\pi^0 \to \gamma\gamma$. This is an incorrect argument. SU(3) relates $\eta_8 \to \gamma\gamma$ to $\pi^0 \to \gamma\gamma$ but says nothing at all about $\eta_1 \to \gamma\gamma$. What is really assumed is not SU(3) symmetry but the OIZ rule. More precisely, in the context of a $\bar{q}q$ bound state, the assumption is that the wave function at the origin is the same for η_1 and η_8. The assumption that octet and singlet wave functions are approximately equal implies approximately equal binding energies, and this in turn implies ideal mixing. This is a good assumption for the vector mesons, as shown by the success of the ideal mixing sum rule,

$$m_\omega^2 + m_\phi^2 = 2\, m_{K^*}^2 \tag{30}$$

It is however a terrible assumption for the pseudoscalars, for which the corresponding sum rule is badly violated. This failure is the "U(1) problem". It means that η_1 and η_8 have very different binding energies. It is therefore very dangerous to assume they have equal wave functions. In this section I present an analysis which avoids this assumption.

The low energy theorems which follow from pole dominance and the chiral anomaly are

$$\mathcal{F}(\eta \to \gamma\gamma) = -\frac{\alpha}{\sqrt{3}\pi}\left(\frac{\cos\theta}{F_8} - 2\sqrt{2}\xi\frac{\sin\theta}{F_1}\right)$$

$$\mathcal{F}(\eta' \to \gamma\gamma) = -\frac{\alpha}{\sqrt{3}\pi}\left(\frac{\sin\theta}{F_8} + 2\sqrt{2}\xi\frac{\cos\theta}{F_1}\right) \tag{31}$$

where F_1, F_8, and θ are defined in Eqs. (25) and (26). The parameter ξ is defined as $\xi = 1$ in QCD and $\xi = 2$ in the Han Nambu model.[19)] (In the chrial limit the amplitude for $\eta_1 \to \gamma\gamma$ was derived in the previous section. For broken chiral symmetry it can be derived in an analogous way to the familiar derivation of $\pi^0 \to \gamma\gamma$).[20] SU(3) symmetry implies $F_8 \simeq F_\pi$ but tells us nothing abut F_1. At this point we have two unknown parameters, θ and F_1, which we might fit with the two experimental quantities $\Gamma(\eta \to \gamma\gamma)$ and $\Gamma(\eta' \to \gamma\gamma)$. We can do better than this by considering $\Gamma(\eta \to \pi^+\pi^-\gamma)$, which is also fixed by pole dominance and the chiral anomaly.

Before discussing what is learned from $\eta \to \pi\pi\gamma$, I want to add a few words about the most remarkable aspect of the chiral (and trace) anomaly. The anomalies are especially fascinating because they connect low and high energy phenomena. The anomaly in the VVA Ward identity that determines $\Gamma(\pi^0 \to \gamma\gamma)$ arises because of the behavior <u>at infinite momentum</u> of the fermion loop in the VVA triangle graph.[20)] So by measuring a low energy quantity, $\Gamma(\pi^0 \to \gamma\gamma)$, we are learning about properties of the theory at very high energy. This is shown explicitly by Crewther[2)] who related the $\pi^0 \to \gamma\gamma$ amplitude to $K\mathcal{R}$, where $\mathcal{R}$ is the familiar $\mathcal{R}$ of e^+e^- annihilation and K is a quantity measured in deep inelastic election scattering. Similarly, as shown in Eqs. (13a) and (17) above, the trace anomaly is fixed by $\mathcal{R}$ alone.[2]

It is this remarkable dependence on the high energy behavior which makes $\eta \to \gamma\gamma$ sensitive to the color degrees of freedom in the Han Nambu model that give $\xi = 2$ in Eq. (31). (See however footnote (19).) This is in contrast to $\sigma(e^+e^- \to \text{hadrons})$ and $\sigma(\gamma\gamma \to \text{hadrons})$ which are not sensitive to the color degrees of freedom at energies far below color threshold.

There are chiral anomalies not only in the VVA and AAA triangles but also in box and pentagon diagrams with odd numbers of axial currents. In particular, there are low energy theorems for $\eta \to \pi\pi\gamma$ and $\eta' \to \pi\pi\gamma$ which follow from the VAAA and VVA anomalies. The low energy theorems are[8)]

$$\mathcal{F}(\eta \to \pi^+\pi^-\gamma) = \frac{-e}{4\sqrt{3}\pi^2}\,\frac{1}{F_\pi^2}\left(\frac{\cos\theta}{F_8} - \sqrt{2}\,\frac{\sin\theta}{F_1}\right)$$

$$\mathcal{F}(\eta' \to \pi^+\pi^-\gamma) = \frac{-e}{4\sqrt{3}\pi^2}\,\frac{1}{F_\pi^2}\left(\frac{\sin\theta}{F_8} + \sqrt{2}\,\frac{\cos\theta}{F_1}\right) \qquad (32)$$

The low energy theorems (31) and (32) apply at the unphysical low energy point where all four-momenta vanish. For $\eta \to \gamma\gamma$ and $\eta' \to \gamma\gamma$ we assume η and η' pole dominance which implies approximate equality of the physical amplitude to the amplitude at the unphysical low energy point. But for $\eta \to \pi^+\pi^-\gamma$ and $\eta' \to \pi^+\pi^-\gamma$ we must also include the effect of the ρ meson on the extrapolation. The Dalitz plot for $\eta \to \pi^+\pi^-\gamma$ shows that the dipion is dominated by the ρ, even though we are well below the ρ threshold. We incorporate this effect by adding a Breit-Wigner factor to the amplitude which is normalized to unity at zero dipion mass as dictated by the low energy theorem. So we replace Eq. (32) by

$$\mathcal{F}(\eta \to \pi^+\pi^-\gamma) \longrightarrow \mathcal{F}(\eta \to \pi^+\pi^-\gamma)\cdot \frac{-m_\rho^2 + i m_\rho \Gamma_\rho}{p_{\pi\pi}^2 - m_\rho^2 + i m_\rho \Gamma_\rho} \qquad (33)$$

where $p_{\pi\pi}$ is the four-momentum of the dipion.

For $\eta' \to \pi\pi\gamma$ we are above the ρ threshold and in fact the decay is completely dominated by $\eta' \to \rho\gamma$. In this case the relevance of the low energy theorem Eq. (32) is unclear, since we must extrapolate across the ρ pole. In addition, there may be two separate components: an elementary ρ amplitude, $\eta' \to \rho\gamma$, which has nothing to do with Eq. (32) as well as $\eta' \to \pi\pi\gamma$ component given by Eq. (32). Because of these uncertainties I will not include $\eta' \to \pi^+\pi^-\gamma$ in the analysis.

So we have finally three experimental quantities to determine the two parameters F_1 and θ for the cases $\xi = 1$ and $\xi = 2$. The amplitudes in Eqs. (31) and (32) are related to the widths by

$$\Gamma(X \to \gamma\gamma) = \frac{m_X^3}{64\pi} \left| \mathcal{F}(X \to \gamma\gamma) \right|^2 \qquad (34)$$

for $X = \eta, \eta'$ and

$$\Gamma(\eta \to \pi^+\pi^-\gamma) = \frac{3 I_\eta}{2 m_\eta (2\pi)^5} \left| \mathcal{F}(\eta \to \pi^+\pi^-\gamma) \right|^2 \qquad (35)$$

where $I_\eta = 4.75 \cdot 10^{-4}\, m_\eta{}^8$ is the result for the three body phase space including the ρ final state interaction of Eq. (33) (which contributes an enhancement of 1.85 to I_η).

While there is now a large spread in experimental values for the three widths $\Gamma(\eta \to \gamma\gamma)$, $\Gamma(\eta' \to \gamma\gamma)$, and $\Gamma(\eta \to \pi\pi\gamma)$, the ratio $\Gamma(\eta \to \gamma\gamma)/\Gamma(\eta \to \pi\pi\gamma)$ seems well determined. We may use the experimental value[22]

$$\frac{\Gamma(\eta \to \gamma\gamma)}{\Gamma(\eta \to \pi^+\pi^-\gamma)} = 7.94 \pm .027 \qquad (36)$$

and Eqs. (32)-(35) to find an interesting constraint on F_1 and θ:

$$\frac{F_8}{F_1} \tan\theta = \begin{cases} -.21 & \xi = 1 \\ -.06 & \xi = 2 \end{cases} \qquad (37)$$

This should be counted as at least a qualitative success, since it implies that if F_8/F_1 is positive and of order one then θ is small and negative - as we would expect from the naive quark model calculation of $\eta - \eta'$ mixing which gives $\theta = -11^0$.

The next step is to use the constraint of Eq. (37) and $F_8 \simeq F_\pi$ to determine $\Gamma(\eta \to \gamma\gamma)$ in terms of θ alone:

$$\Gamma(\eta\to\gamma\gamma) = \begin{cases} 432\ \text{eV}\ \cos^2\theta & \xi = 1 \\ 304\ \text{eV}\ \cos^2\theta & \xi = 2 \end{cases} \qquad (38)$$

Experimentally, the old Cornell value[23)] (from Primakoff photoproduction) is 324 ± 46 eV while from $\gamma\gamma$ scattering the Crystal Ball has 560 ± 120 ± 100 eV and JADE has 560 ± 50 ± 80.[24)] For small θ the prediction for $\xi = 1$ is strategically located between the experimental values while the prediction for $\xi = 2$ would not survive confirmation of the higher value.

Finally we include $\Gamma(\eta' \to \gamma\gamma)$ in the analysis. We impose the constraint Eq. (37) and then compute $\Gamma(\eta \to \gamma\gamma)$ and $\Gamma(\eta' \to \gamma\gamma)$ for a range of values of F_8/F_1. For $F_8/F_1 = 1/2, 1, 2$ the results are shown in Table 1.

Table 1: Predicted values for θ, $\Gamma(\eta \to \gamma\gamma)$ and $\Gamma(\eta' \to \gamma\gamma)$ as a function of $F_8/F_1 = 1/2, 1, 2$. Predictions are given for fractional ($\xi = 1$) and integral ($\xi = 2$) charge quarks.

	F_8/F_1 =	$\frac{1}{2}$	1	2
$\xi = 1$	θ =	-23^0	-12^0	-6^0
	$\Gamma_{\eta\to\gamma\gamma}$ =	366	413	427 eV
	$\Gamma_{\eta'\to\gamma\gamma}$ =	0.75	6	27 KeV
$\xi = 2$	θ =	-7°	$-3\frac{1}{2}^0$	$-1\frac{1}{2}^0$
	$\Gamma_{\eta\to\gamma\gamma}$ =	300	300	300 eV
	$\Gamma_{\eta'\to\gamma\gamma}$ =	$6\frac{1}{2}$	28	115 KeV

The experimental situation for $\Gamma(\eta' \to \gamma\gamma)$ is also very unclear,[24] with results quoted between ~3 and ~6 KeV. We see from Table 1 that the Han Nambu model, $\xi = 2$, could fit the data for $F_8/F_1 \le \frac{1}{2}$ if $\Gamma(\eta \to \gamma\gamma)$

$\leq$ 300 eV. The QCD predictions could be consistent with $\Gamma(\eta \rightarrow \gamma\gamma)$ ~400 eV and $\Gamma(\eta' \rightarrow \gamma\gamma)$ ~$4\frac{1}{2}$ KeV for F_8/F_1 a little smaller than 1 and θ a little smaller than -11°. It will be interesting to see the final experimental determinations of $\Gamma(\eta \rightarrow \gamma\gamma)$ and $\Gamma(\eta' \rightarrow \gamma\gamma)$.

I conclude this section by briefly reviewing a second analysis of the radiative η and η' decays.[9] This analysis uses vector meson dominance to get at the parameter ξ and is independent of the values of F_1, F_8, and θ. The main point is that vector vector meson dominance with ρ, ω, ϕ only applies to the color singlet part of the photon. Therefore the vector dominance relation is

$$\mathcal{F}(\eta_1 \rightarrow \gamma\gamma) = \xi \sum_{V=\rho,\omega,\phi} \frac{e}{f_V} \mathcal{M}(\eta_1 \rightarrow V\gamma) \tag{39}$$

For QCD, ξ = 1, and this is just the usual statement of vector dominance. For the Han-Nambu model, ξ = 2, $(\eta_1 \rightarrow \gamma\gamma)$ has two contributions,

$$\langle \eta_1 | J_{EM}^{HN} J_{EM}^{HN} | 0 \rangle = \langle \eta_1 | J_{EM}^{(1)} J_{EM}^{(1)} | 0 \rangle + \langle \eta_1 | J_{EM}^{(8)} J_{EM}^{(8)} | 0 \rangle \tag{40}$$

Here J_{EM}^{HN} is the electromagnetic current in the Han Nambu model,

$$J_{EM}^{HN} = J_{EM}^{(1)} + J_{EM}^{(8)} \tag{41}$$

$J_{EM}^{(1)}$ is the color singlet current which is equal to the usual electromagnetic current of QCD, and $J_{EM}^{(8)}$ is a color octet current that gives the quarks their integer charge assignments. From the anomaly at the low energy point we know that

$$\langle \eta_1 | J_{EM}^{(1)} J_{EM}^{(1)} | 0 \rangle = \langle \eta_1 | J_{EM}^{(8)} J_{EM}^{(8)} | 0 \rangle \tag{42}$$

and I assume (42) is still approximately correct on the mass shell. Vector dominance with ρ, ω, ϕ applies only to $J_{EM}^{(1)}$, so the right side of Eq. (39) is multiplied by $\xi = 2$ to include the contribution of $<\eta_1|J_{EM}^{(8)}J_{EM}^{(8)}|0>$.

It is an easy exercise in SU(3) flavor symmetry to show that Eq. (39) implies

$$\mathcal{F}(\eta_1 \to \gamma\gamma) = \frac{4}{3}\frac{e}{f_\rho}\mathcal{M}(\eta_1 \to \rho\gamma) \tag{43}$$

Next we use the theorem

$$\sin^2\theta + \cos^2\theta = 1 \tag{44}$$

to write

$$|\mathcal{F}(\eta_1 \to \gamma\gamma)|^2 = |\mathcal{F}(\eta \to \gamma\gamma)|^2 + |\mathcal{F}(\eta' \to \gamma\gamma)|^2 - |\mathcal{F}(\eta_8 \to \gamma\gamma)|^2 \tag{45}$$

$$|\mathcal{M}(\eta_1 \to \rho\gamma)|^2 = |\mathcal{M}(\rho \to \eta\gamma)|^2 + |\mathcal{M}(\eta' \to \rho\gamma)|^2 - |\mathcal{M}(\eta_8 \to \rho\gamma)|^2 \tag{46}$$

By SU(3) flavor symmetry,

$$|\mathcal{F}(\eta_8 \to \gamma\gamma)|^2 = \frac{1}{3}|\mathcal{F}(\pi^0 \to \gamma\gamma)|^2 \tag{47}$$

and

$$|\mathcal{M}(\eta_8 \to \rho\gamma)|^2 = \frac{1}{3}|\mathcal{M}(\omega \to \pi^0\gamma)|^2 \tag{48}$$

Finally we can solve for ξ in terms of six experimentally measurable quantities:

$$\xi^2 = \left(\frac{3f_\rho}{4e}\right)^2 \frac{|\mathcal{F}_{\eta'\to\gamma\gamma}|^2 + |\mathcal{F}_{\eta\to\gamma\gamma}|^2 - \frac{1}{3}|\mathcal{F}_{\pi^0\to\gamma\gamma}|^2}{|\mathcal{M}_{\eta'\to\rho\gamma}|^2 + |\mathcal{M}_{\rho\to\eta\gamma}|^2 - \frac{1}{3}|\mathcal{M}_{\omega\to\pi^0\gamma}|^2} \qquad (49)$$

Fortunately the present uncertainties in the total η' and η widths are correlated in the numerator and denominator of (49) and therefore tend to cancel. The principal uncertainty in ξ^2 is due to $B(\eta \to \rho\gamma)$ which figures only in the denominator.[28] The extraction of the $\mathcal{F}$ and $\mathcal{M}$ amplitudes from the experimental widths is straightforward except for $\mathcal{M}(\eta' \to \rho\gamma)$, for which it is essential to include the effect of the large ρ total width, $\Gamma_\rho = 154 \pm 5$ MeV. This is done by evaluating the three body phase space, $\eta' \to \pi^+\pi^-\gamma$, using a Breit-Wigner rho pole amplitude[21] as in Eq. (35).

The 1980 evaluation of the data gave[9] $\xi^2 = 1.15 \pm .25$. With the present experimental situation I find

$$\xi^2 = .87\,^{+.50}_{-.22} \qquad (50)$$

in good agreement with QCD, $\xi^2 = 1$, and in sharp disagreement with the naive expectation for integrally charged quarks, $\xi^2 = 4$. To arrive at this value of ξ^2, I crudely summarized the experimental situation for η and η' by $\Gamma(\eta \to \gamma\gamma) = 440 \pm 120$ eV and $\Gamma(\eta' \to \gamma\gamma) = 4\frac{1}{2} \pm 1\frac{1}{2}$ KeV and took $B(\rho \to \eta\gamma)$ from Ref. (25). As in Ref. (9) I followed the Yennie prescription[26] for the extrapolation of f_ρ from $q^2 = m_\rho^2$ to $q^2 = 0$, according to which self energy effects cause $f_\rho^2/4\pi = 1.93$ to be replaced by $f_{\rho\pi\pi}^2/4 = 2.96$. Had I used the unrenormalized $f_\rho^2/4 = 1.93$, I would have found $\xi^2 = .51^{+.32}_{-.22}$ still consistent with QCD at the one "sigma" level.

For QCD Eq. (49) is just the vector meson dominance relationship between $\eta \to \gamma\gamma$ and $\eta_1 \to (\rho, \omega, \phi)\gamma$. It is therefore just the analogue of the Gell Mann-Sharpe-Wagner relation between $\pi^0 \to \gamma\gamma$ and $\omega \to \pi^0\gamma$, which in my notation is

$$|\mathcal{F}(\pi^0\to\gamma\gamma)|^2 = \frac{4}{9}\frac{e^2}{f_\rho^2}\,|\mathcal{M}(\omega\to\pi^0\gamma)|^2 \qquad (51)$$

How well does this work? Using the prescription of Ref. (24), $f_\rho \to f_{\rho\pi\pi}$, it works embarassingly well, the left side being $6.42 \cdot 10^{-10}$ MeV^{-2} and the right side $6.49 \cdot 10^{-10} MeV^{-2}$. If instead we use $f_\rho^2/4\pi = 1.93$, the right side overestimates the left by ~55%, just as ξ^2 drop from .87 to .57.

5. ξ/ζ and the Higgs Hypothesis

$\xi(2220)$ and $\zeta(8320)$ are fascinating creatures, which both stand in need of experimental confirmation. If either (or both) is a real effect and if their widths turn out to be too small to be hadronic, then it is natural to consider the possibility that they might be Higgs bosons. Neither can be the Higgs boson of the standard model, since $\Gamma(\psi \to \gamma\xi)$ and $\Gamma(\Upsilon \to \gamma\zeta)$ are respectively ~10 and ~50 times too large. The simplest-variation is to consider two doublet models[27,28,29] in which one double Φ_1 couples to the weak isospin +1/2 fermions and Φ_2 couples to -1/2. Then for instance ξ could be dominantly from Φ_1 and the factor ~10 enhancement in $\psi \to \gamma\xi$ can be accomodated by having the vacuum expectation $v_1 \simeq v/3$. Here v is the vacuum expectation value of the standard one doublet model, and to get M_W right we need

$$v_1^2 + v_2^2 = v^2 \qquad (52)$$

Similarly ζ could be from Φ_2 with $v_2 \simeq v/7$. Because of (52) these two hypotheses are not simultaneously tenable.

It turns out that this explanation of ξ or ζ is extremely difficult to test in a conclusive way. For instance, to measure the spin of ξ would require $\geq$ 20,000,000 ψ decays, which is $\geq$ 2 1/2 times the present largest sample. To improve the limits on the widths is also very difficult. For ξ, the best test would be to look for toponium $\to \gamma\xi$.

Study of the two photon channel can make an important contribution to this problem. If ξ or ζ are the Φ_1 or Φ_2 of a two doublet model, then their $\gamma\gamma$ couplings must be far too small to be observable in $\gamma\gamma$

collisions. Therefore the detection of either in $\gamma\gamma$ scattering would rule out this interpretation.

The decay of a Higgs boson to two photons occurs by intermediate W-boson and fermion loop diagrams.[30] For fermions these are the same diagrams that give rise to the trace anomaly, and as discussed in preceding sections they have a remarkable sensitivity to the high energy structure of the theory. In particular all fermions heavier than the Higgs can contribute to the $\gamma\gamma$ width. In the standard one doublet model, the width is

$$\Gamma(H\to\gamma\gamma) = \frac{\alpha^2 G_F m_H^3}{8\sqrt{2}\,\pi^3}\left| I_W + \sum_f I_f \right|^2 \tag{53}$$

where $I_W = -7/4$ and I_f goes from zero at $m_f \ll m_H$ to $R_f/3$ for $m_f \gg m_H$, R_f being the contribution of $\bar{f}f$ to the R of e^+e^- annihilation.

In two doublet models the fermion contribution to the $\gamma\gamma$ width of a boson predominantly from doublet Φ_i will be enhanced by (v/v_i) in amplitude. Therefore, assuming three generations of fermions, we have in the two doublet interpretation of ξ or ζ

$$\Gamma(\xi\to\gamma\gamma) \cong \frac{\alpha^2 G_F m_\xi^3}{8\sqrt{2}\,\pi^3}\left| I_W + 3\,(I_c + I_t)\right|^2 \cong .02\ \text{eV} \tag{54}$$

and

$$\Gamma(\Upsilon\to\gamma\gamma) \cong \frac{\alpha^2 G_F m_\Upsilon^3}{8\sqrt{2}\,\pi^3}\left| I_W + 7 I_b \right|^2 \cong 1\ \text{eV}. \tag{55}$$

Both are unobservably small. If ξ or ζ can be observed in $\gamma\gamma$ scattering, it will mean they cannot be the Higgs bosons of two doublet models. It would in fact make any Higgs interpretation seem very unlikely, since to make the $\gamma\gamma$ widths observably large would require a dubious tuning of vacuum expectation values in more complicated models.

6. Rogues Gallery

The main points in this talk are:

(1) By measuring the stickiness, i.e., comparing $\Gamma(X \to \gamma\gamma)$ and $\Gamma(\psi \to \gamma X)$, we probe the quark and glue content for all mesons X except possibly the J=0 states.

(2) Contrary to intuition, light J=0 glueballs may have substantial $\gamma\gamma$ widths.

(3) $\Gamma(\eta \to \gamma\gamma)$ and $\Gamma(\eta' \to \gamma\gamma)$ seem well described by QCD and not by integral charge quark models, though the experimental situation needs clarification. Everyone is to desist from claims that $\eta \to \gamma\gamma$ and $\eta' \to \gamma\gamma$ are related to $\pi^0 \to \gamma\gamma$ by SU(3) symmetry.

(4) Observation of $\xi(2220)$ or $\zeta(8320)$ in $\gamma\gamma$ scattering would contradict the difficult-to-contradict hypothesis that ξ or ζ are Higgs bosons in two doublet models.

Table 2 is a rogues gallery of interesting objects which can profitably be studied in $\gamma\gamma$ scattering. It is very important to bounds on $\iota(1440)$ and $\theta(1700)$ in order to evaluate their status as possible glueballs. $\zeta(1270)$ is a possible $J^{PC} = 0^{-+}$ $\eta\pi\pi$ resonance[31] which is important in understanding whether $\iota(1440)$ is a glueball or a radially excited $\bar{q}q$ state.[32] It has the same mass and $\eta\pi\pi$ decay mode as the $J^{PC} = 1^{++}$ D(1270). $\gamma\gamma$ collisions are a good place to look for $\zeta(1270)$, since the D(1270) will not couple to $\gamma\gamma$ because of the Landau-Yang theorem. G(1590) is an interesting object seen at Serphukov,[32] for which a glueball interpretation has been advanced.[34] And in view of Section III and the report of Kück,[6] it is important to study $\gamma\gamma \to \pi\pi$ and $\psi \to \gamma\pi\pi$ at the lowest possible dipion masses.

The Rogues Gallery

Table 2: Resonances and resonance candidates

	Γ(MeV)	J^{PC}	Production/Decay	γγ Limit (KeV)
ι(1440)	76 ± 10	0^{-+}	$\psi \to \gamma\iota$, $p\bar{p}/K\bar{K}\pi, \rho\gamma(?)$	< 7-8/B(KKπ)
θ(1720)	130 ± 25	2^{++}	$\psi \to \gamma\theta/K\bar{K}, \eta\eta$	< 0.14/B(KK)
ζ(1270)	70 ± 25	0^{-+}	$\pi^- p \to \zeta n/\delta\pi \to \eta\pi\pi$	
ξ(2320)	< 40		$\psi \to \gamma\xi/K\bar{K}$	< .5/B(K_SK_S)
G(1590)	210 ± 40	0^{++}	$\pi^- p \to GX/\eta\eta, \eta\eta$	
?(2.1)			$\psi \to \gamma ?/\pi^+\pi^-$	
ϕϕ(2-2.3)		2^{++}	$\pi p \to \phi\phi n$	
ρρ		$0^{++}/2^{++}$	γγ	seen
ρρ/ωω 1.8-1.9		0^{-+}	$\psi \to \gamma\rho\rho, \gamma\omega\omega$	
ζ(8320)	< 80		ϒ→γζ/multihadron	
Low mass dipion		0^{++}	$\gamma\gamma \to \pi\pi$ & $\psi \to \gamma\pi\pi$?	

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