RESONANCES OF CLOSED MODES IN THIN ARRAYS OF COMPLEX PARTICLES

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Abstract. Numerical analysis of reflection properties of double-periodic two-element thin strip arrays is carried out. This analysis indicated extremely high quality band stop and band pass resonances appeared due to the excitation of non-symmetric current mode.

1. Introduction

Controlling the reflection and/or the transmission frequency properties of surfaces is an important problem of applied electromagnetics. For various microwave applications, there is a need to use active material layers with thicknesses extremely small in comparison to the wavelength. These frequency selective surfaces (FSS) are boundary surfaces consisting of some metal or dielectric bodies or the surfaces separated volumes in which there is a need to obtain electromagnetic fields possessing radical different characteristics.

There are several well known periodic arrays of different shapes (rectangular, circular) metal patches (e.g. [3], [4]) and the self-resonant grids such as grids of Jerusalem conducting crosses [1] which are used as FSS. Practically, the first low frequency resonance of such structure appears for a wavelength a bit greater than the array period. As a consequence of this, the transversal size of the whole FSS must be larger in comparison to the wavelength. The quality factor of such structures resonances is not high.

However, the situation is different with the arrays of complex shape resonating particles in that the array resonance is practically the same as the resonance of an individual particle. Conducting particles with resonant size of the order $\lambda/10$ are well known. So the total size of the array may be approximately the same as a wavelength. Recently, great attention has been paid to the study of frequency selective properties of complex shaped particles arrays, such as for example a bianisotropic Ω-shaped planar conducting particle [7], [12], a plano-chiral S-shaped particle [8] and C-shaped particle [10]. The properties of arrays of Ω, S, C-shaped particles and some other ones are well known now. In short, they have simple resonance characteristics and low quality factors.

If we want to design very thin structures having resonant band reflecting or transmitting characteristics with a high quality factor, the next step is to focus on the way of the structure complexity. Multi-particle arrays have these desired properties.

The problems of producing artificial dielectric possessing higher values of effective permittivity and photonic band gap structures are very close to the FSS problem mentioned above.

Commonly, complex materials are fabricated by randomly embedding most often simple metal inclusions like disks, spheres, ellipsoids or needles into a dielectric matrix. These particles have small sizes compared to the wavelength in the host medium and are located so that the distance between them is also small with respect to λ. Such materials possess an effective permittivity of a small value of the order of few units within a wide frequency band. In some applications, however, one needs very high values of effective permittivity at least within a narrow frequency band [2]. The introduction of high quality factor resonating particles in the layer is only one of the ways allowing to solve the problem.

Besides, the required curing process of the host materials and the fixed shape, size, and concentration of the samples are disadvantages. To avoid these difficulties, one can use layered materials with planar periodic arrays of conducting particles on the surface of each layer produced by inexpensive lithography process. Thus, the main purpose of this work is the study of resonance properties of multi-particle arrays which can possess band pass or band stop frequency characteristics with high quality factors.

2. Resonances of Closed Modes

Generally speaking, the resonance high quality factor and the layer small thickness are contradictory requirements. Actually, very thin open structure usually cannot have inner resonating volumes and on the other hand, resonating inclusions are strongly coupled with free space. Consequently, their resonance quality factor is low.

Nevertheless, there are ways to produce thin structures showing high quality frequency resonances thanks to the use of both the field excitation

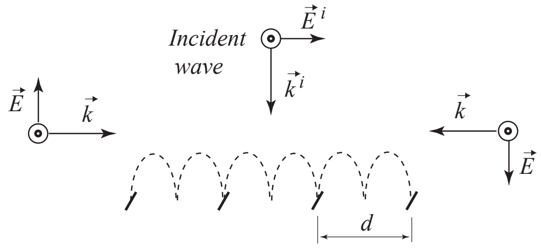

Figure 1. A grating of narrow inclined strips.

in large resonance volume and extremely reducing the coupling between resonating inclusions and free space. This has been achieved by a resonance regime of a so-called *closed modes.*

2.1. FULL REFLECTION FROM THIN STRIP GRATING DUE TO LARGE RESONANCE VOLUME

As an example of a very thin structure with *large resonance volume*, let us consider one-periodic planar grating of very thin narrow infinitely long metal strips (see Figure 1). Let us firstly consider the case of strips placed in grating so that their planes are orthogonal to the grating plane. If a normally incident plane wave polarized orthogonally impinges on the strips, the reflection coefficient of such a knife grating is equal to zero for any frequency.

Let us now incline the strip planes with regard to their plane in the knife grating. The strip width is much smaller than the wavelength. There is now weak interaction between the incident wave and the grating. If the wavelength value is close to the grating period but the shade is larger than it $\lambda \gtrsim d$, a sharp resonance of full reflection occurs. The reason behind the resonance reflection is an indication of a standing wave along the grating plane. The nodes of standing waves are placed in the strip positions. The field of the standing wave occupies a large volume and it is weakly coupled to the field of the incident wave. If the strip width or the strip plane inclination angle decrease, the resonance quality factor of rises.

This full reflection effect was analyzed in [11]. Normal incidence is a requirement for full reflection. Resonance reflection takes place in the case of inclined incidence also but the value of the reflection coefficient is less than one in this case. If we increase the angle of incidence, the first spatial partial wave appears and the resonance reflection vanishes.

The disadvantage for a practical use of this effect is the nearness of resonance frequency of the full reflection to the frequency of the grating first partial waves producing side lobes.

2.2. EIGEN MODES OF TWO-ELEMENT PARTICLES

Resonance regime of closed modes in double periodic arrays of multi-element plane particles is more convenient for microwave applications.

There is a well known the work which analyzed resonance properties of two-element bi-helix particle [6]. Let us mention also the work [5] which is very close to this subject and which studied two-slot waveguide diaphragms.

A two-element particle may be considered as a reciprocal two-port network. The properties of particle consisting of coupled elements are determined by a matrix of complex impedances

$$\mathbf{Z} = \begin{pmatrix} Z_1 & Z_c \\ Z_c & Z_2 \end{pmatrix} \tag{1}$$

where Z_1 and Z_2 are proper impedances of first and second element respectively, Z_c is a mutual coupling impedance. These impedances are frequency dependent values. Eigenfrequencies of two element particles can be found by solving the following equation,

$$\det(\mathbf{Z}) = 0. \tag{2}$$

If the elements of particle are different and have different values of proper impedances

$$Z_2 = Z_1 + \zeta \tag{3}$$

The solution of equation (2) can be found in the form

$$Z_1 = -\zeta/2 \pm \sqrt{(\zeta/2)^2 + Z_c^2}. \tag{4}$$

Thus, two element particle has two lower eigenfrequencies as follows from (4).

Eigen currents in elements of particle are satisfying the set of equations

$$Z_1 I_1 + Z_c I_2 = 0 \tag{5}$$

$$Z_c I_1 + Z_2 I_2 = 0. \tag{6}$$

Two eigenmodes have currents

$$I_2 = -I_1 \left(\sqrt{1 + \eta^2} - \eta \right) \tag{7}$$

$$I_2 = I_1 \left(\sqrt{1 + \eta^2} + \eta \right) \tag{8}$$

corresponding to upper and lower sign in expression (4). Here $\eta = \zeta/(2Z_c)$.

If elements of particle are identical, an exact solution of equation (2) can be obtained

$$Z_1 = \pm Z_c. \tag{9}$$

Eigenmodes have currents $I_2 = -I_1$ and $I_2 = I_1$ respectively.

In the case ζ has a small value, the particle has eigenfrequencies and eigenmodes close to those of particle of identical elements. The difference between the resonant frequencies of particles consisting of two identical elements and the resonant frequency of the same single element resides in the fact that the latter one corresponds to the greatest value of the mutual coupling impedance. If the coupling of identical elements is small, the resonant frequency as for the symmetric current distribution so as for the non-symmetric one (closed mode resonance) is approximately the same as the resonant frequency of the single element of particle.

3. Two-Element Arrays of Complex Shaped Particles

Very sharp resonances of reflection from infinitely thin double-periodic *multi-element* FSS can appear due to resonant properties of strip particles of periodic cell. *Multi-particle* structure of array cell is essential to existence of higher quality resonance of closed mode. For the sake of simplicity we restrict ourselves only to the case of array of two-element particles.

The method of moments is used to solve the problem of electromagnetic scattering by arrays of thin narrow curvilinear strips.

3.1. TWO-ELEMENT ARRAY WITH IDENTICAL ELEMENTS

For example let us consider first a double-periodic two-element array. Each cell of this array contains two identical strip elements opposite one to another. The left split between the strips is a little different from the right split, so that the unit cell is dissymmetric with regards to Oy axis (see Figure 2). The frequency dependence of the reflection coefficients magnitudes are shown in Figure 2. In the same figure the reflection coefficients magnitudes corresponding to an array of one-element C-shaped particles are shown for comparison.

If a normal incident wave is polarized in y direction, a sharp reflection resonance occurs (see curve 4). This resonance corresponds to a closed mode (non-symmetric current mode in two-element particle) because equal and opposite directed currents in the two elements complex particle radiate a little in free space. If the incident is x-polarized wave, a symmetric current mode is exited only. The corresponding resonance has low quality factor. The mutual coupling between the particle elements is not large so closed mode resonance and resonance of symmetric current distribution exited by

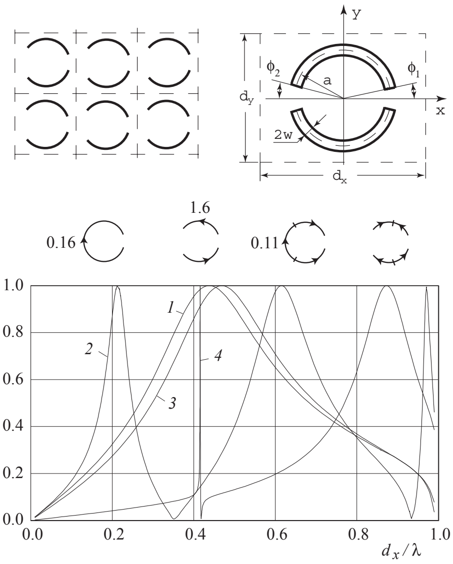

Figure 2. Absolute values of the reflection coefficients of arrays without substrate : $d_x = d_y = 3$ mm, $a = 1.25$ mm, $\phi_1 = 10°$, $2w = 0.1$ mm, r_{xx} (curve 1) and r_{yy} (curve 2) of an array of C-shaped particles ($\phi_2 = 0°$); r_{xx} (curve 3) and r_{yy} (curve 4) of a two-element particles array: the elements of the unit particle are identical, ($\phi_2 = 15°$: the unit cell is dissymmetric with regards to Oy axis).

x-polarized incident wave have approximately equal frequencies. However, the quality factors of these resonances are essentially different. Current distributions and maximum conventional values of current are shown above in Figure 2. They are concerned with resonance frequencies in the case of incident wave polarized along Oy axis. The current maximum value corresponding to the closed mode resonant frequency largely exceeds the current values at usual resonances.

If non-symmetry encreases, the quality factor of the closed mode resonance decreases (see Figure 3).

Reflection characteristics of an array placed on a substrate are similar. Resonant frequencies are shifted to low values. In general the level of reflection is higher in comparison to the one corresponding to an array without substrate (see Figure 4).

3.2. TWO-ELEMENT ARRAY WITH DIFFERENT ELEMENTS

Let us now consider reflection by an array of two different length elements in each cell. They are placed symmetrically to Oy axis as shown in Figure 5. The properties of an array with symmetrically placed different length elements are qualitativly quite different from the array with identical

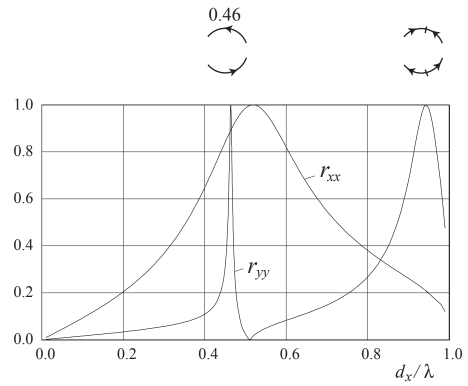

Figure 3. Absolute values of the reflection coefficients of an array without substrate: the elements of the unit particle are identical, $d_x = d_y = 3$ mm, $a = 1.25$ mm, $2w = 0.1$ mm, $\phi_1 = 10°$, $\phi_2 = 30°$ (the unit cell is dissymmetric with regards to Oy axis).

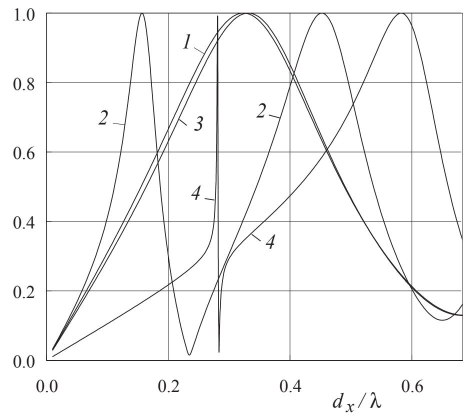

Figure 4. Absolute values of the reflection coefficients of arrays on substrate: $d_x = d_y = 3$ mm, $a = 1.25$ mm, $\phi_1 = 10°$, $2w = 0.1$ mm, $\varepsilon = 3$, $h = 0.25$ mm, r_{xx} (curve 1) and r_{yy} (curve 2) of an array of C-shaped particles ($\phi_2 = 0°$); r_{xx} (curve 3) and r_{yy} (curve 4) of a two-element particles array: the elements of the unit particle are identical, ($\phi_2 = 2.5°$: the unit cell is dissymmetric with regards to Oy axis).

elements. In these arrays non-symmetric high quality current mode can appear against the excitation of the usual symmetric mode.

There are two closely located reflection maxima and a very sharp resonance of full transmittance between frequencies of full reflection. Each reflection resonance appears, roughly speaking, due to the excitation of one of the elements of complex particle. A full transmittance resonance appears due to non-symmetric current mode in the two-element particle. It is a high quality closed mode resonance. Current amplitudes in the case of non-symmetric current mode are approximately equal in each part of

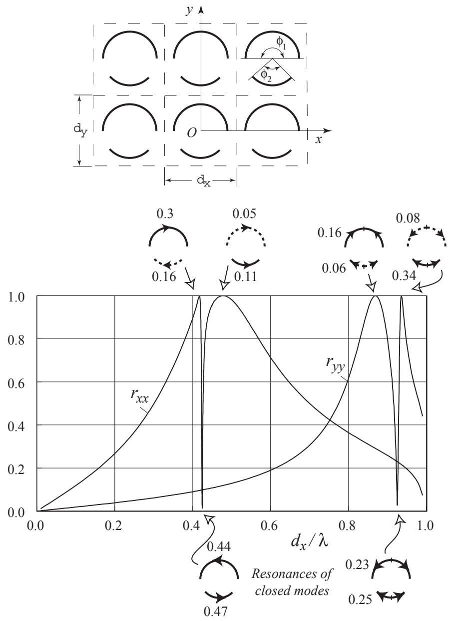

Figure 5. Absolute values of the reflection coefficients of an array without substrate: the elements of the unit particle have different lengths, $d_x = d_y = 3$ mm, $a = 1.25$ mm, $2w = 0.1$ mm, $\phi_1 = 160°$, $\phi_2 = 148°$, (the unit cell is symmetric with regards to Oy axis).

the two-element particle. In Figure 5, resonant and non-resonant current distributions are shown respectively by solid and dashed lines.

So one can obtain thin narrow-band filter of full transmittance. The narrow-band transparent properties of the array of different-element particles are quite similar to the rejection properties of a two-aperture iris in a rectangular waveguide studied in [5].

3.3. CLOSED MODES OF THE GRATING OF WAVY STRIPS

The next example of structures showing closed mode resonance is a grating of wavy strips (see Figure 6). The scattering of electromagnetic waves by gratings of wavy strips was analyzed in [9].

Let us note that the grating of narrow straight strips is approximately fully transparent for incident wave orthogonally polarized with respect to strips. In the case of gratings of wavy strips and the same y-polarization of incident wave, the currents on the two halves of the strip in the boundaries of the grating cell are equal in values but oppositely directed along the Ox axis (see Figure 7) owing to the symmetry of the grating. We can consider the grating of wavy strips as a two-element array. Actually, a current on

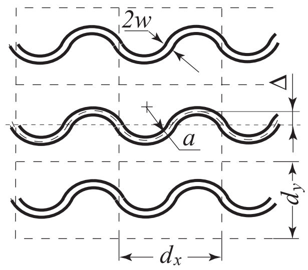

Figure 6. A grating of wavy strips.

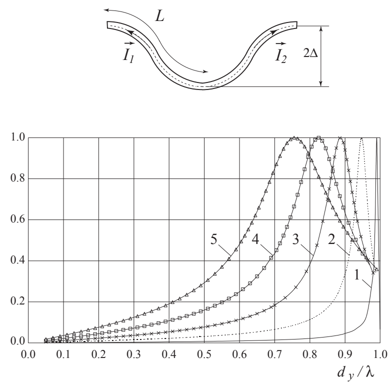

Figure 7. Absolute values of the reflection coefficients: grating of wavy strips without substrate, $d_x = d_y$, $2w/d_y = 0.05$, $\Delta/d_y = 0.05$ (curve 1), 0.1 (curve 2), 0.15 (curve 3), 0.2 (curve 4), 0.25 (curve 5).

the strip has zero values for such excitation in the points where a tangent to the strip is parallel to Ox axis. We can imagine strips cut in these points. Now the grating may be considered as a two-element array of curvilinear dipoles. Thus, such a two-element array consists of identical elements but placed at different locations and its reflection properties are similar to the properties of the array considered in Section 3.1.

If half of length of stretched strip placed in the period cell of the grating is approximately equal to half of wavelength $L = \lambda/2$ a resonance reflection appears. The set of dependencies magnitude of reflection coefficient r_{yy} versus normalized frequency is shown in Figure 7 for different values of amplitude of wavy strip Δ/d_y. If the amplitude of the wavy strip decreases the quality factor of reflection resonance increases.

4. Conclusions

Thus, two-element arrays of strip particles are very thin structures which can possess extremely high quality reflection or transmission resonances due to the excitation of non-symmetric current mode. Because such current mode weakly couples with free space, these resonances are similar to resonances of closed modes in finite widening extension of single-mode waveguide. If elements of stretched lengths placed in one array cell are different from each other, very sharp transmission resonances can appear in the rejection frequency band.

One can expect to obtain more complex resonance frequency characteristics due to use of multi-element arrays.

Electron devices such as photo diodes and *p-i-n* diodes may be used for the array properties control. Due to switching diodes one can modify a geometry of array particle from one-element to two-element by connecting or disconnecting elements in complex particles of array.

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