SLAC-PUB-2869
January 1982
(T/E)

RESONANT ABSORPTION OF COSMIC RAY NEUTRINOS BY THE RELIC NEUTRINO BACKGROUND*

T. Weiler
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

and

Physics Department
University of California, San Diego
La Jolla, California 92093

Within the framework of standard big-bang cosmology and the standard electroweak model, it is shown that $\nu\bar{\nu}$ annihilation on the Z-resonance is the only cosmic ray process sensitive to relic neutrinos. For massive ($m_\nu \gtrsim 10^{-3}$ eV) neutrinos originating from a $z \sim 3$ red-shifted source, a 15% to 50% absorption dip is predicted at $E_\nu \sim 10^{11}$ GeV.

(Submitted to Physical Review Letters)

---

*Work supported by the Department of Energy, contracts DE-AC03-76SF00515 (SLAC) and DE-AT03-81ER-40029 (UCSD).

Big-bang cosmology predicts the existence of a background gas of free photons and neutrinos. The measured 3°K black body photon spectrum supports the applicability of standard cosmology back to the photon decoupling temperature of 1/2 eV, which corresponds to an age measured from the initial singularity on the order of one hundred thousand years. The relic neutrinos have not been measured. Neutrino decoupling is calculated to have occurred when the universe had a temperature of 1 MeV and an age of just one second. Thus, measurement of relic neutrinos would support the validity of standard cosmology back twelve orders of magnitude in time beyond the age of photon decoupling. In this Letter we present an effect proportional to the relic neutrino density: absorption of cosmic ray neutrinos by the relic neutrinos.

It is mentioned in Ref. 1 that a Fermi-Dirac distribution for massive neutrinos, while not a solution to the Boltzmann equation in an expanding universe, should nevertheless be a good approximation if $m \ll T$. Thus we assume that at decoupling, each light (meaning $m \ll T_d$) flavor of neutrino, labeled by i, was isotropically and homogeneously distributed according to

$$f_{\nu_i}(p) = \left[e^{p/T_d - \bar{\mu}_i} + 1 \right]^{-1} . \qquad (1)$$

For antineutrinos make the replacement $\bar{\mu}_i \to -\bar{\mu}_i$. $T_d \sim 1$ MeV is the decoupling temperature,[2] and $\bar{\mu}_i$ is the degeneracy parameter. After decoupling, $p(R) \sim 1/R$, where R is the scale size of an isotropic, homogeneous Friedmann universe. Substituting $p(R_d) = p(R)\, R/R_d$ into Eq. (1) yields the distribution for later times:

$$f_{\nu_i}(p) = \left[e^{\beta p - \bar{\mu}_i} + 1 \right]^{-1} \qquad (2)$$

where $\beta \equiv R/T_d R_d$. βp is R-independent; hence the neutrino number density, $n_{\nu_i}(\bar{\mu}_i) = (2\pi)^{-3} \int d^3\vec{p}\; f_{\nu_i}(p)$, drops as $1/R^3$, as expected for free expansion.

β can be simply related to the relic photon temperature. For massless particles, $T \sim 1/R$ so $T_\gamma = \eta(R_d T_d/R)$; $\eta = (11/4)^{1/3}$ is a "correction" factor accounting for reheating of the photons as e^+e^- annihilated at $T \sim m_e$.[2] Thus $\beta^{-1} = T_\gamma/\eta$. Denoting present time values by a subscript zero, $\beta_o^{-1} = (T_{\gamma_o}/2.7^o\text{K}) \times 1.66 \times 10^{-4}$ eV, with $2.7^o\text{K} \le T_{\gamma_o} \le 3.0^o\text{K}$.[3] A relic neutrino is (non)relativistic if its mass is (large) small compared to the mean momentum, of order $(\bar{\mu}+1)\beta_o^{-1}$.

$\bar{\mu}$ and m are not known. A cosmological bound on $\bar{\mu}(m)$ or $m(\bar{\mu})$ results from requiring that the neutrino energy density ρ_ν (a monotonically, increasing function of m and $\bar{\mu}$) not exceed the total energy density of the universe, ρ. Observational inference[2] gives $\rho_o \lesssim 4 \times 10^{-29}$ g/cm^3. The constraint for neutrinos is

$$\sum_i \int \sqrt{p^2 + m_i^2} \; \frac{d^3\vec{p}}{(2\pi)^3} \left[f_{\nu_i}(p) + f_{\bar{\nu}_i}(p) \right] \le \rho_o \quad . \tag{3}$$

The sum is over light families. For a degenerate (meaning $|\bar{\mu}| >> 1$) neutrino gas, the bracket in Eq. (3) may be approximated by $\Theta(|\bar{\mu}_i| - p\beta_o)$, where $\Theta(x)$ equals unity if $x \ge 0$, and zero otherwise. One easily finds $|\bar{\mu}_i| \lesssim \beta_o(8\pi^2\rho_o)^{1/4} \lesssim 60$ for $m_i\beta_o \lesssim 1 << |\bar{\mu}_i|$, and $|\bar{\mu}_i| \lesssim \beta_o(6\pi^2\rho_o/m_i)^{1/3} \lesssim 10(m_i[\text{eV}])^{-1/3}$ for 0.01 eV $<< m_i <<$ 1 KeV and $1 << |\bar{\mu}_i|$. The neutrino masses are themselves bounded by Eq. (3) with the $\bar{\mu}_i$ set to zero. That is, $\sum_i m_i \lesssim \rho_o/2n_\nu(0) \sim 200$ eV. The factor of 2 includes antineutrinos, and $n_\nu(0) = 3\zeta(3)/4\pi^2\beta_o^3 = 53\ \text{cm}^{-3}$ for $T_\gamma = 2.7^o$K has been used. A more complicated and probably less trustworthy argument relates the observed ^{4}He/H abundance ratio to the n/p ratio when nucleons decoupled, and leads to a more restrictive bound[4] on the electron neutrino degeneracy parameter, $|\mu_{\nu_e}| \le 2$. Families other than ν_e are not bounded by the ^{4}He/H ratio.

It is easy to see that with standard cosmology the scattering of cosmic rays by relic neutrinos[5] is infinitesimal. The mean free path (mfp) of a cosmic ray through the relic neutrinos is roughly $1/n_\nu\sigma_W$. The weak cross section is $\sigma_W \simeq (G_F^2/\pi)[s/(1+s/M_W^2)] \lesssim (G_F^2/\pi)s$. G_F is the Fermi constant, M_W the W-boson mass and $\sqrt{s}$ the center-of-mass energy. For a primary cosmic ray of energy E, impinging on a relic neutrino with mean energy $\langle\varepsilon\rangle$, $\sigma_W \lesssim (2G_F^2/\pi)E\langle\varepsilon\rangle$. But $\langle\varepsilon\rangle n_\nu$ is just the neutrino energy density, certainly less than the total energy density ρ_o. Therefore $\lambda_{mfp} > \pi/2G_F^2E\rho_o$. For the scattering rate to be significant, the mfp must be less than or of order of the Hubble radius $H_o^{-1} = h^{-1} \times 10^{28}$ cm, with $1 \leq h^{-1} \leq 2$ the observational uncertainty.[3] Thus one requires $E > \pi/2G_F^2\rho_o H_o^{-1}$, which in turn is $\gtrsim 10^{14}$ GeV. But the universe is opaque to electrons, nucleons and photons at such energies: radio and thermal photon backgrounds degrade electrons via inverse Compton scattering and e^+e^- pair creation,[6] the nucleons via photomeson production,[7] and absorb primary photons via e^+e^- production.[8] In addition, for the electron such energies are also disallowed due to synchrotron losses occurring inside or outside the galaxy, or even in the earth's magnetosphere.[6,9] The universe (but not the earth) is transparent to cosmic ray neutrinos, but at energies in excess of 10^{14} GeV, the flux may well be negligible. We may also use[10] $\sigma_W \lesssim (G_F^2/\pi)M_W^2$ to deduce $\lambda_{mfp}/H_0^{-1} \gtrsim \pi/G_F^2M_W^2n_\nu H_0^{-1} \gtrsim 10^4/(n_\nu/50\ \mathrm{cm}^{-3})$, which says that regardless of incident energy, cosmic ray scattering on relic neutrinos is negligible unless the relic density is several orders of magnitude larger than the big-bang value predicted for $\bar{\mu} = 0$.

Finally, consider resonant absorption of a cosmic ray lepton by a relic neutrino. Integration over the relic momenta or over the universe's expansion is equivalent, by a change of variable, to integration over the resonance width. Thus the relevant weighted cross section for a Breit-Wigner form is $\bar{\sigma} \equiv \int ds\ \sigma(s)/M_R^2 = 16\pi^2 S\ \Gamma(R \to \ell\nu)/M_R^3$. S is the ratio of resonance spin states to incident lepton spin states. This time, the condition $\lambda_{mfp} \lesssim H_o^{-1}$ becomes

$$\frac{\Gamma(R \to \ell\nu)}{M_R} \gtrsim \frac{G_F\ M_R^2}{(n_\nu/50\ \text{cm}^{-3})S} \quad . \tag{4}$$

We have replaced $10^{-5}\ \text{GeV}^{-2}$ with G_F to make it clear that unless the relic neutrino density is several orders of magnitude larger than the value expected for $\bar{\mu} = 0$, $\Gamma(R \to \ell\nu)/M_R$ must be of order $G_F M_R^2$ rather than $(G_F M_R^2)^2$. This leaves the $W^{\pm}$ and Z as the only significant resonance candidates. Since the universe is opaque to electrons near the resonant energy $E_R \sim M_R^2/2\langle\varepsilon\rangle$, we are left to conclude that in the standard SU(2) × U(1) model, $\nu\bar{\nu}$ annihilation on the Z resonance is the only cosmic ray process having sensitivity to the relic neutrino density.[11] Since the neutrino mean free path for this process is comparable to the Hubble radius, the effects of an expanding universe must be included in our calculation.

The antineutrino cosmic ray transmission probability from time t to t_0 is $e^{-\tau}$, where

$$\tau = \int_t^{t_0} dt \int \frac{d^3\vec{p}}{(2\pi)^3}\ f_\nu(p)\ \sigma_Z\left(1 - p\cos\theta\Big/\sqrt{p^2 + m^2}\right) \quad . \tag{5}$$

θ is the incident angle of collision and σ_Z is the annihilation cross section. Introducing the red-shift variable for cosmological expansion from time t to present, $\omega(t) \equiv R_o/R(t) - 1$, there results a simple change of variable, $d\omega = -(\omega + 1)H dt$. $H \equiv \dot{R}/R$ is the Hubble parameter, itself a function of time. The Einstein equations for a matter-dominated (pressure

$p << \rho$) era[12] relate the Hubble parameter at time t to its present value (we assume zero cosmological constant): $H(t) = H_0(\omega+1)\sqrt{1+\Omega_0\omega}$. Ω_0 is the present energy density of the universe in units of the critical value $3H_0^2/8\pi G$, G being Newton's constant. The bounds from observational astronomy are $0.02 \lesssim \Omega_0 \lesssim 2$.[2]

The limits of Eq. (5) for relativistic and nonrelativistic relic backgrounds are easily obtained by introducing the corresponding limit of $\int ds\, \delta\left[s - 2\left(m^2 + E(\sqrt{p^2+m^2} - p\cos\theta)\right)\right]$ and substituting $p = p_0(\omega+1)$, $E = E_0(\omega+1)$, E being the cosmic ray energy. We also approximate $s = M_Z^2$ in the argument of δ, and use the result of standard electroweak theory, $\int ds\, \sigma_Z(s) = 2\pi\sqrt{2}\, G_F M_Z^2$. The relativistic result is

$$\tau(E,\bar{\mu},z) = \frac{G_F M_Z^4}{4\pi\sqrt{2}\, HE^2} \int_0^z \frac{d\omega}{(\omega+1)^3\sqrt{1+\Omega\omega}} \int_0^\infty dp\, f_\nu(p)\, \Theta\left(p(\omega+1)^2 - M_Z^2/4E\right) . \quad (6)$$

Subscript zeros have been dropped since every variable is now either a present time variable or an integration variable. The maximum red-shift value, z, is the cosmological red-shift of the extragalactic antineutrino source. Unfortunately, unless the degeneracy is large, (nearly) massless neutrinos will not show an absorption dip; the threshold is smeared over a large energy range. Furthermore, it is clear from the theta function of Eq. (6) that the absorption maximum occurs at excessive energies of order $M_Z^2/(1+z)^2\langle p_\nu\rangle$, i.e., $E > 10^{13}$ GeV for $z \lesssim 4$ and $\bar{\mu} \lesssim 50$. Prospects for detection are poor for (nearly) massless neutrinos.

In grand unified models, neutrino masses in the eV range arise naturally.[13] Furthermore, 30 eV neutrinos are a panacea for unresolved questions concerning the large scale structure of the universe.[3,14] Since the relic neutrino mean momentum is of order $(\bar{\mu}+1) \times 10^{-4}$ eV, eV neutrinos are certainly nonrelativistic. The nonrelativistic limit of Eq. (5) is

$$\tau\left(x \equiv \frac{M_Z^2}{2mE}, \bar{\mu}\right) = \frac{x^2 \tau(1)}{\sqrt{1+\Omega(x-1)}} \quad , \quad 1 \le x \le 1+z \qquad (7)$$

with $\tau(1) = 2\pi\sqrt{2}\, G_F n_\nu H^{-1} = .017\ h^{-1}(n_\nu/50\ cm^{-3})$. The allowed range of x has a simple interpretation: a neutrino received with energy E left its red-shifted source with energy E(1 + z), and was a candidate for annihilation only if the resonant energy, xE, lay within this energy range. The transmission probability $e^{-\tau}$ is plotted as a function of received energy in Fig. 1. Curves are characterized by values of $(h^{-1},\Omega,\bar{\mu},T_\gamma)$. The absorption dip begins at an energy (in units of $M_Z^2/2m$) of 1/1+z. The position of the dip for various z values is shown on the top axis of the figure. It is clear that an absorption dip of 15% to 50% can be expected for neutrinos from a z = 3.5 quasar source. If the degeneracy parameter is nonzero, the dip will be much larger. For a 90 GeV Z-boson mass and a z = 3.5 source, the dip energy is 9×10^{10} GeV (10 eV/m). Each species of nonrelativistic neutrino should have its own dip, characterized by the mass and degeneracy of the species. With massive neutrinos there exists the possibility of gravitational clustering of relic neutrinos. If the mfp between such clusters is small compared to the Hubble radius, results are unchanged. If the mfp is comparable to or larger than the Hubble radius, absorption in the direction of a neutrino cluster will be enhanced relative to the homogeneous result; perhaps our own galaxy can serve to amplify the absorption.

We conclude with the hope that neutrinos do have mass in the eV range. Then a sizeable absorption dip at $10^{11\pm1}$ GeV is unambiguously predicted for neutrino cosmic rays from a red-shifted source. Detection feasibility will of course depend on the magnitude of flux from neutrino sources. Quasars, active galactic nuclei, pulsars, supernovae and accreting black holes are suggested sources.[15] Although their ultrahigh energy emission spectrum is unknown, it is implausible that a single source spectrum can be measured at the energies required here. A more realistic approach is to convolute the transmission probability given in this letter with a trial spectrum summed over all sources. A break in the spectrum of the integrated flux is anticipated. This approach is currently under investigation.

ACKNOWLEDGEMENTS

The ideas in this paper originated while I was a visitor at the Aspen Center for Theoretical Physics. I wish to particularly acknowledge discussions with J. Bernstein, J. Fry, H. Partovi, C. Schmid and S. Wolfram. This work was supported by the Department of Energy, contracts DE-AC03-76SF00515 (SLAC) and DE-AT03-81ER40029 (UCSD).

REFERENCES

1. J. Bernstein and G. Feinberg, Phys. Lett. 101B, 39 (1981); 103B, 470 (E).

2. See e.g., S. Weinberg, Gravitation and Cosmology, Wiley, New York, (1972).

3. G. Steigman, Proc. of Neutrino - '81, Honolulu, Hawaii, and references therein.

4. R. V. Wagoner, W. A. Fowler and F. Hoyle, Astrophys. J. 148, 3 (1967); A. Yahil and G. Beaudet, Astrophys. J. 206, 26 (1976); A. D. Linde, Phys. Lett. 83B, 311 (1979); S. Dimopoulos and G. Feinberg, Phys. Rev. D 20, 1283 (1979).

5. First considered by J. Bernstein, M. Ruderman, G. Feinberg, Phys. Rev. 132, 1227 (1963); B. P. Konstantinov and G. E. Kocharov, JETP 19, 992 (1964); R. Cowsik, Y. Pal and S. N. Tandon, Phys. Lett. 13, 265 (1964).

6. V. L. Ginzburg and S. I. Syrovatskii, The Origin of Cosmic Rays, Pergamon Press Ltd., Oxford (1964).

7. K. Greisen, Phys. Rev. Lett. 16, 748 (1966); G. T. Zatsepin and V. A. Kuzmin, JETP Lett. 4, 78 (1966); F. W. Stecker, Phys. Rev. Lett. 21, 1016 (1968).

8. A. I. Nikishov, JETP 14, 393 (1962); P. Goldreich and P. Morrison, JETP 45, 344 (1963); R. J. Gould and G. Schreder, Phys. Rev. Lett. 16, 252 (1966), Phys. Rev. 155, 1404, 1408 (1967); J. V. Jelly, Phys. Rev. Lett. 16, 479 (1966).

9. G. Khristiansen, G. Kulikov and J. Fomin, Cosmic Rays of Superhigh Energies, Verlag Karl Thiemig, Munich (1980).

10. Fot cosmic ray nucleons, the weak cross section is slightly enhanced by the factor $\frac{1}{2}F_2(x=0,\ Q^2=M_W^2)\ \ell n\,(1+s/M_W^2)$, but this does not change conclusions. For the enhancement, see V. S. Berezinskii and A. Z. Gazizov, Yad. Fiz. 29, 1589 (1979).

11. The opacity of the universe to high energy electrons and photons also obviates consideration of the Compton-like processes $\gamma\nu_\ell \rightarrow \ell W$ and $e\bar{\nu}_e \rightarrow \gamma W$.

12. In the unlikely prospect that the universe is in fact radiation-dominated, then $H(t) = H_o(\omega+1)\sqrt{1+\Omega_o\omega(\omega+2)}$ and the upper bound on Ω_o is halved.

13. P. Ramond, Proc. of the First Workshop on Grand Unification, eds., P. Frampton, S. Glashow, A. Yildiz, U. N. H. (1980); E. Witten, ibid.

14. J. R. Bond and A. S. Szalay, Proc. of Neutrino—'81, Honolulu, Hawaii.

15. S. Margolis, D. Schramm and R. Silberberg, Astrophys. J. 221, 990 (1978); D. Eichler, Astrophys. J. 222, 1109 (1978); 232, 106 (1979); also see Proc. DUMAND Summer Workshop, Vol. 2, Ultrahigh Energy Interactions & Neutrino Astronomy, ed. A. Roberts (1978).

FIGURE CAPTION

Fig. 1. Transmission probability for massive ($m >> (\bar{\mu}+1)\beta^{-1}$) cosmic ray antineutrinos as a function of their energy. Unstated values of ($h^{-1},\Omega,\bar{\mu},T_\gamma$) are ($2,1,0,2.7^{\circ}K$). Transmission probabilities for incident neutrinos are obtained by taking $\bar{\mu} \to -\bar{\mu}$.

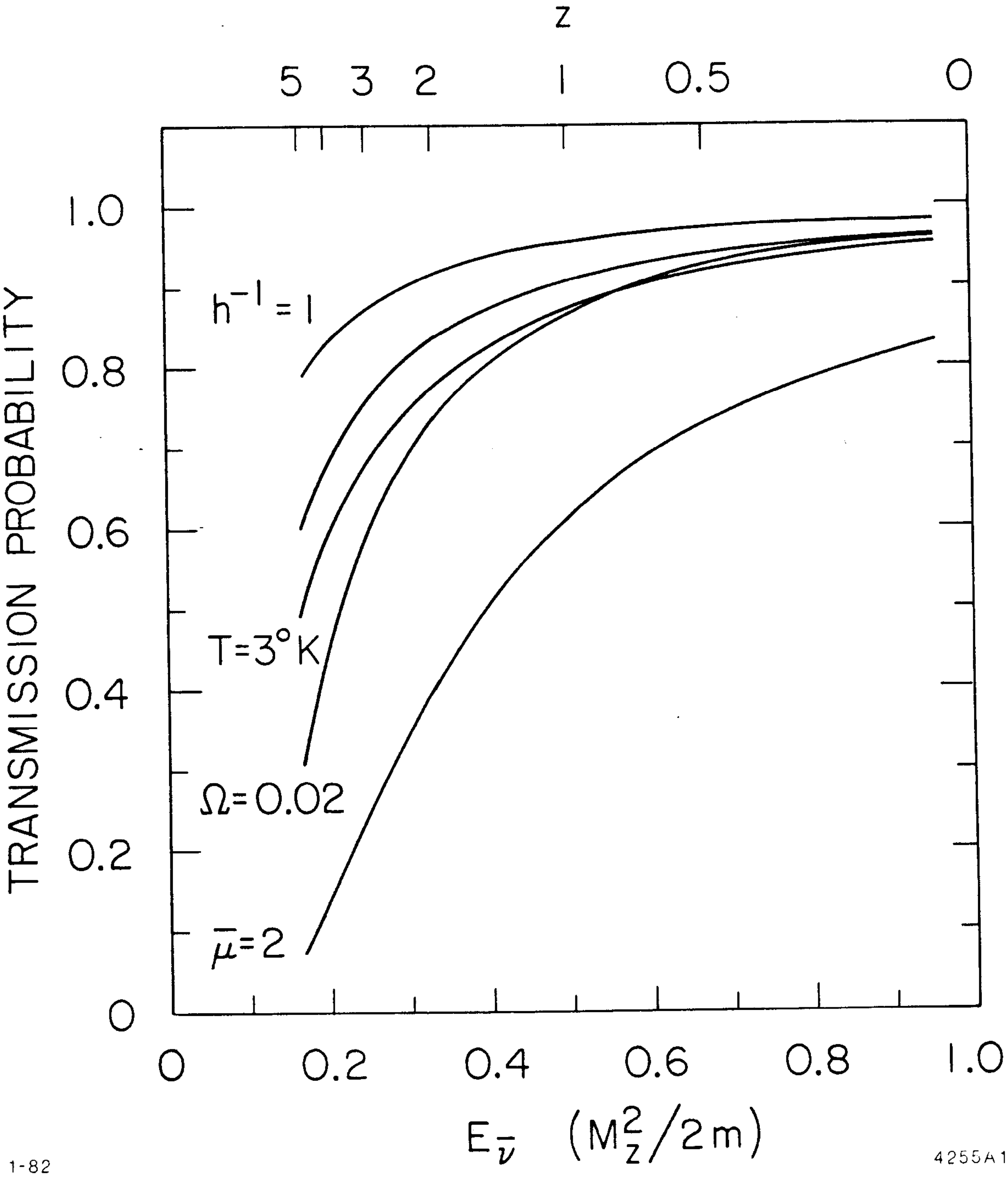
z
5
3
2
1
0.5
0
TRANSMISSION PROBABILITY
1.0
0.8
0.6
0.4
0.2
0
$h^{-1}=1$
$T=3^{\circ}K$
$\Omega=0.02$
$\bar{\mu}=2$
0
0.2
0.4
0.6
0.8
1.0
$E_{\bar{\nu}}$ $(M_Z^2/2m)$
1-82
4255A1

Fig. 1