CHAPTER 32

RESONANT INTERACTIONS FOR WAVES BREAKING ON A BEACH

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ABSTRACT

A laboratory and theoretical study of the transition from strongly reflected surging to dissipative plunging breakers on a relatively steep plane beach (1:8) has revealed the following: (1) The run-up and offshore variation of sea surface elevation of surging waves are well predicted by linear theory. (2) The fluctuating part of the run-up (related to the amplitude of the reflected incident wave) reaches a maximum value; a further increase in incident progressive wave energy results in increased dissipation. (3) Subharmonic edge waves (the growing instabilities of surging waves) are driven primarily by the swash motion, which does not increase with increasing incident breaking wave height. However, the turbulence accompanying incident wave breaking, and the effective eddy viscosity, rapidly increases with increasing breaker height. As a result, subharmonic resonances do not occur with spilling or steep plunging waves; very strong viscous effects suppress the nonlinear instabilities. (4) edge waves generated by a surging incident wave can be suppressed by superimposing an additional breaking wave of different frequency on the incident wave field. Thus, any excited edge waves are likely to have length scales at least the order of a surf zone width.

INTRODUCTION

The shoaling and breaking of a regular train of surface waves has been the subject of many extensive experimental investigations. However, the primary interest has been in waves which break by spilling or plunging and surprisingly little attention has been given to surging, or collapsing, waves. Surging occurs when the reflection from the beach is strong, and the interference between the incoming and reflected waves results in complex patterns of elevation and velocity markedly different from the essentially monotonic changes in wave height associated with spilling waves.

Surging waves, and the transition from surging to plunging, are of interest for several reasons. One is the suggestion that steady, wave-induced, bottom currents having convergences associated with either nodes or antinodes of a standing wave component, are responsible for the formation of offshore bars. However, except for the field work of Suhayda (1974) involving a complicated spectrum of waves, there is little data to support the theoretical location of nodes and antinodes. A second motivation for a more detailed study of wave surging involves the resonant generation of edge waves. It has been shown theoretically that completely reflected, monochromatic, incident waves are unstable to edge wave perturbations (Guza and Davis, 1974; Guza and Bowen, 1975). Qualitative laboratory studies (Galvin, 1967; Guza and Inman, 1975) show that the edge waves excited by nonlinear interaction may have amplitudes at the shoreline more than twice as large as surging incident waves. However, as the incident wave amplitude is increased and the waves break by plunging, the resonance ceases and the edge waves disappear. In order to understand why the resonance ceases, it is clearly necessary to have a detailed understanding of the incident wave behavior.

Before considering the complexities which arise in three-dimensions due to edge waves, it is useful to first consider the two-dimensional case. In section II, a linear model is formulated for normally incident waves, partially reflected from a plane beach. This theoretical model is found to be in close agreement with laboratory measurements of sea surface elevation, seaward of the break point, when the coefficient of reflection, r, is greater than about 0.3. For lower reflections, that is steeper incident waves, nonlinear effects become significant.

In the experiments described in section III, the width of the wave basin is adjusted so that edge waves generation becomes possible. The two-dimensional, monochromatic, incident waves can now force edge waves at the subharmonic $\sigma/2$ of the incident wave frequency σ. Measurements show that these edge waves also have an offshore variation very close to that predicted by linear theory. The equilibrium amplitude of the edge waves was determined for various incident wave conditions, and the disappearance of the resonance with increasing incident wave amplitude was investigated in detail. Additional experiments superimposed a second wave train on a resonant situation so that the effects of a surf zone, not associated directly with the primary incident wave of the resonance, could be examined.

II. TWO-DIMENSIONAL WAVES

Linear Theory

When waves approaching the shore are reflected by a beach they propagate from water which is effectively infinitely deep, to zero depth and then back out into deep water. Although exact linear solutions for this problem are known, they are extremely cumbersome, especially for small beach slopes (Stoker, 1947). Simpler linear theories exist, but none provide a satisfactory description of the total motion even if frictional effects are neglected; different approximations apply for deep and shallow water.

Clearly, at the shoreline and for some distance offshore the appropriate approximation is provided by linear, shallow water theory. The solution for a wave, frequency σ, partially reflected from a plane beach of slope $\tan\beta$ is (Lamb, 1932; Suhayda, 1974)

$$\phi^s = \frac{a_s g}{\sigma}\left[J_0(\chi)\sin\sigma t + Y_0(\chi)\cos\sigma t + r\{J_0(\chi)\sin(\sigma t-\theta) - Y_0(\chi)\cos(\sigma t-\theta)\}\right] \quad (1)$$

where $\chi^2 = \frac{4\sigma^2 x}{g\tan\beta}$, a_s is the amplitude of the incoming wave, r the reflection coefficient and θ a phase shift in the reflected wave, x is positive in the offshore direction with the shoreline at $x = 0$.

Offshore, in deeper water, the slowly varying Stokes solutions are valid, where in water depth h

$$\phi^d = \frac{a_d g}{\sigma}\frac{\cosh k(z+h)}{\cosh kh}\left[\cos\left(\int_0^x k\,dx + \psi + \sigma t\right) + r\cos\left(\int_0^x k dx + \psi - \sigma t + \theta\right)\right] \quad (2)$$

provided $\sigma^2 = gk\tanh kh$, and $a_d = a_\infty(\tanh kh + kh\,\mathrm{sech}^2\,kh)^{-1/2}$; where a_∞ is the amplitude of the incoming wave far offshore ($kh \to \infty$) and ψ is a constant.

Friedrichs (1948) showed that the shallow water limit of the Stokes solution and the offshore limit of the shallow water solution smoothly match together at intermediate depths if appropriate values of a_s and ψ are chosen. Then

$$\phi^{lim} = \lim_{\chi\to\infty}\phi^s = \lim_{h_*^{-1}\to\infty}\phi^d = \frac{a_\infty g h_*^{-1/4}}{\sigma\sqrt{2}}\left[\cos(\chi - \tfrac{\pi}{4} + \sigma t) + r\cos(\chi - \tfrac{\pi}{4} - \sigma t + \theta)\right]$$

$$\text{where}\quad h_* = \frac{\sigma^2 h}{g} = \frac{\sigma^2 x\tan\beta}{g} \quad (3)$$

if $a_s = a_\infty(\pi/2\tan\beta)^{1/2}$ and $\psi = -\pi/4$. In the matching regions $\chi >> 1$, $h_* << 1$, so that

$$\tan^2\beta << h_* << 1 \quad (4)$$

For a wave totally reflected at the shore, a purely standing wave, the ratio of the wave amplitude at the shoreline to that in deep water, given by equation 3, is equivalent to the amplication factor known from the exact solution (Stoker, 1947) if the beach slope is small ($\beta \simeq \tan\beta \simeq \sin\beta$).

A general solution on a sloping beach, valid everywhere is given by

$$\Phi = \phi^s + \phi^d - \phi^{lim} \quad (5)$$

where if Φ^s or Φ^d are outside their range of validity they are cancelled by Φ^{lim} leaving the other as the only contribution in the relevant region. We note that the linear Stokes progressive wave solution (equation 2, $r = 0$) may be improved upon by adding a linear correction term of $O(\tan\beta)$ (Chu and Mei, 1970). This correction term smoothly matches to a higher order expansion of the shallow water Bessel function solutions (eq. 1), and it appears possible to produce a matched asymptotic solution which is an arbitrarily good approximation to the exact linear solution. However, a comparison of the sea surface elevation given by the exact linear (Stoker, 1947) and smoothly matched solutions on a 6^0 slope shows insignificant differences between the two, and suggests that little is to be gained by linear improvements on equation 5. The marked profile asymetry attributed to linear $O(\tan\beta)$ corrections by Biesel (1952) and Gaughan and Komar (1975) do not appear in the exact solution. The applicability of eq. 5 is limited because nonlinearities are neglected, and it is not likely to be a particularly good representation of the wave field right in to the break point unless r is large. The larger wave steepnesses which lead to smaller values of r produce plunging or spilling breakers whose size is generally under-estimated by linear theory (Komar and Gaughan, 1972). However, the present experiments show that eqs. 1-5, are satisfactory up to breaking if $r > 0.3$, which does include part of the regime in which waves break by plunging. Figure 1 shows the normalized local sea surface displacement for a fully reflected wave ($r = 1$, $\theta = 0$) calculated from both the shallow water and Stokes solutions for three different beach slopes. Offshore the shallow water solution diverges from the correct solution, errors in amplitude > 5% occurring for $h_* > 0.25$. The phase of this solution diverges at even smaller values of h_* (phase errors tend to be cummulative) and on small slopes may become large where the amplitude is still well predicted. The parameter which limits the offshore validity of the shallow water solutions is $h_* = \sigma^2 h/g$, not the scale depth which actually appears in the solution $h_* \tan^{-2}\beta$. Consequently the shallow water solutions are valid much further seawards on beaches of gentle slope. As a rough rule, the shallow water solutions are valid for $h_* < 0.1$.

It is evident in Figure 1 that, regardless of beach slope, the linear Stokes solution will correspond very closely to the matched solution except in the immediate vicinity of the shoreline. Here, however, the discrepancy is serious; the solution has a singularity of order $h_*^{-\frac{1}{4}}$ and cannot provide a reasonable value for the amplitude at the shoreline in terms of deep water conditions. Higher order solutions (both linear and nonlinear) for the Stokes wave have even stronger singularities at the shoreline.

It is perhaps worth noting that although the linear shallow water and Stokes' solutions match smoothly together, their higher order, nonlinear terms do not. To provide a uniformly valid non-linear solution an additional intermediate zone governed by nonlinear equations of the Kortweg-De Vries type on a sloping beach is probably needed. Fortunately, waves which are strongly reflected are of low steepness and nonlinearities are negligible everywhere except on the beach face. Figure 2(a), discussed more fully later, shows the measured values of sea surface elevation for totally reflected waves of three different periods, which are in very good agreement with the theoretical estimation derived from the linear theory (eq. 5).

Partial Reflection

Carrier and Greenspan (1958) used the fully nonlinear, inviscid, shallow water equations to study the maximum possible size a standing wave can attain on an impermeable sloping beach. A review of their work, and of the general problem of waves on a sloping beach is given by Meyer and Taylor (1972). Carrier and Greenspan found that a standing wave solution is possible if

$$\varepsilon = \frac{a_0 \sigma^2}{g \tan^2\beta} \leq 1 \qquad (6)$$

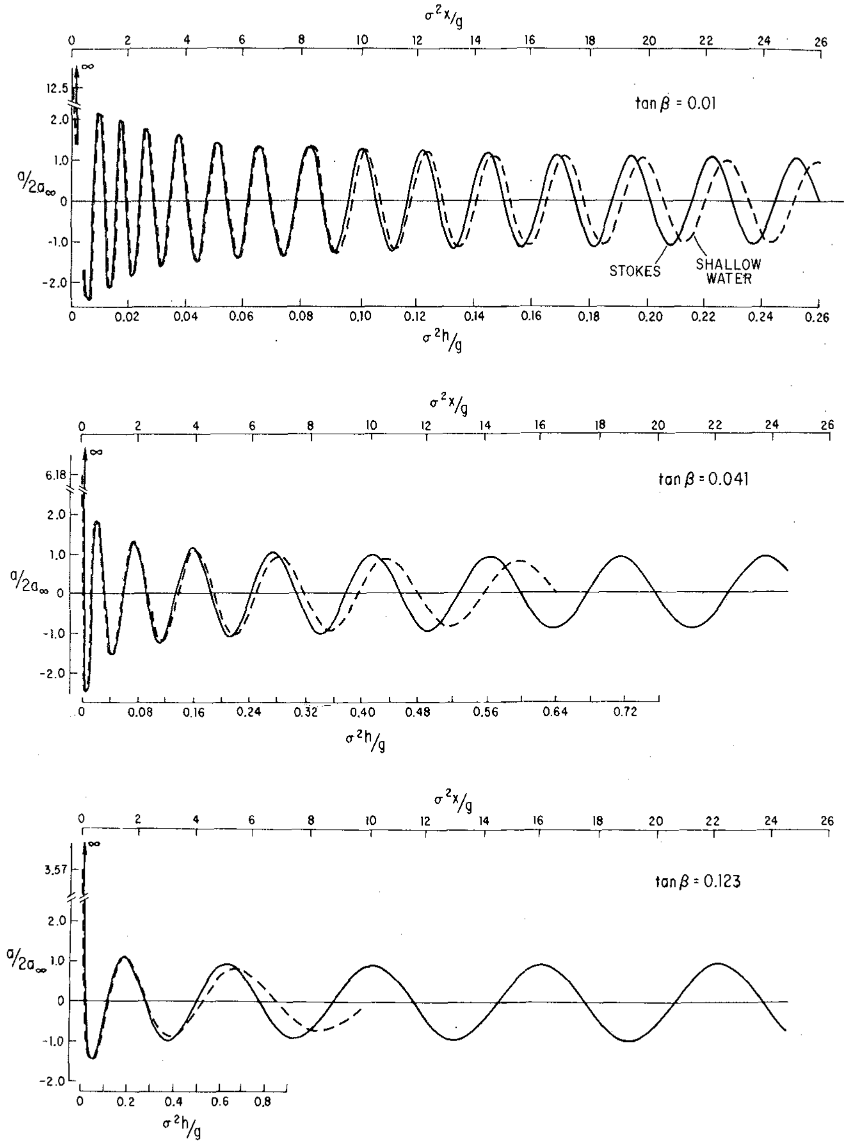

Figure 1. Theoretical local standing wave displacement (a) normalized by offshore amplitude ($2a_\infty$). Stokes solutions are valid offshore, while shallow water solutions properly describe the run-up.

where $2a_0/\tan\beta$ is the total horizontal excursion of the swash, and $a_0 = 2a_s$ (eq. 1; $r = 1.0$, $\theta = 0$) is the standing wave amplitude away from the immediate vicinity of the shoreline. Munk and Wimbush (1969), coincidentally, obtain the same limit condition (eq. 6) using linear theory. If $\varepsilon = 1$, the standing wave is of maximum possible size, the nonlinearities producing considerable distortion of the wave profile; in the run-up the Fourier amplitude at frequency 2σ is theoretically about a quarter of the amplitude of the motion at σ. However, for nonbreaking waves the maximum horizontal displacement of the shoreline, $2a_0/\tan\beta$, is exactly the same as presented by the linear standing wave solution with amplitude a_0. Thus, if the waves are nonbreaking and the effects of viscosity and percolation on the beach face are small, a measurement of the horizontal displacement of the shoreline provides an appropriate value of a_0 to compare to offshore measurements where the wave is essentially linear. Away from the immediate vicinity of the shoreline, the nonlinear shallow water standing wave does not differ appreciably from the linear solution and (3) provides the appropriate patching. The deep water condition for a standing wave which will not break at the shoreline is therefore that

$$\frac{2a_\infty\sigma^2}{g}\left(\frac{\pi}{2}\right)^{\frac{1}{2}}\tan^{-5/2}\beta \le 1 \tag{7}$$

that is, incoming progressive waves amplitude a_∞ in deep water will be totally reflected provided (Meyer and Taylor, 1972).

$$\varepsilon_i = \frac{a_\infty\sigma^2}{g}(2\pi)^{\frac{1}{2}}\tan^{-5/2}\beta \le 1 \tag{8}$$

This result differs by a constant factor of about two from the formula of Miche (1951) which has been commonly used (Moraes, 1970; Suhayda, 1974; and others) to estimate the maximum amplitude of an incoming wave which will be totally reflected

$$\frac{a_\infty\sigma^2}{g}(2\pi)^{\frac{1}{2}}\beta^{-\frac{1}{2}}\sin^{-2}\beta \le 2 \tag{9}$$

For small β, Miche's formula therefore implies that complete reflection may occur provided $\varepsilon_i < 2$. For $\varepsilon_i > 2$ Miche assumes the wave is partially reflected and that the reflection coefficient r decreases with increasing ε_i

$$r = \frac{2}{\varepsilon_i}, \quad \varepsilon_i > 2$$

Now the offshore amplitude of the wave reflected from the beach is ra_∞ so

$$\varepsilon_r = \frac{2ra_\infty}{g}\left(\frac{\pi}{2}\right)^{\frac{1}{2}}\tan^{-5/2}\beta = r\varepsilon_i \tag{10}$$

and the Miche hypothesis is

$$\varepsilon_r = \begin{matrix}\varepsilon_i\\ 2/\varepsilon_i\end{matrix}\ ; \quad \varepsilon_i \begin{matrix}<\\>\end{matrix} 2. \tag{11}$$

Battjes (1974) has suggested a similar expression based on a shallow water parameter similar to ε (eq. 6). There is, however, no allowance for the $\tan^{-\frac{1}{2}}\beta$ (theoretical) amplification in shallow water (3), so Battjes deep water reflection parameter differs from (10, 11) by $\tan^{-\frac{1}{2}}\beta$

$$\varepsilon_r = \frac{\varepsilon_i}{0.787 \quad \tan^{-\frac{1}{2}}\beta} \; ; \quad \varepsilon_i \lesssim 0.787 \tan^{-\frac{1}{2}}\beta \tag{12}$$

The formulas (11, 12) are numerically identical when $\beta = 8.8^0$, and both follow from the assumption that when wave breaking occurs, the wave field outside the breakpoint (or surge line) consists of an incoming progressive component plus a standing wave of the maximum possible amplitude which can occur without breaking.

Field Observations

The idea that the surf zone can be represented as the sum of a standing wave of some maximum amplitude, and an incoming progressive component which decays shoreward of the break point, suggests that the motion at the shoreline is determined primarily by the standing component and should be given by

$$\varepsilon = \frac{a_0\sigma^2}{g \tan^2\beta} = \text{constant} = \varepsilon_c$$

Therefore, if an incident wave field consists of several narrow banded components, the energy spectrum of the wave run-up (vertical)

$$E(\sigma) \approx a_0{}^2 = g^2\tan^4\beta \; \sigma^{-4} \; \varepsilon_c^2 \tag{13}$$

provided the wave components behave, at lowest order, as independent (linear) waves. Although the assumption of independence seems a gross approximation with the breaking bores observed in real surf zones, field observations do seem to show a region in the run-up spectrum where the energy varies as σ^{-4} (Huntley, Guza, and Bowen, in press).

An immediate question is whether this agreement provides real support for the conceptual model of the surf zone and run-up as a simple sum of standing and progressive waves which is implicit in the formula for reflection coefficients. If so, then measurements of shoreline displacement should give the size of the standing wave component and hence the amplitude of the wave components reflected from the beach. The critical relation is then between the wave amplitude at the shoreline and the wave amplitude seen in the reflected wave outside the breakers. Clearly, a first look at this relationship will be most easily obtained in the laboratory.

Laboratory Experiments

To investigate the relevance of the various theoretical suggestions, detailed laboratory measurements were made to: (i) check the limits of applicability of the matched linear solutions (eq. 5) for fully and partially reflected waves; (ii) determine the criteria for the onset of breaking in terms of the parameters ε and ε_i; and, (iii) to compare the behavior at the shoreline with the reflection coefficients determined from the observations offshore.

Experiments were conducted in the 15.2 m x 18.2 m wave basin at the Hydraulics Laboratory at SIO. The beach was a concrete sloping section ($\beta = 7^o$) extending from the shoreline end of the basin for 8.7 m, the depth being constant (65 cm) for the 5.1 m between the toe of the beach and the plunger type wavemaker. The width of the working area was adjusted so that no subharmonic edge waves could be excited by the incident waves. Incident periods of 2.39, 2.76 and 3.39 sec. were studied in detail.

Measurements of sea surface elevation were made with a sensitive resistance gauge from the beach toe to as shallow water ($\sim$ 1.0 cm) as the gauge design allowed. The records were either filtered to remove higher harmonics before the wave amplitude was measured or, equivalently, Fourier analyzed to obtain spectral coefficients. The offshore measurements, therefore, relate linear theory to that part of the wave field at the primary frequency. Ideally, it is possible to measure only the total wave height, and to allow for the harmonics with higher order Stokes theory (Goda and Abe, 1968; Moraes, 1970). However, in real laboratory experiments, a wavemaker producing the primary wave at σ, will in general, generate <u>free</u> waves at harmonics 2σ, 3σ, ..., even if the wavemaker motion is perfectly simple harmonic (Madsen, 1971). As the free harmonic waves and Stokes corrections are dynamically distinct, having different wavenumbers, they are affected differently by reflection and viscosity, and the total motion a 2σ may therefore be a complicated combination of standing and progressive waves of two different wavenumbers. Fortunately, the total Fourier amplitude of 2σ was generally small compared to σ, and the harmonic free waves probably had little effect on the overall wave dynamics. They may, however, provide some errors in the run-up and run-down measurements (done with a meter stick).

To eliminate as far as possible the effects of general boundary dissipation not associated with wave breaking, estimates of reflection are based on measurements of sea surface elevation made in shallow water on a rather steep beach ($\tan\beta = 0.123$). On very gentle beach slopes viscous effects will invalidate the amplification factors predicted by inviscid theory, and measurements in relatively deep water will always indicate less than complete reflection. Moraes (1970) clearly shows lower coefficients of reflection at small β for similar values of ε_i. We did some qualitative experiments with 1.5 sec waves on a 15 m long 2.3^o beach and found (based on offshore measurements) relatively low coefficients of reflection no matter what the value of ε_i. This is to be expected since gentle beach slopes result in a large zone of shallow water waves with high shearing (short wavelengths, Figure 1) and dissipation.

Figure 2a shows some typical measured amplitudes of the primary frequency wave normalized by the amplitude at the shoreline, a_o, where

$$2a_o = R \tan\beta \qquad (14)$$

and R is the horizontal displacement on the beach face. These very closely fit the theoretical profile for a completely reflected wave, as expected since $\varepsilon \lesssim 1$. The values of a and hence ε_i, were computed by fitting the offshore points to eq. 5 (here $r = 1$, $\theta = 0$). The close correspondence between ε and ε_i shows that for completely reflected waves, the amplification at the shoreline relative to deep water, is correctly given by

$$a_o = 2a_s = 2a_\infty\left(\frac{\pi}{2\tan\beta}\right)^{\frac{1}{2}} = 7.14\, a_\infty \qquad (15)$$

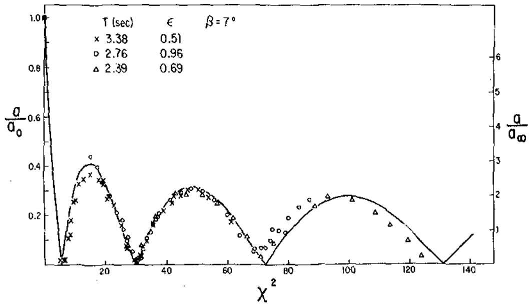

Figure 2a. Measured standing wave displacements (a) compared to linear theory (eqs. 1-5, $r = 1$, $\theta = 0$). Complete reflection occurs because $\varepsilon < 1$. $\varepsilon_i = 0.49$, 0.99, 0.66 for T = 3.38, 2.76, 2.39 sec.

This remains true and the waves are essentially completely reflected provided ε, $\varepsilon_i < 1$ as predicted by Carrier and Greenspan (1958). When $\varepsilon_i > 1$ the incoming waves are partially reflected. Figure 2b shows data from several experiments where $r \approx 0.4$, the agreement between observations and linear theory is emphasized by the superposition which occurs when experiments of different wave heights and periods are properly scaled. There is some variation of ε_i with T for the same value of r. Figure 3 shows a series of experiments for wave period 2.39 sec. where the deep water wave amplitude is the only variable which is changing and provides the corresponding increase in ε_i. Generally, the results are well predicted by linear theory for $\varepsilon_i < 6$. At larger values the amplitude variation with distance from the shore departs markedly from the theory both in the position and in the relative size of the antinodes outside the surf zone. It is clear that finite amplitude effects must eventually become important both directly in the solutions to the wave equations and indirectly through second order effects such as wave set-up. However, there is a suggestion in Figure 3 that the phase shift of the nodes and antinodes increases roughly as the width of the surf zone. Interactions between incident and reflected bores may also contribute to the discrepancy.

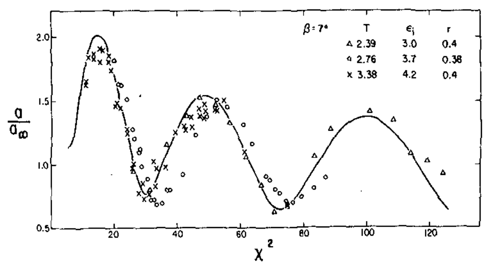

Figure 2b. Measured sea surface displacements (a) of partially reflected waves compared to theory (eqs. 1-5, $r = 0.4$, $\theta = 0$). The agreement is good right up to the break point, $X_b^2 \simeq 10$.

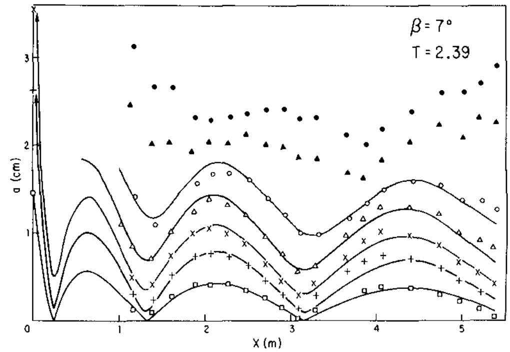

Figure 3. Measured wave displacements (a) for T = 2.39 sec, and various a_∞, ε_i.

	▭	+	X	Δ	0	▲	●
ε_i	0.66	1.3	2.1	3.0	4.24	6.9	8.2
r	1.0	0.8	0.55	0.4	0.26	-	-
θ	0	0	0	0	0.25	-	-

The reflected wave amplitudes are shown in Figure 4 in terms of ε_r, see eqs. 11-12, with $r = \varepsilon_r/\varepsilon_i$. The Miche reflection prediction (11) is frequently reduced by an "intrinsic" reflection factor which varies according to roughness of the reflecting surface, ∿ 0.8 for smooth surfaces, the maximum ε_r is then 1.6. It is emphasized that although Carrier and Greenspan (1958) provides sound theoretical insight into the maximum possible amplitude completely reflected wave, there is no basic theory which predicts the variation in the reflection coefficient with increasing ε_i (eqs. 11-12 are essentially empirical). The experimental data (Figure 4) suggests that the offshore standing wave amplitude continues to increase with increasing ε_i past the range of total reflection, $\varepsilon_i \approx 1$, reaching a slight maximum when $\varepsilon_i \approx 2.5$. It is clear from Figure 4 that Miche's concept of a standing wave of constant amplitude provides a reasonable approximation to the data in this range of ε_i. However, although the wave begins to break at $\varepsilon_i \approx 1$, this does not impose an immediate upper limit on the amplitude of the reflected wave and the standing wave of constant amplitude with $2<\varepsilon_i < 6$ is not the amplitude of the maximum size standing wave which can occur without breaking, $\varepsilon_r \sim 1$.

The horizontal displacement at the shoreline R gives a measure of a_0 (eq. 14) and ε (eq. 6) as a function of ε_i (Figure 5). For $\varepsilon_i < 1$, $\varepsilon = \varepsilon_i = \varepsilon_r$ (Figure 2a) and the motion at the shoreline is completely explained by the excursion of the standing wave at the shoreline; for $1 < \varepsilon_i < 2.5$ the swash motion continues to be appropriate for the offshore standing wave associated with the partially reflected wave. However, for $\varepsilon_i > 2.5$ the motion at the shoreline (ε) continues to slowly increase although the reflected wave (ε_r) estimated from wave conditions outside the surf zone tends to decrease slightly in amplitude (Figure 4). For very large values of ε_i, ε appears to reach a maximum value of about 4 (Fig. 5). The odd ε_i axis on Fig. 5 (linear when $\varepsilon_i \leq 10$) emphasizes the detailed studies for $\varepsilon_i \leq 10$ and is intended to show qualitative behavior for larger ε_i. Although the measurement of run-up and particularly run-down is somewhat subjective, these results are substantially different from those reported by Battjes (1974) where $\varepsilon \simeq 1.26$ is suggested as a reasonable fit to data obtained on steep beaches. However, at the same time Battjes has proposed that reflection takes place as a standing wave where

$$\varepsilon_r = 0.787 \tan^{-\frac{1}{2}}\beta$$

For very steep beaches this may be small but even when $\tan\beta = 0.123$ (1:8) this requires $\varepsilon_r \simeq 2.3$ (Figure 4). For gentler slopes, Battjes requires ε_r to become large, reaching 4.0 for the 2.3° slope used for some of the data in Figure 5.

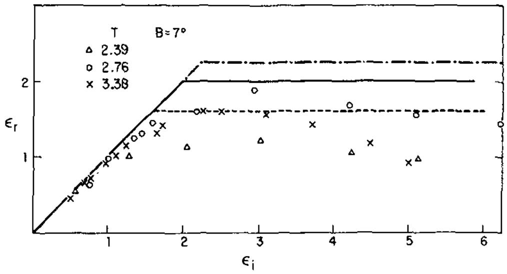

Figure 4. Nondimensional reflected wave amplitude (ε_r) as a function of incident wave (ε_i), based on data seaward of breakpoint as in Figures 2a, b.

- —— - —— Battjes (eq. 12)

- - - Miche (eq. 11)

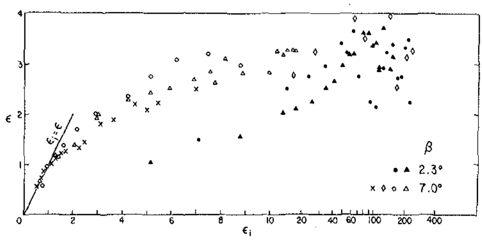

Figure 5. Swash parameter $\varepsilon = a_0\omega^2/g\beta^2$ versus offshore incident wave parameter ε_i (eq. 8). Note the ε_i axis is linear for $\varepsilon_i \le 10$, and log for $\varepsilon_i > 10$.

	▲	●	X	O	Δ	◊
T (sec)	3.06	2.48	3.38	2.76	2.39	1.00
tanβ	0.04	0.04	0.123	0.123	0.123	0.123

In the present experiments ε and ε_r are very much the same size for $\varepsilon_i < 3$, the run-up and down being primarily determined by a standing wave at the incoming wave frequency. For $\varepsilon_i > 3$, ε tends to increase while ε_r remains steady, or slightly decreases. The ratio of the wave amplitude at the shoreline to the amplitude offshore will be effected by set-up, but other non-linear effects are probably more significant and ε may reflect the contributions of the various free and forced harmonic frequencies 2σ, 3σ, etc. to the swash on the beach.

III. EDGE WAVE EXCITATION

Normally incident waves strongly reflected at the shoreline are known to be unstable to perturbation by edge waves; edge waves can grow by extracting energy from the primary incident wave via a nonlinear interaction (Guza and Davis, 1974). The edge wave with the most rapid theoretical growth rate, and the experimentally observed wave, is a zero mode subharmonic having velocity potential

$$\phi^e = \frac{a_e g}{\sigma_e} e^{-k_e x} \cos k_e y \sin(\sigma_e t + \theta) \tag{16}$$

where $$\sigma_e = \frac{\sigma}{2}, \text{ and } \sigma_e^2 = g\, k_e \tan\beta$$

Various experimenters (Galvin, 1965, 1967; Birchfield and Galvin, 1975; Guza and Inman, 1975) have qualitatively determined the final steady state edge wave amplitude as a function of incident wave parameters and beach slope. In all these experiments, the edge wave amplitude was determined through measurements of the swash. At an edge wave antinode, the run-up of the incident wave is alternately in and out of phase with the edge wave, so the difference between successive uprushes gives an approximation of the edge wave amplitude at the shoreline, a_e.

$$a_e = \frac{(R_2 - R_1)\tan\beta}{2}$$

where R_1, R_2, are the maximum (horizontal) shoreward intrusions of successive incident wave uprushes. Thus, the existing edge wave amplitude estimates are based on measurements at the shoreline where nonlinear distortion and viscous effects may be significant. In the present experiments the edge wave amplitude is measured as a function of offshore distance and the exponential ($e^{-k_e x}$) decay verified. The offshore amplitude measurements qualitatively agree with the swash observations, especially at low amplitudes. However, when the incident waves break by plunging the swash measurements tend to give too low a value for the edge wave amplitude.

The incident wave amplitudes were also not well determined in the previous experiment; Guza and Inman (1975) measured the fluctuating part of the run-up, R, (with no edge waves present) and determined a_e as a function of ε. However, ε is not linearly related to ε_i, the incident wave parameter in deep water (Figure 5), so there is some ambiquity as to the amplitude of their incident waves. Galvin (1967) measured only "wave height at the toe of the beach". If reflection is significant, it matters whether the measurement is near a standing wave node or antinode. Birchfield and Galvin (1975) present incident wave data as the "amplitude of the primary wave at the shore" but obtain values of $\varepsilon \approx 9$, which is much larger than observed elsewhere; perhaps they refer to the incident wave amplitude at the break (surge) line. Here, we have taken the incident waves studied in detail in II, altered the basin width to allow edge wave excitation, regenerated the same incident waves, and measured the wave field with edge waves present.

The incident waves, periods 2.39, 2.76, and 3.39 sec. studied in II now generate subharmonic edge waves, periods 4.78, 5.52, and 6.78 sec. whose wave-numbers k_e satisfy the boundary conditions of no flow through the side walls separated by distance b,

$$k_e = \frac{m\pi}{b} \quad , \text{ m integer} \tag{17}$$

for $m = 2$, 3, and 4 if $b = 8.8$ m. Measurements were made at various distances offshore at a longshore position corresponding to an elevation antinode of the edge wave. The wavemaker conditions were identically of those of II, so the form of the incident wave prior to edge wave growth, is known; theoretically, the incident wave may be substantially modified by the interaction with the edge wave.

The measurements of edge wave amplitude show the $e^{-k_e x}$ decay expected from (16), and hence provide a measurement of a_e, the edge wave amplitude at the shoreline. It is useful to also express a_e in a dimensionless form ε_e where

$$\varepsilon_e = \frac{a_e \sigma_e^2}{g \tan^2 \beta} \tag{18}$$

Figure 6a shows the observed variation of ε_e as a function of the incident wave conditions defined by ε_i. The edge wave resonance disappears when $\varepsilon_i > 8$ for wave period 2.39 secs. The paddle was not powerful enough to make incident waves of the longer periods large enough to suppress the resonance. The $T_i = 1.0$ sec point from Galvin (1967) is based on the amplitude at the beach tow, acceptable in this case of large ε_i, and minimal reflection.

Galvin's (1968) classification of breaker type is (Battjes, 1974, gives similar criterion)

$$\begin{array}{lll} \dfrac{H_o}{L_o\beta^2} < 0.09 & & \text{surging-collapsing} \\[2ex] 0.09 < \dfrac{H_o}{L_o\beta^2} < 4.8 & & \text{plunging} \\[2ex] 4.8 < \dfrac{H_o}{L_o\beta^2} & & \text{spilling} \end{array} \tag{19}$$

and when $\varepsilon_i \approx 10$; $H_o/L_o\beta^2 \approx 0.44$ and the resonance ceases near the low steepness end of the plunging wave regime.

Figure 6b shows ε_e versus ε (the incident wave run-up parameter) and indicates that resonance ceased ($T_i = 2.39$) when $\varepsilon \approx 3.2$. Guza and Inman (1975) present similar results for a range of periods and beach slopes, their edge wave amplitudes are qualitative, being based on a swash measurement rather than detailed measurements offshore. Some maximum values of ε for which they observed resonance are shown (⊢a⊣, ⊢b⊣) indicating that ε_{max} decreases with decreasing slope and period.

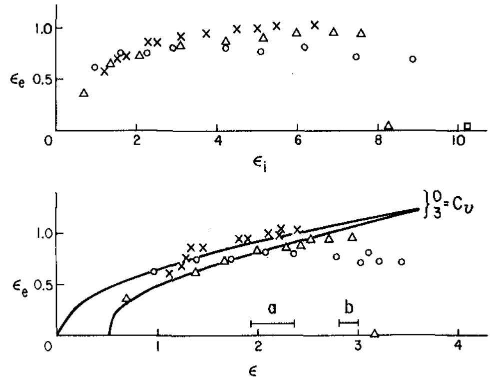

Figure 6a, b. Nondimensional edge wave amplitude ε_e (eq. 18) versus nondimensional incident wave offshore (ε_i) and swash (ε) parameters. Solid line is theory (eq. 21), independent of T_i for $C_\nu = 0$; $C_\nu = 3$ corresponds to T = 2.7 sec.

	x	0	Δ	▭	a	b
T(sec)	3.39	2.77	2.39	1.0	1.9-2.4	2.7-3.11
tanβ	0.123	0.123	0.123	0.149	0.1	0.1

The theory of edge wave excitation by completely reflected incident waves is based on a weak nonlinear interaction formalism using the shallow water approximation for the incident wave (Guza and Davis, 1974; Guza and Bowen, 1976). The forcing of an initially small, zero mode, subharmonic edge wave by the incident wave results in an initial inviscid edge wave amplitude growth

$$a_e = a_e(t = 0)\, e^{\alpha\varepsilon\sigma_i t}, \quad \alpha = \int_0^\infty f(x)\, dx = 0.0169 \qquad (20)$$

where $f(x)$ is a complicated function which expresses the spatial coupling bewteen incident and edge waves. Guza and Bowen (1976) have theoretically determined the equilibrium amplitude, based on the assumption that the incident wave is totally reflected ($\varepsilon = \varepsilon_i$). For a basin width and edge wave frequency satisfying (17)

$$\varepsilon_e^2/\varepsilon = 7.87\left[- 0.203\, d^{\frac{1}{2}} + (2.9 \times 10^{-3} - 3.79 \times 10^{-2}\, d)^{\frac{1}{2}}\right] \qquad (21)$$

where $d = \nu C_\nu^2/\sigma_e a_o^2$

and viscous effects have been modeled with laminar boundary layers and an effective viscosity, ν', where

$$\nu' = C_\nu^2 \nu \qquad (22)$$

For clean water, smooth bottoms, etc., $C_\nu = 1.0$ and bottom boundary layers account for most of the damping. If ν' is some constant, independent of ε_e, viscous effects determine the minimum incident wave amplitude which will excite edge waves but do not limit edge wave growth if it occurs. The edge wave growth is finally curtailed by finite amplitude effects, radiation and finite amplitude detuning (Guza and Bowen, 1976). The condition that nonlinear forcing can overcome viscous damping and initiate resonant growth is

$$\varepsilon > \frac{\sigma_i^2}{g\beta^2}\left(\frac{C_\nu^2 \nu}{\sigma_i}\right)^{\frac{1}{2}} 7.36 \qquad (23)$$

Figure 6b shows a comparison between the laboratory measurements and theory (eq. 21) for $C_\nu = 0$ (inviscid) and $C_\nu = 3$; using the observed shoreline values for a_0 to calculate ε. The agreement is good for $\varepsilon \lesssim 2.4$ (corresponding to $\varepsilon_i \lesssim 5$, $H_0/L_0\beta^2 \lesssim 0.22$). Above $\varepsilon = 2.4$ the edge waves are generally smaller than predicted, the resonance disappearing altogether at large values.

The existing theory is for totally reflected incident waves which do not break and it is not surprising that the observed edge wave amplitudes diverge from theory when the assumption of total reflection is violated. The presence of the progressive incident wave component, and the turbulence which dissipates its energy, have been neglected. We now consider these factors.

The forcing of the edge wave by the incident waves, whether standing or progressive, is expressed as an integral over the entire fluid (eq. 20). It can readily be shown, however, that most of the resonant forcing occurs quite close to shore where the edge wave is large. When only a standing wave is present, $\alpha(20)$ reaches 50% of its total value when $\chi^2 = 8.0$, and 70% when $\chi^2 = 12.8$. Now when $\varepsilon_i \gtrsim 3.5$, $r \gtrsim 0.4$, $\varepsilon \gtrsim 1.8$, the surge line is about $\chi^2 = 10$ (Figure 2b). Therefore, when ε_i is so large that significant dissipation occurs, most of the edge wave forcing by the standing wave occurs inside the breakpoint, and is concentrated in the swash. Hence, ε determined by the value of a_0 measured at the shoreline will be assumed to give the edge wave forcing (eq. 20) by the standing component of a partially reflected incident wave. If the progressive incident wave component (whose amplitude after breaking is proportional to the depth) is superimposed on the standing wave the integral in (20) might be recalculated. This simple, surf zone model is clearly related to Miche's concept of reflection, but is probably a gross oversimplification of the actual conditions. However, because the resonant forcing tends to occur very close to the shoreline where the progressive wave vanishes the integral in (20) might be almost unaffected and it seems that the edge wave forcing is not greatly altered by the progressive wave component. The hypothesis that the edge wave forcing is determined primarily by the swash parameter (ε) implies that the forcing increases almost linearly with ε_i when $\varepsilon_i \lesssim 2$, increases more slowly when $\varepsilon_i > 2$, and eventually reaches a maximum corresponding to $\varepsilon \gtrsim 4$ (Figure 5).

The edge wave damping, however, might be expected to increase dramatically with increasing ε_i, when $\varepsilon_i > 2$. Significant incident wave breaking is beginning to occur, and the associated turbulence results in an "eddy viscosity" much larger than molecular viscosity. If the edge wave dissipation increases much more rapidly than forcing, then it would be expected that there is some maximum ε_i for which resonance can occur; a simplistic model shows that this is indeed the case.

There is no generally accepted form for the eddy viscosity (A) in the surf zone, but the model of Battjes (1976) is certainly plausible;

$$A = Mh(D/\rho)^{1/3}$$

where D is the rate of energy dissipation of incident waves and M is a constant. For completely progressive waves

$$D_p = 0.31\ \rho g^{3/2}\ h^{3/2}\ \beta\gamma^2\ ;\quad x < x_B$$

where the constant ratio between progressive wave height and water depth, $\gamma = H/h \approx 1.0$ on steep beaches. For partially reflected waves, the dissipation rate depends on the progressive component, and we take $D = D_p(1 - r)^2$ where $r = 2/\varepsilon_i$ (Miche's form, Figure 4). The average (across the surf zone) eddy viscosity is then

$$\bar{A} = 0.4M\left[0.31\ \gamma^2(1 - r)^2\right]^{1/3} \beta^{4/3}\ x_B(gh_b)^{1/2} \qquad (24)$$

Assuming that the eddy viscosity is determined by the breaking incident waves (eq. 24), and that the edge wave damping can be modeled (allbeit grossly) by replacing molecular with eddy viscosity, the average edge wave dissipation per unit longshore length is then

$$D_e = \frac{\rho a_e^2\ \sigma_e^{5/2}}{2\sqrt{2}\ \beta^2}\ \bar{A}^{-1/2} \int_0^{x_B} e^{-2kx}\, dx \qquad (25)$$

The surf zone width, and hence average eddy viscosity (eq. 24) and edge wave dissipation (eq. 25), depend on ε_i. We take as an approximate fit to our data

$$x_B^2 = 0 \qquad \varepsilon_i < 2$$

$$x_B^2 = 4(\varepsilon_i - 2) \quad \varepsilon_i > 2$$

which results in

$$D_e = \rho a_e^2\ g^2(8\sigma_i)^{-1}(0.4M)^{1/2}\ (0.31\gamma^2)^{1/6}\ \beta^{2/3}\left(1 - \frac{2.}{\varepsilon_i}\right)^{1/3}(\varepsilon_i - 2.)^{3/4}\left(1 - e^{-(\varepsilon_i - 2)/2.}\right) \qquad (26)$$

The condition for any subharmonic edge wave excitation is that the rate of energy input from the incident waves (eq. 20) exceed the rate of dissipation. Thus, the maximum ε_i which can generate edge wave occurs when (assuming $\gamma = 1.$)

$$\varepsilon = 3.8\beta^{-1/3}(\varepsilon_i - 2)^{3/4}\left(1 - e^{-(\varepsilon_i - 2)/2}\right)\left(1 - \frac{2}{\varepsilon_i}\right)^{1/3} M^{1/2} \qquad (27)$$

Now the present observations (Figure 6) with $\beta = 0.123$ suggest the resonance ceases when $\varepsilon \approx 3$, and $\varepsilon_i \approx 10$, which implies that $M^{\frac{1}{2}} \sim 0.1$. Battjes (1976) has computed values of M from 20 longshore current experiments with fully developed breakers on steep beaches and finds $0.55 < M^{\frac{1}{2}} < 1.34$. Since the edge wave dissipation rate (26) is proportional to $M^{\frac{1}{2}}$, $M^{\frac{1}{2}} \approx 0.1$ implies a dissipation rate an order of magnitude less than with fully developed surf zones, but an order of magnitude greater than laminar damping. It is possible to use the eddy viscosity model to predict ε as a function of ε_i (analagous to eq. 2, Fig. 6b) but this seems pointless in view of the unutterable crudeness of the model. The crux of the matter is that if edge wave forcing is approximately constant (or at least the same order) for breaking and non-breaking waves, then the orders of magnitude increase in viscous effects due to wave breaking effectively suppresses the resonance.

To look more closely at the idea that increased damping, rather than the changed form of the incident wave, is responsible for the absence of resonance, experiments were made in which an additional wave was superimposed on a resonant situation. Waves of period 2.76 sec of constant amplitude ($\varepsilon_i = 2.16$, $\varepsilon = 1.72$), strongly reflected from the beach ($r = 0.77$) generated a subharmonic resonance $\varepsilon_e = 0.75$ in the absence of any further waves, in good agreement with theory (Figure 6). Waves of 1.0 sec period were then superimposed, the wave amplitude being measured in deeper water and a value of the breaker index, $H_0/L_0\beta^2$ obtained, the beach slope being 0.123. Table 1 shows the disappearance of the basic resonance, in terms of the edge wave amplitude, as the size of the 1.0 sec wave is increased. Other experiments show this to be a general result, the particular mix of incident wave frequencies and amplitude in Table 1 but one of a multitude of possibilities.

Table 1. Subharmonic edge wave amplitudes in the presence of high frequency waves of 1 sec period

$H_0/L_0\beta^2$ (T = 1 sec)	0	0.74	1.6	2.2	3.0	3.8
ε_e (T_e = 5.56 sec)	0.75	0.76	0.73	0.62	0.18	0

The question of how much incident wave breaking at other frequencies is needed to suppress a given surging wave resonance is unanswered, and presents great theoretical difficulties. However, it is clearly an important component in the problem of edge wave excitation by an incident wave spectrum. It seems likely that the most important factor will be the ratio between the width of the surf zone (regardless of which breaking waves introduce the turbulence) and the offshore length scale of the edge wave. The instabilities of relatively very long surging waves will be unaffected by very short breakers, where the surf zone, and hence band of increased viscosity associated with the chop is of small extent relative to the incident surging wave and excited edge wave wavelengths. Field situations with offshore breaker bars, and a shoreward zone of reformed potential waves present an even complex problem. Very short edge waves will not "feel" the offshore turbulence!

IV CONCLUSIONS

The model of a surf zone as a simple combination of a standing wave of fixed amplitude plus a progressive wave (decaying shorewards from the breakpoint) seems to provide an accurate representation of the sea surface elevations and run-up for large reflectivities ($r > 0.3$).

The nondimensional amplitude of the standing wave is not, however, determined completely by the condition of the onset of breaking, correctly predicted by $\varepsilon = 1$, but continues to increase to a value of about 1.6 (Figure 4). This agrees quite well with Miche's (1951) empirical suggestion. As the incoming wave height increases, finite amplitude effects seem to complicate the general picture and the relationship between the swash motion ε and ε_r the reflected wave becomes less clear.

The generation of subharmonic edge waves is known to be dependent on the breaker characteristics, the resonance disappearing when the incoming wave breaks cleanly (Galvin, 1965). Simple calculations suggest that the change is due to an increase in damping and not an alteration in the forcing itself, which seems primarily associated with the standing component of the incoming wave field. Experiments in which a surf zone is generated by a wave unconnected with the resonance, while the forcing remains constant, further supports the idea that the resonance is suppressed by the increase in the effective viscosity of the nearshore region.

This has substantial implications for the existence of any edge waves in surf. To exist at all they must be strongly forced and are most likely to survive if their offshore length scales are large in comparison to the width of the surf zone.

Acknowledgements

The laboratory study would not have been possible without the cooperation of Douglas L. Inman, Reinhard E. Flick and John Powell. Financial support was provided by a National Science Foundation Grant (Robert T. Guza), a Killam Foundation Research Professorship (Anthony J. Bowen), and by the Office of Naval Research under contract N00014-75-C-0300.

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