Resource Efficiency in Low-Power Wide-Area Networks for IoT Applications

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***Abstract*—In this paper, we investigate the resource efficiency of uplink transmission for low-power wide-area (LPWA) networks. LoRa is adopted as an example network of focus, however the work can be easily generalized to other radios. We first formulate resource allocation in LPWA networks as a joint optimization problem of channel assignment and power allocation, with guaranteeing throughput fairness among LoRa users with limited spectrum resources, especially for the case with a large number of connected devices in LPWA networks. Specifically, we formulate channel assignment as a many-to-one matching game by treating LoRa users and channels as two sets of selfish players aiming to maximize their own utilities. We then propose a low-complexity matching channel assignment algorithm (MCAA) through distributing the channel access decision making local to LoRa users. For LoRa users assigned to the same channel, we further develop an optimal power allocation algorithm to maximize the achieved minimal transmission rate in LPWA networks. Moreover, simulation results demonstrate that the proposed MCAA can achieve near-optimal performance with much lower computational complexity.**

I. Introduction

It is predicated that the Internet of Things (IoT) brings together a massive number of devices connected to implement applications such as smart cities. Therefore, efficiency in allocating limited resources to a huge number of devices becomes a critical challenge. In many IoT applications, e.g. smart infrastructure monitoring, the amount of data generated by each device can be relatively small even though the number of connected devices are large. This feature of IoT enables the potential of massive connectivity and low-power wide-area (LPWA) networks have been proposed as a promising solution for such types of IoT applications [1–3]. Compared with traditional wireless techniques, LPWA techniques aim to offer a trade-off between power consumption, coverage, and data rates to address the more diverse needs of IoT applications. To achieve long range transmission with low energy consumption, LPWA technologies can normally operate with low data rates, which makes them more suitable for delay-tolerant applications with small amounts of data.

So far, many standard organizations and industrial alliances are working towards open standards for LPWA networks to achieve long range transmission, i.e., few to tens of kilometers, with a battery life of ten years and beyond. Examples are the Third Generation Partnership Project (3GPP), European Telecommunications Standards Institute (ETSI), and LoRa Alliance. These promising technologies mainly operate in the sub-1GHz band due to its good propagation properties.

Among the existing LPWA technologies including LoRa [4], Sigfox [5], and NB-IoT [6], LoRa is regarded as one of the most promising LPWA approaches, and attracting extensive attentions from both industry and academics [7–12]. The core of LoRa's success comes from its chirp spread spectrum technology, which allows flexible long-range communications by adopting different spreading factors (SF) with low power consumption. To maximize the battery life of LoRa devices and the network capacity, LoRaWAN [4] has been proposed to support the physical layer operation, in which the pure ALOHA is adopted to reduce power consumption on listening and sensing.

LoRa and most of the promising LPWA technologies work in the unlicensed spectrum that may be densely accessed for IoT applications. In ultra dense deployment scenarios, LoRa networks will inevitably become interference-limited, rather than noise-limited. As a result, coexistence issues become important for LPWA networks. The performance analysis of LPWA networks has been carried out in [11], in which the interference from both LoRa and other LPWA radios sharing the same frequency have been considered to show its impact on the scalability of LoRa networks. The interference caused by LoRa devices adopting the same SF has been investigated in [12].

Typically, SFs are configured by the server in LoRaWAN and adaptive data rate is enabled. Specifically, based on duty cycle restriction, i.e., no more than 1%, the end-device selects a channel to communicate with server by a pre-configured SF. To optimize data rates, airtime and energy consumption in the network, the server will reduce SF and increase/reduce transmit power based on the required signal-to-noise ratio (SNR). The required SNR for different SFs are given in Table I. The updated SF and transmit power will be contained in the next downlink message and be sent to the end-device. End-device will try to increase the SF if the acknowledgement is missed. This centralized scheme fails to manage the channel conflicts and the transmit power is updated by 3 dBm in each step, which makes the resource efficiency relative low, especially for urban-scale IoT applications with massive number of devices to connect. Therefore, a more advanced resource allocation scheme is desired to avoid channel conflicts and improve resource efficiency as the number of devices scales.

In this paper, we consider uplink transmission as it is preferred by LPWA networks. To the best of our knowledge, this is the first paper investigating the resource efficiency

TABLE I
REQUIRED SNR FOR DIFFERENT SPREADING FACTORS.

Spreading factor	7	8	9	10	11	12
Required SNR (dB)	-7.5	-10	-12.5	-15	-17.5	-20

problem for LPWA networks, with particular focus on LoRa. The contributions of this paper are summarized as follows:

1) We formulate the resource efficiency in LPWA networks as a joint optimization problem including channel assignment and power allocation to guarantee transmission rate fairness among LoRa users. To tackle this non-convex mixed-integer problem, we decouple it into two phases: i) assigning LoRa users into different channels; ii) power allocation for LoRa users sharing the same channel.
2) For channel assignment, we propose a low-complexity matching channel assignment algorithm, named MCAA, to enable LoRa users self-matching themselves with the proper channels. Each LoRa user cares not only the channel it is assigned, but also other users assigned into the same channel, as they introduce interference. We exploit matching theory to guarantee transmission rate fairness by considering LoRa users and channels as two sets of players to be matched.
3) Within each channel, we propose a centralized power allocation algorithm to be implemented at LoRa gateways to obtain optimal transmit power for LoRa users sharing the same channel.
4) We analyze the proposed resource efficiency framework in terms of stability, convergence, complexity, and optimality. Numerical results show that our proposed resource efficiency framework can achieve near-optimal performance but with much lower computational complexity.

II. SYSTEM MODEL AND PROBLEM FORMULATION

In this paper, we focus on the uplink communications of the LoRa radio in LPWA networks. We consider a network with M channels for access. Each LoRa device wakes up for data transmission with satisfying the duty cycle restriction. The number of active LoRa users is N, which are located uniformly across the network. The set of channels and LoRa users are denoted as $\mathcal{CH} = \{CH_1, \ldots, CH_m, \ldots, CH_M\}$ and $\mathcal{U} = \{U_1, \ldots, U_n, \ldots, U_N\}$, respectively. The bandwidth of the m-th channel, CH_m, is denoted as B_m Hz. We assume that both the LoRa gateway and LoRa user are equipped with a single antenna, as defined in LoRa specification [4].

The channel between the LoRa gateway and an arbitrary LoRa user, U_n, is modelled as Rayleigh fading. Therefore, the channel gain $g_{m,n}$ can be expressed as

$$g_{m,n} = h_{m,n}\eta_m \|d_{m,n}\|^{-a}, \tag{1}$$

where $h_{m,n} \sim \exp(1)$ refers to the small-scale fading of CH_m when it is measured at U_n, η_m is a constant related to the path loss of radio link in CH_m, $d_{m,n}$ is the distance between the LoRa gateway and LoRa user, U_n, and a is the path loss exponent that generally depends on the carrier frequencies and environments, such as urban or suburban. Consequently, the received signal power at the LoRa gateway over CH_m can be expressed as

$$y_m = \sum_{n=1}^{N} \alpha_{m,n} p_{m,n} g_{m,n} + \sigma_m^2, \tag{2}$$

where σ_m^2 is the power of additive white Gaussian noise (AWGN), $p_{m,n}$ denotes the uplink transmission power of U_n when transiting over CH_m , and $\alpha_{m,n}$ is used to indicate whether CH_m is allocated to U_n, which can be given by

$$\alpha_{m,n} = \begin{cases} 1, & U_n \text{ occupies } CH_m, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

Denoting L_m as the number of assigned users within CH_m, i.e., $L_m = \sum\limits_{n=1}^{N} \alpha_{m,n}$, the uplink signal-to-interference-plus-noise ratio (SINR) for U_n transmitting over CH_m can then be expressed as

$$\gamma_{m,n} = \frac{p_{m,n} g_{m,n}}{\sum\limits_{l=1,\, l\neq n}^{L_m} p_{m,l} g_{m,l} + \sigma_m^2}, \; \forall\, m,\ n. \tag{4}$$

Then the data rate achieved for U_n over CH_m can be given by

$$R_{m,n} = B_m \log_2\left(1 + \gamma_{m,n}\right), \; \forall\, m,\ n. \tag{5}$$

The utility of U_n, which refers to the minimal transmission rate of U_n among the channels it occupies $\mathcal{J}_n$, can be expressed as

$$R_{U_n} = \min\left(B_m \log_2\left(1 + \gamma_{m,j}\right)\right), \; \forall\, j \in \mathcal{J}_n. \tag{6}$$

Moreover, the utility of CH_m, which refers to the minimal transmission rate of the set of LoRa users, $\mathcal{L}_m$, sharing CH_m, can be described as

$$R_{CH_m} = \min\left(B_m \log_2\left(1 + \gamma_{m,l}\right)\right), \; \forall\, l \in \mathcal{L}_m. \tag{7}$$

Our objective is to maximize the utilities of LoRa users and channels with SINR constraints to guarantee transmission rate fairness among LoRa users. Then the problem can be formulated as

$$
\begin{aligned}
(\mathbf{P1}) \quad & \max_{\alpha_{m,n}} \min_{p_{m,n}} \alpha_{m,n} R_{m,n}, && \text{(8a)}\\
\text{subject to} \quad & C_1:\ 0 \le p_{m,n} \le p^{\max},\ \forall\, m,\ n, && \text{(8b)}\\
& C_2:\ \alpha_{m,n} \in \{0,1\},\ \forall\, m,\ n, && \text{(8c)}\\
& C_3:\ \sum_{m} \alpha_{m,n} \le D,\ \forall\, n, && \text{(8d)}\\
& C_4:\ \sum_{n} \alpha_{m,n} \le 1,\ \forall\, m. && \text{(8e)}
\end{aligned}
$$

where D is the maximum number of users that can be allocated to the same channel to restrict interference. C_1 denotes the transmit power range of U_n over CH_m. In C_2, it is shown that $\alpha_{m,n}$ is either 0 or 1. The maximum number of LoRa users that can be allocated into the same channel is constrained by C_3, and C_4 restricts that at most one channel can be allocated to a LoRa user.

The problem **(P1)** is non-convex due to the binary constraints as well as the interference term in the objective function. Thus no systematic and computational efficient approach is available to solve this problem optimally. Further note that the channel assignment and power allocation variables, i.e., $\alpha_{m,n}$ and $p_{m,n}$, are coupled. Therefore, we propose to decouple the formulated problem into two phases as shown in Fig. 1. In the first phase, the LoRa users are self-matched with different channels. The details are to be explained in Section III. In the second phase, the optimal power allocation algorithm is proposed for LoRa users assigned to the same channel to guarantee user fairness. The details will be introduced in Section IV.

III. Matching Theory based Channel Assignment Algorithm

By assuming the same transmit power at each LoRa user, the channel assignment problem can be formulated as

$$(\mathbf{P2}) \quad \max_{\alpha_{m,n}} \min R_{m,n}, \text{ subject to } C_2, C_3, \text{ and } C_4. \tag{9}$$

Recall that the above channel assignment problem (**P2**) is NP-hard. In order to solve it, we propose a matching theory based channel assignment algorithm with low complexity, named MCAA. The proposed MCAA extends the battery life of LoRa devices by reducing the probability of retransmission as channel conflicts between different LoRa users are reduced. Specially, we consider the set of LoRa users, $\mathcal{U}$, and the set of channels, $\mathcal{CH}$, as two disjoint sets of selfish and rational players aiming to maximize their own utilities, i.e., achieved minimal transmission rates. In this paper, the channel state information (CSI) is assumed to be known [1], i.e., the players are aware of the CSI of others. The more details of the proposed MCAA will be introduced in Section III-B.

A. Many-to-One Matching

In this part, we introduce the fundamental concepts in the many-to-one matching model for channel assignment [13].

1) Matching pair: A matching can be regarded as an assignment of channels in $\mathcal{CH}$ to LoRa users in $\mathcal{U}$, which is formally described as follows.

Definition 1. Given two disjoint sets $\mathcal{CH}$ and $\mathcal{U}$, a many-to-one matching Ψ is a mapping from the set of $\mathcal{CH} \cup \mathcal{U}$ into the set of all subsets of $\mathcal{CH} \cup \mathcal{U}$ such that for each $CH_m \in \mathcal{CH}$ and $U_n \in \mathcal{U}$:

1) $\Psi(CH_m) \subseteq \mathcal{U}$;
2) $\Psi(U_n) \subseteq \mathcal{CH}$;
3) $|\Psi(U_n)| \leq 1$;
4) $|\Psi(CH_m)| \leq D$;
5) $CH_m \in \Psi(U_n) \Leftrightarrow U_n \in \Psi(CH_m)$.

Condition 1) states that each channel is matched with a subset of LoRa users, and condition 2) implies that each LoRa user is matched with a subset of channels. Condition 3) restricts that the number of channels assigned to each LoRa user is no more than one, and condition 4) presents that the number of users sharing the same channel is no greater than D.

[1]The CSI becomes available at the gateway after the proposals from LoRa users are received and can be shared with all LoRa users in the next downlink message.

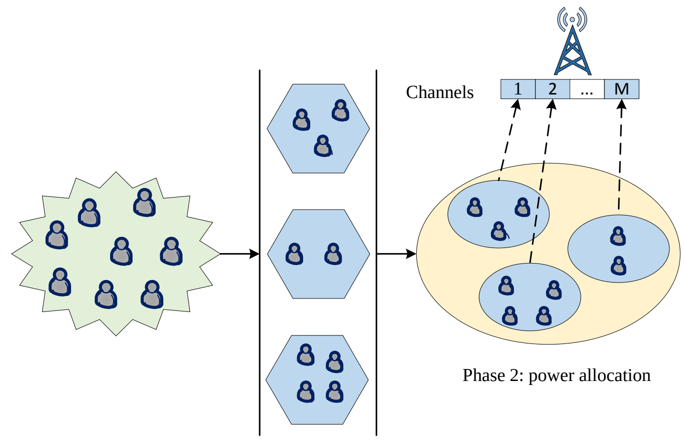

Fig. 1. System model of the proposed two-phase resource allocation framework in low-power wide-area networks.

Remark 1. The matching game formulated above is a many-to-one problem with peer effects.

Proof. Since each user can only be assigned to no more than one channel, and each channel can be assigned with more than one users, this is a many-to-one matching game. Due to the interference component in (4), the rate achieved by an arbitrary LoRa user, U_n, over its occupied channel, CH_m, is related to the set of other LoRa users sharing the same channel. Thus, each LoRa user cares not only about the channel it is matched with, but also the set of LoRa users that are assigned into the same channel. Similarly, for each channel it not only manages the individual LoRa user with which to match with, but also the subset of LoRa users that have inner-relationships through code domain multiplexing by adopting different SFs. Thus, this can be formulated as a many-to-one matching game with peer effects, in which each player tries to maximize their own utilities [14]. □

2) Preference relations: To describe the competition behavior and decision process of each player, we define a preference relation, $\succ$, for both LoRa users and channels. Specifically, for an arbitrary LoRa user, $U_n \in \mathcal{U}$, its preference $\succ_{U_n}$ over any two channels, $CH_m \in \mathcal{CH}$ and $CH_{m'} \in \mathcal{CH}$ with $m \neq m'$, can be expressed as

$$(CH_m, \Psi) \succ_{U_n} (CH_{m'}, \Psi') \Leftrightarrow R_{m,n}(\Psi) > R_{m',n}(\Psi'), \tag{10}$$

where $CH_m \in \Psi(U_n)$ and $CH_{m'} \in \Psi'(U_n)$. This relationship means that user U_n prefers CH_m in Ψ to $CH_{m'}$ in Ψ' if CH_m can achieve higher rate over CH_m than over CH'_m. Similarly, for an arbitrary channel CH_m, its preference $\succ_{CH_m}$

over any two set of LoRa users, i.e., $\mathcal{S} \in \mathcal{U}$ and $\mathcal{S}' \in \mathcal{U}$, can be expressed as

$$\left(\mathcal{S}, \Psi\right) \succ_{CH_m} \left(\mathcal{S}', \Psi'\right) \Leftrightarrow R_{m,n}\left(\Psi\right) > R_{m,n'}\left(\Psi'\right), \quad (11)$$

where $\mathcal{S} \in \Psi\left(CH_m\right)$ and $\mathcal{S}' \in \Psi'\left(CH_m\right)$.

3) Swapping matching: In the matching model, the swapping behaviour of players is considered as where every two players are arranged to exchange their matches without changing any other players' assignment. To better describe the interdependency between players' preference, we introduce the concept of *swap-matching* as follows.

Definition 2. Giving a matching Ψ with $CH_m \in \Psi\left(U_n\right)$, $CH_{m'} \in \Psi\left(U_{n'}\right)$, $CH_m \notin \Psi\left(U_{n'}\right)$, and $CH_{m'} \notin \Psi\left(U_n\right)$, a swap matching $\Psi' = \left\{\Psi \backslash \left\{\left(U_n, CH_m\right), \left(U_{n'}, CH_{m'}\right)\right\}\right\} \cup \left\{\left(U_n, CH_{m'}\right), \left(U_{n'}, CH_m\right)\right\}$ is defined by $CH_m \in \Psi'\left(U_{n'}\right)$, $CH_{m'} \in \Psi'\left(U_n\right)$, $CH_m \notin \Psi'\left(U_n\right)$, and $CH_{m'} \notin \Psi'\left(U_{n'}\right)$.

Note that swap-matching provides a matching generated via a swap operation.

Definition 3. Given a pair of users $\left(U_n, U_{n'}\right)$ that are matched under a given matching Ψ, if there exist $CH_m \in \Psi\left(U_n\right)$ and $CH_{m'} \in \Psi\left(U_{n'}\right)$ such that:

1) $\forall i \in \left\{U_n, U_{n'}, CH_m, CH_{m'}\right\}$, $R_i\left(\Psi'\right) \geq R_i\left(\Psi\right)$ and
2) $\exists i \in \left\{U_n, U_{n'}, CH_m, CH_{m'}\right\}$ such that $R_i\left(\Psi'\right) > R_i\left(\Psi\right)$,

then the swap matching Ψ' is approved, and $\left(U_n, U_{n'}\right)$ is defined as a swap-blocking pair in Ψ.

The definition implies that if a swap matching is approved, then the utility of any player, i.e., R_{U_n} and R_{CH_m} as defined in (6) and (7), will not decrease, and the utility of at least one player will increase. Note that the swap can be initialized by either LoRa users or the controllers of channels, i.e., gateway, since their utilities are all directly related to the data rates.

Definition 4. A matching Ψ is two-sided exchange-stable (***2ES***) if it is not blocked by any swap-blocking pair.

B. Proposed Matching based Channel Assignment Algorithm for LPWA Networks

In this part, we propose a channel assignment algorithm based on matching theory with low complexity, named MCAA, to enable channel selection at LoRa users locally. The key idea of MCAA is to find a ***2ES*** matching for channel assignment after performing a limited number of swap operations. With the low complexity required by MCAA, the battery life of LoRa devices can be extended.

As shown in Algorithm 1, the proposed MCAA can be divided into two steps, including an initialization step and a swap matching step. In the first step, an initialization algorithm, as described in Algorithm 2, is proposed to generate the initial matching, Ψ_0. Here, the transmit power at each LoRa user is assumed to be the same. More specifically, each LoRa user constructs its preference list based on the CSI. For instance, for U_n, the first preferred channel is the one with $m = \arg\max\limits_{\forall m} g_{m,n}$. Gateway initializes the preference list for each LoRa based on its distance to the LoRa gateway for simplification, i.e., the closest LoRa user has the highest preference. The reason is that large-scale fading is the main influence on the achieved transmission rate in LPWA networks. Then each LoRa user proposes to its first preferred channel and each channel will only accept the proposals from the first D users on its preference list. This process continues until all LoRa users are matched with a channel. If there is a channel that is not matched with any user, its first preferred user is forced to match with the vacant channel to improve the achieved minimal transmission rate potentially. Initial matching Ψ_0 is then returned for Algorithm 1.

In the second step of Algorithm 1, each user keeps searching for swap-blocking pairs, which guarantees the utilities of all LoRa users and channels are not decreased and utilities of at least one player (i.e., can be either LoRa user or channel) is increased. If there is one swap-blocking pair in the current matching, the swap operation is carried out between the paired users. This searching and swap operation process continues until the final matching state is reached. The preference lists of active LoRa users are updated according to channel access decision returned by Algorithm 1.

For the proposed MCAA, we have the following theorems.

Theorem 1. Stability: the final matching of proposed MCAA is a ***2ES*** matching.

Proof. If there exists at least one more swap-blocking pair in the final matching Ψ of the proposed MCAA, the utility of at least one player could be improved without degrading the utilities of any other player. However, if there exists a pair of players blocking the matching Ψ, the proposed MCAA will continue. This means Ψ is not the final matching, which causes conflict. Therefore, the final matching Ψ is ***2ES***. □

Theorem 2. Convergence: the proposed MCAA converges to a ***2ES*** matching after a limited number of swap operations.

Proof. In the proposed MCAA, as the number of players is limited and the LoRa users that can be allocated to one channel is restricted, the number of potential swap operation is finite. Moreover, the minimal achievable rate of each channel will increase after each swap operation. Since the achievable rate of each channel has an upper bound due to the limited spectrum resources, the swap operations stop after the transmission rate is saturated in the worst case. Therefore, within a limited number of swap operations, the proposed MCAA reaches its stable state. □

Theorem 3. Complexity: the computational complexity of the proposed MCAA is upper bounded by $\mathrm{O}\left(MN + \frac{1}{2}IDN\left(M-1\right)\right)$.

Proof. The computational complexity of MCAA comes from two parts: the initial matching step and the swap operations. For the initial matching step, the complexity is of the order of $\mathrm{O}\left(MN\right)$ in the worst case, in which all LoRa users will propose to all available channels.

In the search and swap step of MCAA, computational complexity mainly lies in the number of iterations and the attempts of swap matching in each iteration. However, as

Algorithm 1 Matching based Channel Assignment Algorithm (MCAA) for LPWA Networks

Initialization

Generate the initial matching Ψ_0 by Algorithm 2.

Search and Swap

while $\exists\left(U_n, U_{n'}\right)$ blocks current matching **do**
 for $\forall U_n \in \mathcal{U}$ **do**
  for $\forall U_{n'} \in \{\mathcal{U} \backslash U_n\}$ with $CH_m \in \Psi\left(U_n\right)$ and $CH_{m'} \in \Psi\left(U_{n'}\right)$ **do**
   if $\left(U_n, U_{n'}\right)$ is a swap-blocking pair and $C_2 - C_4$ are satisfied **then**
    U_n exchanges its match CH_m with $U_{n'}$'s match $CH_{m'}$.
    Update Ψ.
   end if
  end for
 end for
end while
return final matching Ψ.

Algorithm 2 Initial Matching Algorithm

Initialization Set of unmatched users $\ominus_{\mathcal{UM}} = \mathcal{U}$, $\alpha_{m,n} = 0$, proposal indicator $\beta_{m,n} = 0$, $\forall\, m,\ n$.

Calculate preference list of each user $\mathcal{PL}_{U_n}$, $\forall\, U_n \in \mathcal{U}$.
Calculate preference list of each channel $\mathcal{PL}_{CH_m}$, $\forall\, CH_m \in \mathcal{CH}$.
while $\ominus_{\mathcal{UM}} \neq \emptyset$ **do**
 for $\forall U_n \in \mathcal{U}$ **do**
  U_n proposes to its first preferred channel that it has not been rejected before.
  Update $\beta_{m,n} = 1$ if U_n proposes to CH_m.
 end for
 for $\forall\, CH_m \in \mathcal{CH}$ **do**
  if $\sum\limits_n \left(\alpha_{m,n} + \beta_{m,n}\right) \leq D$ **then**
   CH_m accepts all proposals from LoRa users.
  else
   CH_m accepts proposals from its D most preferred users.
  end if
  Update $\ominus_{\mathcal{UM}}$ by removing all the matched U_n.
  Remove CH_m from $\mathcal{PL}_{U_n}$ if $\beta_{m,n} = 1$.
  Update Ψ_0 with $\alpha_{m,n} = 1$ for all the matched U_n.
 end for
end while
if there are vacant channels, CH_m **then**
 Match CH_m with its first preferred user.
 Update Ψ_0.
end if
return Ψ_0.

proved in Theorem 2, the number of iterations, I, that required by the MCAA to reach the ***2ES*** matching is limited even though it cannot be expressed in a closed form. In each iteration of MCAA, for an arbitrary user U_n, there are $M-1$ possible swap-blocking pairs as there are M channels and each LoRa user can occupy at most one channel. For the selected channel CH_m, at most D users can be assigned with it. Therefore, a swap matching Ψ for U_n has at most $D\left(M-1\right)$ possible combinations. For the proposed MCAA, at most $\frac{1}{2}DN\left(M-1\right)$ swap matchings need to be considered in each iteration. Therefore, complexity of the search and swap step is upper bounded by $\mathrm{O}\left(\frac{1}{2}IDN\left(M-1\right)\right)$.

In summary, the computational complexity of the proposed MCAA is upper bounded by $\mathrm{O}\left(MN + \frac{1}{2}IDN\left(M-1\right)\right)$. $\square$

We can see that the complexity of MCAA is much lower than the brute force exhaustive-search approach, whose computational complexity increases exponentially with the number of active LoRa users, N.

IV. Optimal Power Allocation

After LoRa users are assigned into different channels, we further propose a power allocation scheme for LoRa users sharing the same channel to control the intra-interference and maximize the minimal transmission rate of LoRa users.

Denoting $\mathbf{P_m} = \{p_{m,1}, \ldots, p_{m,L_m}\}$ as the transmit power vector for LoRa users assigned into CH_m, the power allocation problem within CH_m can be transformed as

$$(\mathbf{P3}) \quad \max_{\mathbf{P_m}} \min_{l \in (1,\ldots,L_m)} R_{m,l}, \quad \text{subject to } C_1. \tag{12}$$

Here, (**P3**) is not convex, hence, it is difficult to solve it by standard optimization solvers. In order to make (**P3**) tractable, we transform the problem into a sequence of linear programs and develop an optimal power allocation algorithm to solve it.

Proposition 1. (**P3**) is a quasi-concave problem.

Proof. A maximization optimization problem is quasi-concave when the objective function is quasi-concave and the constraints are convex [15]. It is noted that C_1 is linear, therefore, it is convex. Denoting $\mathbf{\Phi}_\lambda = \left\{\min\limits_l R_{m,l}\left(\mathbf{P_m}\right) \geq \lambda,\ \lambda \in \mathbb{R}\right\}$ as the set of $\mathbf{P_m}$, in which the achieved rate over CH_m is larger than λ. Because of the min operator, we have $\mathbf{\Phi}^*_\lambda = \{R_{m,l}\left(\mathbf{P_m}\right) \geq \lambda,\ \forall\, l\}$. Setting $\mathbf{\Phi}^*_\lambda$ is concave for $\lambda \in \mathbb{R}$, we can have

$$Cp_{m,l}g_{m,l} \geq C\left(2^{\lambda/B_m} - 1\right)\left(\sum_{i=1, i\neq l}^{L_m} p_{m,i}g_{m,i} + \sigma_m^2\right),\ \forall\, l, \tag{13}$$

where C is a constant to guarantee the values of both sides will not be too small to cause unstable results. Note that (13) are linear inequalities, hence, the objective function of (**P3**) is concave. $\square$

Then the power allocation problem can be transformed equivalently into

$$(\mathbf{P4}) \quad \text{Find } \mathbf{P_m},\ \text{subject to } C_1 \text{ and (13)}. \tag{14}$$

As illustrated in Algorithm 3, with the aid of appropriately bounding $\lambda \in \left(\lambda_{LB}, \lambda_{UB}\right)$, a bisection search approach has been proposed to solve (**P4**) for obtaining the optimal transmit power $\mathbf{P_m}$ for each LoRa user sharing CH_m with a desirable accuracy ε. The achieved minimal transmission rate $R^*_{m,l} = \min_l R_{m,l}$ for LoRa users sharing CH_m, which is the optimal objective function value of (12), is returned by Algorithm 3 as well.

Even though the proposed power allocation scheme is a centralized approach, it is implementable as the gateway only needs to know the CSI of the LoRa users assigned to the corresponding channel, and the number of LoRa users sharing the same channel is typically small after channel assignment phase, i.e., no greater than D for LoRa. Moreover, the energy consumption of power allocation is alleviated at LoRa devices by adopting gateway to perform the power allocation algorithm. In combination with the MCAA that matches a large number of LoRa users into different channels to make LoRa network scalable, the proposed resource efficiency framework can achieve a good trade-offs between computational complexity and system performance, which enables the potentials for its practical implementation.

V. Numerical Results

In this section, we first verify our proposed MCAA by comparing it with the baseline approach. With MCAA, performance of the proposed optimal power allocation algorithm is verified by comparing with the random scheme. Finally, performance of the proposed resource allocation framework is presented with different radii of the region where LoRa users are distributed randomly.

In our simulation, the duty cycle is set to 1% to meet the maximal limitations required by ETSI, and the locations of active LoRa nodes are randomly distributed in a circle with the radius of $r = 1$ km unless otherwise specified, as LoRa aims to support long range transmission in terms of kilometers. The simulation parameters are set according to the LoRa specifications unless stated otherwise. There are eight channels supporting multi data rate at 868 MHz for LoRa, and the bandwidth of each of them is $BW = 125$ KHz, which is one of the most common settings. The allowed maximal number of users is $D = 6$ as we set the SF ranges from 7 to 12 for LoRa. The noise is calculated as $\sigma^2 = -174 + 10\log_{10}(BW)$ in dBm level. The path-loss exponent for the communication links is $\alpha = 3.5$. In Algorithm 3, $\varepsilon = 10^{-4}$, $\lambda_{LB} = 0$, and $\lambda_{UB} = BW \log\left(1 + \frac{p^{\max} g_{\max}}{\sigma_m^2}\right)$, where $g_{\max} = \max(g_{m,n}), \forall\, m, n$. The maximal and minimal transmit power are $p_{\max} = 20$ dBm and $p_{\max} = 0$ dBm, respectively.

Fig. 2 shows the effectiveness of the proposed MCAA algorithm with various numbers of active LoRa users. The results of a brute force exhaustive-search approach and random channel assignment approach are also presented for comparison. In this case, the fixed power for each LoRa is set to be $p_{\max}$ to eliminate the influence caused by different transmit power of LoRa users. It is worth noting that our proposed MCAA can achieve about 80% of the optimal performance in comparison with the brute force exhaustive-search approach, however, the computational complexity is reduced significantly as illustrated in Theorem 3. It is also observed that the achieved minimal transmission rate of LoRa users degrades to 0.9 kbps when the number of active LoRa users is increased to 12, which is caused by the aggravated interference as there are more LoRa users sharing the same channel by adopting different SFs.

Algorithm 3 Optimal Power Allocation for Solving (**P4**)

Initialization λ_{LB}, λ_{UB}, and ε.
while $\lambda_{UB} - \lambda_{LB} \geq \varepsilon$ **do**
  Update $\lambda = (\lambda_{UB} + \lambda_{LB})/2$;
  Calculate $\boldsymbol{P_m}$ with the constraints in (**P4**);
  if feasible **then**
    Update $\boldsymbol{P^*_m} = \boldsymbol{P_m}$;
    Calculate $\boldsymbol{R_{m,l}}$ for $l \in (1, ..., L_m)$ by (5) with $\boldsymbol{P^*_m}$;
    Update $\lambda_{LB} = \lambda$, $R^*_{m,l} = \lambda$;
  else
    Update $\lambda_{UB} = \lambda$;
  end if
end while
return $R^*_{m,l}$ and $\boldsymbol{P^*_m}$.

Fig. 3 shows the performance comparison of the proposed optimal algorithm and the fixed power allocation as well as the random power allocation schemes in terms of achieved minimal transmission rate of active LoRa users. In this case, the proposed MCAA is adopted for channel assignment to alleviate the influence caused by different channel access decision. We can observe that the optimal power allocation outperforms both the fixed and random power allocation schemes, which demonstrates the effectiveness of the proposed Algorithm 3. It is also worth noting that the battery life of LoRa nodes can be extended without degrading system performance by adopting the optimal transmit power in comparison with the fixed random power allocation case, in which the maximal transmit power, i.e., $p_{\max} = 20$ dBm, is adopted for each LoRa user.

Fig. 4 shows the performance of the proposed resource allocation framework versus different radii of the region where LoRa users are uniformly distributed. With fixed number of LoRa users, i.e., $N = 20$, the curves with different number of available channels are compared. From the figure, the achieved minimal transmission rate decreases with larger radius of the circle region, r. It is also worth noting that the achieved minimal transmission rate of LoRa users increases when the number of available channels increases. The performance improvement benefits from less interference caused by LoRa users sharing the same channel.

VI. Conclusion

In this paper, we have investigated the resource efficiency problem for uplink transmissions in LoRa representing an example of low-power wide-area (LPWA) networks. Particularly, we have proposed a channel assignment algorithm with low complexity, named MCAA, to make the LoRa network scalable through distributing the channel access decision making local to the LoRa node. For LoRa users sharing the same channel, we have further proposed an optimal power allocation

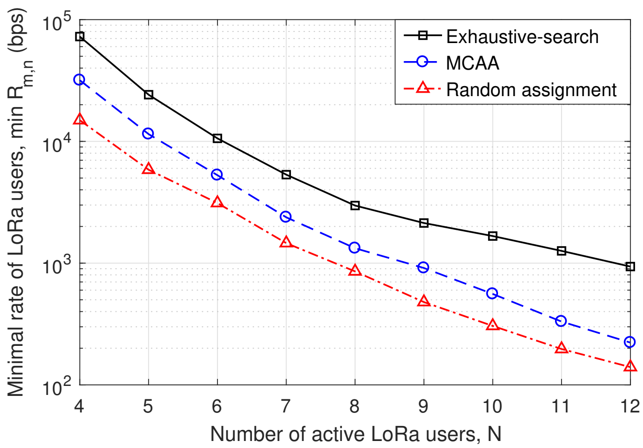

Fig. 2. Performance comparison of the proposed MCAA, exhaustive-search approach and random channel assignment, number of available channels is $M = 3$.

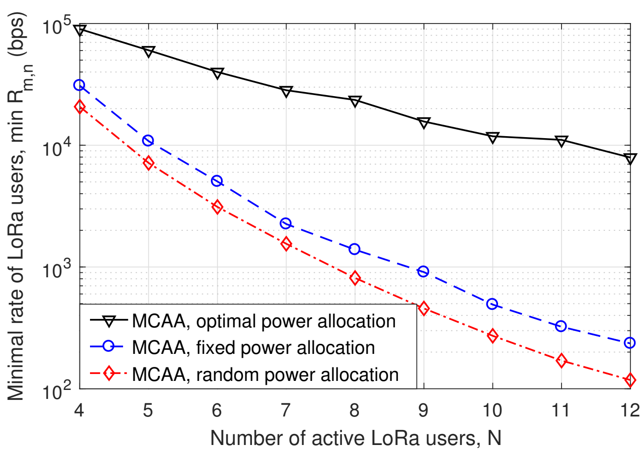

Fig. 3. Performance comparison of proposed optimal power allocation algorithm, the fixed and random power allocation schemes, number of available channels is $M = 3$.

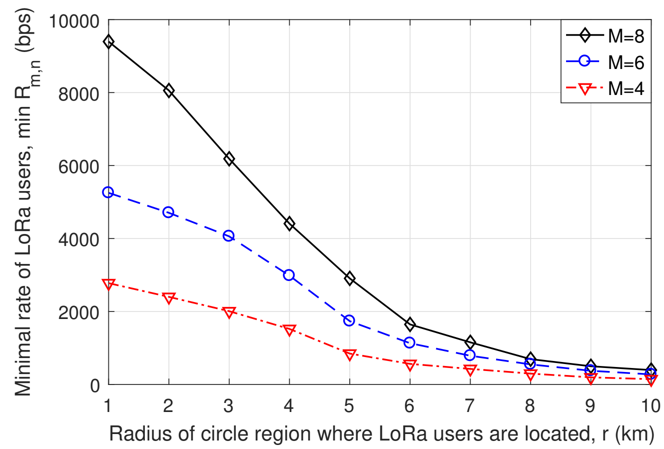

Fig. 4. Minimal transmission rate $\min R_{m,n}$ of the proposed resource allocation framework versus different radii of region that LoRa users are distributed uniformly, number of active LoRa users is $N = 20$.

algorithm to guarantee the user fairness with limited spectrum resources. Simulation results have shown that the proposed resource allocation framework can achieve the near-optimal performance, i.e., more than 80% of the baseline method, but with much lower complexity. Therefore, we can conclude that our proposed resource efficiency framework can achieve a good trade-off between system performance and computational complexity and extend the battery lifetime of LoRa devices essential for real implementation. As part of a smart city project, our proposed framework will be deployed in the Queen Elizabeth Olympic Park in London as a proof of concept.

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