

N 62 - 11704

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Energy Methods

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**The Response of Ribbed Panels to Reverberant
Acoustic Fields**

by

Gideon Maidanik

Bolt Beranek and Newman Inc., Cambridge 38, Massachusetts

ABSTRACT

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where ρ_s is the surface mass density. If we use the relation between strain (or stress) and velocity for a thin plate developed by Hunt,⁹ the ratio of stress spectrum to pressure spectrum (dimensionless) is given by

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$$S_s = 2.5 \times 10^{-4} S_p = 4.4 \times 10^5 (\text{psi})^2/\text{cps}$$

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Of course, the precise stress predicted in Eq. (12) is subject to a fair degree of uncertainty since the value of η is only estimated, and there are several other ambiguities in the form of modal density, plate dimensions, etc. It is gratifying to see the rather simple form of the predicted stress Eq. (11) and its result which is consistent with the information we are given. It will be desirable to see if the "band forming" conditions can be simulated under laboratory conditions in subsequent research.

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- 9) F. V. Hunt, "Stress and Strain Limits on the Attainable Velocity in Mechanical Vibration," J. Acoust. Soc. Am. 32, 9, 1123 (1960).
- 10) Y. K. Lin, "Free Vibration of Continuous Skin-Stringer Panels," J. App. Mech. 27, 4, 669 (1960).

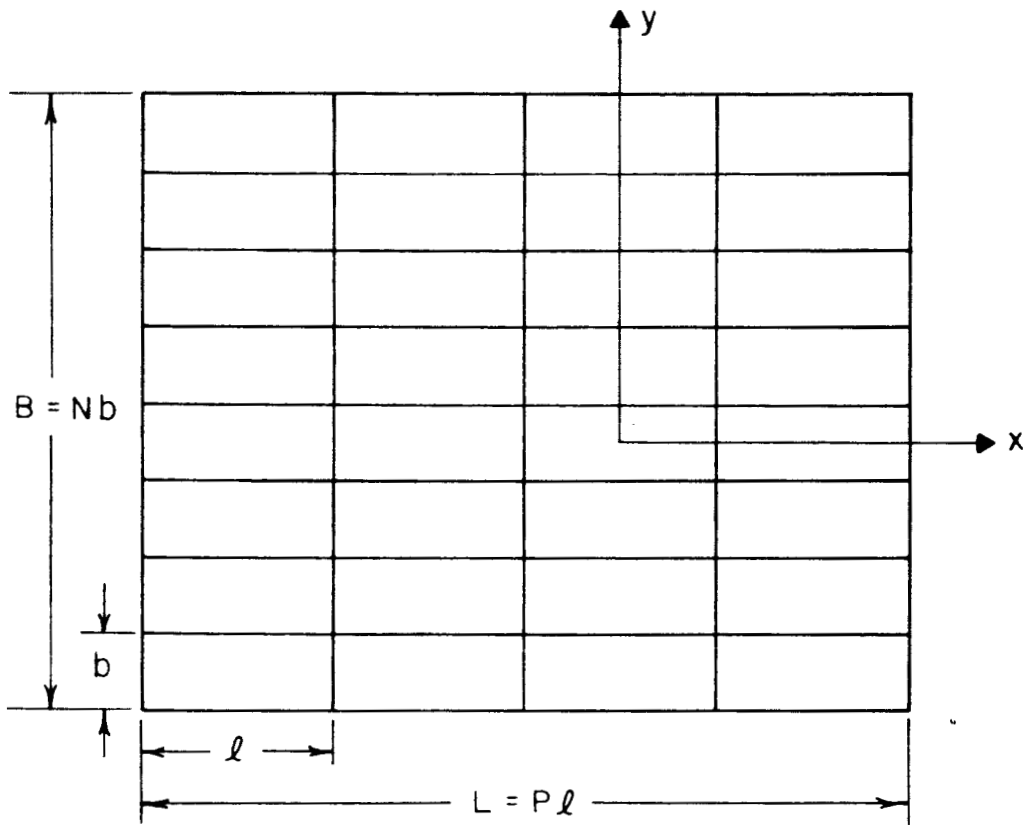


FIG. 1 DIAGRAM OF RIBBED PANEL

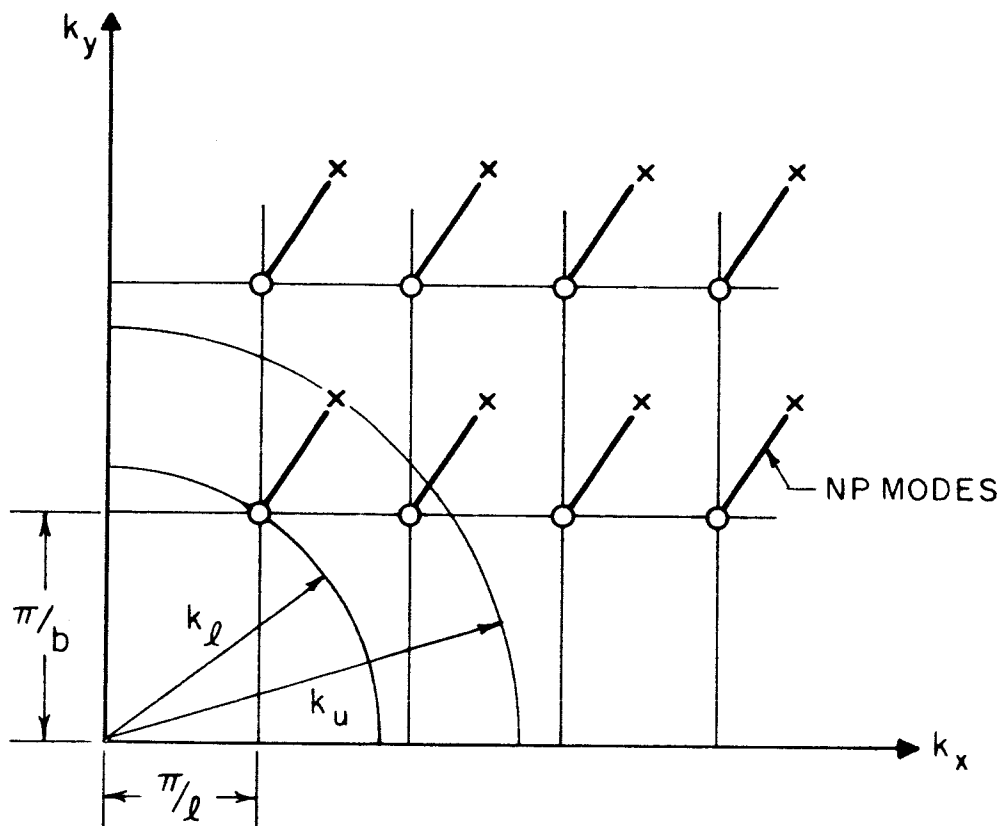


FIG. 2 WAVE NUMBER LATTICE FOR RIBBED PANEL

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- 7) B. L. Clarkson and R. D. Ford, "Experimental Study of the Random Vibrations of an Aircraft Structure Excited by Jet Noise," USAA Report No. 128, University of Southampton, January 1960.
- 8) M. Heckl, R. H. Lyon, G. Maidanik, and E. E. Ungar, "New Methods for Understanding and Controlling Vibrations of Complex Structures," Bolt Beranek and Newman Report No. 875, Contract AF 33(616)-7973, August 1961, p. 20.

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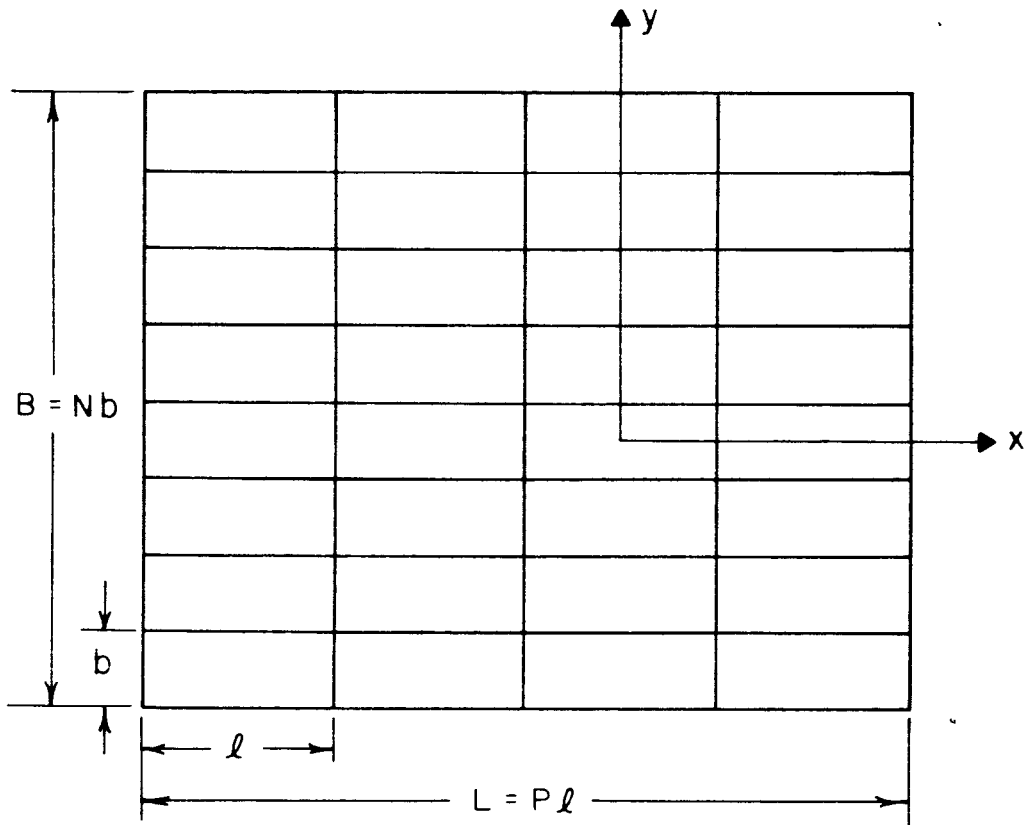


FIG. 1 DIAGRAM OF RIBBED PANEL

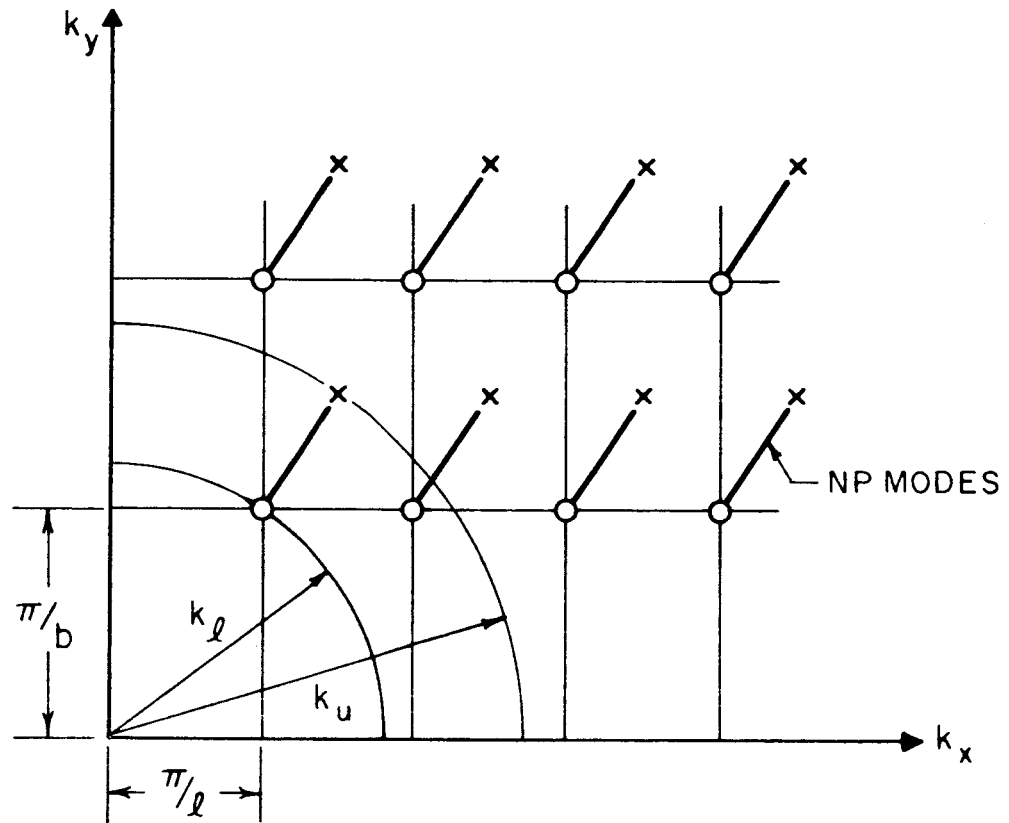


FIG. 2 WAVE NUMBER LATTICE FOR RIBBED PANEL