

# Response of the Water Level in a Well to Earth Tides and Atmospheric Loading Under Unconfined Conditions

STUART ROJSTACZER AND FRANCIS S. RILEY

*U.S. Geological Survey, Menlo Park, California*

The response of the water level in a well to Earth tides and atmospheric loading under unconfined conditions can be explained if the water level is controlled by the aquifer response averaged over the saturated depth of the well. Because vertical averaging tends to diminish the influence of the water table, the response is qualitatively similar to the response of a well under partially confined conditions. When the influence of well bore storage can be ignored, the response to Earth tides is strongly governed by a dimensionless aquifer frequency  $Q'_u$ . The response to atmospheric loading is strongly governed by two dimensionless vertical fluid flow parameters: a dimensionless unsaturated zone frequency,  $R$ , and a dimensionless aquifer frequency  $Q_u$ . The differences between  $Q'_u$  and  $Q_u$  are generally small for aquifers which are highly sensitive to Earth tides. When  $Q'_u$  and  $Q_u$  are large, the response of the well to Earth tides and atmospheric loading approaches the static response of the aquifer under confined conditions. At small values of  $Q'_u$  and  $Q_u$ , well response to Earth tides and atmospheric loading is strongly influenced by water table drainage. When  $R$  is large relative to  $Q_u$ , the response to atmospheric loading is strongly influenced by attenuation and phase shift of the pneumatic pressure signal in the unsaturated zone. The presence of partial penetration retards phase advance in well response to Earth tides and atmospheric loading. When the theoretical response of a phreatic well to Earth tides and atmospheric loading is fit to the well response inferred from cross-spectral estimation, it is possible to obtain estimates of the pneumatic diffusivity of the unsaturated zone and the vertical hydraulic conductivity of the aquifer.

## INTRODUCTION

In some wells which tap unconfined aquifers, the water level in the well responds measurably to aquifer deformation induced by both Earth tides and atmospheric loading [e.g., *Bower and Heaton*, 1973, 1978]. For these wells, the water level change cannot be a direct reflection of the water table response. While the water table may respond to atmospheric loading due to unsaturated zone effects [*Yusa*, 1969; *Weeks*, 1979] or significant gas content in the capillary fringe [*Peck*, 1960; *Turk*, 1975], the water table can be expected to be insensitive to Earth tide induced deformation [*Bredehoeft*, 1967].

Water level fluctuations in phreatic wells produced by changes in both Earth tides and atmospheric loading can be explained if the water level in the well reflects the response of the aquifer vertically averaged over the saturated depth of the well. While the water table is generally insensitive to rock deformation, the aquifer at depth can be largely isolated from water table influences if the vertical hydraulic diffusivity of the aquifer is low. For wells that tap thick unconfined aquifers, the average response of the screened interval will at least be partially influenced by the response of the aquifer under conditions where the water table has little influence; the response will be qualitatively similar to the response of wells under partially confined conditions [*Rojstaczer*, 1988a, b]. As in the partially confined case, the tidal and barometric response will be a function of the length of time or frequency over which the deformation takes place. Water well response due to rapid changes in deformation will be weakly influenced by the water table; the response will approach that which would occur if the aquifer were confined [*Jacob*, 1940; *Bredehoeft*, 1967; *Van der Kamp and*

*Gale*, 1983; *Rojstaczer and Agnew*, 1989]. Water level response to slow changes in deformation will be strongly influenced by water table drainage.

Figure 1 shows an idealized cross section of a phreatic well. As suggested in the figure, fluid flow is an intrinsic part of water well response to aquifer deformation. For Earth tide induced deformation, groundwater flow to and from the water table, as well as flow into and out of the well bore, can influence well response. For the case of atmospheric loading, air flow in the unsaturated zone can also influence well response.

The influence of fluid flow on the response of phreatic wells to Earth tides or atmospheric loading has been examined by others. *Bower and Heaton* [1973] examined the response to Earth tides and atmospheric loading under the assumption that the well was open only at the bottom of the hole, the water table was a fixed boundary, and that well bore storage effects and (for the case of atmospheric loading) unsaturated zone effects were negligible. *Johnson* [1973] examined the theoretical response to atmospheric loading in a spherically shaped aquifer under the assumptions that unsaturated zone effects were negligible and that the water table was a fixed boundary. *Yusa* [1969] and *Weeks* [1979] examined the influence of air flow in the unsaturated zone on well response to atmospheric loading and assumed that well bore storage effects were negligible, the water table was a fixed boundary, and the water table pressure change due to the atmospheric load represented the pressure change throughout the monitored depth of the aquifer.

This study extends the results of the above studies by using theoretical models of water well response to Earth tides and atmospheric loading to examine: (1) the significance of assuming that the well responds to the vertically averaged aquifer pressure change; (2) the appropriateness of assuming that the water table is fixed; (3) the influence of partial penetration on well response. For simplicity, we

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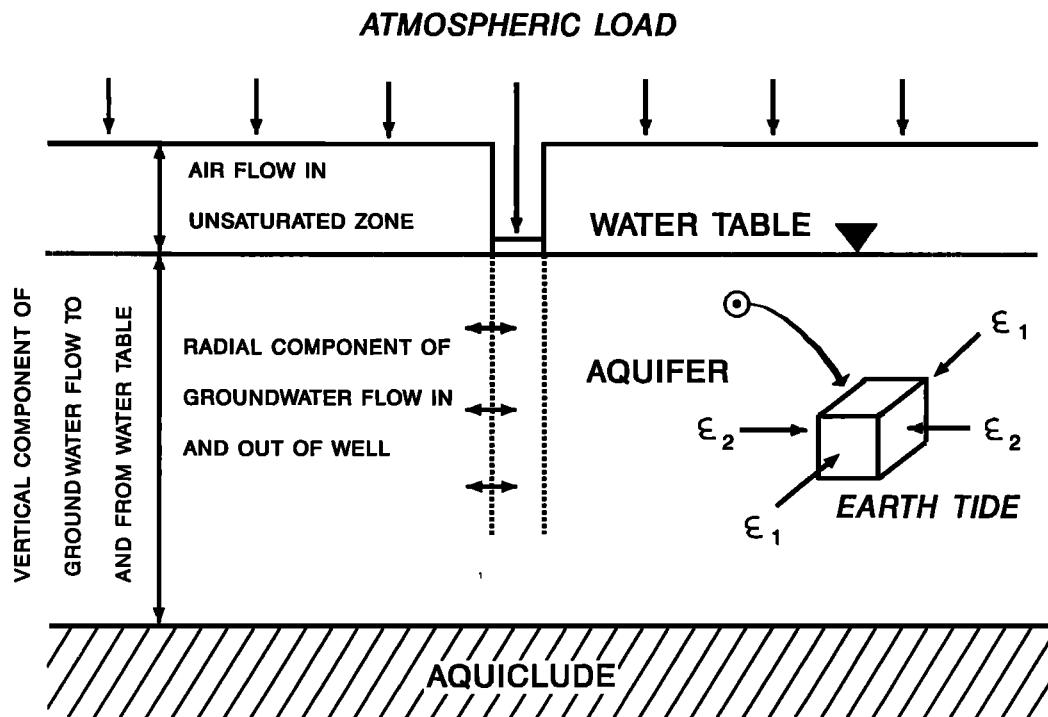


Fig. 1. Cross section of a phreatic well showing the influences of fluid flow on well response to Earth tides and atmospheric loading.

assume that the well taps an unconfined aquifer which has a lateral permeability high enough that well bore storage effects are negligible at the frequencies present in the Earth tide and atmospheric load signals. We also assume that air encapsulation in the capillary fringe is small so that water table fluctuations induced by gas transport in the capillary fringe are negligible. Comparison is made with the theoretical results given elsewhere [Rojstaczer, 1988a, b] for water well response under partially confined conditions. The theoretical model is then applied to the response of a phreatic well to Earth tides and atmospheric loading to yield estimates of the pneumatic diffusivity of the unsaturated zone and the vertical hydraulic conductivity of the unconfined aquifer.

#### SOLUTION TO THE RESPONSE OF AN UNCONFINED WELL TO EARTH TIDES AND PERIODIC ATMOSPHERIC LOADING

The response of a phreatic well to Earth tides and atmospheric loading is governed by several processes which operate simultaneously. For the case of Earth tide response, the following processes take place: (1) deformation of the aquifer due to the imposed strain; (2) vertical diffusion of groundwater pressure through the aquifer; (3) diffusion of groundwater pressure between the aquifer and the well. In addition, the following processes influence atmospheric loading response: (1) pressurization at the water surface of the open well due to the air load and (2) diffusion of pneumatic pressure between the Earth's surface and the water table. As noted elsewhere [Rojstaczer, 1988a, b], these processes also influence the response of well aquifer systems under partially confined conditions.

We can readily obtain closed form solutions to the response of a phreatic well to Earth tides and atmospheric

loading if we assume that (1) the well bore is in quasi-static equilibrium with the vertically averaged aquifer pressure (i.e., well bore storage effects are negligible); (2) the aquifer has uniform material properties; and (3) (for the case of atmospheric loading) the air flow between the Earth's surface and the water table is predominantly vertical. The solution for Earth tide response can be obtained by solving for one-dimensional groundwater flow near a water table in response to periodic deformation of the aquifer. The solution for atmospheric loading response can be obtained by combining the solutions to two separate fluid flow problems: (1) vertical diffusion of pneumatic pressure between the Earth's surface and the water table and (2) vertical diffusion of the atmospheric pressure signal through the unconfined aquifer with concomitant loading. As is noted in detail below, the responses given by these solutions are similar to the response of wells under partially confined conditions to Earth tides and atmospheric loading.

#### Vertical Groundwater Flow Induced by Earth Tides

If we assume that well bore storage effects are negligible, the response of an unconfined aquifer to Earth tides is governed by (compression is positive) [Rojstaczer and Agnew, 1989]

$$D' \frac{\partial^2 p}{\partial z^2} = \frac{\partial p}{\partial t} + \rho g A'_s A \omega \sin(\omega t) \quad (1)$$

where  $p$  is pore pressure,  $D'$  is a hydraulic diffusivity for imposed areal strain under conditions of plane stress [Van der Kamp and Gale, 1983],  $\rho$  is the fluid density,  $g$  is gravitational acceleration,  $A'_s$  is the static-confined areal strain sensitivity of the aquifer or water level rise per unit

strain compression [Van der Kamp and Gale, 1983; Rojstaczer and Agnew, 1989], and  $A$  and  $\omega$  are the amplitude and frequency of the Earth tide signal, respectively. The source term in (1) accounts for the periodic deformation of the aquifer due to the Earth tide.

For completeness, the appropriate boundary conditions should take into account the possible effect of any periodic fluctuations in water table height induced by fluid flow to and from the water table. If the water table boundary condition is imposed at the mean height of the water table ( $z = 0$ ), we obtain the following first-order, linearized approximation of the boundary conditions:

$$\partial p(0, t)/\partial z = -(S_y/K_z)\partial p(0, t)/\partial t \tag{2a}$$

$$\partial p(d, t)/\partial z = 0 \tag{2b}$$

where  $K_z$  and  $S_y$  are the vertical hydraulic conductivity and specific yield of the aquifer, respectively, and  $d$  is the thickness of the aquifer. The above boundary conditions are identical to the first-order approximation used by Neuman [1972] in his analysis of the response of a phreatic aquifer to constant fluid withdrawal from a well. The solution of (1) subject to the boundary conditions given in (2) is given in Appendix A:

$$p = -\rho g A_s' A \exp(i\omega t) [\exp[-(i+1)(Q')^{1/2}]H_1 + \exp[(i+1)(Q')^{1/2}]H_2 - 1] \tag{3}$$

where  $Q'$  is a dimensionless frequency referenced to the saturated thickness at the depth of interest,  $z$  (the depth from the mean height of the water table to the observation point), the specific storage of the aquifer under conditions of imposed horizontal strain [Van der Kamp and Gale, 1983],  $S_s$ , and the vertical hydraulic conductivity of the aquifer,  $K_z$ :

$$Q' = \frac{\omega S_s z^2}{2K_z} = \frac{\omega z^2}{2D'} \tag{4}$$

$H_1$  and  $H_2$  are terms that reflect the presence of an impermeable boundary at depth,  $d$ :

$$H_1 = 1 + \exp[-2(i+1)(\omega d^2/2D')^{1/2}] - \Omega' [1 - \exp[-2(i+1)(\omega d^2/2D')^{1/2}]] \tag{5a}$$

$$H_2 = 1 + \exp[2(i+1)(\omega d^2/2D')^{1/2}] + \Omega' [1 - \exp[2(i+1)(\omega d^2/2D')^{1/2}]] \tag{5b}$$

and  $\Omega'$  is a dimensionless parameter which governs the movement of the water table:

$$\Omega' = (1-i)(S_y K_z / 2S_y^2 \omega)^{1/2} \tag{6}$$

The solution given in (3) is very similar to a solution given elsewhere for pore pressure response to areal strain in a formation of infinite vertical extent or a thin formation bounded above and below by partial confining layers [Rojstaczer, 1988b]:

$$p = -\rho g A_s' A \exp(i\omega t) [\exp(-(i+1)\sqrt{Q'}) - 1] \tag{7}$$

The difference here is that we have allowed for the additional complexities of a fluctuating water table and an impermeable layer at a depth,  $d$ , below the water table. If we assume that

$d$  is at infinity and that the water table parameter  $\Omega'$  is zero, (3) simplifies to (7).

Since water well response is assumed to be driven by the depth averaged pressure change in the aquifer,  $\bar{p}$ , we vertically average the solution in (3) over the saturated well depth to obtain

$$\bar{p} = -\rho g A_s' A (\bar{U}' + i\bar{V}') \exp(i\omega t) \tag{8}$$

where  $\bar{U}'$  and  $\bar{V}'$  are

$$\begin{aligned} \bar{U}' = & \exp - \sqrt{Q'_u} [-\cos \sqrt{Q'_u} + \sin \sqrt{Q'_u}] / 2 \sqrt{Q'_u} H_1 \\ & + \exp \sqrt{Q'_u} [\cos \sqrt{Q'_u} + \sin \sqrt{Q'_u}] / 2 \sqrt{Q'_u} H_2 \\ & + [1/H_1 - 1/H_2] / 2 \sqrt{Q'_u} - 1 \end{aligned} \tag{9a}$$

$$\begin{aligned} \bar{V}' = & \exp - \sqrt{Q'_u} [\cos \sqrt{Q'_u} + \sin \sqrt{Q'_u}] / 2 \sqrt{Q'_u} H_1 \\ & + \exp \sqrt{Q'_u} [-\cos \sqrt{Q'_u} + \sin \sqrt{Q'_u}] / 2 \sqrt{Q'_u} H_2 \\ & + [1/H_2 - 1/H_1] / 2 \sqrt{Q'_u} \end{aligned} \tag{9b}$$

and  $Q'_u$  is a dimensionless aquifer frequency referenced to the saturated thickness of the well,  $b$ :

$$Q'_u = \frac{\omega S_s b^2}{2K_z} = \frac{\omega b^2}{2D'} \tag{10}$$

### Response of the Water Table to Earth Tides

The conditions necessary for significant water table fluctuations to be induced by Earth tides can be examined through analysis of the water table parameter  $\Omega'$ . The water table parameter,  $\Omega'$ , is an indicator of the ability of the water to rise and fall in response to periodic deformation of the aquifer under conditions where the aquifer is of infinite thickness. For an aquifer of infinite thickness, (3) can be easily reduced to solve for changes in water table height ( $z = 0$ ):

$$p(z = 0)/\rho g = -A_s' A [1/(1 - \Omega') - 1] \exp(i\omega t) \tag{11}$$

Figure 2 shows the modulus of (11),  $|p/\rho g|$ , divided by the term  $A_s' A$  as a function of  $\Omega'/(1 - i)$ . It indicates that significant water table fluctuations (changes in water table height with amplitude in excess of  $0.1 A_s' A$ ) occur when the parameter  $\Omega'/(1 - i)$  exceeds 0.1. If we limit our analysis to peak tidal frequencies (about 1–2 cycles/d), then the upper bound of 0.1 for  $\Omega'/(1 - i)$  indicates that the term  $S_s K_z / S_y^2$  must be greater than  $2 \times 10^{-7} \text{ s}^{-1}$  for water table fluctuations to be significant relative to  $A_s' A$ . As might be expected, significant water table fluctuations are favored in aquifers with high specific storage, high hydraulic conductivity and low specific yield. Formations with these physical properties will have an ability to yield large amounts of fluid when deformed and rapidly transport that fluid to and from the water table; because specific yield is low, fluid mass transfer to and from the water table will cause relatively large changes in water table height.

It is useful to examine whether  $\Omega'/(1 - i)$  can realistically exceed 0.1 in formations which are sensitive to Earth tides. Earth tides produce areal strains with amplitudes of the order of  $10^{-8}$  [Melchior, 1978] and aquifers can be expected to be sensitive to Earth tides under confined conditions if  $A_s'$

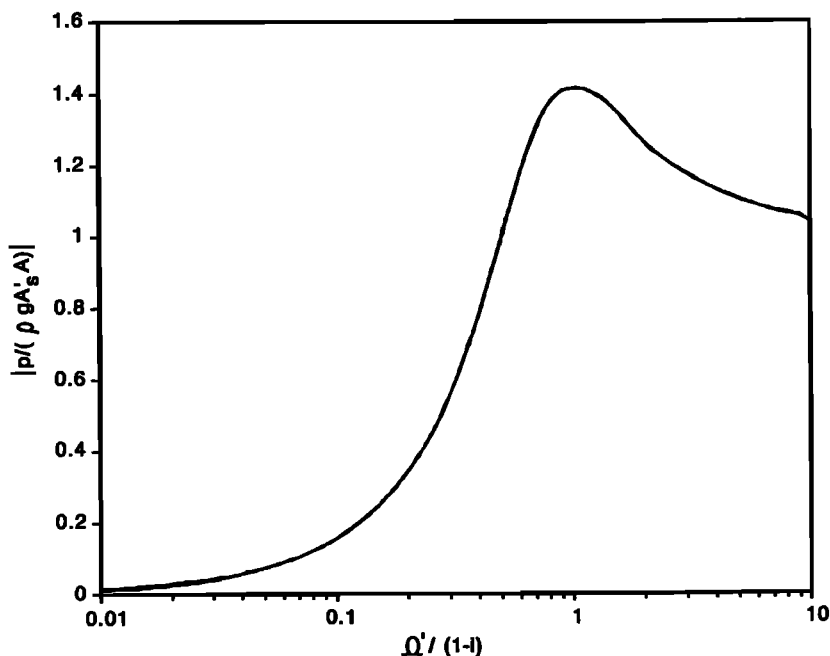


Fig. 2. Amplitude of the response of the water table to Earth tides (in terms of  $p/[\rho g A_s A]$ ) as a function of the water table parameter  $\Omega'$ .

exceeds  $0.02 \text{ cm}/n\epsilon$ . This lower bound on  $A_s'$  indicates that  $S_s$  is generally less than  $6 \times 10^{-8} \text{ cm}^{-1}$  in these formations [Rojstaczer and Agnew, 1989]. Given this bound on  $S_s$  and assuming that  $\Omega'/(1-i)$  must be greater than 0.1,  $K_z$  must be greater than  $3S_y^2 \text{ cm/s}$  for significant water table fluctuations to be produced by Earth tides at peak tidal frequencies. Thus we cannot expect significant Earth tide induced water table fluctuations unless the formation has a very high hydraulic conductivity and low specific yield. Furthermore, this hydraulic conductivity bound is derived from (11), which optimistically assumes that the aquifer is infinitely thick. By analogy to heat flow [Carslaw and Jaeger, 1959, p. 66], (11) is appropriate when the aquifer thickness exceeds the diffusive depth,  $d_p$ :

$$d_p = (4\pi K_z / S_y \omega)^{1/2} \tag{12}$$

Given the parameters discussed above,  $d_p$  is  $7 \times 10^7 S_y^2 \text{ cm}$ . Hence the aquifer must be unrealistically thick to drive water table fluctuations unless  $S_y$  is of the order of 0.01. We can generally assume that unless  $S_y$  is unusually low and  $K_z$  unusually high,  $\Omega'$  is essentially zero. This analysis of the dynamic response of the water table to Earth tides is consistent with the conclusions derived from a static analysis given elsewhere [Bredehoeft, 1967].

*Response of a Phreatic Well to Earth Tides*

The response of a well to Earth tides can be obtained, in the absence of well bore storage effects, through the use of (8). The areal strain sensitivity,  $A_s$ , and phase,  $\theta$ , of the response are

$$A_s(\omega) = \left| \frac{\bar{p}_0}{\rho g A} \right| \tag{13a}$$

$$\theta(\omega) = \arg(\bar{p}_0) \tag{13b}$$

where  $\bar{p}_0$  is the depth-averaged pore pressure within the aquifer,  $\bar{p}$ , divided by  $\exp(i\omega t)$ :

$$\bar{p}_0 = \bar{p} \exp(-i\omega t) \tag{14}$$

The vertical bars in (13a) denote the modulus of the complex function; and  $\arg$  in (13b) denotes the inverse tangent of the ratio of the imaginary component to the real component of the complex function. Equations (8) and (13) indicate that the areal strain sensitivity,  $A_s$ , and phase,  $\theta$ , of the response are a function of two dimensionless parameters: (1)  $Q'_u$ , the dimensionless frequency of the aquifer under conditions of imposed horizontal strain and (2) the well penetration ratio  $b/d$ . As noted above, the dimensionless parameter  $\Omega'$  can be assumed generally to be zero.

Figure 3 shows the response of a well to Earth tides as a function of  $Q'_u$  and  $b/d$  for the case of an aquifer with a static-confined areal strain sensitivity,  $A_s'$ , of  $0.05 \text{ cm}/n\epsilon$ . In the figure,  $\Omega'$  is assumed to be zero. The response given here for the case where  $b/d$  approaches 0 is qualitatively very similar to the response to imposed strain of a piezometer tapping a formation of infinite vertical extent (or a thin aquifer bounded above and below by partial confining layers) given elsewhere [Rojstaczer, 1988b]. The difference here is that  $Q'$  has been replaced by  $Q'_u$  and because the well is screened over its entire saturated thickness, the rate of attenuation and phase advance occur more rapidly as a function of dimensionless frequency.

Independent of the ratio  $b/d$ , sensitivity to areal strains such as that produced by Earth tides rapidly attenuates as  $Q'_u$  decreases from 10 to 0.1 due to the increasing influence of water table drainage. The phase relations, however, are highly dependent upon  $b/d$ . For the case of full penetration ( $b/d = 1$ ) there is rapid phase advance with decreasing dimensionless frequency,  $Q'_u$ . When this ratio is small ( $<0.1$ ), there exists an intermediate frequency band where phase advance asymptotically approaches  $45^\circ$ . Both the

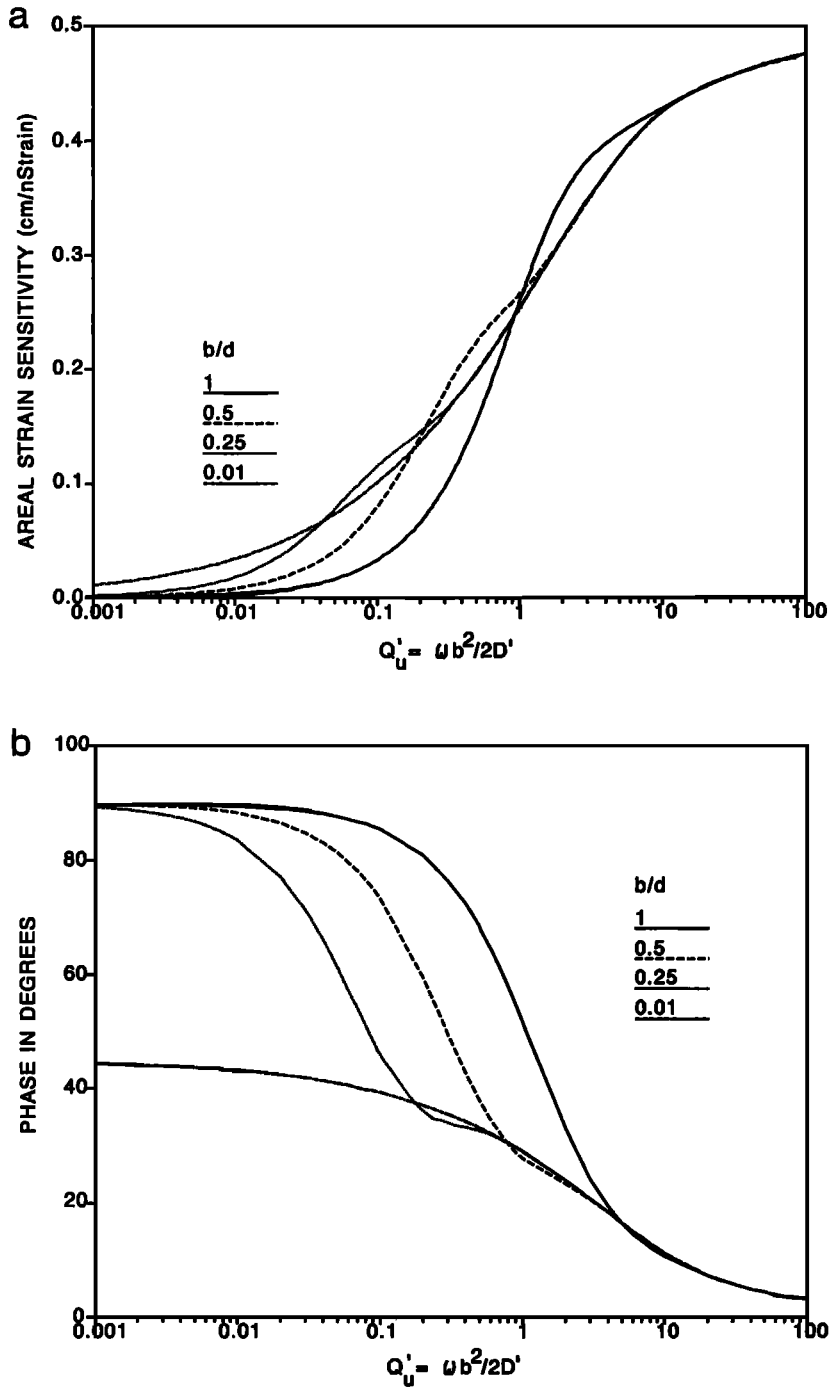


Fig. 3. (a) Areal strain sensitivity and (b) phase of the response of a phreatic well to Earth tides as a function of  $Q'_u$  and  $b/d$ . The static confined areal strain sensitivity of the well is 0.05 cm/nε.

areal strain sensitivity,  $A_s$ , and phase,  $\theta$ , asymptotically approach the static-confined response for dimensionless frequencies,  $Q'_u$  greater than 100.

*Vertical Air Flow Induced by Atmospheric Loading*

As was noted above, the response of water levels in wells to atmospheric loading differs from that due to Earth tides principally because of the influence of pneumatic diffusion through the unsaturated zone. We can easily account for this added complexity if we assume that periodic vertical flow of air between the Earth's surface and the water table is

governed by a simple diffusion equation [Yusa, 1969; Weeks, 1979]:

$$D_a \frac{\partial^2 p_a}{\partial z^2} = \frac{\partial p_a}{\partial t} \tag{15}$$

subject to the following boundary conditions:

$$p_a(-L, t) = A \cos(\omega t) \tag{16a}$$

$$p_a(L, t) = A \cos(\omega t) \tag{16b}$$

where  $p_a$  is the air pressure and  $D_a$  is the pneumatic diffusivity. The boundary  $L$  is taken to be the Earth's

surface; the zone from depth 0 to depth  $-L$  is simply an artifice to assure that at the water table,  $z = 0$ , there is no air flux. The solution for air pressure at the mean height of the water table ( $z = 0$ ),  $p_a$ , is [Rojstaczer, 1988b]

$$p_a = (M - iN)A \exp(i\omega t) \quad (17)$$

where  $M$  and  $N$  are

$$M = \frac{2 \cosh(\sqrt{R}) \cos(\sqrt{R})}{\cosh(2\sqrt{R}) + \cos(2\sqrt{R})} \quad (18a)$$

$$N = \frac{2 \sinh(\sqrt{R}) \sin(\sqrt{R})}{\cosh(2\sqrt{R}) + \cos(2\sqrt{R})} \quad (18b)$$

and  $R$  is a dimensionless frequency referenced to pneumatic diffusivity,  $D_a$ , and the depth,  $L$ , from the Earth's surface to the water table:

$$R = L^2 \omega / 2D_a \quad (19)$$

#### Vertical Groundwater Flow Induced by Atmospheric Loading

In the absence of well bore storage effects, the response of the unconfined aquifer to periodic atmospheric loading is governed by [Rojstaczer and Agnew, 1989]

$$D \frac{\partial^2 p}{\partial z^2} = \frac{\partial p}{\partial t} + \gamma A \omega \sin \omega t \quad (20)$$

where  $D$  is the vertical hydraulic diffusivity of the unconfined aquifer under conditions of surface loading,  $p$  is pore pressure,  $A$  is the amplitude of the atmospheric load and  $\gamma$  is the surface loading efficiency of the aquifer [Rojstaczer and Agnew, 1989]. The source term in (20) accounts for the deformation of the aquifer due to the imposed surface load.

As in the Earth tide case, we take into account the possible effect of any periodic fluctuations in water table height. Taking the water table boundary condition at the mean height of the water table ( $z = 0$ ), we obtain the following first-order, linearized approximation of the boundary conditions:

$$\begin{aligned} \partial p(0, t) / \partial z &= -(S_y / K_z) \partial p(0, t) / \partial t \\ &+ (S_y / K_z) [MA \sin(\omega t) - NA \cos(\omega t)] \end{aligned} \quad (21a)$$

$$\partial p(d, t) / \partial z = 0 \quad (21b)$$

The first term on the right-hand side of the water table boundary condition is identical to the first-order approximation used in the Earth tide case. The second term on the right-hand side of the water table boundary condition is obtained from the solution given in (17) and accounts for the influence of pneumatic pressure diffusion on the well response. The above boundary conditions ignore the influence of any water table fluctuations induced by significant gas content in the capillary fringe. This capillary fringe effect, which has been noted in wells which tap shallow water table aquifers is discussed in detail elsewhere [Peck, 1960; Turk, 1975]. The solution of (20) subject to boundary conditions given in (21) is given in Appendix B:

$$\begin{aligned} p &= (M - iN - \gamma)A \exp(i\omega t) [\exp(-(i+1)\sqrt{Q})/H_1 \\ &+ \exp((i+1)\sqrt{Q})/H_2] + \gamma A \exp(i\omega t) \end{aligned} \quad (22)$$

where  $\Omega$  is a water table parameter analogous to  $\Omega'$ :

$$\Omega = (1 - i)(S_a K_z / 2S_y \omega)^{1/2} \quad (23)$$

$Q$  is a dimensionless frequency analogous to  $Q'$ :

$$Q = \frac{\omega S_a z^2}{2K_z} = \frac{\omega z^2}{2D} \quad (24)$$

and  $S_a$  is the specific storage of the aquifer under conditions of surface loading [Rojstaczer and Agnew, 1989].

It should be noted that (22) is nearly the same as the solution given elsewhere [Rojstaczer, 1988a] for diffusion of the atmospheric load through a partial confining layer:

$$\begin{aligned} p &= (M - iN - \gamma)A \exp(i\omega t) [\exp(-(i+1)\sqrt{Q}) \\ &+ \gamma A \exp(i\omega t) \end{aligned} \quad (25)$$

As in the Earth tide case, the difference here is that we have allowed the water table to periodically fluctuate and included the effects of an impermeable layer at saturated depth,  $d$ .

Because the processes which govern fluid flow in response to atmospheric loading are very similar to those which govern Earth tide response, the solution given in (22) is also very similar to that given in (3) for pressure diffusion in response to imposed areal strain. The added complexity of pneumatic diffusion is included in the terms  $M$  and  $N$ . The water table term  $\Omega$  and the dimensionless frequency  $Q$  differ from  $\Omega'$  and  $Q'$  because  $S_s$  has been replaced by  $S_a$ . As is discussed elsewhere,  $S_s$  and  $S_a$  may differ by 20% or less in aquifers which are sensitive to Earth tides; in aquifers which are highly compressible and are underlain by basement rock, the difference between  $S_s$  and  $S_a$  will be very small [Rojstaczer and Agnew, 1989]. As a result, in the absence of significant air transport in the unsaturated zone, pressure diffusion in response to atmospheric loading can be expected to be very similar to pressure diffusion in response to Earth tide induced deformation. Unlike the Earth tide case, however, the water table parameter  $\Omega$  cannot be assumed generally to be zero. This point is discussed in the next section.

Since we assume that water well response is driven by the depth averaged pressure change in the aquifer,  $\bar{p}$ , we vertically average the solution in (22) over the saturated well depth to obtain

$$\bar{p} = (M - iN - \gamma)(\bar{U} + i\bar{V})A \exp(i\omega t) + \gamma A \exp(i\omega t) \quad (26)$$

where  $\bar{U}$  and  $\bar{V}$  are

$$\begin{aligned} \bar{U} &= \exp - \sqrt{Q_u} [-\cos \sqrt{Q_u} + \sin \sqrt{Q_u}] / 2\sqrt{Q_u} H_1 \\ &+ \exp \sqrt{Q_u} [\cos \sqrt{Q_u} + \sin \sqrt{Q_u}] / 2\sqrt{Q_u} H_2 \\ &+ [1/H_1 - 1/H_2] / 2\sqrt{Q_u} \end{aligned} \quad (27a)$$

$$\begin{aligned} \bar{V} &= \exp - \sqrt{Q_u} [\cos \sqrt{Q_u} + \sin \sqrt{Q_u}] / 2\sqrt{Q_u} H_1 \\ &+ \exp \sqrt{Q_u} [-\cos \sqrt{Q_u} + \sin \sqrt{Q_u}] / 2\sqrt{Q_u} H_2 \\ &+ [1/H_2 - 1/H_1] / 2\sqrt{Q_u} \end{aligned} \quad (27b)$$

and  $Q_u$  is a dimensionless aquifer frequency referenced to the saturated thickness of the well,  $b$ :

$$Q_u = \frac{\omega S_a b^2}{2K_z} = \frac{\omega b^2}{2D} \quad (28)$$

#### Water Table Fluctuations Induced by Atmospheric Loading

Under some conditions, significant water table fluctuations can be induced by atmospheric loading. This may occur even in the absence of significant gas content in the capillary fringe. The conditions under which water table fluctuations can be induced by atmospheric loading can be found through an examination of the parameter  $\Omega$ . Unlike the Earth tide case,  $\Omega$  cannot be assumed generally to be zero. This difference is principally due to two factors: (1) aquifers need not have low values of specific storage to respond to atmospheric loading and (2) the atmospheric loading signal has considerable energy at periods much longer than 24 hours. The presence of considerable energy in the atmospheric signal at seasonal and monthly periods [Rojstaczer, 1988b] indicates that in comparison to the Earth tide case, permeabilities required to cause water table fluctuations need not be quite as high. Since formation sensitivity to atmospheric loading is not limited to low storage materials, aquifers do not have to be unrealistically thick to drive water table fluctuations.

We can derive hydraulic conductivity and thickness bounds for atmospheric loading induced water table fluctuations by using the same approach as in the Earth tide analysis. For an aquifer of infinite vertical extent and assuming  $R$  is infinite (i.e., the water table is pneumatically isolated from the Earth's surface), (22) can be readily reduced to solve for changes in water table height ( $z = 0$ ):

$$p = -(\gamma A/\rho g)[1/(1 - \Omega) - 1] \exp(i\omega t) \quad (29)$$

Similar to the Earth tide case, significant water table fluctuations (greater than 0.1  $\gamma A/\rho g$ ) occur when  $\Omega/(1 - i)$  exceeds 0.1. If we examine well response at monthly periods, then the term  $S_a K_z/S_y^2$  must be greater than  $5 \times 10^{-9} \text{ cm}^{-1}$  for water table fluctuations to be significant relative to  $\gamma A/\rho g$ .

As is noted elsewhere, formations which have a high compressibility possess a high loading efficiency,  $\gamma$  [Rojstaczer and Agnew, 1989]; independent of porosity, formations with sedimentlike compressibilities will possess loading efficiencies which exceed 0.8. For a typical unconsolidated sand ( $S_a \approx 1 \times 10^{-6} \text{ cm}^{-1}$ ,  $S_y \approx 0.2$ ),  $K_z$  must exceed  $2 \times 10^{-4} \text{ cm/s}$  for the parameter  $\Omega/(1 - i)$  to exceed 0.1 at monthly frequencies; this bound is generally lower than that given in the Earth tide case. The bound on aquifer thickness is also generally lower. Following our approach in the Earth tide analysis, the aquifer thickness would have to exceed

$$d > d_p = (4\pi K_z/S_a \omega)^{1/2} \quad (30)$$

For the parameters discussed above, the aquifer thickness,  $d$ , would have to exceed 1 km. Hence if an unconsolidated formation was very thick and possessed moderately high hydraulic conductivity, fluctuations in water table height at monthly periods would be possible. If both of these condi-

tions are not met, however, the water table parameter,  $\Omega$ , will be close to zero.

#### Response of a Phreatic Well to Atmospheric Loading

The response of an open well to atmospheric loading can be obtained through the use of (26). We assume that in the frequency range of interest, well bore storage effects are negligible. The relation between the amplitude of the water level change in an open well,  $x_0$ , and the amplitude of the atmospheric load wave,  $A$ , is then

$$x_0 = -A/\rho g + \bar{p}_0/\rho g \quad (31)$$

where  $\bar{p}_0$  is the depth-averaged pore pressure within the aquifer,  $\bar{p}$ , divided by  $\exp(i\omega t)$ :

$$\bar{p}_0 = \bar{p} \exp(-i\omega t) \quad (32)$$

Equation (31) describes the response of the well in the frequency domain and states that the change in water level in the well plus the atmospheric load (in terms of equivalent change of water level) equals the depth-averaged pore pressure change (in terms of equivalent water level).

The barometric efficiency,  $E_b$ , and phase,  $\theta$ , of the response are

$$E_b(\omega) = \left| \frac{x_0 \rho g}{A} \right| = |\bar{p}_0/A - 1| \quad (33a)$$

$$\theta(\omega) = \arg(x_0 \rho g/A) \quad (33b)$$

Equations (28) and (33) indicate that the barometric efficiency,  $E_b$ , and phase,  $\theta$ , of the response are a function of four dimensionless parameters: (1)  $R$ , the dimensionless unsaturated zone frequency; (2)  $Q_u$ , the dimensionless frequency of the aquifer; (3)  $b/d$ , the penetration ratio; and (4) the water table parameter  $\Omega$ .

Figure 4 shows the response of a water well as a function of dimensionless aquifer frequency  $Q_u$  and dimensionless unsaturated zone frequency  $R$ . The static barometric efficiency of the well under confined conditions is 0.5, the ratio  $b/d$  is assumed to be unity (i.e., there is full penetration), and the parameter  $\Omega$  is assumed to be zero. The later two assumptions will be relaxed below. Water well response is a strong function of both  $R$  and  $Q_u$ . When the ratio  $R/Q_u$  is  $10^{-4}$  or less, attenuation of air flow has little influence on response and the barometric efficiency gradually attenuates (relative to the static response under confined conditions) with decreasing frequency; the phase shows a monotonic advance with decreasing frequency. The response at  $R/Q_u$  at  $10^{-4}$  is functionally identical to the Earth tide response shown in Figure 3 when  $b/d$  equals unity; the only difference is that  $Q_u'$  has been replaced by  $Q_u$ .

For larger values of  $R/Q_u$ , however, the water table response to periodic atmospheric loading is attenuated by unsaturated zone influences. As a result, the barometric efficiency curves exceed the static-confined response over much of the frequency band analyzed and the phase curves show a slight lag. When  $R/Q_u$  is large, the water table can be effectively isolated from the atmospheric load at the soil surface and the barometric efficiency can approach unity over a wide frequency band. It should be noted that when  $R/Q_u$  is greater than 10, the barometric efficiency at the resonance frequency of the system actually exceeds unity.

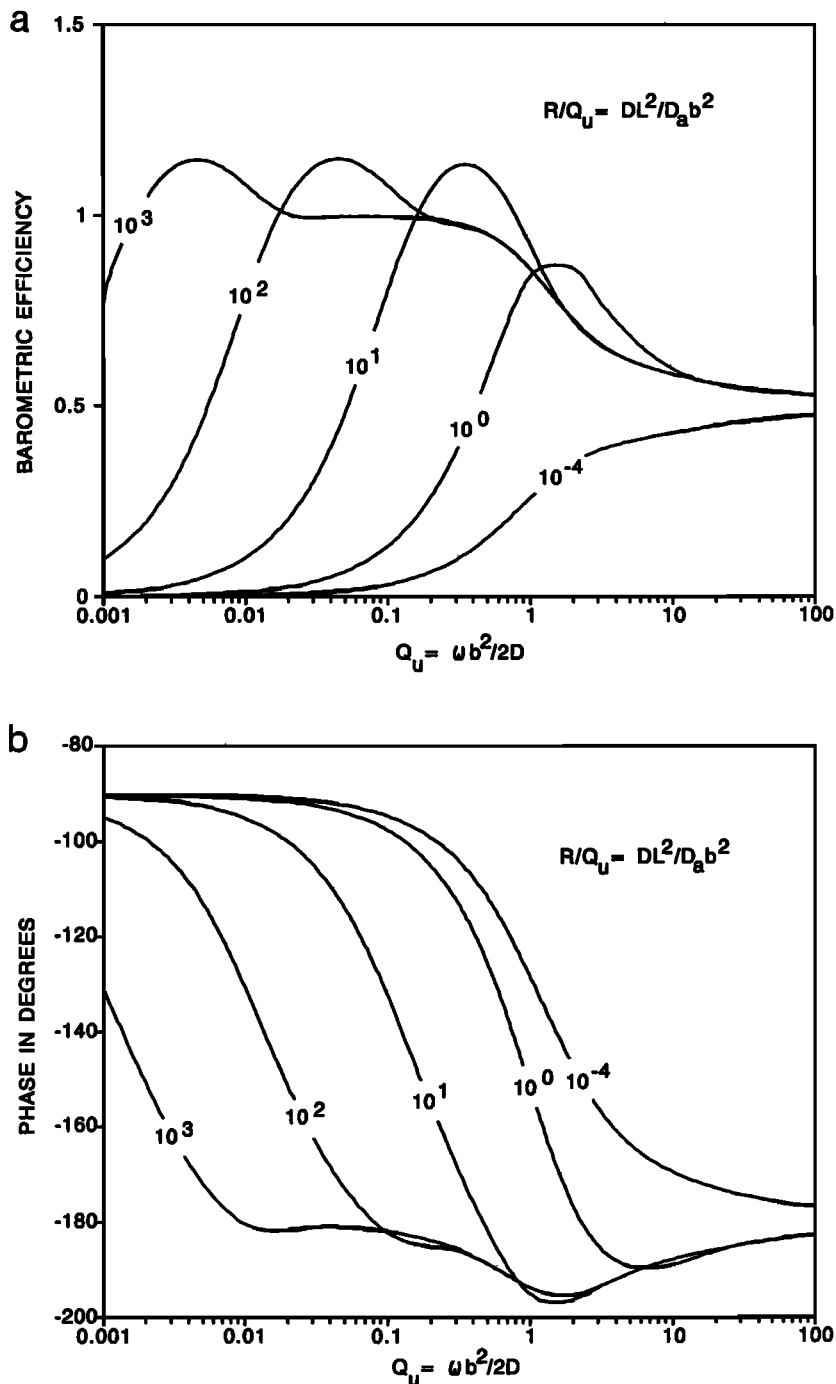


Fig. 4. (a) Barometric efficiency and (b) phase of the response of a phreatic well to atmospheric loading as a function of  $R/Q_u$  under conditions of full penetration ( $b/d = 1$ ). The water table parameter  $\Omega$  is assumed to be zero. Static confined barometric efficiency of the well is 0.5.

The influence of partial penetration can be seen in Figure 5, where we have assumed that the ratio  $R/Q_u$  is unity. As in the Earth tide case, the effect of partial penetration on the amplitude of the response is minor in nature and there is a significant change in the phase response. The effect of partial penetration is to retard phase advance at intermediate values of dimensionless frequency. For the case of  $b/d$  approaching zero (i.e., the aquifer is of infinite vertical extent), the response is qualitatively similar to the "low-frequency" response of a well tapping a partially confined aquifer to atmospheric loading given elsewhere [Rojstaczer, 1988a].

The differences in response strictly reflect the differences between the vertically averaged unconfined aquifer response and the response of a piezometer (or the response of a thin aquifer bounded above and below by partial confining layers). As in the Earth tide case, attenuation and phase shift due to water table drainage occur more rapidly under unconfined conditions.

The effects of water table fluctuations on well response are shown in Figure 6 for the case when the penetration ratio  $b/d$  equals 0 and  $R/Q_u$  equals unity. As noted earlier, water table fluctuations induced by atmospheric loading can be signifi-



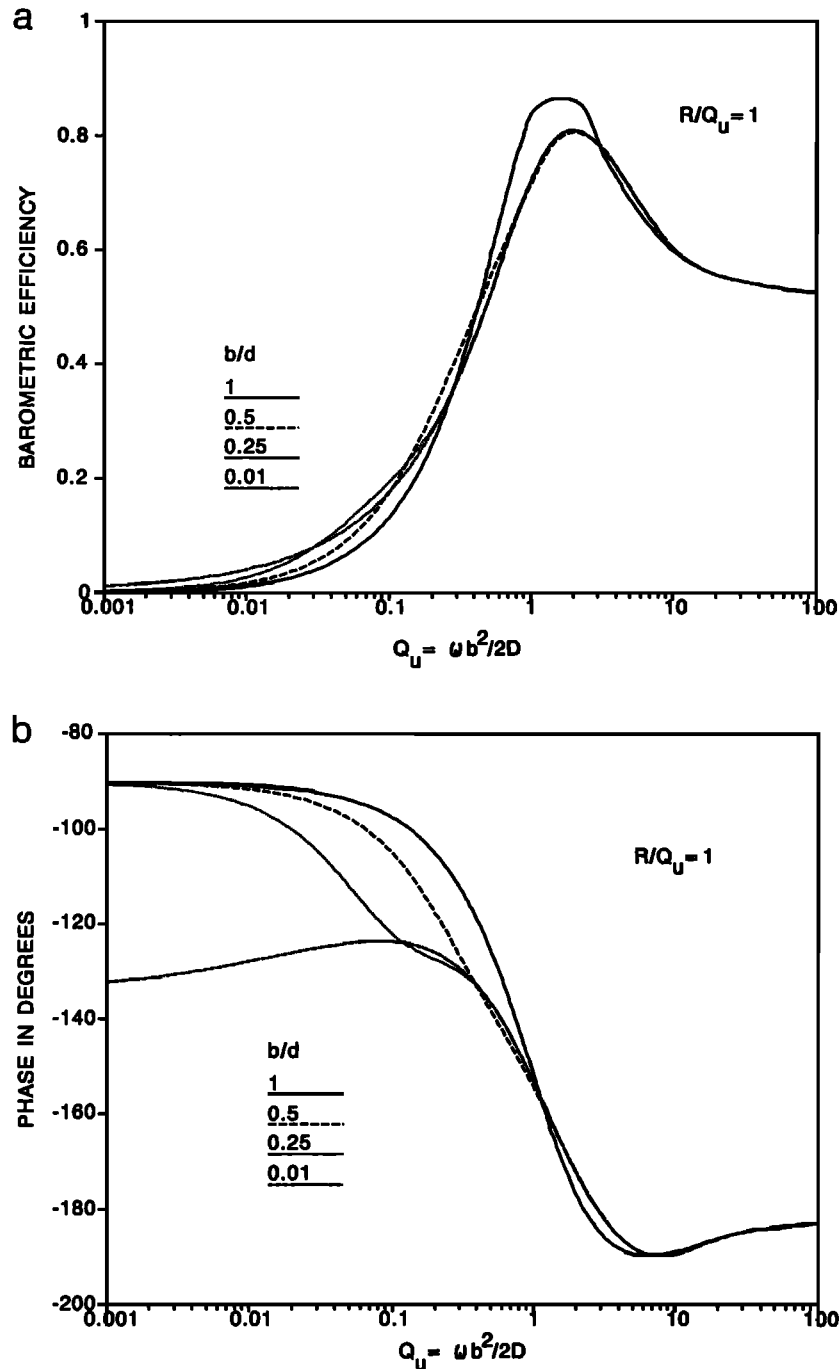


Fig. 5. Influence of partial penetration on the (a) barometric efficiency and (b) phase of the response of a phreatic well to atmospheric loading.  $R/Q_u$  is assumed to be unity,  $\Omega$  is assumed to be zero and the static confined barometric efficiency of the well is 0.5.

cant if the aquifer has a low specific yield, high hydraulic conductivity and high specific storage and if the frequency of the atmospheric loading is low. Consistent with the results noted above, water table fluctuations significantly influence well and aquifer response when the parameter  $\Omega/(1-i)$  exceeds 0.1. When this parameter exceeds a value of 10, the response approaches the static-confined response over the entire frequency band shown. As noted earlier, however, the conditions under which the parameter  $\Omega/(1-i)$  exceeds a value of 0.1 are not very common.

#### APPLICATION OF THEORETICAL RESPONSE

The theoretical results given above indicate that water well response to Earth tides and atmospheric loading can be strongly dependent on several dimensionless parameters which are a function of formation material properties and system geometry. If the response of a well can be fit to the theoretical solutions, it is possible to make estimates of or place bounds on these dimensionless parameters.

Because the Earth tide response is governed by an inher-

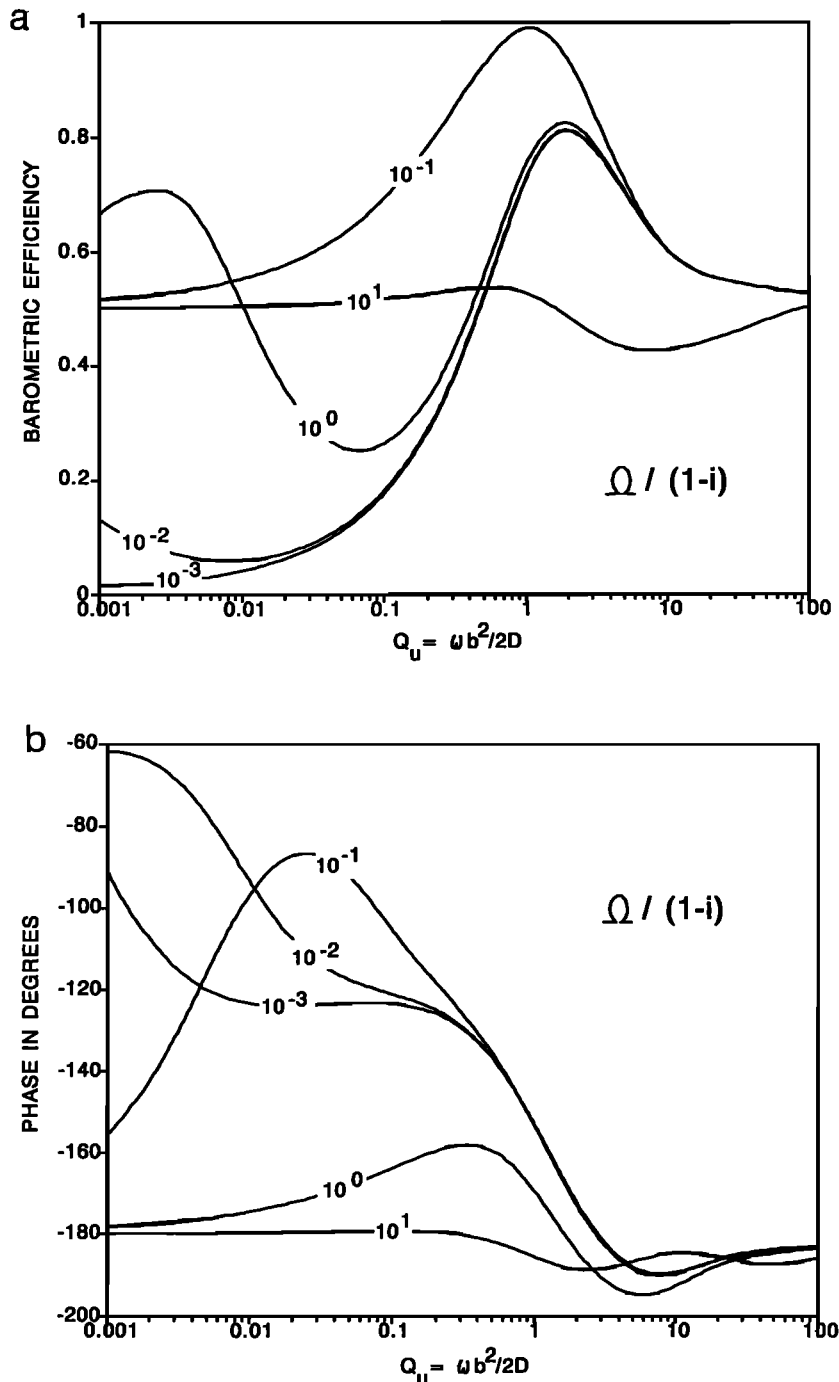


Fig. 6. Influence of water table fluctuations on the (a) barometric efficiency and (b) phase of the response of a phreatic well to atmospheric loading.  $R/Q_u$  is assumed to be unity,  $b/d$  is assumed to be zero and the static confined barometric efficiency of the well is 0.5.

ently simpler process, it would be best to analyze the frequency response to Earth tides, prior to the atmospheric loading response, to obtain the dimensionless parameters  $Q'_u$  and  $b/d$ . The next step would be to analyze the frequency response to atmospheric loading to refine an estimate of  $b/d$  and obtain estimates of  $Q_u$ , the unsaturated zone dimensionless frequency,  $R$  and possibly the parameter  $\Omega$ . While this approach would be attractive, it is not generally feasible. The first problem is that the energy contained in the Earth tide signal is generally confined to a narrow frequency band and as a result, it is difficult, if not impossible, to identify any

trends in response as a function of frequency. The second problem is that the phases of the Earth tidal constituents are not well known a priori [Beaumont and Berger, 1975; Berger and Beaumont, 1976]; as a result, it is difficult to derive anything meaningful out of the phase characteristics of the frequency response to Earth tides unless actual measurements of the Earth tide signal are made. Because of these problems inherent in the analysis of the frequency response to the Earth tide, we must rely principally on the frequency response to atmospheric loading to estimate or place bounds on both the saturated and unsaturated dimensionless param-

TABLE 1. Description of Well GD

Parameter	Value
Lateral hydraulic conductivity	$3 \times 10^{-5}$ cm/s
Depth to water table	18 m
Open interval	18–88 m

eters. If these parameters can be identified, it is possible to make estimates of the vertical pneumatic diffusivity of the unsaturated zone and vertical hydraulic diffusivity of the aquifer.

As is noted elsewhere [Rojstaczer, 1988a], the process of fitting well response as a function of frequency to dimensionless theoretical curves is analogous to the standard practice of fitting water level declines as a function of time in response to pumpage to "type curve" plots. The essential difference is that because the solutions given here are a function of frequency, there are two "type curves" which are fit simultaneously: one for admittance or barometric efficiency and one for phase.

A description of the well (GD) examined in this paper is given in Table 1. The well is located on Gold Hill near Parkfield, California. Figure 7 shows a 10-day hydrograph of this well and the corresponding atmospheric load and theoretical tidal strain at the site. The tidal strain was computed from a homogeneous Earth model with no corrections made for oceanic loading and geological inhomogeneities [Beau-mont and Berger, 1975]. Well GD taps an unconfined granodiorite aquifer of unknown but presumably considerable vertical extent. The lateral aquifer hydraulic conductivity at GD was determined from its response to a slug test. An earlier paper [Hsieh *et al.*, 1987] has analyzed the response of this well in terms of the effect of well bore storage on

Earth tide response. If it is assumed that the influence of well bore storage on the response of unconfined wells can be approximated by the theoretical response of confined or partially confined aquifers to periodic loading [Hsieh *et al.*, 1987; Rojstaczer, 1988a], the lateral hydraulic conductivity of this well indicates that well bore storage effects will be very small (attenuation less than 0.95 and phase lag less than  $10^\circ$  due to well bore storage) at frequencies less than 2 cycles/d for an aquifer with a specific storage typical of crystalline rock.

In order to compare a water well's response to the theoretical solutions, we need to determine its frequency response to Earth tides and atmospheric loading. The frequency responses or transfer functions for the well were determined from cross-spectral estimation [e.g., Bendat and Piersol, 1986] and details are discussed in Appendix C. For GD the length of the water level record examined is roughly 5 months. As shown elsewhere [Rojstaczer, 1988b], atmospheric loading and Earth tides have small signals at frequencies greater than 2 cycles/d, and we limit our analysis to frequencies no higher than this bound. The low end of the frequency band analyzed for each well was determined from the coherence squared,  $\Gamma^2$ , of the relationship between water level and atmospheric loading where the coherence squared is defined as [e.g., Bendat and Piersol, 1986]

$$\Gamma^2(\omega) = \frac{|BW(\omega)|^2}{BB(\omega)WW(\omega)} \quad (34)$$

It should be noted that  $BW$  is the cross spectrum between air pressure and water level, and  $BB$  and  $WW$  are the power spectra of the atmospheric load and the water level, respectively. This coherence squared is analogous to  $r^2$  in linear regression and represents the ability of a linear relationship

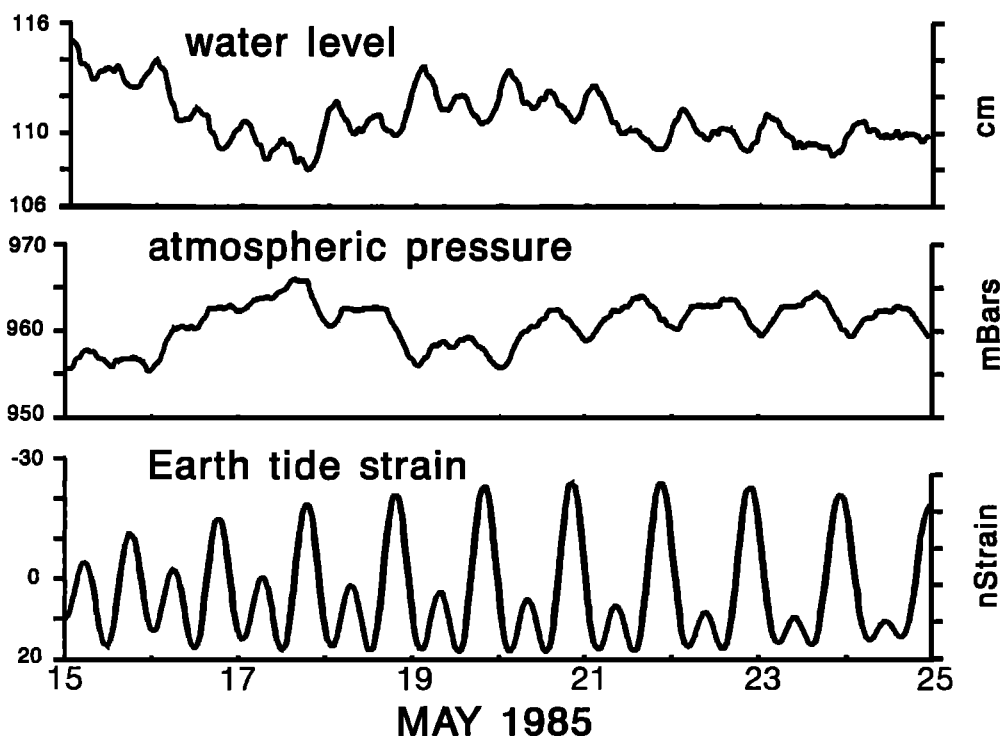


Fig. 7. Ten-day hydrograph of well GD and accompanying barograph and theoretical areal strain.

TABLE 2. Response of Well GD to Earth Tides

Component	Phase, deg	Admittance, cm/nε
$O_1$	-13 ± 10	0.24 ± 0.04
$M_2$	-1.4 ± 8	0.30 ± 0.04

Error estimates are at the 95% confidence interval.

between atmospheric load and water level to account for the water level signal at a given frequency. For the well response analyzed here, we excluded frequencies at which the coherence squared was less than 0.7; this limited analysis to frequencies greater than 0.09 cycles/d. We also excluded frequencies at which the value of the water level power spectrum was less than 0.1 cm<sup>2</sup> d/cycle because transfer function estimates at frequencies when the value of the water level spectrum was below this limit were implausible: the barometric efficiency and phase appeared to be a random function of frequency and sometimes had values which had no theoretical basis.

The admittance and phase relations for water well response to the theoretical tidal strain at the  $M_2$  and  $O_1$  frequencies are shown in Table 2. These two tidal constituents compose much of the energy in the tidal signal [Melchior, 1978] and are at frequencies where the atmospheric load signal has little energy. The sensitivity to the  $O_1$  constituent is slightly less than the sensitivity to the  $M_2$  constituent. The phase relations suggest that the theoretical homogeneous Earth tide inadequately describes temporal relations in deformation due to tidal forcing at this site. The  $M_2$  constituent is roughly in phase with the homogeneous Earth tide while the  $O_1$  constituent lags by roughly 10°. A previous analysis of the response of this well to tidal strain indicated an ambiguous phase relation for the  $O_1$  constituent and indicated that the  $M_2$  constituent led the measured Earth tide signal by roughly 10° [Hsieh *et al.*, 1987]. The difference in these two results reflects the difference in phase between the theoretical tidal strain and the actual tidal strain measured near the well [Roeloffs *et al.*, 1989].

The transfer function for the response of well GD to atmospheric loading is shown in Figure 8. Barometric efficiency is a strong function of frequency and ranges from 0.3 to 0.6 in the frequency band examined. The phase indicates that the water level lags the atmospheric load over much of the frequency band analyzed, but this phase lag diminishes with decreasing frequency. The figure also shows the model fit to the observed transfer function. The theoretical model suggests that the response in the observed frequency band is dominated by water table influences. The depth-averaged response of the aquifer never approaches the static response under confined conditions. The phase response suggests that the well only partially penetrates the granodiorite aquifer. There is no evidence in the frequency response of significant water table fluctuations in the observed frequency band.

The key parameters indicated by the model are a static-confined barometric efficiency of 0.10 and a value for both dimensionless frequencies  $R$  and  $Q_u$  of  $4.5\omega$  where frequency is in terms of cycles per day. The pneumatic and hydraulic diffusivities estimated from these values of  $R$  and  $Q_u$  are shown in Table 3. The specific storage for the aquifer under conditions of surface loading is estimated elsewhere

[Rojstaczer and Agnew, 1989] and is determined from the inferred static-confined barometric efficiency and areal strain sensitivity of the well. The specific storage is used in conjunction with the hydraulic diffusivity to estimate the aquifer's vertical hydraulic conductivity. The vertical hydraulic conductivity is a factor of 3 less than the inferred lateral hydraulic conductivity of the aquifer, suggesting that the rock tapped by the well possesses modest, if any, hydraulic anisotropy. The lower bound for  $b/d$  indicated by the model fit is 10, suggesting the presence of significant permeability down to depths of at least 800 m. The upper bound for  $\Omega/(1-i)$  is unity; this bound is consistent with the value for vertical hydraulic conductivity estimated here and the values for specific storage and porosity estimated elsewhere [Rojstaczer and Agnew, 1989].

## CONCLUSIONS

The water level response of wells which tap water table aquifers to Earth tides and atmospheric loading is qualitatively similar to the partially confined response detailed elsewhere [Rojstaczer, 1988a, b]; it is dependent on the elastic and fluid flow properties of the aquifer as well as the air flow properties of the material overlying the aquifer. Water well response can be dependent on the frequency of the deformation and reflects the response of the unconfined aquifer averaged over the saturated depth of the well. As in the partially confined response, attenuation and amplification relative to the static-confined response of the aquifer can occur in theory and is observed in the well examined; phase lags and advances observed in response to atmospheric loading also have a theoretical basis.

Because the response of water table wells to aquifer deformation is so similar to the response of wells which tap partially confined aquifers, it is not possible to unambiguously identify the type of aquifer tapped by the well on the basis of this response. This determination can only be made by pump tests or by a thorough knowledge of the hydrogeologic setting.

The theoretical response of a well to Earth tides and atmospheric loading can be used in conjunction with the observed response of a water well as a function of frequency to yield estimates of or place bounds upon the vertical fluid flow properties of the aquifer and the pneumatic diffusivity of the unsaturated zone. The response of the well examined here to atmospheric loading indicates that wells need not tap aquifers with particularly low values of hydraulic conductivity to achieve partial isolation from the water table response over periods of days to weeks. Wells which are open over a thick interval can be influenced by the confined response of the aquifer to Earth tides and atmospheric loading even if vertical permeabilities are quite high.

While the well examined here responds strongly to Earth tides, it is difficult to utilize this information to infer aquifer fluid flow properties unless the tidal strain is also measured. These measurements are difficult and expensive to obtain [Agnew, 1986]. However, theoretical tidal strain calculations are helpful in using well sensitivities to tidal strain to infer formation compressibilities as is shown elsewhere [Rojstaczer and Agnew, 1989].

For the well examined, the barometric response is a strong function of frequency and estimates of the controlling fluid flow parameters can be somewhat readily made. It should be

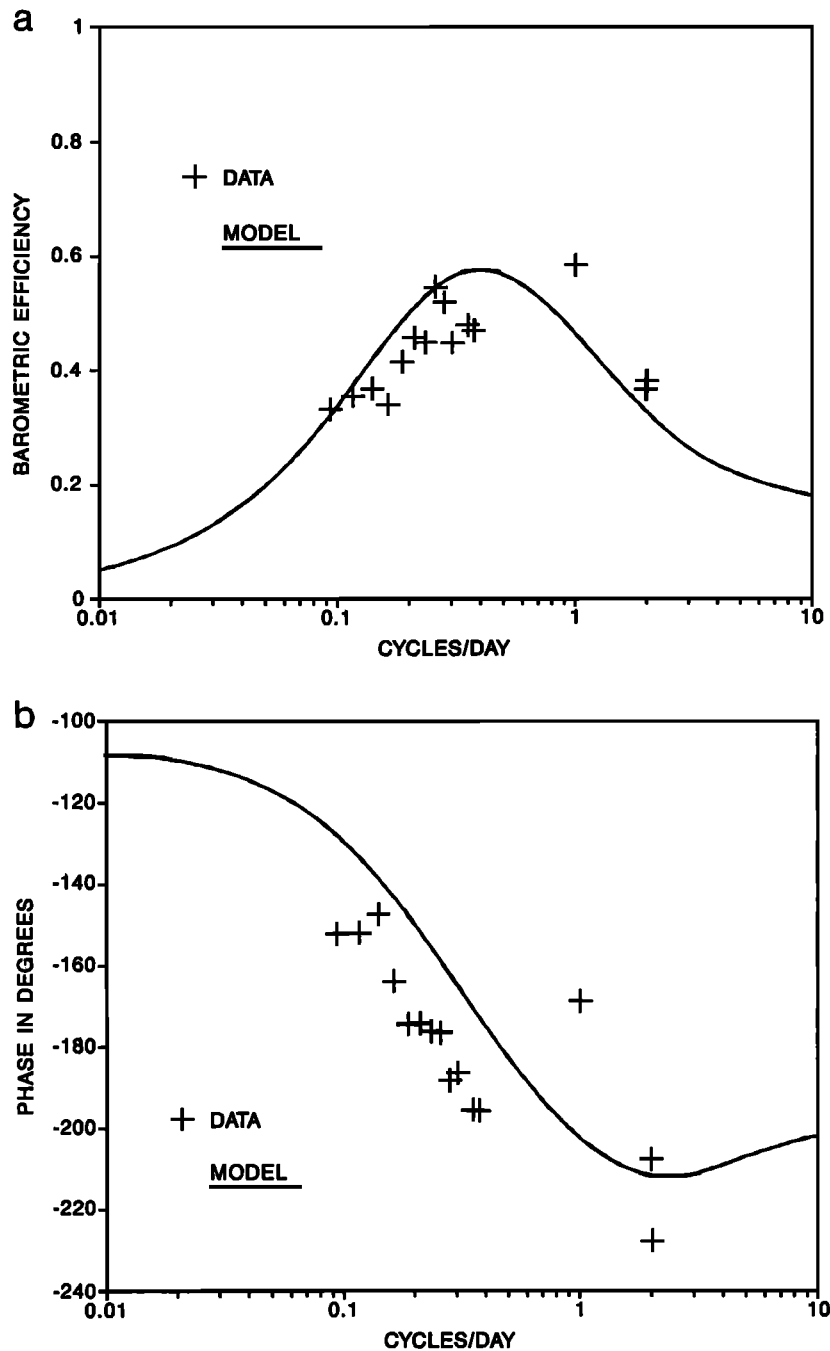


Fig. 8. Response of GD to atmospheric pressure in terms of (a) barometric efficiency and (b) phase. Fit to data is solid curve denoted as MODEL.

noted that the parameters which control response may not always be identifiable. When the depth to the water table is shallow, it may be possible to place only a lower bound on the pneumatic diffusivity. Under conditions where the vertical hydraulic conductivity of the aquifer is relatively high and the saturated depth of the well is relatively thin, the depth-averaged response of the aquifer may only be weakly influenced by the static-confined response of the aquifer in the frequency band analyzed; for these situations it may only be possible to place a lower bound on the vertical hydraulic diffusivity of the aquifer. Under conditions where the vertical hydraulic conductivity of the aquifer is very low and the

saturated well depth is very thick, the barometric response may be largely independent of water table influences throughout the frequency band examined here; for these situations it may be possible to place only an upper bound on the vertical hydraulic diffusivity. Finally, if the well taps a zone utilized for water supply, the influence of pumpage will likely mask the tidal or barometric response of the well. Although cross-spectral estimation of the response of wells to Earth tides and atmospheric loading may have limited application, the results given here indicate that under certain conditions it can yield some useful information about the material properties of unconfined aquifers and the unsaturated zone.

TABLE 3. Estimate of Fluid Flow Properties of Aquifer Tapped by Well GD

Parameter	Value
$R/Q$	1
Unsaturated zone pneumatic diffusivity, $\text{cm}^2/\text{s}$	4
Vertical hydraulic diffusivity, $\text{cm}^2/\text{s}$	60
Vertical hydraulic conductivity, $\text{cm}/\text{s}$	$1 \times 10^{-5}$

#### APPENDIX A: SOLUTION TO THE RESPONSE OF AN UNCONFINED AQUIFER TO DEFORMATION INDUCED BY EARTH TIDES

Aquifer response to periodic areal strain,  $A \cos(\omega t)$ , is governed by the equation [Rojstaczer and Agnew, 1989]:

$$D' \frac{\partial^2 p}{\partial z^2} = \frac{\partial p}{\partial t} + \rho g A_s' A \omega \sin(\omega t) \quad (\text{A1})$$

If the aquifer is phreatic, of thickness,  $d$ , and underlain by an aquiclude, the appropriate linearized boundary conditions are

$$\partial p(0, t)/\partial z = -(S_y/K_z) \partial p(0, t)/\partial t \quad (\text{A2a})$$

$$\partial p(d, t)/\partial z = 0 \quad (\text{A2b})$$

The solution of (A1) subject to the boundary conditions (A2) is easily solved by employing complex notation. Taking  $p$  to be complex,

$$p(z, t) = F(z) \exp(i\omega t) \quad (\text{A3})$$

and substituting into (A1) and (A2) we obtain

$$D' F'' - i\omega F = -\rho g A_s' A i\omega \quad (\text{A4a})$$

$$F'(0) = -i\omega S_y F(0)/K_z \quad (\text{A4b})$$

$$F'(d) = 0 \quad (\text{A4c})$$

where the prime following  $F$  denotes differentiation, and all exponential terms have been divided out. The solution to (A4) is

$$F(z) = -\rho g A_s' A [\exp(-(i+1)\sqrt{Q'})/H_1 + \exp((i+1)\sqrt{Q'})/H_2 - 1] \quad (\text{A5})$$

Combining (A3) and (A5) yields the solution given in (3).

#### APPENDIX B: SOLUTION TO THE RESPONSE OF AN UNCONFINED AQUIFER TO PERIODIC ATMOSPHERIC LOADING

The response of an unconfined aquifer to periodic atmospheric loading,  $A \cos(\omega t)$ , is governed by [Rojstaczer and Agnew, 1989]

$$D \frac{\partial^2 p}{\partial z^2} = \frac{\partial p}{\partial t} + \gamma A \omega \sin \omega t \quad (\text{B1})$$

The linearized boundary conditions for the response of a phreatic aquifer of thickness  $d$  are

$$\partial p(0, t)/\partial z = -(S_y/K_z) \partial p(0, t)/\partial t$$

$$+ (S_y/K_z)[MA \sin(\omega t) - NA \cos(\omega t)] \quad (\text{B2a})$$

$$\partial p(d, t)/\partial z = 0 \quad (\text{B2b})$$

As in the Earth tide case, we take  $p$  to be complex:

$$p(z, t) = F(z) \exp(i\omega t) \quad (\text{B3})$$

and substitute into (B1) and (B2) to obtain

$$DF'' - i\omega F = -A\gamma i\omega \quad (\text{B4a})$$

$$F'(0) = -i\omega S_y[F(0) - AM + AiN]/K_z \quad (\text{B4b})$$

$$F'(d) = 0 \quad (\text{B4c})$$

The solution to (B4) is

$$F(z) = (M - iN - \gamma)A[\exp(-(i+1)\sqrt{Q})/H_1 + \exp((i+1)\sqrt{Q})/H_2] + \gamma A \exp(i\omega t) \quad (\text{B5})$$

Combining (B3) and (B5) yields the solution given in (22).

#### APPENDIX C: METHOD BY WHICH THE TRANSFER FUNCTIONS OF WATER LEVEL TO EARTH TIDES AND ATMOSPHERIC LOADING WAS DETERMINED

The transfer functions between water level, Earth tides and atmospheric loading were found using cross-spectral estimation [Bendat and Piersol, 1986]. For the water well record examined here, the transfer functions were obtained by (1) removing the mean and the long-term trend from the water level, Earth tide and atmospheric loading time series; (2) determining the power spectra and cross-spectra for the water well record, the theoretical areal strain produced by the Earth tides and the local atmospheric pressure record; (3) solving the following system of complex linear equations for every frequency:

$$\begin{vmatrix} BB & BT \\ TB & TT \end{vmatrix} \begin{vmatrix} HB \\ HT \end{vmatrix} = \begin{vmatrix} BW \\ TW \end{vmatrix} \quad (\text{C1})$$

where  $BB$  and  $TT$  denote the power spectra of the atmospheric pressure and Earth tides, respectively,  $BT$  and  $TB$  denote the cross spectrum and complex conjugate of the cross spectrum, respectively, between atmospheric loading and Earth tides,  $BW$  and  $TW$  denote the cross spectra between atmospheric loading and water level and Earth tides and water level, respectively, and  $HB$  and  $HT$  denote the transfer function between water level and atmospheric loading and water level and Earth tides, respectively. The Earth tides were included in the analysis in the frequency band 0.9–2.0 cycles/d, a band which contains almost all of the energy in the Earth tide signal. At frequencies less than 0.9 cycles/d, the atmospheric loading transfer function  $HB$  was determined simply by taking the ratio  $BW/BB$ . Because energy levels in the Earth tide signal are very small for frequencies less than 0.9 cycles/d, the Earth tide transfer function,  $HT$ , was not estimated at these low frequencies. Further details on how the transfer functions were determined can be found elsewhere [Rojstaczer, 1988b].

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F. S. Riley and S. Rojstaczer, U.S. Geological Survey, MS 439, 345 Middlefield Road, Menlo Park, CA 94025.

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