Response Time of a Thermometer

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The response time of a measuring instrument is often ignored because it is generally much less than that of the system under investigation. In some situations, however, this is not necessarily a good assumption. The basic thermodynamic measuring instrument, the thermometer, provides an interesting example. Not only can the response time be important, but, unlike other measuring instruments, this time is critically dependent upon the nature and state of the system (fluid) being measured.

The response time of a thermometer is governed by Newton's law of cooling,[1,2] a law that continues to be of interest to the science teaching community.[3–5] The following discussion may provide insights for the introductory student on the importance of the characteristics of the measuring instrument in the experimental process. It also provides an interesting application of the first law of thermodynamics together with Newton's law of cooling.

Response Time

Let's consider the quite common situation where a solid, initially at a temperature T_0, is immersed in a liquid at a temperature T_f. The heat transfer occurs *primarily* by convection, and Newton's law of cooling, $Q = hA(T_f - T)$, is applicable. Note that "the heat transfer coefficient, h, is sometimes called the *film conductance* because of its relation to the conduction process in the thin stationary layer of fluid at the wall surface."[1]

Applying the first law in the often-convenient form "Heat Lost = Heat Gained," we can equate the amount of heat transferred by convection
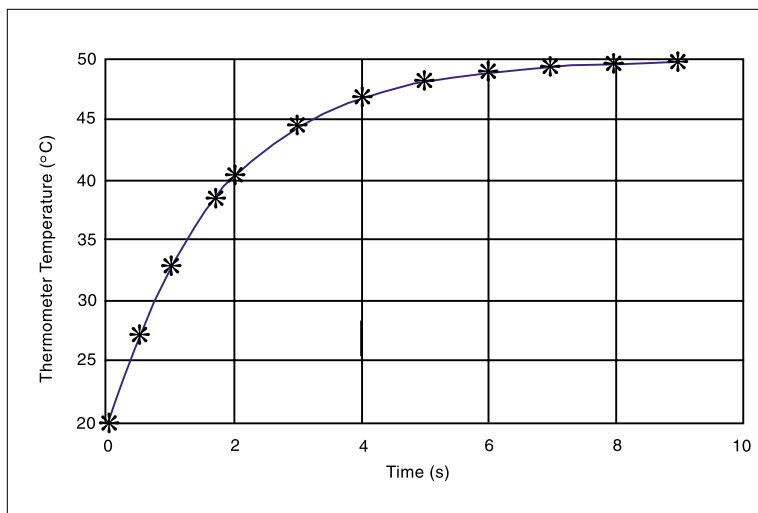

Fig. 1. Response time of thermometer when going from room-temperature air to still water at 50 °C.
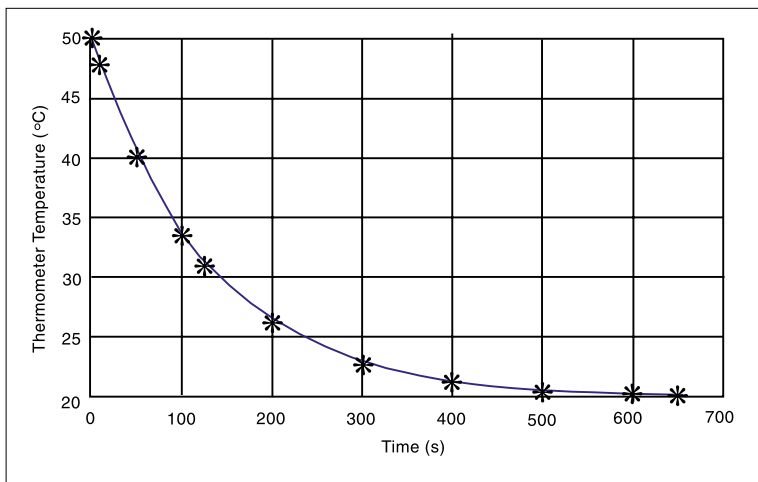
Fig. 2. Response time of thermometer when going from water at 50 °C to still air at 20 °C.

from the fluid in a time interval dt to the thermal energy gained by the solid:

$$hA(T_f - T)\, dt = mC\, dT \qquad (1)$$

where, h is the coefficient of convective heat transfer, A is the surface area of the solid, m is the mass of the solid, and C is its specific heat.

Collect the constants and make the substitution,

$$\tau = mC/hA \qquad (2)$$

Then, $\quad (T_f - T)\, dt = \tau\, dT \qquad (1a)$

or, $\quad dT/dt = (T_f - T) / \tau \qquad (1b)$

The solution of this first-order linear differential equation by separation of variables is accessible to high-school students with calculus. Given the boundary condition that $T = T_f$ at $t = 0$, the solution is,

$$T = T_f - (T_f - T_o) \, e^{(-t/\tau)} \qquad (3)$$

where τ is the "time constant" and a measure of the response time of the system.

The quantity $1/hA$ is called the "thermal resistance," R_t, for convective heat transfer. Also, the quantity mC is usually referred to as the "thermal capacitance," C_t.[1,2] Making the substitutions into Eq.(2),

$$\tau = R_t \, C_t \qquad (4)$$

This formula is more familiar from electrical studies in the form, $\tau = RC$. In an electrical circuit in which a capacitor is charged or discharged through a resistance, the time constant is the time to reach 67% of the final or zero charge, respectively. The time to final charge or discharge is generally approximated at 5τ.

The Thermometer

The solid in question is a mercury in glass thermometer that is immersed in a liquid. The temperature T indicated by the thermometer is the mercury temperature which will only equal T_f under equilibrium conditions.

The cylindrical bulb of a typical laboratory thermometer is about 3 mm in diameter and 12 mm long. This gives a surface area of 1.27×10^{-4} m^2 and a volume of 8.48×10^{-8} m^3. The density of mercury is 1.355×10^4 kg m^{-3}, so that the mass of the bulb is about 1.15×10^{-3} kg. The specific heat of mercury (at 25 ^{o}C) is 139.4 J kg^{-1} K^{-1}.

Let the thermometer be initially at room temperature, 20 ^{o}C, and then placed inside a water bath held at a constant temperature of 50 ^{o}C. The coefficient of convective heat transfer for *still* water (not stirred or otherwise agitated) is approximately 750 W m^{-2} K^{-1}. The time constant, calculated from Eq. (2), is $\tau = 1.7$ s. The response of the thermometer is shown in Fig. 1.

Now consider the opposite scenario in which the same thermometer is initially at equilibrium with the 50 ^{o}C water bath and is removed into still air at a temperature of 20 ^{o}C. The coefficient of convective heat transfer for still air is on the order of 10 W m^{-2} K^{-1}. In this case the time constant is about 125 s. The response of the thermometer is shown in Fig. 2.

Discussion

It is interesting and instructive to note that the response time of thermometer is critically dependent upon the medium in which the measurement takes place. While the mass of mercury, m, its specific heat capacity, C, and the surface area, A, are constants for a particular thermometer, the coefficient of convective heat transfer is not.

This places the thermometer in a rather unique position as a measuring instrument. All other instruments have a response time that is solely a function of the instrument itself.

If the experimental design is such that the temperature to be measured will *not* remain at this temperature for a time greater than about 5τ, then the temperature measured by the thermometer will be in error.

With electrical measuring instruments, for example, the extent to which the system under consideration is affected by the instrument itself is determined by the input resistance (impedance) of the instrument itself. For accurate voltage measurements we desire high input impedances.

The thermometer, however, requires low thermal resistance as characterized by the coefficient of heat transfer of the fluid for the fastest response time and least effect on the system being measured. This particular relationship between response time and input impedance is another interesting feature of the thermometer as a measuring instrument.

Conclusions

The coefficients of convective heat transfer are in many cases not known to better than about $\pm 25\%$.[1] The theoretical discussion presented here can be developed into experimental determinations of such coefficients under various experimental conditions. Such investigations can provide challenging undergraduate research or science fair projects for introductory students.

A better appreciation of the role of the response time of a measuring instrument in the experimental process may be gained by considering the rather anomalous case of the thermometer.

References

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