

Development of Bulk Metallic Glass Matrix Composites (BMGMC) by Additive Manufacturing: Modelling and Simulation – A Review: Part B

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Keywords: bulk metallic glass matrix composites, additive manufacturing, modeling and simulation

Abstract. Bulk metallic glasses (BMGs) and their composites (BMGMC) have emerged as competitive materials for structural engineering applications exhibiting superior tensile strength, hardness along with very high elastic strain limit. However, they suffer from lack of ductility and subsequent low toughness due to the inherent brittleness of the glassy structure which render them to failure without appreciable yielding owing to mechanism of rapid movement of shear bands all throughout the volume of the material. This severely limits their use in the manufacture of structural engineering parts. Various theories and mechanisms have been proposed to counter this effect. Introduction of secondary ductile phase in the form of *in situ* nucleating and growing dendrites from melt during solidification have proved out to be best solution of this problem. Nucleation and growth of these ductile phases have been extensively studied over the last 16 years since their introduction for the first time in Zr-based BMGMC by Prof. Johnson at Caltech. Data about almost all types of phases appearing in different systems have been successfully reported. However, there is very little information available about the precise mechanism underlying their nucleation and growth during solidification in a copper mould during conventional vacuum casting and melt pool of additively manufactured parts. Various techniques have been proposed to study this including experiments in microgravity, levitation, synchrotron light and modelling and simulation. In this report, which is Part B of two parts comprehensive overview, state of the art of development, manufacturing, characterisation and modelling and simulation of BMGMCs is described in detail. Evolution of microstructure in BMGMC during additive manufacturing have been presented with the aim to address fundamental problem of lack in ductility along with prediction of grain size and phase evolution with the help of advanced modelling and simulation techniques. It has been systematically proposed that 2 and 3 dimensional cellular automaton method combined with finite element (CAE) tools programmed on MATLAB® and simulated on Ansys® would best be able to describe this phenomenon in most efficient way. Present part B focuses on methodology by which modelling and simulation can be adopted and applied to describe evolution of microstructure in this complex class of materials.

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2.1.1 Introduction

Present section of this Part B deals with evolution of microstructure during the processing of BMGMC in an incipient transient liquid melt pool formed in AM. This analysis is divided into two sections. The first section deals with the evolution of the melt pool as a result of the interaction of highly localised, focused laser light with matter (metal powder). This results in the formation of a melt pool whose shape, size, geometry and transient behaviour is very much a function of the heat transfer coefficients (HTC) evolving at every step of its formation (melting and homogenisation)

and dissipation (solidification). Solidification in this section is considered by a modified general (classical) nucleation theory (CNT). Once formed, this pool travels as the laser traverses its path all along the powder bed dictated by CAD geometry at the back end. The second section deals with the microstructural evolution during solidification which is primarily a *solute diffusion* and *capillary action* dominated phenomena. This is dealt with by microscopic 2D and 3D probabilistic CA models which model nucleation and equiaxed dendritic growth resulting in the formation of the microstructure within the liquid melt pool as it solidifies (Note: only “Vitrification (glass formation)” effects are taken into account and devitrification (heat treatment) is not considered). The evolution of microstructure is checked against the variation of number density, size and distribution of ductile phase in the glassy matrix. Inoculants for ductile phase formation were selected previously by edge to edge matching (E2EM) [280, 281].

2.1.2 A brief general introduction to modelling and simulation

Although in use since ancient Roman times [302], modelling and simulation picked its interest and achieved pinnacle in modern day scientific and engineering sectors with the advent of computer technology which came not more than two decades ago. Now, it has proved itself to be an important integral part of product and part design, product development as well as production, utilisation and enhancement of properties. Various branches of modelling and simulation, ranging from part scale modelling which involves development of codes of theorems in advanced computing platform such as Java®, C, C++ and MatLab Simulink® to their simulation as in customised simulation packages such as Solidworks®, Ansys® and Catia® to performing complex atomistic simulations in dedicated proprietary software, have now become an integral part of design procedure in major industrial clusters. Its use in research and development is also becoming an important part of whole process to eliminate so called “trial and error” methods which are not only time consuming but are energy, materials and resources extensive. In materials science and engineering mainly two of its branches are routinely used. These are “part scale modelling and simulation” and “atomistic modelling and simulation”. The former is used for the complete design of complex machinery segments, equipment, assemblies, and their materials of fabrication and property prediction in different regions as a function of extrinsic parameters such as heat, velocity, pressure, time whilst the latter is used for prediction, estimation and improvement in atomic-scale properties using theories of atomic configuration and arrangement mainly relying on intrinsic parameters (such as specific heat / latent heat, heat capacity, density, heat of fusion and atomic fit or misfit). The unique ability of atomistic modelling and simulation is that it uses atomic functions and their variables to generate knowledge about their behaviour under various impulses. In both cases, the use of these methods are big help and support in saving time, materials, resources as well as improve functionality and in-service property development and behavior prediction.

The exponential rise in the use of modelling and simulation with the advent and progress of computer technology and increase of computing power of machines gave rise to greater flexibility in the design and development process. Many difficult, or in some cases, impossible to envisage problems can now be simulated using these computing platforms. These include simulation of water flow and its patterns in rivers and channels, simulate of interior of sun, stars and other heavenly bodies, cosmic events and nuclear engineering problems. However, despite of these advantages, there are still situations and applications which limits the use of modelling and simulation techniques. These include, unavailability of strong efficient computing algorithms (with lesser approximations) needed for the replication of actual real world situations, unavailability of real world experimental data (physical constants and thermo-physical properties) needed to simulate a particular problems, unavailability of more accurate deterministic or non-probability based models using actual situations rather than basing their outcome on statistics. Owing to these reasons, there is still need for further investigation and removal of bottlenecks from modelling and simulation problems and it is envisaged that their popularity is still at arm’s length.

2.1.3 Modelling and simulation of heat transfer in liquid melt pool – Solidification

As the microstructure formed during SLM is mostly columnar [303], it is a good indicator that heat flux transfer from melt is highly unidirectional thus heat transfer from bottom is a transient 1D process. Although, heat is lost from the material in x-y plane i.e. perpendicular to the z – direction (perpendicular to build direction), its contribution is so low that it can be safely ignored. However, this was an old concept. New experimental observations have proposed a new concept according to which during SLM, a melt pool is formed, where the shape of this pool is a function of:

- a. Laser power (laser beam intensity).
- b. Presence of thermocapillary convection (marangoni convection).

In even more advanced and recent models, [304, 305] the transfer of heat after its generation is considered by three main parameters:

- a. Heat transfer due to convection.
- b. Evaporation (i.e. formation of plasma) (this results in re-radiation (inverse radiation)).
- c. Conduction from the bottom and the side walls

This is very recent and advanced approach which, however, ignores marangoni convection effects. Overall, the heat transfer phenomena associated with the solidification of metal in a liquid melt pool in AM is associated with three processes:

- Generation of heat (laser matter interaction).
- Assimilation of heat (melting and stages of solidification).
- Extraction of heat.

2.1.3.1 Generation of Heat (Laser-matter interaction)

This is the first stage of AM in which heat is generated. The problem in this stage is related with impingement of light of certain intensity (I) on a solid surface for a certain amount of time which may results in production of heat. This interaction can be explained in terms of law known as the “Beer lambert law”.

2.1.3.1.1 Beer lambert law for AM

Consider a thin layer of powder with thickness d_1 , on a flat disk substrate of refractory metal with thickness d_2 and radius r uniformly illuminated by light of intensity I .

For absorptivity of powder (or melt) assuming uniform temperature throughout the disk, the temperature evolution is

$$(\rho_1 c_1 d_1 + \rho_2 c_2 d_2) \frac{dT}{dt} = A(T)I - Q(T) \quad (11)$$

where

- $A(T)$ = Absorptivity
- $Q(T)$ = Thermal loss (convective and radiative)
- I = Intensity
- ρ_1 = Density of powder
- ρ_2 = Density of substrate
- c_1 = Specific heat of powder
- c_2 = Specific heat of substrate
- d_1 = Thickness of powder
- d_2 = Thickness of substrate

Heat generated by this process is used for melt pool generation (its morphology, homogenisation, and holding (generation of supercooled liquid (SCL) region and its progression)).

2.1.3.2 Assimilation of heat (Melting and stages of solidification)

As the heat generated above interacts with metal powder, it causes its melting and generation of liquid melt pool. The behaviour of a certain metal / alloy in the melt pool can be explained by its cooling curve which is briefly described below.

2.1.3.2.1 General form of cooling curve

A cooling curve of a metal / alloy is a plot of the variation of temperature with time. It has different regions which embodied various types of information. Cooling curves can have different shapes depending on the metal or alloy type. A schematic cooling curve is shown in Fig – 11 for a single component pure metal (without any inoculants).

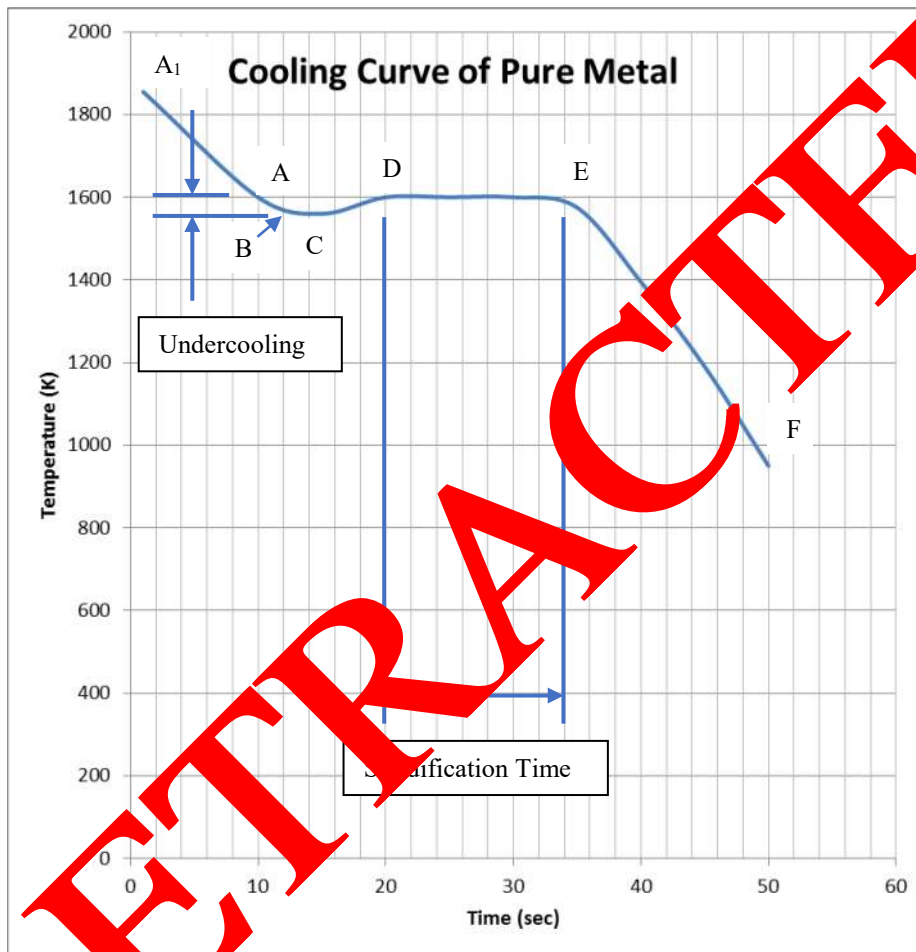


Fig. 11: Cooling curve for a single component pure metal (without any inoculants).

Its distinctive regions are explained as following;

Region above A₁: This is the region in which metal is in its complete liquid-state and can be described by only melting and liquid-state homogenisation. Heat carried by metal in this region is “super heat” only and lost in the form of specific heat ($mc_p\Delta T$). This homogenisation in turn depends on type of melting (gas / solid (coal) / liquid (oil) fired crucible furnace melting, electric (resistance / induction / arc) melting) and subsequent melt treatment. (Note: Homogenisation is required by some external means in case of all modes of melting. Only induction furnace is manifested by self-homogenisation due to phenomena of induction currents).

Region A₁ – A: This is a region which is characterised by the loss of super heat until the first arrest point A. (Point at which the first nucleant form – explained in detail in later sections). This is also called the start of solidification. In pure metals it is a sharp point (melting point) while in alloys, it can be a range (melting range). In BMGMCs / multicomponent alloys, it is also called start of the

super cooled region (SCL). This region is followed by undercooling (ΔT_n) region which is described below

Region A – D: This is the most important region of cooling curve (present case) for pure metals. In this region, metal cools down to a specific temperature characterised by a certain minimum amount of energy (activation energy for nucleation) needed to overcome a barrier of energy (energy barrier to nucleation) to create a liquid – solid (L – S) interface eventually leading to formation of a stable nuclei out of the melt. This region is further divided into two regions. A – C and C – D

Region A – C: This is region in which undercooling occurs, heat is extracted, the temperature drops and shape of cooling curve goes down. This is characterised by two energies described in the above paragraph.

Region C – D: This is the region in which heat energy is absorbed, temperature is gained and shape of curve goes up. This is called recalescence.

Notes:

- (a) Recalescence is gain in temperature as a result of thermal fluctuations caused by phase transformations occurring within solidifying melt / alloy. In present case, phase change is solid formation within undercooled liquid while thermal fluctuations are described by release of heat in the form of heat of fusion.
- (b) Region A – C is characterised by another point, Point “B” occurring in the middle of cooling curve. This is specifically shown in Fig – 11 as intermediate point of Supercooled liquid region (SCL). For the present case model (transient heat transfer conditions will be modelled at this point as well to get better understanding of phenomena occurring in SCL in BMGMCs).

Region D – E: This is the region at which (after passing point D), metal losses all its heat of fusion (mH_f). In this region transformation occurs at constant temperature in such a way that all liquid gets transformed into complete solid (all the equiaxed grains formation at mould wall (Cu mould casting) / at surface of inoculant (heterogeneous nucleation - not present case), “equiaxed – columnar” transition, growth of columnar dendrites, CET and growth of all equiaxed dendrites accomplishes). This is also called the solidification time.

Region E – F: This is the region in which solid cools. That is, after all liquid gets transformed into solid, the solid casting cools down to room temperature. This again occurs after a sharp invariant point (point F) in case of pure metal and after a range in case of multicomponent alloys.

2.1.3.2.2 Cooling curve for well inoculated Zr-based *in-situ* dendrite BMGMCs

Shape of cooling curve changes its form as melt is changed from single component to binary to multicomponent alloys. This can be explained in the form of various cases.

Case 1: Well inoculated single component melt: In these types of alloys, undercooling / undercooled region (ΔT_n) diminishes and is almost absent. Inoculation with potent nuclei serves as active nucleation sites and triggers heterogeneous nucleation as the alloy reaches its first invariant point. Thus, no undercooling happens and solid alloy directly starts cooling as all liquid gets transformed to solid at constant temperature.

Case II: Binary alloys without inoculants (slowly cooled)

In these types of alloys cooling occurs in following steps

1. Distinct undercooling occurs (characterised by drop and gain (recalescence) of temperature)
2. It is followed by region of constant temperature cooling which is called *local solidification*. This is only visible in case of very fluid alloys in which mushy region is very fluid / less viscous (not BMGMCs). This region is absent in most multicomponent (industrial) alloys as their solidification is dominated by mushy zone. (Note: BMGMCs are special case of alloys in which mushy region is extensively dominated but another phenomenon known as “sluggishness” governs the solidification. In these alloys, three laws [7] which describe BMGMC formation and evolution make sure that not only sluggishness dominates kinetics but it also ensures “glass formation” (i.e. retaining supercooled liquid at room temperature).
3. Alloy solidification range (it depends on alloy. In slowly cooled binary alloys (most laboratory conditions), this is very clearly marked (usually bears an intermediate shape))
4. At the end of this range, alloy becomes stable momentarily at constant temperature (usually negligible in most industrial castings) at which nuclei (dendrite and branches) grow and fills interdendritic arm spacing and other small liquid pockets. This is marked by end of solidification. (In some cases, it is also characterised by start of CET and then growth of equiaxed grains)
5. Following this point, solid alloy cools to room temperature or below room temperature (in case of cryogenic cooling).

Note: For theoretical analysis, cooling curve can be of any type of combination between type of alloy (single component, binary and multicomponent), method of cooling (slow or fast) and inoculation (zero inoculation and well inoculated). All cases can be drawn following rules of thermal transitions and kinetics. For simplicity and efficiency, we will jump to cooling curve of Multicomponent alloy (BMGMCs) fast cooled and well inoculated (present case).

Case III: Multicomponent alloys with inoculants (fast cooled) (present case BMGMCs)

In these types of alloys, cooling occurs following below steps. (Fig – 12)

1. No undercooling occurs (as there is sufficient amount (number) of potent nuclei which serve as sites for active nucleation triggering heterogeneous nucleation prior to loss of temperature (drop of cooling curve) and gain of temperature (recalescence – rise of cooling curve)).
2. This is followed by region of constant temperature at which all liquid get transformed into solid. However, in these alloys, this region is very small (because of presence of marked mushy zone).
3. Instantly after this region, alloy enters in alloy “solidification range”. As the alloy is very fast cooled, this region is again not very clearly identified which is typical behaviour in case of fast cooled castings.
4. Following this, again alloy momentarily enters in brief constant temperature zone which marks start of CET and growth of equiaxed grains (B2 CuZr phase equiaxed dendrites) until all liquid gets transformed into solid (end of solidification). This again is not very distinct as other phenomena (suppressing kinetics) dominate.
5. Finally, after this, BMGMC solidifies to room temperature.



Fig. 12: Cooling curve for a multicomponent alloys with inoculants (fast cooled) (BMGMC).

Note: Shape of cooling curve in case of slowly cooled and fast cooled alloys is the slope of curve towards the end of cooling which is very steep in case of very fast cooled alloys (liquid melt pools (present case)).

2.1.3.2.3 Extraction of Heat – Determination of Heat Transfer Coefficients (HTCs)

In the development of model, heat transfer coefficients will be determined at every point of cooling curve following earlier defined one dimensional (1D) schemes [306]. These will ensure, time of solidification calculation during cooling following above cooling curve and helps in determining shape of melt pool and its transient behaviour during cooling.

2.1.3.2.4 Final time of solidification

Final time of solidification is sum of time in each region / section of cooling curve of a particular alloy / melt. It will be determined using standard transport equations and will be used empirically to assess the conformability of AM process. Time of solidification gives other parameters as well such as fraction of mass solidified after a time, t , which is direct measure of microstructure evolved during that time. It can be qualitatively (extrapolation) used to predict further (type (equiaxed, columnar, mix, CET) and amount) evolution of microstructure with time.

2.1.4 Modelling and simulation of nucleation (heterogeneous) in liquid melt pool – Microstructural development

Modelling and simulation of microstructural development in liquid melt pool can be described by macroscopic and microscopic models of heat and mass transfer depending on type of alloy, its nature, number of elements, cooling curve, undercoolings (constitutional (solute / particulate), curvature, interfacial), thermal and kinetic limitations, behaviour of mushy zone, presence or absence of inoculants. These can be broadly divided into macroscopic and microscopic models [307] which are explained as follows;

2.1.4.1 Macroscopic models

By following the regimes of macroscopic models, finite element (FEM) and finite difference methods (FDM) can be used to explain microstructural development both during steady and transient state transport processes.

2.1.4.1.1 Limitations

Both FEM and FDM based models cannot fully describe mushy region, its behaviour and evolution during solidification as they do not account for microscopic

- solute diffusion and
- capillary effects

which are primarily responsible for scale at which microstructure forms (which is very small as compared to macroscopic methods based on average continuity equations [308-311] in which it is assumed that solidification starts at liquidus and finishes at solidus / eutectic temperatures (A case of BMGMCs having good match of GFA and eutectic temperature [64, 65]). In order to overcome these limitations, microscopic models were proposed.

2.1.4.2 Microscopic models of Microstructure evolution / formation during solidification

Stage 1 Model: These models take into account the mechanism of (1) grain nucleation and (2) grain growth in alloys which are solidifying with equiaxed dendrite or eutectic microstructures [312]. These do not account for alloys which are solidifying with columnar dendritic and planar interfaces. A modification of these accounts for equiaxed-columnar (at mould wall) and columnar to equiaxed transition (CET) in bulk of liquid (This will be discussed later). These can be used to “describe microstructures” and “prediction of grain size” in case of eutectic compositions of BMGMC. Majority of these is based on “analytical / deterministic approaches” which can be described as follows;

2.1.4.2.1 Nucleation

- Choose a time t_0 (initially non – zero value)
- At this time t_0 density of grains (which have nucleated in bulk) is a function of undercooling

$$d = f(\Delta T_n) \quad (12)$$

$f(\Delta T_n)$ is difficult to be found from theoretical considerations alone. It needs to be found experimentally i.e. form a set of experiments e.g.

Method 1 Measurement of cooling curve

This has been explained in detail in section 3.4.2.2.1 and 3.4.2.2.2

Method 2 Measurement of grain density (optical micrograph of cross section (using Image J® / manually)) for specimens solidified at various cooling rates [307]

2.1.4.2.2 Growth

As soon as grain has nucleated, and its growth can be explained by special modified case of CNT for BMGMC (A detailed treatment of modified CNT for BMGMC is given in Appendix A) and its distribution can be explained by Constitutional Supercooling Zone / Interdependence theory (propagation of L – S interface / L – S spherical front) (a possibility which is still under investigation by author for suitability for AM processes), it grows with an interface velocity which is also a function of undercooling.

2.1.4.2.3 Velocity of growth

Velocity of growth may be written as

$$V_g = f(\Delta T_n) \quad (1)$$

In this case, there is no need to determine solidification kinetics of dendrite tip/eutectic (spherical front) interface by cooling curve or grain size but it can be determined by theoretical models developed (by using basic laws of physics) [313, 314] as applied to BMGMC only under transient condition.

2.1.4.2.4 Impingement

Impingement of grains as they grow is another important phenomenon which for all practical reasons governs the shape of grain after CET (CET in AM is recently explained by Amrita Basak et. al. [315] which is combined with present model and is explained in detail in Appendix B). This phenomenon is not remarkably present in Bulk Metallic Glass (BMG) and their Composites (BMGMC) due to their sluggish nature and very little formation of crystal grains as compared to huge glassy matrix. However, despite these drawbacks, this is mainly responsible for equiaxed dendritic grain formation even in glassy alloys, especially eutectic compositions which is assumed to be the case for present research.

This has been typically treated by

- Standard J M A K [316, 317] correction or by
- Geometrical [318, 319] or
- Random grain arrangement models [307].

These “microscopic” solidification models have been coupled with “macroscopic” transient one dimensional (1D) heat flow calculations to successfully predict “microstructural features” specially “grain size” at the scale of whole process (part scale) [320, 321].

2.1.4.2.5 Limitations

These deterministic models have their following limitations

a. Grain selection

They cannot account for the “grain selection” which occurs

- Close to mould region / surface giving rise to columnar dendritic microstructure (in case of conventional Cu mould casting / TRC) or
- At surface of external inoculant particles (precursors of heterogeneous nucleation) in case of well inoculated melts (present case)) giving rise to onset of columnar dendritic microstructure (at a very small length scale) since they almost neglect any aspect which is related to crystallographic effects.

b. “Equiaxed – Columnar” Transition

They cannot predict the co-called “equiaxed-columnar” transition which occurs very near to mould wall [322] or variation of transverse size of columnar grains [323] (also known as columnar dendritic arm branching). This is explained in detail in individual cases for each type of metal (crystal structure)

i. Case 1: Cubic Metals

It is well established facts that for cubic metals, this “grain selection” is based upon a criterion of best alignment” of the $\langle 100 \rangle$ crystallographic axes of grain with heat flow direction [322-324]. Thus, this method cannot account for this anisotropic behaviour of heat flow. A solution to this problem could be proposed by determining best fit direction by use of recent developments in crystallography and their application to solidification. *Edge to edge matching (E2EM)*: One way is to use Edge to Edge Matching (E2EM) technique at inoculant – ductile phase level (in case of Zr based BMGMC) (present research). This gives rise to selection of suitable potent nuclei of certain size and specific preferred orientation (i.e. along a defined easy crystallographic plane (e.g. (001)). If this crystallographic plane direction could be used in conjunction with microscopic heat flow models, it can give rise to “prediction or selection of grain”. In other words, if matching crystallographic axes (suitable for a potent inoculant selection) of B2 ductile phase’s preferred precipitation (in case of BMGMC) could be best aligned with heat flow direction (or heat flow direction could be assigned to this preferred matching crystallographic axis) a best “grain selection: could be determined (one of aim of present research – not done previously elsewhere). This type of phenomena is particularly important in

- a. Directional solidification (DS) or
- b. Production of single crystal dendritic arms for aerospace applications or
- c. Production of BMGMC by Bridgman solidification.

Note: This is in addition to use of E2EM for selection of potent nuclei

ii. Case 2: BCC Metals

These methods are also ineffective in predicting “equiaxed-columnar” and then “branching of dendrite arms” in bcc metals (i.e. grain selection) as best alignment between heat flow and crystallographic direction is not best known. Only assumptions are possible (i.e. in case of bcc best heat flow direction could be assumed to close packed direction)

iii. Case 3: FCC Metals

These methods are again ineffective in predicting the “equiaxed – columnar”, “CET” and then branching of dendrite arms in fcc metals (i.e. grain selection) as best alignment between heat flow and close packed direction (111) could only be assumed (to a satisfactory qualitative level). More quantitative experimentation is needed to determine best directions along which heat flow occurred or revert to more advanced models.

c. Extension of a grain into an open region of liquid.

They cannot account for extension of a grain into an open region of liquid.

d. Columnar – to – Equiaxed Transition (CET)

Finally, when very fine equiaxed grains at a region very close to mould wall / right at the interface of inoculant and melt are converted to columnar grains, which when grow, there comes a point / plane at which columnar grains gets converted to not so fine equiaxed grains. This point is known as Columnar to Equiaxed Transition (CET)). These equiaxed grains finally extend towards centre of casting (wedge shape / melt pool centreline in case of AM). CET primarily happens as a result of thermal fluctuations which happen at melt (liquid) and solid (solidified melt) interface which are

triggered by solutal effects as well as heat extraction or absorption due to phase changes occurring at a micro-scale (explained in subsequent sections). CET is dominant when thermal gradient is small.

2.1.4.3 Evolution of probabilistic models

The solution to above four problems is presented first by Brown and Spittle [325, 326]. They developed probabilistic models. They used Monte Carlo (MC) procedure for explaining solidification phenomena developed in earlier research [327]. MC method is based upon minimising of interfacial energy (which is practically calculated by using physical properties of material (Zr- and Fe-based BMGMC)) from literature and earlier published data or inference from extrapolation or interpolation of data as needed). Procedurally, this minimisations is achieved by

- (a) Considering the energy of “unlike sites” (e.g. (a) “liquid / solid sites” or (b) “sites belonging to different grains” and
- (b) By allowing transition between these states to occur according to randomly generated numbers

By using this method, Brown and Spittle merely able to produce computered 2D microstructures which resembled very closely to those observed in real micrographic cross section. In particular

- a. The selection of grains in the columnar zone and
- b. CET

were nicely reproduced using this technique also

- a. the effect of solute concentration or
- b. melt superheat upon the resultant microstructure

was determined “qualitatively” in a nice way. Their quantitative representation was not achieved.

2.1.4.3.1 Limitations

These methods suffer consistently from lack of physical basis and thus cannot be used to analyse quantitatively the effect of various physical phenomena (happening within the phase transformations). For example, to illustrate this, consider the following example.

- a. During one MC time step, Consider N sites where N is number of sites whose evolution is calculated and is chosen from a further N (total number) sites. Therefore, not all sites of interest (i.e. those located near to solid – liquid interface) are investigated. This in turn, leads to algorithm predicted grain competition in columnar region, which does not at all reflect the physical mechanisms observed in organic alloys.
- b. Furthermore, the results are sensitive to type of Monte Carlo network itself which is used for computation. Thus a single powerful model is presented in present work which combines “advantages of probabilistic methods with those of deterministic approaches” to predict more accurately the grain structure in a casting.

2.1.4.4 Two dimensional Cellular Automaton (CA) Method

For this purpose, for now, a 2D Cellular Automaton model is developed which is based upon physical mechanisms of nucleation and growth (NG) of dendritic grains. Its salient features are as follows.

1. Heterogeneous Nucleation; which was modelled by means of a nucleation site distribution in deterministic solidification models, is treated in a similar way in present probabilistic approach.
2. If total density of grains which nucleate at a given undercooling is obtained from an average distribution ($d_c = \text{average (distribution)}$), the location of these sites is chosen randomly

$$d_c = \frac{a_1 + a_2 + a_3 + a_4 + a_5 + \dots + a_n}{n} \quad (14)$$

where $a_1, a_2, a_3, a_4, \dots, a_n$ are distributions of grains 1, 2, 3 to n

where $n = R$ ($R = \text{Real numbers}$)

3. Crystallographic orientation of a newly nucleated grain is also taken into account at random.
4. The growth kinetics of (a) dendrite tip and (b) of side branches are also incorporated into the model in such a way that final simulated microstructure is independent of the “cellular automaton network” which is used for computations.

Although, it produces micrographic cross sections very much similar to those already obtained by Brown and Spittle, present model has a “sound physical basis” and can thus reflect effect of (a) cooling rate” or (b) “solute concentration” quantitatively.

2.1.4.4.1 Detailed description (Phase 1 – Application of CAFE to Conventional Casting)

Physical background: Consider a BMGMC wedge shape casting as shown in Fig. 13 below

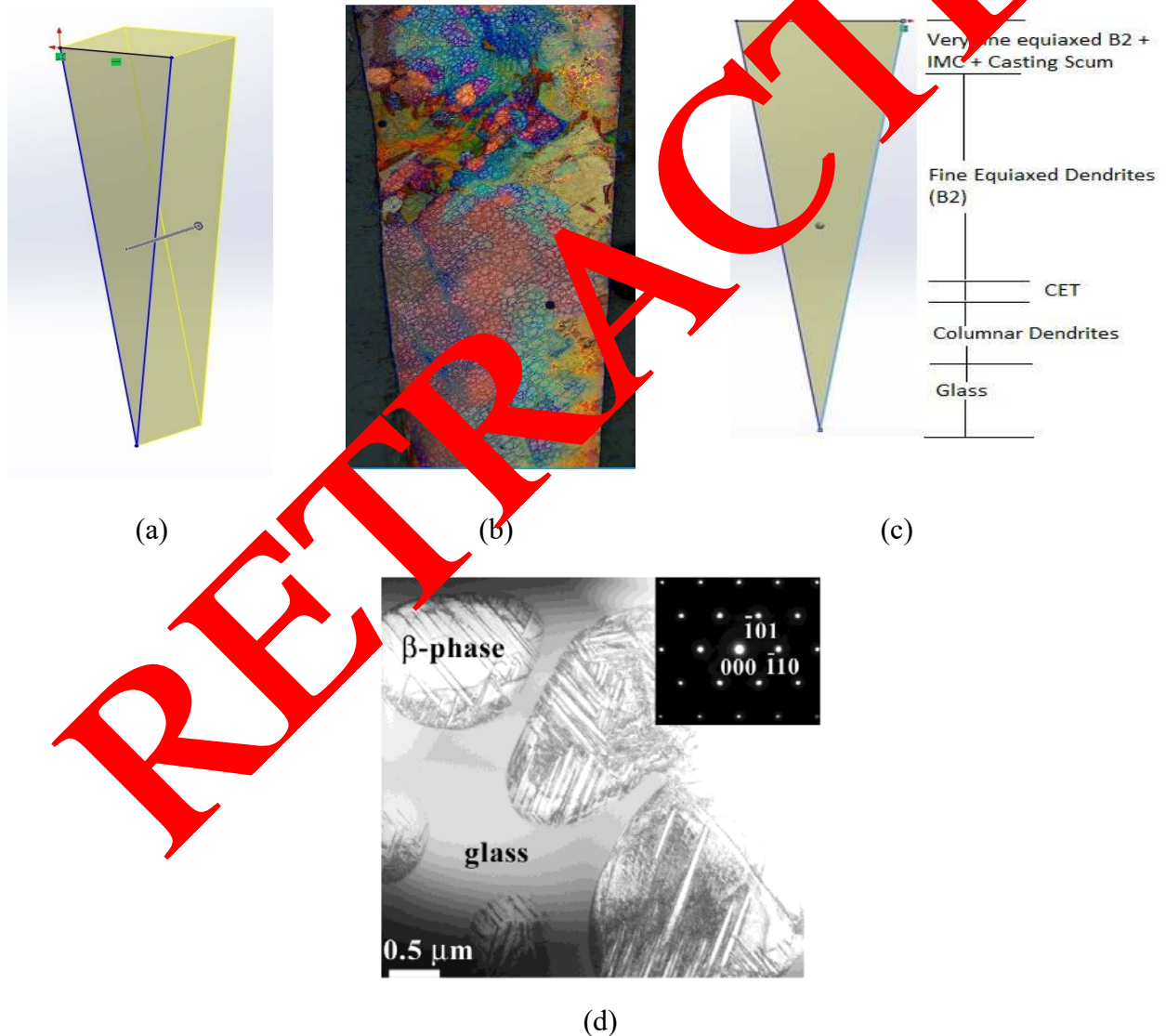


Fig. 13: (a) Schematic 3D (b) Optical Micrograph of cross section (etched) (c) 2D Schematic showing regions (d) A specific region (from B2 dendrites) showing B19' twins (B2 – B19' TRIP) [328].

Fig – 13 (c) is typical 2D cross section of cast eutectic Zr-based BMGMC solidified in water cooled wedge shape Cu – mould [255, 329]. Their dendritic grains which have various crystallographic orientations appear as zones of different colours (Fig – 13 (b)). Most common regions encountered in any casting appear here [322, 323] and are marked all along cross-section. On the top end of wedge shape ingot coarse grains are present as this region was exposed to air. Its more detailed explanation will follow after characterizing region chronologically from bottom to top.

2.1.4.4.2 Characterization

Bottom region Glass: The tip of casting is 100% glass (monolithic BMG). This region is classified as glass and no crystal structure is observed here because cooling rate is maximum here which results in extraction of heat at a very high rate resulting in retaining supercooled liquid state at room temperature.

Bottom region Columnar Dendrites: This region marks the beginning of “equiaxed to columnar” first transition. This consists of very fine layer in which this transition happens and then columnar grains grow (primarily) in random 3D orientation) because of still rapid rate of heat transfer which is complemented by sluggish nucleation on growth mechanisms of BMGMC. These grains are not very long as heat flow pattern is somewhat exponential because of wedge shape casting which triggers next transition too quickly before extension of growth as predicted by kinetics. This helps in retaining glassy matrix all throughout the casting. Otherwise 100% crystallization would have happened.

Bottom region Columnar to Equiaxed Transition (CET): This is the region in which columnar dendritic grains which have developed / grown to a satisfactory level, transit to equiaxed grains, known as CET. This is triggered by various phenomena such as solute diffusion, solute – solvent partitioning, shape of liquid – solid propagation front, thermal fluctuations happening at the tip of L – S propagating interface.

Fine equiaxed dendrites (B2): Once CET happens, equiaxed dendrites are formed all throughout the casting. Only their shape differs. In this region, they are fine sized while in *Top Region*, their size is even more reduced due to presence of scums. Casting scum and other impurities coupled with faster cooling rate from open top (convection and radiation) and side walls (conduction).

NOTE: In case of BMGMC no impurities particles serve as sites for heterogeneous nucleation but grain boundaries also serve this purpose [216, 330]. Other defects and solidification microstructure also serve as sites for heterogeneous nucleation. (Their effects in total solidification (nucleation and growth model) need to be taken into account in final model).

2.2 Appendix A

Heterogeneous nucleation and growth in very fluid alloys (as per CNT) [331]

Heterogeneous Nucleation rate per unit volume is defined as

$$I = N_s v \exp\left(-\frac{\Delta G_d}{kT}\right) \exp\left(-\frac{\Delta G_c}{kT}\right) \quad (15)$$

Where

N_s = No. of atoms in contact with substrate

v = vibrational frequency

ΔG_c = Activation energy for nucleation (Critical energy of nucleus formation (i-e creation of liquid – solid interface)

ΔG_d = Activation energy of diffusion (Diffusional activation energy)

Rearranging equation (15) using definition of v vibrational frequency

$$I = (N_o - N) I_o \exp\left(-\frac{\Delta G_c}{kT}\right)$$

$$I = (N_o I_o - N I_o) \exp\left(-\frac{\Delta G_c}{kT}\right) \quad (16)$$

Proof

$$I = N_o I_o e^{\left(\frac{-\Delta G_c}{kT}\right)} - N I_o e^{\left(\frac{-\Delta G_c}{kT}\right)} \quad (17)$$

$$I = N_o \left(\frac{N_s}{t}\right) e^{\left(\frac{-\Delta G_c}{kT}\right)} - N \left(\frac{N_s}{t}\right) e^{\left(\frac{-\Delta G_c}{kT}\right)} \quad (18)$$

$$I = N_o \left(\frac{N_s}{t}\right) e^{\left(\frac{-\Delta G_c}{kT}\right)} - N \left(\frac{N_s}{t}\right) e^{\left(\frac{-\Delta G_c}{kT}\right)} \quad (19)$$

$$I = (v \times t) \left(\frac{N_s}{t}\right) e^{\left(\frac{-\Delta G_c}{kT}\right)} - (v \times t) \left(\frac{N_s}{t}\right) e^{\left(\frac{-\Delta G_c}{kT}\right)} \quad (20)$$

$$I = (N_s \times v) e^{\left(\frac{-\Delta G_c}{kT}\right)} - (N \times v) e^{\left(\frac{-\Delta G_c}{kT}\right)} \quad (21)$$

$$I = v e^{\left(\frac{-\Delta G_c}{kT}\right)} (N_s - N) \quad (22)$$

$$I = v e^{\left(\frac{-\Delta G_c}{kT}\right)} N_s \quad (23)$$

N can be neglected as during initial stages there is no nucleation event

According to CNT, a minimum energy value is needed to create a solid – liquid interface eventually leading to stable nuclei out of melt. This is known as “activation energy”. This activation energy is the energy to overcome ΔG^* - the energy barrier to nucleation. Now, as solid – liquid interface grows to form stable nuclei, atoms must be transported through liquid thus another temperature dependent activation energy must be overcome known as ΔG_d (activation energy for diffusion)

The net effect is that CNT predicts a nucleation rate given by

$$I = I_o \exp \left[-\frac{(\Delta G^* + \Delta G_d)}{k_\beta T} \right]$$

it is the nature of difference between ΔG^* and ΔG_d that dictates whether solidification will be crystalline or glassy. For crystalline solids, ΔG_d has a significant value while for glassy solids there is no diffusion thus ΔG_d can be neglected. Thus

$$I = I_o \exp \left[-\frac{\Delta G^*}{k_\beta T} \right]$$

where K_β is constant dictated by nature and type of liquid composition and measured experimentally. ΔG_d is also zero for small undercooling (i-e well inoculated liquids / multicomponent alloys (Metallic Glasses inoculated with / without potent nuclei (present research))). [332]

Notes:**1) Vibrational frequency**

$$\frac{N_s}{t} = \frac{\text{occurrence of Total number of heterogeneous substrate particles}}{\text{total time}}$$

$$\frac{v \times N_s}{N_o} = \frac{N_s}{t}$$

where

$$N_s = I_o \times t$$

or

$$I_o = \frac{N_s}{t}$$

i-e

$$\text{initial nucleation rate} = \frac{\text{Total No. of atoms in contact with substrate}}{\text{total time}}$$

Definition used in eq. 17

- 2) The difference between frequency and rate is that frequency is “occurrence of an event per unit time” while rate is total number of that event (in terms of numerical value) per unit time.

Thus, from equation 16

N_o = Total number of heterogeneous substrate particles originally available per unit volume

N = Number that have already nucleated

I_o = constant

Value of I_0 can be calculated from equation 15 using another term known as “liquid diffusion coefficient”.

$$D \approx a^2 x v \exp\left(-\frac{\Delta G_d}{kT}\right) \tag{24}$$

where

$$a = \text{atomic diameter} = 0.4 \text{ nm}$$

$$v = \text{frequency}$$

which gives

$$I_0 = 10^{18} - 10^{22} / s$$

for small values of undercooling (well inoculated melts / multicomponent alloys)

$$\Delta G_c \propto 1/(\Delta T)^2$$

where ΔT = undercooling

Thus, Nucleation rate is equation 16

$$I = [N_0 10^{20} - N 10^{20}] \exp\left(-\frac{1}{kT(\Delta T)^2}\right)$$

or

$$I = [N_0 - N] 10^{20} \exp\left(-\frac{u}{(\Delta T)^2}\right) \tag{25}$$

where u is a constant

$$u = \frac{1}{kT}$$

The value of u can be measured from

Method 1: T (heterogeneous nucleation temperature). This is defined as temperature, where there is an initial nucleation rate of one nucleus / cm^3 / sec.

Method 2: Second method to calculate u is

$$u = -(\Delta T_N)^2 \ln(N_0 x 10^{20})$$

Proof

Taking natural log of equation (25) both sides

$$\ln I = \ln(N_0 - N) 10^{20} \left(-\frac{u}{(\Delta T_N)^2}\right)$$

$$-u = \frac{(\Delta T_N)^2 [\ln I - \ln(N_0 - N) 10^{20}]}{(\Delta T_N)^2}$$

$$-u = (\Delta T_N)^2 [\ln I - \ln N_0 10^{20} + \ln N 10^{20}]$$

because $\ln I$ and $\ln N 10^{20}$ can be neglected

$$u = -(\Delta T_N)^2 \ln I - (N_0 x 10^{20})$$

where ΔT_N = undercooling at heterogeneous nucleation temperature

Time is user defined input and temperature comes from user defined value initially as well. Then its every new value is assigned back to equation 15. With temperature and time, k changes and assigned back to equation (25). Also, with time, v (vibrational frequency) changes and assigned back to original equation (15). Similarly, the value of u also changes with time and temperature. Below table (Table 5) summarises the values which are user defined and which change as a function of transience programs runs.

Table 5: Summary of user defined and program determined functions used in CNT modified for BMGMC.

Sr. No.	User defined value	Time	Temp(t)
1	Time		
2	Temp(i)		
3	Temp(t)	✓	
4	k	✓	✓
5	Y	✓	✓
6	U	✓	✓

Note:

1. In BMG, in some cases due to slow motion of large atoms, only nucleation happens and growth never happens. In these cases a new phenomenon known as soft impingement effects of crystals must be considered. These could be solutal / thermal. However, this is quite rare.
2. In general, in case of BMG, CNT cannot be applied alone to describe NG.
3. CSZ and Interdependence models cannot be applied because of very high (η) viscosity of BMG (and their sluggish nature). CSZ and Interdependence theories are for less viscous / more fluid alloys.
4. A new concept, known as complex inter diffusion tensor [332] is much more helpful to describe NG in BMG
5. Fick's law in its native form (i-e linear form) is not sufficient.

2.3 Appendix B**1. Special case of growth of “Columnar microstructures”**

The growth of columnar dendrites, which is initiated by nuclei that form at the mould interface (Cu mould casting / TRC of BMGMC) (only if constitutional supercooling zone (CSZ) is suppressed – not present case) is usually simulated in a much simpler way. Again, in this case, there is no need to use cooling curve measurements or grain size measurement but solid growth kinetics models [7, 8] can be used to determine.

Undercooling of eutectic front (ΔT_n eutectic) or

Undercooling of dendrite tips (ΔT_n dendrite tip) as well as

Undercooling of lamellae or dendrite trunk spacing (ΔT_n lamellae / trunk spacing)

This undercooling is determined by direct measurement

- (a) Thermal gradient and
- (b) Speed of corresponding isotherm (eutectic / liquidus, respectively, i.e. speed of eutectic isotherm and speed of liquidus isotherm)

The later values are obtained from a macroscopic (part scale) heat flow calculations [306, 312]. The secondary arm spacing of both equiaxed and columnar dendritic microstructures are deduced from a local solidification time.

2. **Columnar structure growth in well inoculated BMGMC:** Growth of columnar dendrites can also occur at surface of external inoculants (well inoculated deeply undercooled melts – present case of BMGMC development. However, it should also be noted that another condition for growth of columnar dendrite to occur is suppression of CSZ which is clash with aforementioned condition for onset of this phenomena at external potent nuclei of inoculant. That's why still there is dispute about application of this concept to deeply undercooled well inoculated melts (BMGMC) whose solution is under investigation.

3. Columnar to equiaxed transition (CET) [315]

Growth rate of solid – liquid interface

$$V = S \cos \theta \quad (14)$$

where S = Scan speed

Temperature gradient parallel to dendrite growth direction can be calculated using

$$G_{hkl} = G / \cos \psi \quad (15)$$

where ψ = Angle between “normal vector” and “possible dendrite growth orientation” at the solid – liquid interface. This is evaluated by CFX – Post in Ansys®

A modification known as Rappaz modification is applied to predict CET. This is as follows

$$\frac{G_{hkl}^n}{V_{hkl}} \geq a \left[3 \sqrt{\frac{-4\pi}{3 \ln(1 - \phi)}} \sqrt{\frac{N_o}{n + 1} \left(1 - \frac{\Delta T_n^{n+1}}{\Delta T_{tip}^{n+1}} \right)} \right] \exp(n) \quad (16)$$

where

$$V_{hkl} = \text{dendrite growth velocity} = \frac{S \cos \theta}{\cos \psi}$$

n = material constant determined from literature [238, 333]

ϕ = equiaxed fraction (critical value = 0.066%)

N_0 = nucleation density

ΔT_{tip} = tip undercooling

ΔT_n = nucleation undercooling

This will be incorporated in present model at point where CET is determined. However, this model does not give true 3D representation output.

NOTE: In general Phase Field (PF) Methods are for microstructure evolution (its type (planer front, spherical front), morphology (precipitates, dendrites etc.)) while Cellular Automaton (CA) methods are for grain size determination (equiaxed / columnar dendritic) and its prediction. If both are combined [334-336], it is possible to get full map of microstructure evolution and grain size.

2.4 Comparison

Below a comparison of “strengths and capabilities” and “evolution of different theories over time” which have enabled a better understanding of nucleation and growth phenomena in bulk metallic glass matrix composites, is tabulated. The aim is to present research with a complete smart workable data for first hand use and reference for solving nucleation and growth problems in bulk metallic glass matrix composites by modelling and simulation. This will help professional programmer, working engineer and a researcher to effectively find previously done research till now with its strengths and capabilities at one platform.

2.4.1 Strengths and capabilities

Below (Table 6) a comparison of strength, capabilities and shortcomings of both deterministic and probabilistic methods are described. It highlights and brings out parameters and certain segments of each technique which could possibly advantageously used over others for modelling and simulation of bulk metallic glass matrix composites.

Table 6: Comparison of strength and capabilities of modelling and simulation techniques as applied to nucleation and growth problem of bulk metallic glass matrix composites (BMGMCS).

Sr. No.	Phenomena / Property		Deterministic Models		Probabilistic Models		Comments	References
			Ductile Phase	Glass	Ductile Phase	Glass		
1	Nucleation (Heterogeneous)		✓	N/A	✓	N/A		[312]
2	Growth		✓	✓	✓	✓		[331, 332]
3	Growth Mechanism (Interdependence theory / Complex inter diffusion tensor)		✓	N/A	X	X		[332, 337]
4	Different types of undercoolings (M-H Model)		✓	✓	N/A	N/A		[338]
5	Growth Kinetics		X	X	✓	✓		[307, 314]
6	Velocity of Growth		✓	✓	✓	✓		[307, 314]
7	CET		✓	N/A	✓	N/A	Deterministic models can model ductile phase in 2D only	[307, 315, 339]
8	Impingement after CET		✓	N/A	✓	N/A	Deterministic models can model ductile phase by M A K Correction, Geometrical and Random Grain Arrangement Models only	[307, 316-319]
9	Grain Selection	Qualitatively	X	X	✓	N/A	Probabilistic models can model ductile phase by MC only	[307]
		Quantitatively			✓		Probabilistic models can model ductile phase by CA only	
10	Columnar dendrite arm branching		X	X	✓	✓		[307, 340]
11	Extension of grain		✓		✓	✓		[340, 341]
12	CET in 3D		✓	N/A	✓	N/A		[341-343]
13	Physical Basis		N/A	N/A	✓	✓	Probabilistic models can form the basis of modelling by MC	[307, 340]
					✓	✓	Probabilistic models form the basis of modelling by CA	
14	Quantitative			✓	✓	✓	Probabilistic models can only model quantitatively employing CA method	[307, 340]
15	Liquid – Liquid Transition (LLT)		✓	✓	N/A	N/A		[173, 344, 345]
16	Devitrification		✓	✓	✓	✓	Probabilistic models can model ductile and glass phase by 2D CA	[163, 346]

Note: N/A is an abbreviation to “Not Applicable”

2.4.2 Evolution of theories

Table 7: Evolution of theories of modelling and simulation as applied to nucleation and growth problem of bulk metallic glass matrix composites (BMGMCS).

Sr. No.	Method / Approach	Theory / Action Explanation	and Limitation to Explanation	Group / Institute	Year	Reference
Part Scale Modelling						
Analytical Modelling						
1	Deterministic Continuum Model	Non-random methods, produce same types of exact results	Does not depend on initial state / point		1993	[348]
2	Probabilistic / Stochastic Models	Randomised result based methods	Does depend on initial state		2016	[349]
Computational Modelling						
3	Lattice Boltzmann Methods (LBM)	Solution of basic "Continuity" and "Navier Stokes" Equations for CFD based on Ludwig Boltzmann's Kinetic theory of gases	Limited to CFD type problems	Ge, D (Maastricht University, Dusseldorf)	2004	[350-356]
4	Phase Field Method (PFM)	Solution of phase field parameter ϕ to describe physical state (liquid / solid) of material	Limited by type ϕ for a particular situation	Napolitano, R. E (Iowa State)	2002 and 2012	[357, 358]
5	Cellular Automaton (CA) Method	Division of entire volume into finite cells and solution of transport equations applied to individual cells	Large initial Capital (Processor / RAM)	Rappaz, M (EPFL)	1993	[307, 341, 359-361]
6	Virtual Front Tracking (VFT) Method	Dendritic growth in low Péclet number systems	Best in 2D	Stefanescu, D.M (OSU)	2007	[362]
7	Sharp Interface (SIF) Method	Evolution of Interface as a function of time	Best in simple cases	Vermolen, F. J (Delft)	2006	[363]
8	CAFE	Combine Cellular Automaton (CA) Scheme with Finite Element (FE) Method		Rappaz, M (EPFL)	1994	[340]
9	PFFE	Combine Phase Field (PF) with Finite Element (FE) Method		Britta Nestler (KIT)	2011	[364]

10	PFCA	Combine Phase Field (PF) with Cellular Automata (CA) Regime		Shin, Y. C (Purdue)	2011	[336]
Atomistic Modelling						
10	Classical MD	Exact Solutions	Computing power	Alder and Wainwright (Lawrence Livermore)	1957	[54, 365, 366]
11	Monte Carlo (MC) Simulation	Set of probability based possible outcomes	Range of Solutions	Metropolis, Nicholas and co-workers (LANL)	1953 and 1963	[367-370]
12	Ab initio Method / First Principle Calculation	Based on Solution of Schrödinger Equation	Works well for H atom only. For all other atoms, approximations are needed	Robert Parrinello (Caltech)	1970	[371]
13	Hartree Fork Method and Slater Determinant	Uses the variational theorem (which is wavefunction based approach using mean field approximation)	Approximate solution is obtained. It is a form of Ab initio method			[372]
14	Evolution of Hartree Fork Method	Self-Constrained Field (SCF) Method	Evolution of HF Method	Approximate solutions		[372]
		Møller Plesset (MP2) perturbation (MP2)	Hamiltonian is divided into two parts $\hat{H} = \hat{H}_0 + \lambda \hat{V}$ and solved	ψ and energy are HF ψ and HF energy		[372]
		MP2	ψ remain same, energy is changed	ψ is treated by the help of summations		[372]
		Density Functional Theory (DFT)	Energy of system is obtained from electron density	Approximation based		1996
15	Interatomic Potential	Explain Interaction of atoms in a system in terms of potentials	Limited by Accuracy, Transferability and Computational Speed of System	Multi (Many) Body Potentials Daw Baska (Sandia National Labs)	1984	[376]

4. Conclusion

Nucleation and growth phenomena in single component (pure metals), binary and multicomponent alloys is rather well understood. CNT [316] provides many answers to the behaviour of these melts. BMGs and their composites (BMGMCs) are relatively new class of materials which have recently emerged on the surface of science and technology and gained attention due to their unique properties [21, 34, 127, 377]. Traditionally, they were produced using conventional methods (Cu mould casting [220, 222, 378] and TRC [226]) in which their metastable phase (glass) and any *in-situ* ductile precipitates (stable phase) are nucleated based on their ability to surpass activation energy barrier. In addition, these processes, impart very high cooling rate to castings which is essential for retention of supercooled liquid (glass) at room temperature explained by phenomena of confusion [17], ordering [16, 379, 380], frustration [15], vitrification [381, 382].

Very recently, with the advent and popularity of AM, interest has sparked to explore the inherent and fundamental advantages present in this unique process to produce BMG and BMGMCs. AM techniques are useful in achieving this objective as very high cooling rate in fusion liquid melt pool is already present inherently to assist the formation of glassy structure which is suppression of “kinetics” and prolonging of undercooling (“thermodynamics”) – two main phenomena responsible for any phase transformation. However, the *in-situ* nucleation of second phase equiaxed dendrites during solidification and then microstructural evolution (*solute diffusion* and *capillary* assisted) is not satisfactorily explained by CNT alone.

Either some modifications are needed in CNT or more reliable probabilistic microstructure evolution models (e-g J-M-A-K Correction [332]) are needed to explain nucleation and growth (and other phenomena e-g LLT [173, 344, 383] and phase separations [159]) in BMGMCs. In this work, an effort has been made to meet both requirements. Following are propositions;

- 4.1 At present scenario, there is no single hybrid / combined model which explain phenomena of heat transfer (liquid melt pool formation – result of laser – matter interaction and its evolution – solidification) and coupled this with nucleation and growth (NG) (solute diffusion [384] and capillary action driven at microscale to predict microstructure and grain size in BMGMC as melt cools in liquid pool of AM.
 - 4.1.1 Only one study has been conducted to model the same phenomena (solidification only) during Cu mould casting which will serve as base [385] in addition to very recent attempts [332] in which emphasis is laid on development of generalised theory rather than solving a problem.
 - 4.1.2 Only one study has been reported on microstructure formation during TRC using CAFE [227] but that is not aimed at BMGMCs, is carried out using commercial software package and does not involve any mathematical modelling at the back end. Software embedded (NG and heat transfer) models are used only.
 - 4.1.3 Four prominent studies namely by Zhou et al. [386], Zhang et al. [387], Zinoviev et al. [388] and a group at Shenyang, China [389, 390], have been reported very recently using CAFE but these are based on modelling microstructure evolution in modified AM (HDMR [386], LAMP [391] on 316L SS [387], 2D CAFE [388], cladding [389, 390]) processes.
 - 4.1.4 Few studies in the past have been conducted employing SLM using CAFE [389, 390, 392, 393], CAPF, CAFVM [394], modified CAFE [395] etc. approaches but none have been conducted on BMGMC.
- 4.2 No effort has been made to correlate the effect of E2EM with assigning direction of easy heat flow and easy crystallographic growth.
- 4.3 No substantial study has been reported about evolution of microstructure in three dimensions in BMGMCs in AM.
- 4.4 No effort has been made to combine the effect of changing properties with decrease of temperature (transient conditions). Most of models till now predict solutions in terms of steady state processes.

4.5 Very few studies have been carried out to combine CA with FE in case of AM while it is routine approach to predict grain size in case of other processes (casting, welding).

In this review consisting of two parts, an effort has been put together to overcome these shortcomings and propose a methodology for the modelling and simulation of solidification phenomena during additive manufacturing of BMGMCs. A model system $Zr_{47.5}Cu_{45.5}Al_5Co_2$ has been proposed owing to its ductile nature and tendency to show shape memory effect (exhibiting two types of martensitic phase evolution from B2 ductile phase). Further, the method is proposed to be applied to conventional wedge shape casting geometry along with its final application to melt pool in AM making use of powers of deterministic, probabilistic and their coupled modelling approaches. This route is proposed to get maximum benefit from application of modelling and simulation to understand nucleation and growth phenomena during solidification both in conventional as well as modern processing technologies (AM). It is envisaged that application of hybrid CAFE model by MatLAB® and Ansys® will help understand solidification of BMGMC in much better way not done elsewhere previously.

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