

Retrial Queuing System with Starting Failure, Single Vacation and Orbital Search

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ABSTRACT

This paper is concerned with the analysis of a single server retrial queue with vacation and orbital search. The server is subject to starting failure and repair. At the completion epoch of each service, the server may take a single vacation. After vacation completion, the server searches for the customers in the orbit or remains idle. Retrial times, service times and vacation times are assumed to be arbitrarily distributed. Various performance measures are derived and numerical results are presented.

1. INTRODUCTION

Vacation models had been the subject of interest to queue theorists of deep study in recent years because of their applicability and theoretical structures in real life congestion situations such as manufacturing and production, computer and communication systems, service and distribution systems, etc. The most remarkable work done by some researchers on vacation models include Doshi (1990), Takagi (1991), Ke (2003), Baba (2005) and Wu and Takagi (2005).

One of the most important characteristic in the service facility of a queueing system is its starting failures. An arriving customer who finds the server idle must turn on the server. If the server is started successfully the customer gets the service immediately. Otherwise the repair for the server begins and the customer must join the orbit. The server is assumed to be reliable during service. Such systems with starting failures have been studied as queueing models by Yang and Li (1994), Krishna kumar et al. (2002), Mokaddis et al. (2007) and Ke and Chang (2009).

Retrial Queues considered by researchers so far have the characteristic that each service is preceded and followed by an idle period. Artalejo et al. (2002) have considered a retrial queue in which immediately after a service completion the server searches for customer from the orbit or remains idle. If a search is made a service is followed by another service, and if not, a service is followed by an idle time. Research work on retrial queueing system with orbital search is seldom found in literature. This motivates to study a single server retrial queue with vacation, starting failure and orbital search.

2. MODEL DESCRIPTION

Consider a single server retrial queue with the server subject to starting failure. Customers arrive at the system according to Poisson process with rate λ . If the server is found to be busy the arriving customers enter the retrial queue to try again for their requests after random intervals. Successive interretrial times of any customer are governed by an arbitrary probability distribution function $A(x)$ and Laplace-Stieltjes transform $A^*(s)$. If the server is free, an

arriving customer must start the server, which takes negligible time. If the server is started successfully, the customer gets service immediately. Otherwise the repair for the server commences immediately and the customer must leave for the orbit and make a retrial at a later time. Successive repair times are independent and identically distributed with probability distribution function $G(x)$, Laplace-Stieltjes transform $G^*(s)$ and the first two moments g_1 and g_2 . The probability of successful commencement of service is assumed to be α .

Successive service times are independent with common probability distribution function $B(x)$, Laplace-Stieltjes transform $B^*(s)$ and the first two moments b_1 and b_2 . At the completion of each service the server may take a vacation with probability β or waits for the next customer with complementary probability δ . After completing vacation, the server searches for customer in the orbit (if any) with probability θ or remains idle with probability $\bar{\theta} (= 1 - \theta)$. The search time is assumed to be negligible. Vacation times are general with common probability distribution function $V(x)$, Laplace-Stieltjes transform $V^*(s)$ and the first two moments v_1 and v_2 .

The state of the system at time t can be described by the Markov process, $\{N(t); t \geq 0\} = \{C(t), X(t), \xi_0(t), \xi_1(t), \xi_2(t), \xi_3(t), t \geq 0\}$ where $C(t)$ denotes the server state 0, 1, 2 or 3 according as the server being free, providing the service, under repair or on vacation respectively and $X(t)$ corresponds to the number of customers in the orbit at time t . If $C(t) = 0$ and $X(t) > 0$ then $\xi_0(t)$ represents the elapsed retrial time, if $C(t) = 1$ and $X(t) > 0$ then $\xi_1(t)$ corresponds to the elapsed time of the customer being provided service, if $C(t) = 2$ and $X(t) > 0$ then $\xi_2(t)$ represents the elapsed repair time and if $C(t) = 3$ and $X(t) > 0$ then $\xi_3(t)$ represents the elapsed vacation time at time t .

Let the functions $\eta(x)$, $\mu(x)$, $r(x)$ and $\gamma(x)$ be the conditional completion rates at time x respectively for repeated attempts, service, repair and vacation. Then

$$\eta(x) = a(x)/(1 - A(x)); \mu(x) = b(x)/(1 - B(x)) \\ r(x) = g(x)/(1 - G(x)) \text{ and } \gamma(x) = v(x)/(1 - V(x)).$$

3. STABILITY CONDITION

The following theorem provides the necessary and sufficient condition for the system to be stable.

Theorem 1.

The embedded Markov chain $\{N_n, n \in \mathbb{N}\}$ is ergodic if and only if

$$\lambda(\alpha\beta v_1 + ab_1 + \alpha g_1(1 - \theta\beta)) + \bar{\alpha} < A^*(\lambda) + \theta\beta(1 - A^*(\lambda)).$$

Proof:

$\{N_n, n \in \mathbb{N}\}$ is an irreducible aperiodic Markov chain. To prove ergodicity, we shall use the Foster's criterion, an irreducible and aperiodic Markov chain is ergodic if there exists a non negative function $f(j), j \in \mathbb{N}$ and $\xi > 0$ such that the mean drift $x_j = E[f(N_{n+1}) - f(N_n) | N_n = j]$ is finite for all $j \in \mathbb{N}$ and $x_j \leq -\xi$ for all $j \in \mathbb{N}$ except for a finite number of j s. Here we consider the function $f(j) = j$. Then we have

$$x_j = \begin{cases} \lambda(\alpha\beta v_1 + ab_1 + \alpha g_1(1 - \theta\beta)) + \bar{\alpha} - A^*(\lambda) - \theta\beta(1 - A^*(\lambda)), j = 1, 2, 3, \dots \\ E(x), j = 0 \end{cases}$$

Clearly the inequality $\lambda(\alpha\beta v_1 + ab_1 + \alpha g_1(1 - \theta\beta)) + \bar{\alpha} < A^*(\lambda) + \theta\beta(1 - A^*(\lambda))$ is a sufficient condition for ergodicity.

The same inequality is also necessary for ergodicity. As noted in Sennot et al. (1983), we can guarantee non ergodicity, if the Markov chain $\{N_n, n \in \mathbb{N}\}$ satisfies Kaplan's condition, namely $x_j < \infty$ for all $j \geq 0$ and there exists $j_0 \in \mathbb{N}$ such that $x_j \geq 0$ and $j \geq j_0$. In our case Kaplan's condition is satisfied because there is $k \in \mathbb{N}$ such that $r_{ij} = 0$ for $j < i - k$ and $i > 0$, where (r_{ij}) is the one step transition matrix of $\{N_n, n \in \mathbb{N}\}$. Then $\lambda(\alpha\beta v_1 + ab_1 + \alpha g_1(1 - \theta\beta)) + \bar{\alpha} > A^*(\lambda) + \theta\beta(1 - A^*(\lambda))$ implies the non ergodicity of the Markov chain.

4. STEADY STATE DISTRIBUTION

Define

$$I_0(t) = P\{C(t) = 0, X(t) = 0\}$$

$$I_n(x, t)dx = P\{C(t) = 0, X(t) = n, x \leq \xi_0(t) < x + dx\};$$

$$t \geq 0, x \geq 0, n \geq 1$$

$$W_n(x, t)dx = P\{C(t) = 1, X(t) = n, x \leq \xi_1(t) < x + dx\};$$

$$t \geq 0, x \geq 0, n \geq 0$$

$$R_n(x, t)dx = P\{C(t) = 2, X(t) = n, x \leq \xi_2(t) < x + dx\};$$

$$t \geq 0, x \geq 0, n \geq 1$$

$$V_n(x, t)dx = P\{C(t) = 3, X(t) = n, x \leq \xi_3(t) < x + dx\};$$

$$t \geq 0, x \geq 0, n \geq 0$$

The system of steady state equations that governs the model is given below

$$\lambda I_0 = (1 - \beta) \int_0^\infty W_0(x) \mu(x) dx + \int_0^\infty V_0(x) \gamma(x) dx \quad (1)$$

$$\frac{d}{dx} I_n(x) = -(\lambda + \eta(x)) I_n(x), n \geq 1 \quad (2)$$

$$\frac{d}{dx} W_0(x) = -(\lambda + \mu(x)) W_0(x) \quad (3)$$

$$\frac{d}{dx} W_n(x) = -(\lambda + \mu(x)) W_n(x) + \lambda W_{n-1}(x), n \geq 1 \quad (4)$$

$$\frac{d}{dx} R_1(x) = -(\lambda + r(x)) R_1(x) \quad (5)$$

$$\frac{d}{dx} R_n(x) = -(\lambda + r(x)) R_n(x) + \lambda R_{n-1}(x), n \geq 2 \quad (6)$$

$$\frac{d}{dx} V_0(x) = -(\lambda + \gamma(x)) V_0(x) \quad (7)$$

$$\frac{d}{dx} V_n(x) = -(\lambda + \gamma(x)) V_n(x) + \lambda V_{n-1}(x), n \geq 1 \quad (8)$$

with boundary conditions

$$I_0(0) = (1 - \beta) \int_0^\infty W_n(x) \mu(x) dx + (1 - \theta) \int_0^\infty V_n(x) \gamma(x) dx + \int_0^\infty R_n(x) r(x) dx, n \geq 1 \quad (9)$$

$$W_0(0) = \alpha \lambda I_0 + \alpha \int_0^\infty I_1(x) \eta(x) dx + \theta \int_0^\infty V_1(x) \gamma(x) dx \quad (10)$$

$$W_n(0) = \alpha \lambda \int_0^\infty I_n(x) dx + \alpha \int_0^\infty I_{n+1}(x) \eta(x) dx + \theta \int_0^\infty V_{n+1}(x) \gamma(x) dx, n \geq 1 \quad (11)$$

$$R_1(0) = \alpha \lambda I_0 + \alpha \int_0^\infty I_1(x) \eta(x) dx \quad (12)$$

$$R_n(0) = \alpha \lambda \int_0^\infty I_{n-1}(x) \eta(x) dx + \alpha \int_0^\infty I_n(x) \eta(x) dx, n \geq 2 \quad (13)$$

$$V_n(0) = \beta \int_0^\infty W_n(x) \mu(x) dx, n \geq 0 \quad (14)$$

The normalizing condition is

$$I_0 + \sum_{n=1}^\infty \int_0^\infty I_n(x) dx + \sum_{n=0}^\infty \int_0^\infty W_n(x) dx + \sum_{n=1}^\infty \int_0^\infty R_n(x) dx + \sum_{n=0}^\infty \int_0^\infty V_n(x) dx = 1 \quad (15)$$

Define the probability generating functions

$$I(x, z) = \sum_{n=1}^\infty I_n(x) z^n; \quad W(x, z) = \sum_{n=0}^\infty W_n(x) z^n$$

$$R(x, z) = \sum_{n=1}^\infty R_n(x) z^n \quad \text{and} \quad V(x, z) = \sum_{n=0}^\infty V_n(x) z^n$$

The following theorem discusses the steady state distribution of the system.

Theorem 2.

If

$$\lambda(\alpha\beta v_1 + ab_1 + \alpha g_1(1 - \theta\beta)) + \bar{\alpha} < A^*(\lambda) + \theta\beta(1 - A^*(\lambda))$$

then

The probability generating function of the orbit size when the server is idle is

$$I(z) = (1 - A^*(\lambda))((1 - \beta)B^*(\lambda - \lambda z) + (1 - \theta)\beta B^*(\lambda - \lambda z)V^*(\lambda - \lambda z)\alpha z) + (z - \theta\beta B^*(\lambda - \lambda z))V^*(\lambda - \lambda z)(\alpha z G^*(\lambda - \lambda z) - 1)I_0 / D(z) \quad (16)$$

The probability generating function of the orbit size when the server is busy is

$$W(z) = \alpha A^*(\lambda)(B^*(\lambda - \lambda z) - 1)I_0 / D(z) \quad (17)$$

The probability generating function of the orbit size when the server is under repair is

$$R(z) = \alpha A^*(\lambda)(z - \theta\beta B^*(\lambda - \lambda z)V^*(\lambda - \lambda z))(G^*(\lambda - \lambda z) - 1)I_0 / D(z) \quad (18)$$

The probability generating function of the orbit size when the server is on vacation is

$$V(z) = \alpha\beta A^*(\lambda)B^*(\lambda - \lambda z)(V^*(\lambda - \lambda z) - 1)I_0 / D(z) \quad (19)$$

The steady state probability that the system is empty is

$$I_0 = ((1 - \theta\beta)(A^*(\lambda) - \alpha\lambda g_1 - 1) + \alpha - \alpha\lambda(b_1 + \beta v_1)) / \alpha A^*(\lambda) \quad (20)$$

where

$$D(z) = z - \{\alpha(z + (1 - z)A^*(\lambda))[(1 - \beta) + (1 - \theta)\beta V^*(\lambda - \lambda z)]B^*(\lambda - \lambda z) - \theta\beta B^*(\lambda - \lambda z)V^*(\lambda - \lambda z)[1 - \alpha G^*(\lambda - \lambda z)](z + (1 - z)A^*(\lambda))\} - z\alpha(z + (1 - z)A^*(\lambda))G^*(\lambda - \lambda z)$$

Proof:

Multiplying equations (1) – (14) by z^n and summing over $n, n=0,1,2,3,\dots$, we obtain the partial differential equations

$$\left[\frac{d}{dx} + \lambda + \eta(x) \right] I(x, z) = 0 \quad (21)$$

$$\left[\frac{d}{dx} + \lambda(1 - z) + \mu(x) \right] W(x, z) = 0 \quad (22)$$

$$\left[\frac{d}{dx} + \lambda(1 - z) + r(x) \right] R(x, z) = 0 \quad (23)$$

$$\left[\frac{d}{dx} + \lambda(1 - z) + \gamma(x) \right] V(x, z) = 0 \quad (24)$$

$$I(0, z) = (1 - \beta) \int_0^\infty W(x, z)\mu(x)dx + (1 - \theta) \int_0^\infty V(x, z)\gamma(x)dx + \int_0^\infty R(x, z)r(x)dx - \lambda I_0 \quad (25)$$

$$W(0, z) = \alpha \lambda I_0 + \alpha \lambda \int_0^\infty I(x, z)dx + \frac{\alpha}{z} \int_0^\infty I(x, z)\eta(x)dx + \frac{\theta}{z} \int_0^\infty V(x, z)\gamma(x)dx \quad (26)$$

$$R(0, z) = \alpha \lambda z \int_0^\infty I(x, z)dx + \alpha \int_0^\infty I(x, z)\eta(x)dx + \alpha \lambda z I_0 \quad (27)$$

$$V(0, z) = \beta \int_0^\infty W(x, z)\mu(x)dx \quad (28)$$

Solving the partial differential equations (21) – (24), we obtain

$$I(x, z) = I(0, z)e^{-\lambda x} (1 - A(x)) \quad (29)$$

$$W(x, z) = W(0, z)e^{-\lambda(1-z)x} (1 - B(x)) \quad (30)$$

$$R(x, z) = R(0, z)e^{-\lambda(1-z)x} (1 - G(x)) \quad (31)$$

$$V(x, z) = V(0, z)e^{-\lambda(1-z)x} (1 - V(x)) \quad (32)$$

Using equations (29) – (32) in equations (25) – (28) and after some algebraic manipulation, we get

$$I(0, z) = (((1 - \beta)B^*(\lambda - \lambda z) + (1 - \theta)\beta B^*(\lambda - \lambda z))V^*(\lambda - \lambda z)\alpha z) + (z - \theta\beta B^*(\lambda - \lambda z)V^*(\lambda - \lambda z))(\alpha z G^*(\lambda - \lambda z) - 1)I_0 / D(z) \quad (33)$$

$$W(0, z) = \alpha A^*(\lambda)(z - 1)\lambda I_0 / D(z) \quad (34)$$

$$R(0, z) = \alpha A^*(\lambda)(z - \theta\beta B^*(\lambda - \lambda z)V^*(\lambda - \lambda z))(z - 1)\lambda I_0 / D(z) \quad (35)$$

$$V(0, z) = \alpha\beta A^*(\lambda)B^*(\lambda - \lambda z)(z - 1)\lambda I_0 / D(z) \quad (36)$$

By substituting the expressions (33) to (36) in equations (29) to (32), we get the expressions for $I(x, z), W(x, z), R(x, z), V(x, z)$. Integrating these with respect to x from 0 to ∞ , we obtain the equations (16) to (19). Using normalizing condition $I_0 + I(1) + W(1) + R(1) + V(1) = 1$ and solving we get the analytical expression for I_0 .

Corollary 1.

The steady state probabilities of the server being idle, busy, repair and on vacation are

$$I = I(1) = (1 - A^*(\lambda))(\alpha - \theta\beta + \lambda(\alpha\beta v_1 + \alpha b_1 + \alpha g_1(1 - \theta\beta))) / \alpha A^*(\lambda)$$

$$W = W(1) = \lambda b_1$$

$$R = R(1) = \alpha(1 - \theta\beta)\lambda g_1 \quad \text{and}$$

$$V = V(1) = \beta\lambda v_1.$$

Corollary 2.

The mean number of customers in the orbit and system under steady state conditions are

$$L_q = N' / D' \quad \text{and} \quad L_s = L_q + \lambda b_1$$

$$\text{where } N' = \alpha A^*(\lambda)I_0,$$

$$D' = (1 - \theta\beta)(A^*(\lambda) - \alpha\lambda g_1 - 1) + \alpha - \alpha\lambda(b_1 + \beta v_1)$$

$$D'' = 2(1 - A^*(\lambda))[\lambda b_1(\alpha - \theta\beta) + \beta\lambda v_1(\alpha - \theta) + \alpha(1 + \lambda g_1(1 - \theta\beta))] + \alpha\beta(2\lambda^2 b_1 v_1 + \lambda^2 v_2) + 2\alpha\lambda g_1(1 - \theta\beta)(\lambda v_1 + \lambda b_1) + \lambda^2 b_2(1 - \alpha\theta\beta)$$

Proof:

The probability generating function of the number of customers in the orbit is

$$H(z) = I_0 + I(z) + W(z) + R(Z) + V(z)$$

$$L_q = \lim_{z \rightarrow 1} \frac{d}{dz} H(z)$$

Similarly, the probability generating function of the number of customers in the system is

$$K(z) = I_0 + I(z) + zW(z) + R(Z) + V(z)$$

$$L_s = \lim_{z \rightarrow 1} \frac{d}{dz} K(z)$$

Theorem 3.

The steady state availability of the server is

$$A = I_0 + I(1) + W(1) = 1 - \lambda\beta v_1 - \alpha\lambda g_1(1 - \theta\beta) / \alpha.$$

5. PARTICULAR CASES

If $\alpha = 1$, the model under study reduces to M/G/1 retrial queue with vacation and orbital search.

If $\beta = 1$, the model reduces to M/G/1 retrial queue with starting failure and orbital search.

If $\theta = 0$, the model under study reduces to M/G/1 retrial queue with vacation and starting failure.

If $A^*(\lambda) \rightarrow 1$, the system reduces to M/G/1 queue with vacation, starting failure and orbital search.

6. NUMERICAL RESULTS

The numerical calculations are carried out by assuming the service time, retrial time, repair time and vacation time to follow exponential distribution with respective parameters μ, η, r and γ . As expected it is observed that

- I_0 increases with increase in $\beta, \theta, \alpha, \mu, \eta$ and r .
- I, L_s and L_q decrease with increase in μ, θ, α and η and increase with increase in β .
- Increase in λ increases I, W, R, V, L_s and L_q .

Table 1: Effect of λ on the performance measures
($\beta = \theta = \alpha = 0.5, \mu = r = \gamma = 10$ and $\eta = 6$).

λ	I_0	I	W	R	V	L_s	L_q
1.2000	0.5760	0.0630	0.1200	0.0900	0.0600	0.2618	0.1418
1.4000	0.4948	0.0828	0.1400	0.1050	0.0700	0.3329	0.1929
1.6000	0.4107	0.1053	0.1600	0.1200	0.0800	0.4260	0.2660
1.8000	0.3235	0.1305	0.1800	0.1350	0.0900	0.5601	0.3801
2.0000	0.2333	0.1583	0.2000	0.1500	0.1000	0.7857	0.5857

Table 2 : Effect of μ on the performance measures
($\lambda = 1, \beta = \theta = \alpha = 0.5, r = \gamma = 10$ and $\eta = 6$)

μ	I_0	I	W	R	V	L_s	L_q
15.0000	0.6931	0.0403	0.0667	0.0750	0.0500	0.1728	0.1061
20.0000	0.7125	0.0375	0.0500	0.0750	0.0500	0.1548	0.1048
25.0000	0.7242	0.0358	0.0400	0.0750	0.0500	0.1433	0.1033
30.0000	0.7319	0.0347	0.0333	0.0750	0.0500	0.1354	0.1020
35.0000	0.7375	0.0339	0.0286	0.0750	0.0500	0.1295	0.1010

Table 3: Effect of η on the performance measures
($\lambda = 1, \beta = \theta = \alpha = 0.5, \mu = r = \gamma = 10$)

η	I_0	I	W	R	V	L_s	L_q
1.2000	0.1708	0.2292	0.1000	0.0750	0.0500	4.4878	4.3878
1.4000	0.2571	0.1964	0.1000	0.0750	0.0500	2.5417	2.4417
1.6000	0.3219	0.1719	0.1000	0.0750	0.0500	1.7670	1.6670
1.8000	0.3722	0.1528	0.1000	0.0750	0.0500	1.3507	1.2507
2.0000	0.4125	0.1375	0.1000	0.0750	0.0500	1.0909	0.9909

Table 4: Effect of r on the performance measures
($\lambda = 1, \beta = \theta = \alpha = 0.5, \mu = \gamma = 10$ and $\eta = 6$)

r	I_0	I	W	R	V	L_s	L_q
15.0000	0.6833	0.0417	0.1000	0.0500	0.0500	0.2474	0.1474
20.0000	0.6979	0.0396	0.1000	0.0375	0.0500	0.2678	0.1678
25.0000	0.7067	0.0383	0.1000	0.0300	0.0500	0.2796	0.1796
30.0000	0.7125	0.0375	0.1000	0.0250	0.0500	0.2873	0.1873
35.0000	0.7167	0.0369	0.1000	0.0214	0.0500	0.2928	0.1928

Table 5: Effect of θ on the performance measures
($\lambda = 1, \beta = \alpha = 0.5, \mu = r = \gamma = 10$ and $\eta = 6$)

θ	I_0	I	W	R	V	L_s	L_q
0.2000	0.5867	0.0483	0.1000	0.0900	0.0500	0.2281	0.1281
0.4000	0.6317	0.0467	0.1000	0.0800	0.0500	0.2113	0.1113
0.6000	0.6767	0.0450	0.1000	0.0700	0.0500	0.1968	0.0968
0.8000	0.7217	0.0433	0.1000	0.0600	0.0500	0.1841	0.0841
1.0000	0.7667	0.0417	0.1000	0.0500	0.0500	0.1728	0.0728

Table 6: Effect of β on the performance measures
($\lambda = 1, \theta = \alpha = 0.5, \mu = r = \gamma = 10$ and $\eta = 6$)

β	I_0	I	W	R	V	L_s	L_q
0.2000	0.6217	0.0383	0.1000	0.0900	0.0200	0.2016	0.1016
0.4000	0.6433	0.0433	0.1000	0.0800	0.0400	0.2031	0.1031
0.6000	0.6650	0.0483	0.1000	0.0700	0.0600	0.2045	0.1045
0.8000	0.6867	0.0533	0.1000	0.0600	0.0800	0.2058	0.1058
1.0000	0.7083	0.0583	0.1000	0.0500	0.1000	0.2071	0.1071

Table 7: Effect of α on the performance measures
 ($\lambda = 1, \beta = \theta = 0.5, \mu = r = \gamma = 10$ and $\eta = 6$)

α	I_0	I	W	R	V	L_s	L_q
0.2000	0.0167	0.1083	0.1000	0.3000	0.0500	13.4000	13.3000
0.4000	0.5479	0.0563	0.1000	0.1125	0.0500	0.2707	0.1707
0.6000	0.7250	0.0389	0.1000	0.0500	0.0500	0.1701	0.0701
0.8000	0.8135	0.0302	0.1000	0.0187	0.0500	0.1362	0.0362
1.0000	0.8667	0.0250	0.1000	0.0000	0.0500	0.1192	0.0192

7. REFERENCES

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