Retrofitting O'Raifeartaigh models with dynamical scales

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We provide a method for obtaining simple models of supersymmetry breaking, with all small mass scales generated dynamically, and illustrate it with explicit examples. We start from models of perturbative supersymmetry breaking, such as O'Raifeartaigh and Fayet models, that would respect an R symmetry if their small input parameters transformed as the superpotential does. By coupling the system to a pure supersymmetric Yang-Mills theory (or a more general supersymmetric gauge theory with dynamically small vacuum expectation values), these parameters are replaced by powers of its dynamical scale in a way that is naturally enforced by the symmetry. We show that supersymmetry breaking in these models may be straightforwardly mediated to the supersymmetric standard model, obtain complete models of direct gauge mediation, and comment on related model building strategies that arise in this simple framework.

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I. INTRODUCTION AND GENERAL IDEA

Dynamical supersymmetry (SUSY) breaking [1] and the mediation of SUSY breaking to the supersymmetric standard model (SSM) have been studied extensively. A particularly important objective is to identify simple models of dynamical SUSY breaking that may be straightforwardly mediated to the SSM, yielding predictive and phenomenologically attractive superpartner spectra [2–4]. In early examples with gauge-mediated SUSY breaking [5], the problems of SUSY breaking and its mediation were addressed by postulating separate SUSY breaking and messenger sectors. These models motivated many advances [6–8], culminating in a few genuinely simple and viable models of direct gauge mediation [9–15], in which fields of the SUSY breaking sector also play the role of messengers, transmitting SUSY breaking to the SSM.

In this paper, we develop a straightforward method for obtaining simple models of SUSY breaking in which all small scales are generated dynamically. We show further that SUSY breaking in these models may be rather simply communicated to the SSM, providing new avenues for direct gauge mediation and gravity mediation. To illustrate the method, we work through two complete gauge mediation examples that are representative of large classes of models, and we discuss the method's application to more general model building problems.

The basic strategy in its simplest realization can be summarized as follows:

(1) Start with a model of perturbative SUSY breaking, such as an O'Raifeartaigh or Fayet model, whose small input parameters m_i break an R symmetry that would be restored if the m_i transformed as the superpotential does.

(2) Couple the system to a SUSY preserving sector with a dynamically small operator vacuum expectation value (VEV). Our prototypical example will be pure $SU(2)$ Yang-Mills theory, with gauge field strength superfield W_α and dynamical scale Λ. Replace dimensional parameters m_i in the superpotential by $W_\alpha W^\alpha$ suppressed by appropriate powers of a high scale M_*. At low energies, $W_\alpha W^\alpha \sim \Lambda^3$. This renders the m_i dynamically small in a way naturally enforced by the symmetries and preserves a local SUSY breaking minimum.

We will refer to this procedure (1)–(2) as *retrofitting* the old fashioned perturbative SUSY breaking models. Elementary ingredients suffice to bring such models up to modern model building standards of naturalness, while preserving some of the simplicity of early constructions [16]. In effect, we consider a *supersymmetric* hidden sector to obtain dynamically small scales, which allows the SUSY *breaking* sector to be more directly coupled to the SSM.

If desired, the couplings to $W_\alpha W^\alpha$ can arise from purely renormalizable interactions by integrating out massive flavors in the $SU(2)$ SUSY gauge theory [17]. In any case, the coupling to the $SU(2)$ sector does not destroy the local SUSY breaking minimum of the perturbative model (1), though it often introduces SUSY vacua far away in field space. As discussed, for example, in [8,11,12,18–20], we need not impose that the SUSY breaking configuration be the global minimum of the potential.

Indeed, one element of many successful models is metastability. In field theory models of dynamical SUSY breaking, the requirement that SUSY be broken in the global minimum is very restrictive, and allowing for metastable vacua greatly simplifies the problems of model building, especially for gauge mediation. This point was emphasized clearly, for example, in [7,8,11,12]; more recently it has found application in the problem of moduli stabilization [18] and dynamics [21–23], in the vacuum structure of large N gauge theories arising in generalizations of AdS/CFT [19], and in supersymmetric QCD [20].

As we will see, simple constructions lead almost trivially to a large class of dynamical SUSY breaking models and suggest an array of further model building possibilities. It is worth remarking that the models need not be chiral and can have nonvanishing Witten index, like the models of [19,20]. They can possess interesting (discrete) symmetries, which naturally protect the structures required for model building goals. This simple method allows construction of theories with direct gauge mediation as well as gravity-mediated models with appropriately large (nonloop-suppressed) gaugino masses.

As discussed recently in [20], some basic classes of supersymmetric gauge theories reduce at low energies to infrared-free O'Raifeartaigh models with metastable SUSY breaking. In some circumstances, the direct mediation models of the type we consider here may be UV completed by asymptotically free quantum field theory. In other circumstances, the models may be completed by string theory, where metastable SUSY breaking [18] has played a crucial role.

From the perspective of weakly coupled string theory, one might worry that there are additional approximate moduli that affect the value of the gauge coupling. It is worth noting in this connection that the current state of the art in string moduli stabilization—via a combination of a tree-level potential, orientifolds, and Ramond-Ramond fluxes—can fix the dilaton and other moduli at a high scale. In the context of low energy supersymmetric models, this allows for a gaugino condensate which does not vary with extra moduli beyond those evident in the low energy field theory of interest here, whose couplings are fixed by discrete symmetries.

In the next section, we consider retrofitting a class of O'Raifeartaigh models and work through a simple example in detail. We next simplify the model further to extract some lessons about the role of chirality and symmetry. We follow this in Sec. III with another general class of models including a Fayet-Iliopoulos parameter. In the final section, we summarize and discuss further model building applications.

II. RETROFITTING O'RAIFEARTAIGH MODELS

In this section we will implement the procedure outlined above in concrete examples and comment on model building lessons that arise in this framework. We begin with a brief review of O'Raifeartaigh models and their challenges. Next we consider a simple explicit example which we retrofit to render its scales dynamical in a way consistent with symmetries. This model is complete in that it readily incorporates messengers appropriate for gauge mediation, generating standard model superpartner masses. In the final subsection we extract lessons illustrated by even simpler systems, emphasizing the role of metastability in avoiding the unnecessary constraints of chirality and vanishing Witten index.

Consider O'Raifeartaigh models, with n fields $Z_1, \ldots, Z_n$, n' fields $\phi_1, \ldots, \phi_{n'}$, $n' < n$, and superpotential

$$W = \sum_i Z_i f_i(\phi_a). \tag{2.1}$$

This class of models breaks SUSY classically for generic choices of functions f. At tree level, its main shortcomings are (i) there is automatically a flat direction in its potential, (ii) it does not automatically provide messengers and R symmetry breaking as required to mediate SUSY breaking to the SSM, and (iii) its scales are input by hand, with some couplings set to zero without a symmetry reason. [With regard to point (ii), for definiteness we here consider gauge-mediated SUSY breaking and consider more general applications in the later discussion.]

We will address each of these, illustrating the technique with perhaps the simplest version of (2.1). Let us first summarize the method. With regard to point (i), the Coleman-Weinberg potential expanded about an appropriate point in field space generically lifts the flat direction; one can explicitly check for self-consistent metastable solutions as in [12,16]. Point (ii) can be addressed by coupling in messengers and including their contribution to the Coleman-Weinberg potential self-consistently. Finally, point (iii) can be addressed by coupling in an otherwise supersymmetric $SU(2)$ gaugino condensate, or any other more general SUSY sector with a dynamically small operator VEV.

A. A complete, simple example

As a very simple illustrative example, consider a model with messengers η and $\tilde{\eta}$ in, say, the $\mathbf{5}$ and $\bar{\mathbf{5}}$ of $SU(5)$, and three fields Z_1, Z_2, and ϕ. A natural superpotential based on the O'Raifeartaigh paradigm (2.1) is

$$W = Z_1 \frac{\phi^3}{3M_*} + Z_2\left(\lambda \frac{\phi^2}{2}\left[1 + \lambda_1 \frac{Z_2}{M_*}\right] - \lambda \frac{\mu^2}{2} + \frac{\phi\eta\tilde{\eta}}{M_*}\right) + \lambda\phi\eta\tilde{\eta} + \lambda_2 \frac{(\eta\tilde{\eta})^2}{M_*}, \tag{2.2}$$

where M_* is a high scale corresponding to new(er) physics, such as a grand unified or Kaluza-Klein scale. We will obtain the parameter $\mu^2 \sim \Lambda^3/M_*$ dynamically from a coupling $\int d^2\theta W_\alpha W^\alpha \frac{\lambda Z_2}{M_*}$ between the $SU(2)$ sector and the O'Raifeartaigh model.

This theory is invariant under the following two symmetries: a discrete $\mathbb{Z}_{2N}$ R symmetry, with $N > 2$, under which the superpotential transforms with charge 2 and the fields ϕ, Z_1, Z_2, W_α, and $\eta\tilde{\eta}$ have charges 1, -1, 0, 1, and 1; and a continuous R symmetry, under which ϕ is neutral and Z_1, Z_2, and $\eta\tilde{\eta}$ transform, which governs the renormalizable terms, but is broken by the M_*-suppressed operators. The superpotential of (2.2) is the most general one respecting these symmetries, up to terms higher order in M_*^{-1}.

In the absence of the messengers, the model has a massless combination of Z_1 and Z_2, and a ϕ VEV

$$\phi_0^2 \approx \frac{\mu^2}{1 + 2\lambda_1 Z_2/M_*} - \frac{2\mu^4}{3M_*^2[1 + 2\lambda_1 Z_2/M_*]^4\lambda^2}. \quad (2.3)$$

Plugging in this solution yields $F_{Z_{1,2}}$ terms of order

$$F_{Z_1} \approx \frac{\mu^3}{3M_*}, \qquad F_{Z_2} \approx -\frac{\mu^4}{3M_*^2\lambda}, \quad (2.4)$$

plus corrections down by powers of μ/M_* and Z_2/M_*. In the full model, F_{Z_2} couples to the messengers η, $\tilde{\eta}$, suppressed by an additional power of $\phi_0/M_* \approx \mu/M_*$. (This suppression is forced on the model by the discrete symmetries it respects.) An F term for Z combined with a VEV for ϕ will produce naturally small superpartner masses as we will discuss further below.

The Coleman-Weinberg potential obtained by integrating out ϕ, η, $\tilde{\eta}$ yields a metastable minimum for Z at the origin in a self-consistent expansion about $\phi = \phi_0$, $\eta = \tilde{\eta} = 0$. Let us begin by integrating out the fluctuations of ϕ; we will show that these dominate over messenger loops in this model.[1] Writing $\phi = \phi_0 + \delta\phi$, the mass terms for the fluctuations $\delta\phi \equiv \delta\phi_1 + i\delta\phi_2$ are of the form

$$\lambda^2\delta\phi\delta\bar{\phi}(\mu^2 + |Z_2|^2) + (\delta\phi_1^2 - \delta\phi_2^2)\frac{\mu^4}{3M_*^2}, \quad (2.5)$$

plus contributions subleading in the regime $\mu/M_* \ll 1$. The fermion loops cancel the $\delta\phi\delta\bar{\phi}$ contribution here, and so the leading contribution to the potential from the ϕ multiplet is

$$\begin{aligned}\Delta V &\approx \mathrm{Tr}\,\log\left[(\mu^2\lambda^2 + |Z_2|^2\lambda^2 + p^2)^2 - \left(\frac{\mu^4}{3M_*^2}\right)^2\right] \\ &\quad - \mathrm{Tr}\,\log[(\mu^2\lambda^2 + |Z_2|^2\lambda^2 + p^2)^2] \\ &= \frac{1}{32\pi^2}\left(\frac{\mu^4}{3M_*^2}\right)^2 \log[\lambda^2(\mu^2 + |Z_2|^2)/M_*^2], \quad (2.6)\end{aligned}$$

plus subleading contributions. The messenger loops are subleading relative to the ϕ loops; the η, $\tilde{\eta}$ mass terms are of the form $(|\eta|^2 + |\tilde{\eta}|^2)\mu^2|\lambda + Z_2/M_*|^2$ plus a SUSY breaking term proportional to $\eta\tilde{\eta}\mu^5/(M_*^3\lambda)$, which will be much smaller than that in (2.5).

Although subleading in the Coleman-Weinberg potential, the messengers provide the dominant transmission of SUSY breaking to the SSM. As we just noted, the leading contribution to the messenger masses is from the supersymmetric $\lambda\phi\eta\tilde{\eta}$ coupling, giving $m_{\eta,\tilde{\eta}} \sim \lambda\mu$, while the leading SUSY breaking contribution to their masses is $\Delta m_{\eta\tilde{\eta}}^2 \sim \mu^5/(M_*^3\lambda)$. In application to gauge mediation, this yields gaugino and squark masses of the order of

$$\tilde{m} \sim \frac{g^2}{16\pi^2}\frac{\mu^4}{M_*^3\lambda^2}, \quad (2.7)$$

where g represents SSM gauge couplings.

For $F_{Z_i} \le 10^{20}$ GeV^2, the gravity-mediated contribution to superpartner masses is suppressed relative to the gauge-mediated contribution. Imposing this, we find that, for example, $m_{\eta,\tilde{\eta}} \sim \lambda\mu \sim 10^{11}$ GeV and $M_* \sim 10^{15}$ GeV produces a viable model, with $\lambda \sim 0.1$. Of course, if M_* were much lower than the grand unified theory (GUT) scale, then the messenger scale could be lower as well. As we will discuss further in the next subsection, another application of our method is to models where gravity mediation dominates.

To retrofit the model, as discussed above, we couple in a pure SUSY Yang-Mills sector with gauge superfield W_α, replacing the superpotential (2.2) with

$$\begin{aligned}W &= Z_1\frac{\phi^3}{3M_*} + \left(-\frac{1}{4g^2} - \frac{\lambda Z_2}{M_*}\right)W_\alpha^2 \\ &\quad + Z_2\left(\frac{\lambda\phi^2}{2}[1 + \lambda_1 Z_2/M_*] + \frac{\phi\eta\tilde{\eta}}{M_*}\right) \\ &\quad + \lambda\phi\eta\tilde{\eta} + \frac{\lambda_2(\eta\tilde{\eta})^2}{M_*}. \quad (2.8)\end{aligned}$$

Integrating out the gauge interactions yields

$$\begin{aligned}W &= Z_1\frac{\phi^3}{3M_*} + \lambda\Lambda^3 e^{-12Z_2/b_0M_*} \\ &\quad + Z_2\left(\lambda\frac{\phi^2}{2}[1 + \lambda_1 Z_2/M_*] + \frac{\phi\eta\tilde{\eta}}{M_*}\right) \\ &\quad + \lambda\phi\eta\tilde{\eta} + \frac{\lambda_2(\eta\tilde{\eta})^2}{M_*}. \quad (2.9)\end{aligned}$$

Expanding in Z_2 yields at leading order a model of the form (2.2), with $\mu^2 \propto \Lambda^3/M_*$. It is self-consistent to integrate out the gauge degrees of freedom because they have M_*-suppressed couplings to the rest of the system, too weak to compete against the forces in the Yang-Mills sector proper, which appear at the scale Λ.

Including the Z_2 dependence in solving for ϕ_0 yields

$$F_{Z_2} \propto \frac{\Lambda^6}{M_*^4\lambda}e^{-24Z_2/b_0M_*} + \ldots, \quad (2.10)$$

generalizing (2.4). This Z_2 dependence can lead to the presence of supersymmetric minima far away for appropriate ranges of parameters, but it does not destabilize our local minimum, as we can see easily as follows. Expanding in Z_2, the term $|F_{Z_2}|^2$ in the effective potential produces a tadpole of order $Z_2\Lambda^{12}/(M_*^9\lambda^2)$. The Coleman-Weinberg potential (2.6) produces a mass term of order

[1] In the model of Sec. III, the messengers themselves will play a leading role in stabilizing the scalar fields, providing a particularly direct mediation mechanism.

$\lambda^2|Z_2|^2\Lambda^9/M_*^7$, sufficient to stabilize Z_2 close to its original minimum at the origin.

B. Remarks on retrofitting O'Raifeartaigh models

In the previous subsection, we implemented the retrofitting procedure in a complete model, which was natural, given the specified symmetries, and incorporated messengers generating sparticle masses. The method has wider applicability, and it is interesting to extract and separate some of the essential elements of the procedure and consider independently the role of symmetry, chirality (or lack thereof), and metastability.

A simple example illustrates some of the main points. Consider a model with singlets Z, A, and B, and superpotential

$$W = MAB + \lambda Z(A^2 - \mu^2). \tag{2.11}$$

This model breaks supersymmetry. For $M > \sqrt{2}\lambda\mu$, there is a minimum in the A direction at $\langle A\rangle = 0$. At the classical level, there is a flat direction; the expectation value of Z is undetermined. However, at one loop, the standard Coleman-Weinberg calculation gives $\langle Z\rangle = 0$. The potential grows quadratically near the origin and logarithmically for $Z \gg M$.

Before rendering the mass parameters dynamical, note that a small deformation of the model makes the SUSY breaking minimum merely metastable. If we write

$$W = MAB + \lambda Z(A^2 - \mu^2) + \epsilon MZ^2, \tag{2.12}$$

for sufficiently small ϵ, there is still a metastable minimum near the origin. There is also a global SUSY preserving minimum at $Z = \lambda\mu^2/(2\epsilon M)$. (One can check that there is still a massless goldstino in the metastable minimum.)

Now we can retrofit the model and generate the small parameters dynamically. First, replace the μ^2 coupling by a coupling of Z to a strongly interacting gauge theory. This can be simply a pure supersymmetric gauge theory, leading to

$$W = MAB + \lambda ZA^2 + \left(-\frac{1}{4g^2} + \frac{Z}{M^*}\right)W_\alpha W^\alpha. \tag{2.13}$$

We are assuming g is fixed. If there are other moduli-like fields contributing to the gauge coupling, we assume that they are fixed at a higher scale, e.g., by fluxes or other dynamics. Now μ^2 is related to the dynamical scale of the hidden sector theory; integrating out the gauge interactions, the superpotential is

$$W = MAB + \lambda ZA^2 + \Lambda^3 e^{12Z/b_0M_*}. \tag{2.14}$$

Expanding the exponential in powers of Z, the linear term reproduces the original O'Raifeartaigh model. Near the origin, the Coleman-Weinberg corrections still generate a positive curvature. This still leads to a local minimum, provided $M \ll M_*$. As $Z \to -\infty$ (with $A = B = 0$), the energy tends to zero and SUSY is restored, though other effects may come in depending on the UV completion of the system.

This model closely parallels the O'Raifeartaigh models arising in the low energy limit of certain SUSY QCD theories [20] in a number of ways. With the small mass term for Z, $M\epsilon Z^2$, if $M\epsilon$ is sufficiently small, there is still a local minimum near the origin, but there is a supersymmetric minimum for $Z \sim \lambda\mu^2/(2M\epsilon)$. If the gauge group of the strongly interacting sector is $SU(N)$, the index can be computed for nonzero ϵ, and it is equal to N. The analogous statements hold for the models of [20] for small quark mass.

With $\epsilon = 0$, this model is the most general consistent with a discrete $\mathbb{Z}_{2N}$ R symmetry, under which the fields Z, A, and B, have charges 0, 1, and 1, respectively. The low energy theory has an *approximate, continuous* R symmetry under which Z, A, and B have charges 2, 0, and 2.

So far, this model has an additional scale M. But we can make this scale dynamical as well, without introducing any new scales beyond M^* and Λ. Simply introduce two other singlets, χ and C, with couplings

$$W_\chi = CAB + \lambda\chi C^2 + a\frac{\chi}{M^*}W_\alpha W^\alpha. \tag{2.15}$$

The parameter a is naturally of order one, if χ is neutral under the discrete R symmetry. In contrast to our complete models in Sec. II A and III, this structure is not enforced by symmetries, but it is meant only to be illustrative. The addition of small, symmetry-preserving couplings does not alter its basic features.

All of this illustrates that it is easy to construct metastable models of dynamical SUSY breaking with nonvanishing Witten index, which are not (necessarily) chiral. (These features also appear in the O'Raifeartaigh models in the infrared limit of some recent SUSY QCD examples [20] and earlier models of gauge-mediated SUSY breaking.)

Unlike in our complete example of Sec. II A, in this case we did not include messengers for gauge mediation. A simple coupling $Z\phi\tilde{\phi}$ would not suffice here since $Z \sim \Lambda^3/M_*^2$ is extremely small in the minimum obtained above (including the small tadpole introduced by the Z dependence of the $SU(2)$ gauge coupling). In Sec. II A, we solved this problem via a natural superpotential leading to spontaneous breaking of a discrete R symmetry. That example was limited to high or intermediate scale messenger masses, and it is of interest to explore this vast class of retrofit O'Raifeartaigh models in search of models with lower mass messengers. In models of this type, the SUSY breaking scale would be arbitrary. If the approximate R symmetries are broken at a scale of order the SUSY breaking scale, then the messenger mass scale is arbitrary as well. This may allow the construction of gauge-mediated models with scales of SUSY breaking as low as 10 TeV.

On the other hand, it is also a very simple to consider these theories as hidden sectors for gravity mediation.

These models are promising from this viewpoint since no symmetry forbids a coupling of Z to the SSM gauginos. In this case, the scalar and gaugino masses are of the same order, rather than being suppressed by a loop factor, as in anomaly mediation.

III. RETROFITTING FAYET MODELS

Another class of illustrative examples includes Fayet models, another of the classic models of perturbative SUSY breaking. We will start by describing a version with two input parameters, at least one of which needs to be small for natural SUSY breaking. We then upgrade the model to obtain the necessary small scale dynamically. This class of examples has the feature that the fields generating the leading contributions to the Coleman-Weinberg potential also can play the role of messengers of gauge-mediated SUSY breaking.

A. The perturbative SUSY breaking model

Begin with gauge group $U(1)$ and chiral fields X, ϕ, and $\tilde{\phi}$ with charges 0, 1, and -1, respectively. The model has superpotential

$$W = \phi X \tilde{\phi} + M^2 X - \frac{\lambda}{3} X^3, \tag{3.1}$$

and D term

$$D = e|\phi|^2 - e|\tilde{\phi}|^2 - r. \tag{3.2}$$

So far the model has two parameters input by hand: r and M.

Taking ϕ, $\tilde{\phi}$ to be messengers, a SUSY breaking configuration with $\langle X\rangle, \langle F_X\rangle \neq 0$ would transmit SUSY breaking to the standard model a la gauge mediation. This model has such a minimum, as follows. The potential energy of the model is

$$V(\phi, \tilde{\phi}, X) = |X|^2(|\phi|^2 + |\tilde{\phi}|^2) + |\phi\tilde{\phi} + M^2 - \lambda X^2|^2 + \tfrac{1}{2}(e|\phi|^2 - e|\tilde{\phi}|^2 - r)^2 + \Delta V, \tag{3.3}$$

where ΔV is the Coleman-Weinberg potential expanded about the point of interest in field space.

To obtain the structure described above, let us expand the theory about $\phi = \tilde{\phi} = 0$ and $X \approx M/\sqrt{\lambda}$. We assume $X^2 \gg eD$, and also take $eD \gg F_X \equiv M^2 - \lambda X^2$; we will verify that the latter assumption is self-consistent at the end. With these hierarchies, the ϕ, $\tilde{\phi}$ origin is stable, with $m_\phi^2 = |X|^2 + eD$ and $m_{\tilde{\phi}}^2 = |X|^2 - eD$. Setting $\phi = \tilde{\phi} = 0$, we find the following potential for X:

$$V_{\rm eff}(X) = |M^2 - \lambda X^2|^2 + \tfrac{1}{2}D^2 + \Delta V, \tag{3.4}$$

where, at the present level, $D = r$ is an input constant. In the dynamical version to follow, we will render D dynamically small in the vacuum.

The Coleman-Weinberg potential ΔV is straightforward to calculate here, particularly given $eD \gg F_X$. It is

$$\Delta V(X) = {\rm Tr}\,\log((|X|^2 + p^2)^2 - e^2D^2) - {\rm Tr}\,\log(|X|^2 + p^2)^2 + \mathcal{O}(F_X^2). \tag{3.5}$$

Here the first term comes from the ϕ, $\tilde{\phi}$ loops, and the second comes from the fermion loops which must cancel the first term up to the subdominant F-breaking effects. Performing the integration over momentum gives the result

$$\Delta V(X) = \frac{e^2D^2}{16\pi^2}\log(|X|^2/M_*^2) + \mathcal{O}(F_X^2) + \mathcal{O}(D^4e^4/X^4). \tag{3.6}$$

This potential (3.4) has extrema at

$$X_\pm^2 = \frac{M^2}{2\lambda}\left(1 \pm \sqrt{1 - \frac{e^2D^2}{8\pi^2M^4}}\right), \tag{3.7}$$

of which $X_+ \equiv X_0$ is a metastable minimum. In the regime defined above, this yields

$$X_0^2 \approx \frac{M^2}{\lambda} - \frac{e^2D^2}{32\pi^2\lambda M^2}, \qquad F_X \approx \frac{e^2D^2}{32\pi^2M^2}. \tag{3.8}$$

As a self consistency check, for $M \gg eD$, we have $F_X \ll eD$, as assumed above in the calculation of the Coleman-Weinberg potential.

It is worth noting that the result (3.8) for F_X follows from a simple scaling argument, which could be useful in more complicated examples. Before including the D-term breaking effect and resulting Coleman-Weinberg potential, the theory had a supersymmetric vacuum at $X = M/\sqrt{\lambda}$, with X mass $m_X = M$. The perturbative correction to the potential produces a tadpole $\partial\Delta V/\partial X$ evaluated at $X_0 \sim M$, which shifts the field by an amount $\Delta X \sim (\partial\Delta V/\partial X)/m_X^2$. The resulting F term is then of order

$$F_X \sim \frac{\partial F}{\partial X}\Delta X, \tag{3.9}$$

which agrees with the solution (3.8) in the present example.

Altogether, we have recovered the standard structure of gauge mediation in a simple model of perturbative SUSY breaking. In this example, the messengers participate directly in the SUSY breaking dynamics, in that their radiative effects generate the Coleman-Weinberg potential. Hence this constitutes a model of direct mediation. So far we have two input parameters, eD and M. The former is the only very small input scale required in the model, and we will render it dynamically small in the next subsection. Tying M to a dynamical scale would be somewhat more complicated.

B. Dynamical D

To render eD dynamically small, we first trade it for a superpotential term using the original Fayet model. Add two chiral fields a, $\tilde{a}$ of charge ± 1 under the $U(1)$ symmetry, and a superpotential

$$W_{a0} = m_{a0} a\tilde{a}. \tag{3.10}$$

As above we will be interested in large X, where $\phi = \tilde{\phi} = 0$. In this regime, for $er \geq m_{a0}^2$, the minimization in a, $\tilde{a}$ yields a vacuum

$$e|a|^2 = r - m_{a0}^2/e, \qquad eD = m_{a0}^2. \tag{3.11}$$

Thus the input Fayet-Iliopoulos parameter r itself can be of order the large scale M_*, and the problem of obtaining a naturally small eD reduces to that of obtaining m_{a0} dynamically.

This can be done as follows. First, note that the model would respect a $\mathbb{Z}_2$ R symmetry under which a, $\tilde{a}$ are neutral (and under which X is neutral, with $\phi\tilde{\phi}$ transforming nontrivially), if m_{a0}, M^2, λ were replaced with a dynamical operator which transforms nontrivially under the symmetry. Introduce a pure $SU(2)$ sector, with kinetic term $\int d^2\theta W_\alpha W^\alpha$. Here $W_\alpha W^\alpha$ transforms nontrivially under the $\mathbb{Z}_2$ R symmetry so that this kinetic term is invariant under the symmetry. Imposing this symmetry, we cannot write down a bare $m_{a0} a\tilde{a}$ term, but we can write

$$W_{a\Lambda} = a\tilde{a} W_\alpha W^\alpha / M_*^2 \propto a\tilde{a}\Lambda^3/M_*^2, \tag{3.12}$$

which weakly couples the $SU(2)$ degrees of freedom to the O'Raifeartaigh/Fayet SUSY breaking sector. In the last step in (3.12), we replaced $W_\alpha W^\alpha$ with its holomorphic VEV Λ^3. As in the O'Raifeartaigh case discussed above, it is consistent to integrate out the Yang-Mills degrees of freedom, since they couple weakly via M_*-suppressed couplings to the rest of the theory.

By the same token, the above symmetry prevents the pure superpotential $M^2X - \lambda X^3/3$ from appearing, but this times $W_\alpha W^\alpha / M_*^3$ can appear, along with an MX^2 term. (Adding an additional symmetry-respecting term proportional to $\phi\tilde{\phi}$ has no effect as it can be absorbed by a shift in X.) This modification leaves fixed the scaling of the X VEV found above, $X_0 \sim M$, and the scaling (3.9) of the resulting F term. Altogether this produces a theory in which the small parameter eD has been effectively replaced with $m_a^2 \sim \Lambda^6/M_*^4$. This leads to $F_X \sim \Lambda^9/M_*^7$.[2]

This much is sufficient to obtain very high scale gauge mediation naturally, with weak scale SUSY breaking obtained via the above method for rendering eD dynamically small, and with M an order of magnitude or two below $M_* \equiv M_{\rm GUT}$ as the only input parameter. If we take $M \sim 10^{-1} M_{\rm GUT}$, then we obtain a high scale gauge mediation model with a naturally small SSM gaugino mass arising from the dynamically small eD we obtained via the retrofitting procedure.

[2] One could also consider the symmetry $X \to -X$ under which $\phi\tilde{\phi}$ is invariant.

IV. DISCUSSION AND FUTURE DIRECTIONS

In this paper we combined simple ingredients in a straightforward way to obtain SUSY breaking models with all hierarchically small scales naturally explained dynamically. This procedure of retrofitting simple models can, of course, also be applied to more intricate examples; for example, one can similarly retrofit the model of [20] to render the input quark mass scale dynamically small, as was done recently in a footnote in [24]. In retrospect, however, perhaps the simplest possibility for model building is to obtain the small scale as a supersymmetric but dynamically small VEV, while obtaining the breaking of SUSY the old fashioned way.

There are several future directions to pursue. Here we focused on perhaps the very simplest models of perturbative SUSY breaking, but there are more general classes containing gauge fields for which one can systematically analyze the vacuum structure and retrofitting. It also will be interesting to investigate the realization of these models in string compactifications and to investigate retrofitting models to yield low scale messenger masses.

Gauge mediation models with messenger masses below $\sim 10^7$ GeV have the desirable feature that they do not require nonstandard cosmology to avoid overclosing the Universe with gravitinos, and they predict the spectacular prompt photon and multilepton collider signals usually associated with gauge mediation. Most direct gauge mediation models discussed previously predict intermediate or high scale messenger masses, in part because their extra particle content would otherwise force couplings to Landau poles well below the GUT scale. The explicit examples of Sec. II A and III. also yielded intermediate and high scale messenger masses. However, as noted in Sec. II B, low scale models may be possible, especially given the simplicity of the class of models discussed here.

Realistic application of these models requires an assessment of their cosmological stability. The metastable vacua themselves are very long lived, but whether the Universe finds its way into them cosmologically is an *a priori* separate question. This is very plausible given the symmetries governing our system [25]. It is under investigation in a similar class of models along the lines of [20] in [26] and may be affected by the process described in [27].

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