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SYMPOSIUM

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Revealing information in auctions: the allocation effect

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Abstract When there are two bidders, releasing independent information in an English auction with private values makes the seller worse off. However, this is no longer true with more bidders: when there is enough competition, revelation benefits the auctioneer. In three examples the dividing case is shown to be three bidders. This allocation effect applies to other standard auctions and parallels the bundling decision in a multi-unit auction.

Keywords Auctions · Information revelation · Bundling

JEL Classification Numbers D44 · D82 · L12

1 Introduction

When selling a good, one of the auctioneer's key choice variables is the amount of information to reveal to prospective bidders. Sotheby's success as an auction house owes much to their reputation for accurate appraisal and the timely release of this information. Similarly, prior to an oil auction, the Department of the Interior has to decide how much exploratory drilling to do, and whether it should make this information public. The auctioneer can also choose whether or not to allow bidders to acquire valuable information. On U.S. wildcat leases, bidders can gather seismic information, but no on-site drilling is allowed. Likewise, before an initial public

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offering or a takeover, the board of the target firm has to decide how much access to give potential bidders.

The theoretical analysis of information revelation was initiated by Milgrom and Weber's celebrated linkage principle, which says that expected revenue rises if the seller commits to reveal information about the value of a good. Intuitively, by making this new information public, bidders have less private information and make less rent.

This, however, is only half the story. Revealing information has another, potentially more important, effect which is orthogonal to the linkage principle. When the new information can change the order of bidders' valuations, an *allocation effect* may increase or reduce revenue. This allocation effect is at work across all auction formats, under common and private value auctions, and irrespective of whether information is released publicly or privately.

Aspects of the allocation effect have been analysed by a number of influential papers. Perry and Reny (1999) consider an example with two bidders, where the allocation effect outweighs the linkage principle.[1] More recently, Milgrom (2004, p. 199) constructs a simple private-value example, where information reduces revenue.[2] Bergemann and Pesendorfer (2003) analyse a multiple-signal model where the seller's information affects each bidder independently, as in Example 2. They characterise the optimal information structure while allowing the auctioneer to simultaneously choose the sales mechanism. Closer to the current paper, Ganuza (2004) considers a model where agents are distributed around the edge of a circle but do not know where the good is located, as in Example 3. When revealing information is costly, he shows that the revenue-maximising degree of information revelation increases in the number of bidders, converging towards the efficient level as $N \to \infty$.[3]

The purpose of this paper is threefold. First, we construct a general model that isolates the allocation effect, and identify under which conditions it will affect revenue. The allocation effect has been attributed to having asymmetric bidders or multiple units, yet none of this is necessary. Rather, we show the crucial feature is that new information changes the order of bidders' valuations.

Second, we analyse the impact the number of bidders has on the allocation effect. We show that, with two bidders, releasing information reduces revenue; and that, with many bidders, releasing information raises revenue. These results are derived with minimal structure on the model and follow from the allocation effect alone.

[1] Perry and Reny's example is essentially as follows. Agents have private signals $\theta_1, \theta_2 \sim U[0, 1]$ while the seller considers releasing a public signal $z \sim U[0, 1]$. The two agents have valuations $v_1(\theta_1, \theta_2) = \theta_1 + \alpha(\theta_2 + z)$ and $v_2(\theta_1, \theta_2) = \theta_2$ with $\alpha \in (0, \frac{2}{3})$. This single-unit version comes from Krishna (2002, p.115).

[2] Milgrom's example is as follows. Two bidders either have valuations $(v_1, v_2) = (1, 3)$ or $(v_1, v_2) = (3, 1)$, with equal probability. If the seller reveals which bidder has which value, revenue equals 1. However, if the seller hides the information, revenue equals 2.

[3] There are a number of other related papers. Vagstad (2006) asks whether the auctioneer should provide information to agents before they enter the auction in a multiple-signal model. Engelbrecht-Wiggans (1988), Persico (2000) and Rezende (2005) consider information acquisition in multiple-signal models. Eso and Szentes (2006) analyse the optimal mechanism when the auctioneer can sell their information.

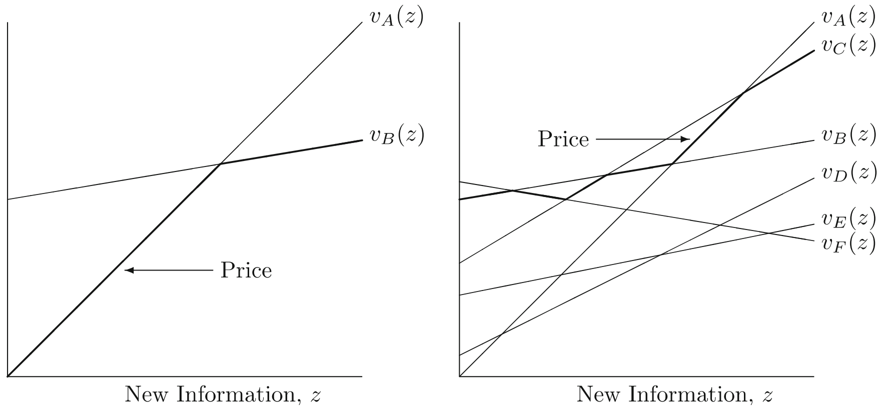

Fig. 1 Releasing Information with **a** two agents and **b** six agents

Third, we show there is a close analogy between releasing information in a single-unit auction and the bundling decision in a multi-unit auction. By observing that the two theories share the same formal structure, we hope to bring together two branches of auction theory which have hitherto been treated separately.

Suppose Sotheby's sells a painting to two bidders via an English auction. Before the auction, Sotheby's has to decide how much information to release, such as the history of the picture and their financial appraisal. Of the two bidders, agent A's valuation is greatly affected by this new information, while agent B likes the picture for its own sake and is less sensitive. This is shown in Fig. 1a.

Suppose the auctioneer publicly releases the information. A high signal helps A, causing her to win the auction; a low signal hurts A, causing her to lose the auction. Since the price is set by the losing bidder, the price is insensitive to public information when the signal is high, and sensitive when the signal is low. That is, good information leads to a small price rise, while bad information leads to a large price fall. Thus the average effect of releasing information is negative, reducing revenue.

This logic relies on there being only two bidders. More generally, an increase in the public signal may still lead the first and second agents to swap places, reducing the sensitivity of the price to public information. However, an increase in the public signal may also lead the second and third agents to swap, increasing the sensitivity of the price to information. As the number of bidders grows large, this second effect becomes more important and information revelation will tend to increase revenue (see Fig. 1b).

Naive intuition might suggest that when $N = 3$, the price setting bidder has one agent below and one agent above, so the two effects should cancel each other out. That is, revealing information will tend to lower the price when $N < 3$, and raise the price when $N > 3$. In three examples this intuition is shown to have some merit.

The paper is organised as follows. Section 2.1 shows that, with two bidders, revealing information always reduces revenue. The result is original to this paper, although examples have been provided by Perry and Reny (1999) and Milgrom (2004). Section 2.2 shows that, with enough bidders, revealing information increases revenue. A version of this result has been derived by Ganuza (2004), albeit with more structure on agents' preferences. Section 2.3 examines the dividing line in the context of three examples. Section 3 develops the bundling analogy, while Sect. 4 concludes.

2 Model

In order to examine the allocation effect, while parsing out the linkage principle, consider the release of information in single-item, private-value model. We suppose N agents compete in an English auction. Each agent i has private value $v(t_i, z)$, where $t_i \in \mathbb{R}^L$ represents agent i's type and $z \in \mathbb{R}^M$ represents new information the auctioneer may reveal. Assume that the new information, z, is independent of agents' private information. This is satisfied if (a) information z is independent of types t_i, or (b) types t_i are common knowledge. Denote the kth highest valuation from $\{v(t_i, z)\}_{i=1}^N$ by $[v(t_i, z)]_{k:N}$.

This model is quite general. The information, z, can be multi-dimensional, generated by an arbitrary distribution, and may or may not be observed by the auctioneer herself. Types may also be multi-dimensional, either private or public, generated by any distribution, and possibly correlated.

2.1 Two bidders: how information reduces revenue

When the new information is revealed, it is a weakly dominant strategy for i to bid $b_i^R = v(t_i, z)$. Similarly, when new information is hidden, it is a weakly dominant strategy for i to bid $b_i^H = E_z v(t_i, z)$, since z is independent of agents' private information. Taking expectations over this new information, denote the expected price when information is revealed (hidden) by P_N^R (P_N^H). When $N = 2$, Jensen's inequality implies

$$\begin{aligned} P_2^R &= E_z \min\{v(t_1, z), v(t_2, z)\} \\ &\leq \min\{E_z v(t_1, z), E_z v(t_2, z)\} = P_2^H \end{aligned} \tag{1}$$

since "min" is a concave function from $\mathbb{R}^2 \to \mathbb{R}$. Hence revealing information reduces revenue. This is true for every realisation of types, although revealing information only strictly reduces revenue if the identity of the winner depends upon z, as shown in Fig. 1a.

Welfare, which equals $\max\{v(t_1, z), v(t_2, z)\}$, is convex and hence increases with revelation. That is, when information is hidden the wrong bidder may sometimes win, reducing welfare.

Rents also increase under revelation. To see this notice that rents equal $\max\{v(t_1, z), v(t_2, z)\} - \min\{v(t_1, z), v(t_2, z)\}$, which is convex. While total rents increase under revelation, any single bidder's rent may decrease. Of course, if the agents are ex-ante symmetric, then both agents' ex-ante rents rise. To summarise,

Proposition 1 *Suppose $N = 2$ and consider any types, (t_1, t_2).*

(a) *Revealing independent information in a private-value English auction weakly reduces revenue.*

(b) *Revealing independent information in a private-value English auction strictly reduces revenue if and only if*

$$A_1 := \{z : v(t_1, z) < v(t_2, z)\} \quad \textit{and} \quad A_2 := \{z : v(t_1, z) > v(t_2, z)\} \tag{2}$$

have positive measure.

Proof (a) Follows from Eq. (1).

(b) First, suppose A_i has zero measure, for $i \in \{1, 2\}$. Then $P_2^R = E_z v(t_j, z) = P_2^H$, where $j \neq i$. Second, suppose both A_1 and A_2 have positive measure and let $E_z v(t_i, z) \leq E_z v(t_j, z)$. Then

$$P_2^H - P_2^R = E_z v(t_i, z) - E_z[v(t_i, z) + (v(t_j, z) - v(t_i, z))\mathbf{1}_{A_j}] > 0$$

as required. □

Proposition 1 says that the allocation effect can impact revenue if and only if the new information alters the order of bidders' valuations. This means that we should be concerned about the allocation effect only where there is sufficient horizontal differentiation among bidders.

A comparison with Milgrom and Weber (1982) is illuminating. Milgrom and Weber do not get the allocation effect because their *joint* assumptions of symmetry and monotonicity mean that the order of valuations coincides with the order of types, for any possible z. In practice either of these assumptions may be violated: in Sect. 2.3, Example 1 drops monotonicity, Example 2 drops symmetry, while Example 3 drops both. In comparison, the linkage principle is caused by correlation between the new information and bidders' private information. In our model, this is ruled out by assumption.[4]

2.2 Many bidders: how information increases revenue

The result for two bidders depends upon the concavity of the "min" operator. Concavity essentially means that the price is less sensitive to new information when the signal is high, as argued in the Introduction. However, when $N \geq 3$ the price is determined by the second order statistic which is no longer concave. Even when $N = 3$, the effect on revenue may therefore go either way.

1. Suppose $v(t_3, z) \leq \min\{v(t_1, z), v(t_2, z)\}$ for all z. Revenue equals $\min\{v(t_1, z), v(t_2, z)\}$ and is lowered by releasing information.
2. Suppose $v(t_1, z) \geq \max\{v(t_2, z), v(t_3, z)\}$ for all z. Revenue equals $\max\{v(t_2, z), v(t_3, z)\}$ and is raised by releasing information.

These examples illustrate that it is hard to make predictions with small numbers without more structure. Indeed, there is nothing to guarantee the sign of $(P_N^R - P_N^H)$ will even be monotone in the number of bidders. Nevertheless, as the number of bidders grows large revelation will benefit the auctioneer.

[4] To illustrate the linkage principle, suppose there are two bidders, where i's signal θ_i is privately known and IID, $z = \theta_1 + \theta_2$ and $v_i(\theta_i, z) = \theta_i + z$. When z is hidden, agent i bids $3\theta_i$. When z is revealed, i bids $2\theta_i + \theta_j$. Thus the price is higher under revelation.

Proposition 2 *Suppose that* (a) $\lim_{N\to\infty} E_z\big[[v(t_i,z)]_{1:N} - [v(t_i,z)]_{2:N}\big] = 0$ *and* (b) $\lim_{N\to\infty} E_z[[v(t_i,z)]_{1:N}] - [E_z v(t_i,z)]_{1:N} > 0$. *Then there exists* $\hat{N}$ *such that* $P_N^R > P_N^H$ *for* $N \geq \hat{N}$.

Proof From assumptions (a) and (b), there exists an $\hat{N}$ and $\epsilon > 0$ such that $E_z\big[[v(t_i, z)]_{1:N} - [v(t_i,z)]_{2:N}\big] < \epsilon$ and $E_z[[v(t_i,z)]_{1:N}] - [E_z v(t_i,z)]_{1:N} > \epsilon$, for $N \geq \hat{N}$. Hence, when $N \geq \hat{N}$, the benefit from revealing information is

$$\begin{aligned} P_N^R - P_N^H &= E_z[[v(t_i,z)]_{2:N}] - [E_z v(t_i,z)]_{2:N} \\ &\geq E_z[[v(t_i,z)]_{2:N}] - [E_z v(t_i,z)]_{1:N} \\ &> E_z\Big[[v(t_i,z)]_{2:N} - [v(t_i,z)]_{1:N}\Big] + \epsilon \\ &> 0 \end{aligned}$$

as required. □

Proposition 2 makes two assumptions. Assumption (a) states that, as the numbers of bidders rise, the auction becomes competitive and rents disappear. This is satisfied if, for example, types are drawn IID from some common distribution and $v(t_i, z)$ is bounded above. Assumption (b) states that there is horizontal differentiation among bidders. The weak version of this inequality always holds, by Jensen's inequality. The strict version of this inequality is satisfied if there exists an $\epsilon > 0$ and $\tilde{N}$ such that

$$A_j = \{z : [v(t_i,z)]_{1:N} - v(t_j,z) \geq \epsilon\} \tag{3}$$

has measure ϵ when $N \geq \tilde{N}$ $(\forall j)$.[5] Together, assumptions (a) and (b) imply that the auctioneer prefers to reveal information when the number of bidders is sufficiently large.

Intuitively, when the second bidder exchanges place with the first, then releasing information reduces revenue. When the second bidder exchanges places with any other bidder, then realeasing information increases revenue. If the first and second valuations grow close they may still exchange places; however the resultant reduction in revenue will be very small, and the second effect will dominate.

Note that this "closeness" condition in assumption (a) is not necessary. For example, if $v(t_1, z) \geq \max_i v(t_i, z)$ $(\forall z)$ then agent 1 always wins. Thus the first and second agents never swap places, and revealing information will increase revenue.

Propositions 1 and 2 can easily be extended in a number of ways. The analysis applies directly to the first-price auction when types are common knowledge. If types t_i are IID and induce a distribution of valuations that admits a positive, continuous density, then the analysis also extends to the first-price auction under incomplete information, by the revenue equivalence theorem (Krishna 2002, Propositions 2.2 and 3.1). The comparative statics also hold with respect to partial revelation of information, since we merely used Jensen's inequality. Finally, the

[5] To verify that (3) implies assumption (b), observe that $E_z[[v(t_i,z)]_{1:N}] - E_z v(t_j,z) \geq \epsilon^2$ $(\forall j)$ when $N \geq \tilde{N}$. Hence $E_z[[v(t_i,z)]_{1:N}] - [E_z v(t_i,z)]_{1:N} \geq \epsilon^2$ when $N \geq \tilde{N}$.

results still hold if there are K_N goods for sale. When N is large, revealing information will tend to reduce revenue if $K_N/N \approx 1$, and increase revenue if $K_N/N \approx 0$. Examples 1–3 suggest that the dividing line will often be around $K_N/N \approx 1/2$.

2.3 The dividing line

The following three examples explore how many bidders are required for revealing information to benefit the auctioneer.

Example 1 (*Single signal model*) Suppose i's valuation is given by $v_i = x + t_i z$, where x is fixed and large, and $t_i \in [\underline{t}, \overline{t}]$ is privately known and IID. The new information z is assumed to be independent of $\{t_i\}_i$ and can take negative values, where $E_z[z] \geq 0$ wlog. Under the English auction, when information is hidden, revenue is

$$\begin{aligned} P_N^H &= x + E_t[t_{2:N}]E_z[z] \\ &= x + E_t[t_{2:N}]E_z[z\mathbf{1}_{z>0}] + E_t[t_{2:N}]E_z[z\mathbf{1}_{z<0}] \end{aligned}$$

When information is revealed, revenue is

$$P_N^R = x + E_t[t_{2:N}]E_z[z\mathbf{1}_{z>0}] + E_t[t_{N-1:N}]E_z[z\mathbf{1}_{z<0}]$$

Hence the auctioneer prefers hiding when $N = 2$, is indifferent when $N = 3$, and prefers revelation when $N \geq 4$.

One can derive a number of other results in this model (see the Appendix for details). Since the English auction is efficient, revealing information always increases welfare. Revealing information also increases interim rents. As in Bergemann and Pesendorfer (2003) and Ganuza (2004), the intuition is that new information increases differentiation across agents.[6] These results go through with partial revelation of information and, by revenue equivalence, apply to other standard auctions.

Example 2 (*Multiple signal model*)[7] Suppose the auctioneer releases an IID vector $z = (z_1, \ldots, z_N)$ where agent i has valuation $z_i \subset [\underline{Z}, \overline{Z}]$. In the English auction, the price when information is hidden is $P_N^H = E_z[z_i]$. Under revelation the price is $P_N^R = E_z[z_{2:N}]$. If the distribution of z_i is symmetric around its mean, $E_z[z_{2:3}] = E_z[z_i]$, and revenue is reduced by revelation when $N = 2$, unaffected when $N = 3$ and increased when $N \geq 4$. By revenue equivalence this also holds for any standard auction.

To further illustrate, consider the power distribution with $F(z) = z^\alpha$ with $z \in [0, 1]$. Denoting the point where the auctioneer is indifferent between hiding and revealing by N^*, it can be shown that

$$N^* = \frac{3 + \sqrt{5 + 4/\alpha}}{2}$$

[6] This effect, however, relies on the convexity of $[v(t_i, z)]_{1:N} - [v(t_i, z)]_{2:N}$ and is thus model-specific. For example, releasing information lowers rents if $v(t_1, z) \geq \max_i v(t_i, z)$ $(\forall z)$.

[7] This model is analysed by Bergemann and Pesendorfer (2003), who allow the auctioneer to design the information structure and auction mechanism simultaneously. More in the style of our results, Ganuza and Penalva-Zuasti (2006) suppose the auctioneer controls the informativeness of the new information through a single parameter.

When $\alpha = 1$, the distribution of types is uniform and $N^* = 3$, as suggested by the naive intuition in the Introduction. However, when the right-hand tail gets thinner, then $\lim_{\alpha \to 0} N^* = \infty$. Intuitively, when the right-hand tail is very thin, there is one bidder who is much keener to win than all the others. (For example, think of selling the autograph of celebrity z, which only appeals to that celebrity's fans). If the seller reveals the state, the price is set by a bidder who is far less enthusiastic than the winner. If the seller hides the state, agents who are desperate to win in state z compete against those who are desperate to win in state z', raising the price.

Reservation prices reduce the dividing line, N^*. Denote the auctioneer's valuation by z_0. If $z_0 \leq \underline{Z}$ then Bulow and Klemeperer (1996) tells us that introducing the reservation price is similar to adding half a bidder. If the distribution of z is symmetric, the auctioneer strictly prefers to hide information when $N \in \{1, 2\}$ and reveal when $N \geq 3$. When z_0 is larger, N^* is further reduced. In the extreme, if $z_0 = E_z[z_i]$, then the auctioneer should always reveal information. The effect of the reservation price is further analysed by Lewis and Sappington (1994) in the case of one bidder.

Example 3 (*Circle differentiation; Ganuza 2004*)[8] Suppose the position of the good being sold, z, and agents' types, t_i, lie on the boundary of a circle of circumference 1. Let $\mu(t_i, z)$ be the shortest distance the agent must travel, and suppose their valuation is $-E[\tau(\mu(t_i, z))]$ where τ is some travelling cost. When information is perfectly revealed, $P_N^R = -E[\tau(\mu_{N-1:N})]$. When information is hidden, $P_N^H = -E[\tau(\mu_i)]$. Ganuza (2004) assumes that agents are uniformly distributed so $\mu_i \sim U[0, 1/2]$, implying μ_i is a mean preserving spread of $\mu_{2:3}$. If τ is linear, the auctioneer is indifferent between revealing and hiding when $N = 3$. If τ is strictly convex, the auctioneer strictly prefers to reveal information when $N \geq 3$.

3 Bundling information

In this section we observe that, when set up correctly, the auctioneer's information revelation problem parallels the bundling decision of a multi-unit auctioneer. As far as we know, this relationship has been overlooked in both literatures. This section explains where the parallel works, and where it fails. It is hoped that this relationship can then be used to simplify and unify the analysis of both topics.

Consider the information revelation problem in Sect. 2, and define the good being sold in each state of the world, z, as a separate commodity. By hiding information, the auctioneer is bundling different commodities together. By revealing information, the auctioneer is unbundling the commodities.

The seminal paper on bundling is that of Palfrey (1983), who considers an auctioneer who sells K goods which may be bundled or sold separately. Palfrey supposes agent i's valuation of good z, denoted $v_i(z)$, are NK IID random variables. When sold as a bundle, revenue is $[\sum_{z=1}^K v_i(z)]_{2:N}$. When sold separately, revenue is $\sum_{z=1}^K [v_i(z)]_{2:N}$. These equations look remarkably like those in Sect. 2. Indeed, Palfrey shows that with two bidders, bundling increases revenue, but reduces rents and welfare, as shown in Proposition 1. Palfrey also shows that as the

[8] I thank Heski Bar-Isaac for bringing this paper to my attention.

number of bidders grows large, so the auctioneer will sell the goods separately, as in Proposition 2. Moreover, when the distribution of $v_i(z)$ is symmetric, the auctioneer is indifferent between bundling and not when $N = 3$ (Chakraborty 1999).[9]

The reason for these results is that Palfrey's model contains an extreme form of the allocation effect: the ranking of valuations in state z is independent of the ranking in state z'. This is the polar opposite of Milgrom and Weber (1982), where the ranking of valuations is the same for all z.[10]

The parallel between the information revelation decision and the bundling decision has its limits, and depends on two important assumptions.

1. Agent i's value for a bundle of z_1 and z_2 equals the sum of $v_i(z_1)$ and $v_i(z_2)$. In the information model, the value for the bundle is the expectation over the states, so this assumption is automatically satisfied. However, in the bundling model, this additivity assumption is quite strict.[11]
2. The distribution of z is independent of agents' private information. In the bundling model, the set of goods is common knowledge, so this assumption is automatically satisfied. However, in the information model, this assumption rules out the linkage principle where an agent with a higher signal thinks they are bidding on a better bundle.

4 Conclusion

This paper has shown that releasing new information can affect revenue if it changes the order of bidders' valuations. This allocation affect is at work across all auction formats, under common and private value auctions, and irrespective of whether the information is released publicly or privately. The allocation effect is also orthogonal to the linkage principle, which results from new information being correlated with bidders' private information.

From a practical point of view, the weight the seller assigns to the allocation effect and the linkage principle should depend upon the case at hand. For small numbers of bidders, the two effects will go in opposite directions, so it will be important to identify which is most relevant. For example, releasing information about development of W-CDMA technology before a 3G telecoms auction is likely to reduce revenue if firms are using different standards, but increase revenue if firms are using the same standard. For larger auctions, the two effects will go in the same direction, supporting the release of information. As illustrated by Examples 1–3, this is likely to be the case even with a moderate number of bidders. In addition, releasing information may increase rents and promote entry, something outside the scope of this model.

[9] Chakraborty paper is far more general than this: for a wide class of distributions it is shown that there exists an N^* such that bundling is optimal if and only if $N < N^*$.

[10] Chakraborty (1999) and Armstrong (2000) consider an intermediate case where valuations in states z and z' are correlated. With suitable reinterpretation, their results are immediately applicable to the information revelation problem.

[11] See Krishna (2002, p. 288) for a version of Proposition 1 in a multi-unit auction model with general preferences.

Appendix: Analysis of Example 1

Bidder i is endowed with private type $t_i \in [\underline{t}, \bar{t}]$ with density $f(t_i)$. A sales procedure then consists of two stages. First, the auctioneer announces a signal s, correlated with the payoff relevant information, z. Second, the auctioneer holds an English auction (or, by revenue equivalence, any other standard auction).

Let $E_z[z \mid s]^+ := \max\{E_z[z \mid s], 0\}$ and $E_z[z \mid s]^- := -\min\{E_z[z \mid s], 0\}$. We say signal s_H is more informative than s_L if s_H it is a sufficient statistic for s_L.

Lemma 1 *Suppose s_H is more informative than s_L. Then $E_z[E_z[z \mid s_H]^+] \geq E_z[E_z[z \mid s_L]^+]$ and $E_z[E_z[z \mid s_H]^-] \geq E_z[E_z[z \mid s_L]^-]$.*

Proof Apply Jensen's inequality. □

Proposition 3 *Suppose the auctioneer holds an English auction. An increase in the informativeness of the signal:*

(a) *increases welfare;*
(b) *increases each agent's interim utility; and*
(c) *decreases (resp. increases) revenue if $N \leq 3$ (resp. $N \geq 3$).*

Proof (a) Welfare in the English auction is given by

$$\begin{aligned}\text{Welfare} &= x + E_z\big[E_z[z \mid s]^+\big]E_t[t_{1:N}] - E_z\big[E_z[z \mid s]^-\big]E_t\big[t_{N:N}\big]\\ &= x + E_z\big[E_z[z \mid s]^+\big]E_t\big[t_{1:N} - t_{N:N}\big] + E_z\big[z\big]E_t\big[t_{N:N}\big]\end{aligned}$$

Apply Lemma 1.

(b) Consider agent i. Let Y_1 and Y_{N-1} be the highest and lowest of his opponent's types. Agent i's interim rents are

$$\begin{aligned}E_z u_i(t_i) &= E_z\big[E_z[z \mid s]^+\big]E_{Y_1}[(t_i - Y_1)\mathbf{1}_{t_i > Y_1}]\\ &\quad + E_z\big[E_z[z \mid s]^-\big]E_{Y_{N-1}}[(Y_{N-1} - t_i)\mathbf{1}_{Y_{N-1} > t_i}]\end{aligned}$$

Apply Lemma 1.

(c) Revenue is given by

$$\begin{aligned}\text{Revenue} &= x + E_z\big[E_z[z \mid s]^+\big]E_t\big[t_{2:N}\big] - E_z\big[E_z[z \mid s]^-\big]E_t[t_{N-1:N}]\\ &= x + E_z\big[E_z[z \mid s]^+\big]E_t\big[t_{2:N} - t_{N-1:N}\big] + E_z\big[z\big]E_t\big[t_{N-1:N}\big]\end{aligned}$$

Apply Lemma 1. □

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