# Revealing the order of a vortex through its intensity record

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We show that the intensity distribution of an optical vortex contains information of its order. Specifically, the number of dark rings in the Fourier transform of the intensity is found to be equal to the order of the vortex. Based on this property and the orthogonality of Laguerre polynomials, we demonstrate the feasibility of an experimental technique for determining the order of optical vortices. It shows the beauty of going to complementary spaces, which has been employed earlier also to find the information not available in other domains.   © 2011 Optical Society of America

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Optical vortices are singular points in the phase distribution of a light field and observed as dark points on the screen [1]. Light beams with such a feature have found a variety of applications in the optical manipulation of microscopic particles [2] and quantum information and computation [3].

Detection and determination of the charge of optical vortices is one of the basic requirements in singular optics. Most of the techniques to determine the order of vortices are based on interferometry [4–7]. The interferometry has been further extended to find the spatial coherence function that also provides information about the order of vortex [8]. All these methods require a good number of optical elements and their fine alignment. Aberrations in optical elements, scratch and dig, dust particles on these elements, and their misalignments disturb the characteristic interference pattern of the vortex. Therefore, efforts have been made to find the order of the vortex using techniques other than the interferometry [9–13].

In this Letter, we show that the order of a vortex can be obtained from the record of its intensity distribution itself. We know that when the order of the vortex increases, the size of the dark core at the center increases [14]. Moreover, the size of the dark core may vary depending on the resolution of the hologram used in the spatial light modulator (SLM) [15]. Therefore, just by measuring the size of the dark core, it is difficult to discriminate between the orders experimentally.

We provide a method to discriminate between the orders of the vortices by taking the Fourier transform (FT) of the recorded intensity of the vortex and using the orthogonality of the Laguerre polynomials. Our main result is based on Eq. (12) and Fig. 4.

The field of a vortex of order $m$ can be written as

$$E_m(x, y) = (x + iy)^m \exp[-(x^2 + y^2)/\sigma^2], \qquad (1)$$

where $\sigma = 1.69\,\text{mm}$ is the radius of the first diffraction order Gaussian beam at the CCD (see Fig. 1). The beam was generated by placing a grating without fork pattern at the SLM. The intensity of the vortex is given by

$$I_m(x, y) = (x^2 + y^2)^m \exp[-2(x^2 + y^2)/\sigma^2]. \qquad (2)$$

The FT of $I_m(x, y)$ has the expression

$$
\begin{aligned}
\mathcal{F}_m(\omega_1, \omega_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} & (x^2 + y^2)^m \\
& \times \exp[-2(x^2 + y^2)/\sigma^2 \\
& - i(\omega_1 x + \omega_2 y)] \mathrm{d}x \mathrm{d}y,
\end{aligned}
\qquad (3)
$$

where $\omega_1$ and $\omega_2$ are spatial frequencies. Expanding $(x^2 + y^2)^m$ in a binomial series, Eq. (3) can be written as

$$\mathcal{F}_m(\omega_1, \omega_2) = \sum_{n=0}^{m} \binom{m}{n} \mathcal{I}_n(\omega_1) \mathcal{I}_{m-n}(\omega_2), \qquad (4)$$

where

$$\mathcal{I}_n(\omega_1) = \int_{-\infty}^{\infty} x^{2n} \exp(-2x^2/\sigma^2 - i\omega_1 x)\mathrm{d}x. \qquad (5)$$

Using the formulas [16]

$$
\begin{aligned}
& \int_0^{\infty} x^{2n} \exp(-\beta^2 x^2) \cos axdx \\
& = (-1)^n \frac{\sqrt{\pi}}{(2\beta)^{2n+1}} \exp\left(-\frac{a^2}{4\beta^2}\right) H_{2n}\left(\frac{a}{2\beta}\right),
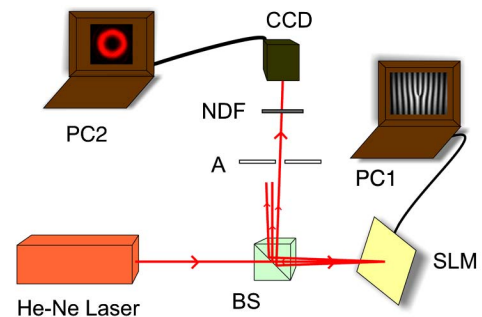\end{aligned}
\qquad (6)
$$



Fig. 1.   (Color online) Experimental setup to generate optical vortices and to find their order.