# Revealing The Parameter of Risk-Aversion From Option Prices When Markets Are Incomplete: Theory and Evidence.

By

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*Very Preliminary and Incomplete*
*Comments are welcome*

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# Revealing The Parameter of Risk-Aversion From Option Prices When Markets Are Incomplete: Theory and Evidence.

*Abstract:* A standing assumption in the literature concerning the estimation of the parameter of relative risk aversion from option prices is that a representative investor exists. This thus assumes that markets are complete. We suggest a new methodology in order to extract the parameter of risk aversion from option prices when markets are possibly incomplete. Our estimates of the parameter of relative risk aversion ranges from 1.6 to 3.1. When it is time varying and only Calls are used, the parameter of risk aversion is shown to vary pro-cyclically with the market while the pricing kernel varies counter-cyclically. When estimated using only Puts, the parameter of risk aversion is show to vary counter-cyclically with respect to the market while the pricing kernel vary pro-cyclically. As a consequence, separating Calls from Puts help understand that the so-called Pricing Kernel Puzzle may not be a puzzle. Finally, since we took all the moneyness range available each day, the reasonable values obtained for the parameter of relative risk aversion show that puts in general, and deep-out of the money Puts in particular, are not mispriced. Market incompleteness provides a relevant explanation of their price behavior.

# 1. INTRODUCTION

Option Markets are ideally suited to the study of many issues that are of interest to both academics and practitioners. Among these issues, the estimation of the market level of risk aversion has been given a great of attention. This stems from the fact that, for a given underlying asset and a given maturity, several prices of contingent claims (options) written on this underlying may be observed. These prices can be helpful in obtaining some information on the pricing function in the financial market. Since this function is intimately related to a Representative Investor's intertemporal marginal rate of substitution, some information on the parameters of the Representative Investor utility function can be inferred from the pricing kernel of the Economy.

Three elements are required by the standard approach that has so far been used to estimate the parameter of risk aversion from option prices: i) a risk neutral probability measure also called a RND, i.e. Risk Neutral Density function, ii) a historical probability measure of the distribution of the returns of the underlying also called the historical density function, and iii) a utility function. These three variables are linked by a well-known relation which states that, for a given state of nature, the probability ratio of this state given by the RND over the probability of this state given by the historical density function is proportional to the marginal utility of the Representative Investor. Once two of these variables are known, the third one can be computed or estimated. The RND is usually estimated from the option data either using parametric, semi-parametric or non-parametric methods. This procedure is a necessary and difficult step which has been given much attention in the Literature[1]. Once the RND has been estimated, there are two ways of recovering the Representative Investor's relative risk aversion parameter. One way is to fit a stochastic process to the historical behavior of the

[1] See Jackwerth (1999) for a survey of the literature dealing with RND extraction from option prices and its relationship with the historical probability density function. Recent works by Jondeau and Rockinger (2000), Bliss and Panigirtzoglou (2002) and Anagnou et al. (2002) test the robustness of parametric and non parametric methods to this end. While previous studies estimate RND at a given point, Panigirtzoglou and Skiadopoulos (2004) suggest a way to model the dynamics of the implied RND over time.

underlying return so as to obtain the historical density function[2]. It is then used to recover the parameter of risk aversion after assuming a particular utility function[3]. A second way consists in postulating a particular utility function and then finding the parameter of relative risk aversion such that the risk adjusted RND is a good estimator (relative to a given criterion) of the historical density function[4].

The main limit of these approaches that aim at estimating the parameter of risk aversion using options is that the relationship linking the two probability measures to the parameter of risk aversion is only valid if markets are complete. In such a case, it is well known (Constantinides (1982)) that a representative agent exists. If markets are incomplete (as a possible result of some market frictions like short sale constraints) the existence of a Representative Investor is not guaranteed and the approach is therefore not valid.

Incompleteness is likely to be a standing feature of financial markets. If it were not, it would be difficult to explain the extensive development of option markets. If markets were complete, options would be redundant and would not thus be so successful. In addition, since the work of Mehra and Prescott (1985) on the equity premium puzzle, the pricing implications of the Representative agent setting have been unable to explain the behavior of asset prices. Oddly enough, the strand of literature briefly reviewed above still relies on this assumption in the case of the option markets. The findings hint at the absence of an Equity Premium Puzzle in option returns which are however far more volatile than their underlying asset. These studies report reasonable values for the

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[2] Most of the time, this historical Density function was estimated independently from the RND like in Britten-Jones and Neuberger (2000). However, others tried to estimate jointly the RND and the historical Density function like Chernov and Ghysels (2000).

[3] See recent contribution by Ait Sahalia and Lo (2000), Engle and Rosenberg (2002) and Jackwerth (2000) among many others. Jackwerth (2000) suggested an approach than does not require an estimation of the shadow price of the Representative Investor budget constraint which is (divided by the risk free gross rate of return) the factor by which the ratio of probabilities is multiplied in the linking relationship between the three ingredients.

[4] This is the approach followed by Bliss and Panagirtzoglou (2004) and Anagnou et al. (2002) among others.

relative risk aversion parameter which is a criterion of the economic validity of the models. For example, Engle and Rosenberg (2002) found a time varying parameter ranging between 2 to 12 while Bliss and Panigirtzoglou (2004) recently reported values for the parameter between 1.97 and 7.91 which are reasonable values from an economic point of view. In other words, these studies suggest that options returns are not subject to the Equity Premium Puzzle. We believe this to be a first puzzle which adds up to a second one which has recently been pointed out by Brown and Jackwerth (2003). They observe that the level of risk aversion is pro-cyclical (it increases with the wealth level), which seems counter intuitive.

Finally, the moneyness range is often restricted in this kind of studies which is problematic since, as shown by Bondarenko (2003), deep-out-of-the-money Put prices are not compatible with standard asset pricing models based on market completeness, or even with improvements of this setting that for instance introduce habit formation. Therefore, these Puts are unfortunately always put aside and are not used in empirical investigations. In this paper, we do use all the moneyness range from which we derive strong evidence that market incompleteness explains the return behavior of options. Such evidence is obtained through our methodology which, we hope, will shed light on the two puzzles we have previously discussed and explain option prices.

In this paper, we suggest an alternative approach for estimating the parameter for risk aversion from option prices when markets are possibly incomplete. Our strategy builds on the seminal contribution of Constantinides and Duffie (1996) (thereafter CD) who determined the necessary conditions for the idiosyncratic risk to matter for asset pricing at the equilibrium when markets are incomplete. The results of CD's analysis may be summarized as follows: in an Economy populated with agents endowed with an identical utility function but different income processes, the equilibrium asset pricing is isomorphic to the equilibrium asset pricing in an Economy with a Representative Consumer whose preferences are different from the preferences of each agent in the economy. Their findings imply that the estimated relative risk aversion parameter using a standard Euler equation with per capita consumption is not the parameter of relative risk aversion in the economy, but a function of this parameter. Two crucial assumptions were laid out by Constantinides and Duffie (1996) to achieve such a result. The first one is that the idiosyncratic risk is persistent and countercylical. A recent contribution by Storesletten et al. (2004) shows that idiosyncratic risks actually have such empirical features. The second assumption states that the pricing kernel under incomplete markets is higher (state by state) than the pricing kernel that will prevail if markets are complete (the pricing kernel condition). In other words, CD assume that market incompleteness makes financial assets, and contingent claims in particular, more expensive. A recent study by Lioui and Malka (2004) using data from the Tel Aviv stock exchange, which is complete four days per week and incomplete one day per week, shows that this assumption is also verified. The main prediction of CD's model is that the dispersion of consumption growth among consumers is a priced factor. Such a feature has received empirical support both at the domestic level (Jacobs and Wang (2004)) and the international level (Sarkissian (2003)). Therefore, we believe that using this model for extracting the parameter of risk aversion implicit in option prices is warranted.

In this paper, we use factor representation of the Stochastic Discount Factor which is projected on a set of traded options. We grouped the options into six moneyness groups and two maturities, one short maturity (the first maturity each day) and one long maturity (the second maturity each day). We take all the moneyness range of Calls and Puts, which constitute a total of 24 assets. We estimate the parameter of risk aversion, in two cases: constant and time varying. The estimated constant parameter ranges between 1.62 and 3.17 while the average time varying parameter of relative risk aversion ranges between 1.61 and 3.08. We provide results both for Calls and Puts, and for short term and long term options. Among many interesting results, we find that option returns are subject to the standard equity premium puzzle when incompletenes is not accounted for. Moreover, when Calls are used to estimate the parameter of relative risk aversion, the latter is shown to be pro-cyclical while it is counter-cyclical when Puts are used. The pricing kernel/Stochastic Discount Factor inferred from Calls is shown to be counter-cyclical while it is pro-cyclical with Puts. Finally, the low level of the estimated parameter of risk aversion, even when deep-out-of-the-money Puts are used, show that market incompleteness is an interesting way of explaining the observed “abnormal” returns.

The approach we suggest, besides accounting for the possibility of market incompleteness, has several other advantages. First and foremost, it is not subject to the “error in variables” problem of the standard approach. Indeed, one essential element for the standard approach for estimating the RRA parameter is the RND which is estimated from option prices and then used to estimate the parameter of risk aversion. Although significant progress has been done in estimating such an RND both parametrically and non parametrically, it still faces a data problem: there are usually up to still 30 options (both calls and puts) in a cross section of relevant prices. These prices are used to generate thousands of other prices, by smoothing for example the implied volatility function. As a consequence, and given the sensibility of the RND to the smoothing scheme used, these problems will impact on the estimated parameter of risk aversion. Another advantage of our approach is that all the traded options, i.e. all the maturities and all the strikes, can be taken into account. Previous studies do not often use all the data and restrict themselves either to at the money options, out of the money options, or only Calls since Puts are “redundant” due to the Call Put Parity, or to a limited range of moneyness. We can use all the options and show that once market incompleteness is accounted for, options with high levels of implied volatilities are not necessarily mispriced options.

Our findings show that without accounting for market incompleteness, options returns are subject to the Equity Premium Puzzle, nevertheless the behavior of the pricing kernel using Calls alone and Puts alone are different. This could explain the fact that previous studies found reasonable values for the parameter of relative risk aversion. The authors use both Calls and Puts for extracting the RND while we explicitly separate these two sets of assets. By doing so, we show that the pricing kernels extracted from

Calls and Puts behave in opposite manners. If the pricing kernel found by using Puts is more convex than the corresponding one using Calls, combining Calls and Puts will probably yields a pricing kernel which is an increasing function of the Market Index. Therefore, the Pricing Kernel Puzzle is not a puzzle but simply a direct consequence of the empirical methodology. Finally, since the curvature of the pricing kernel extracted from Call Prices will be negative and that extracted from Puts will be positive, combining both of them will yield a small parameter of risk aversion in absolute term and give the illusion that these assets are not subject to the Equity Premium Puzzle. This is exactly what occurs in the existing literature.

The rest of the paper is organized as follows. In the next section we set up out pricing framework and the empirical design of the methodology. In section 3 we describe the Data used for the investigation and in section 4 we report the results of our empirical investigation.

## 2. ASSET PRICING IN INCOMPLETE MARKETS: THEORY AND EMPIRICAL DESIGN

When markets are complete, pricing contingent claims, and especially options, is a relatively easy task. This is because when markets are complete and frictionless, it is well known that there exists a risk neutral probability density function (a martingale measure) such that the price of any contingent claim today is simply the expected value under this martingale measure of its future discounted cash flow. Since this martingale measure is unique, the contingent claim price is also unique[5]. Another way to see this feature of complete markets is based on the fact that when markets are complete and frictionless, the equilibrium asset pricing implications of such an economy are isomorphic to the asset pricing implications of an economy with a representative

[5] See for example Harrison and Kreps (1979).

investor[6]. The Stochastic Discount Factor in the Economy is simply the intertemporal marginal rate of substitution of the Representative investor using per capita consumption.

When markets are incomplete, this important feature fails to hold and i) all contingent claims are not necessarily replicable and ii) some contingent claims have more than one strategy than can replicate their cash flows and thus have more than one price. As to the representative investor's approach, the main implication of market incompleteness is that it does no longer necessary exist. In particular, what is relevant for asset prices is no longer only the aggregate consumption risk (systematic risk) which is the only relevant risk in the economy when markets are complete, but also the idiosyncratic risk of the agents.

Many Authors suggested ways to deal with such difficulties inherent in market incompleteness. One approach suggest to price non attainable contingent claims using a super-replicating strategy, meaning building a trading strategy which cash flows in the future is at least equal to the cash flow of the contingent claim[7]. Others suggested to reduce the number of pricing kernels by imposing some restrictions on the first moments. For example, by imposing a limit on its volatility or its Sharpe ratio[8]. While such approaches restrain the set of admissible prices, there is till a part of arbitrary to pick up the Stochastic Discount Factor. One advantage of the equilibrium approach is that it does not require necessarily more assumptions than these approaches while leading to a unique stochastic discount factor.

Among alternative approaches to deal with market incompleteness, we follow the approach developed by Constantinides and Duffie (1996) whose main advantage is that it requires only few additional assumptions related to the standard complete market

[6] See Constantinides (1982) for additional conditions and results.
[7] See Cvitanic (1999).
[8] See for example Cochrane and Saa Requejo (2000).

setting which leads to the existence of a Representative Investor. When asset markets are Arbitrage free, the pricing kernel conveniently summarize the complex stochastic environment in which asset pricing takes place. If an asset j yields a stochastic dividend stream $d_{j,t}$ up to infinity, its price $P_{j,t}$ at date t is such that[9]:

$$P_{j,t} = \frac{1}{M_t} E\left[ \sum_{s=t+1}^{\infty} d_{j,s} M_s \middle| \phi_t \right] \tag{1}$$

where $M_t$ stands for the pricing kernel, E the expectation operator and $\phi_t$ is the information available at time t. The absence of arbitrage opportunities guarantees that at least one $M_t$ exists and it is always positive. In CD's model, agents in the economy are assumed to have the same time-additive utility function over their consumption. Therefore, the utility of agent i from its consumption $C_{i,t}$ at time t is such that:

$$U(t, C_{i,t}) = e^{-\rho t} \frac{C_{i,t}^{1-\alpha}}{1-\alpha} \tag{2}$$

where $\alpha > 0$ is the constant relative risk aversion and $\rho$ the subjective discount rate. The Euler equation for consumption for agent i is written as:

$$E\left[ R_{j,t+1} e^{-\rho} \left( \frac{C_{i,t+1}}{C_{i,t}} \right)^{-\alpha} \middle| \phi_t \right] = 1 \tag{3}$$

where $R_{j,t+1}$ stands from the gross return on asset j from t to t+1. When markets are complete, there exists a representative investor in the economy and the Euler equation associated with her first order condition for optimality that characterizes asset prices in the Economy is written as:

$$E\left[ R_{j,t+1} e^{-\rho} \left( \frac{C_{t+1}}{C_t} \right)^{-\alpha} \middle| \phi_t \right] = 1 \tag{4}$$

[9] For convenience, we use the same notations as CD.

where $C_t$ stands now for the per capita (or aggregate) consumption in the Economy. Thus, when markets are complete, the Stochastic Discount Factor is given by the intertemporal marginal rate of substitution,

$$\frac{M_t}{M_{t-1}} = e^{-\rho}\left(\frac{C_t}{C_{t-1}}\right)^{-\alpha} \tag{5}$$

When markets are incomplete, the Euler equation that characterizes asset prices is:

$$E\left[R_{j,t+1}e^{-\rho}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha} e^{\frac{\alpha(\alpha+1)}{2}\mathrm{Var}\left(\ln\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)\right)}\middle|\phi_t\right] = 1 \tag{6}$$

where $\mathrm{Var}\left(\ln\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)\right)$ stands for the variance of the cross section of consumption growth among the consumers in the economy. This is the way the idiosyncratic risk affects asset prices when markets are incomplete. When markets are complete, such variance is zero.

For our purpose, and since we do not rely on consumption data, we need to lay out additional assumptions on this important variable. First, note that:

$$\mathrm{Var}\left(\ln\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)\right) = \frac{1}{\alpha^2}\mathrm{Var}\left(\ln e^{-\rho}\left(\frac{C_{i,t+1}}{C_{i,t}}\right)^{-\alpha} - \ln e^{-\rho}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\right) \tag{7}$$

We can thus assume that this variance is simply a linear function of aggregate consumption growth rate, i.e.,

$$\mathrm{Var}\left(\ln\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)\right) = a_t + \ln\left(b_t e^{-\rho}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\right) \tag{8}$$

with relevant constraints on the processes a and b for the right hand side of (8) to be well defined. The left-hand side of (8) is the variance of the spread between the intertemporal marginal rate of substitution of an individual agent in the Economy and the aggregate intertemporal marginal rate of substitution. We think it is reasonable to assume a linear relationship of this spread with respect to the aggregate intertemporal marginal rate of substitution. Success of models with external habit formation[10] in explaining several asset pricing anomalies seems to us an important support to the chosen specification.

Substituting for (8) into (6) yields:

$$E\left[ R_{j,t+1} e^{-\hat{\rho}_t} \left( \frac{C_{t+1}}{C_t} \right)^{-\hat{\alpha}} \middle| \phi_t \right] = 1 \tag{9}$$

where

$$\begin{aligned} \hat{\rho}_t &= \rho\left(1 + \frac{\alpha(\alpha+1)}{2}\right) - \frac{\alpha(\alpha+1)}{2} a_t + \frac{\alpha(\alpha+1)}{2} \ln b_t \\ \hat{\alpha} &= \alpha\left(1 + \frac{\alpha(\alpha+1)}{2}\right) \end{aligned} \tag{10}$$

Therefore, the asset pricing equilibrium in this economy is isomorphic to the asset pricing of an Economy with a Representative Investor, whose preferences have perturbed parameters with respect to the parameters of the utility function of the agents in the Economy. An immediate consequence is that the estimated Euler equation that originated in the work of Mehra and Prescott (1985) that showed the Equity premium puzzle is not a genuine estimate of the parameter of risk aversion in the economy but only one that is perturbed by the idiosyncratic risk. An advantage of CD's findings, and of the version (9) of their model that we consider here, is that we can recover the "true" parameter of risk aversion using the estimated parameters from (10).

[10] See Campbell and Cochrane (1999) for example.

Here, a word of caution is needed. Given (10), the risk aversion parameter can be recovered by knowing $\hat{\alpha}$. An a priori difficulty is that the relationship between the two given by (10) is a polynomial relation; as such, there may be up to three possible $\alpha$ for any $\hat{\alpha}$. This will happen for values of $\hat{\alpha}$ between –1.06 and 0.32,. However, this never happens in practice: therefore, there will always be one solution for the parameter of risk aversion. The simple intuition why this never happens is related to the fact that option returns are subject to the "Equity Premium Puzzle", i.e. estimates of $\hat{\alpha}$ from Euler Equation such as (9) are high.

Most empirical investigations of CD have been based on consumption data. In our case, where we deal with asset prices, we refrain from using consumption data and rather use the standard factor representation of the SDF. Consumption growth is approximated by changes in a market index such that:

$$e^{-\hat{\rho}_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\hat{\alpha}} = f(Grm_{t+1}) \tag{11}$$

where Grm stands for the gross return of the market portfolio. Thus (9) is rewritten as:

$$E\left[R_{j,t} f(Grm_t) \middle| \phi_t\right] = 1 \tag{12}$$

We actually follow the strand of the literature which assumes that the underlying asset itself drives the pricing kernel. The measure of the parameter of risk aversion is written as:

$$\hat{\alpha} = -Grm_{t+1} \frac{\partial f(Grm_{t+1})}{\partial Grm_{t+1}} \frac{1}{f(Grm_{t+1})} \tag{13}$$

Another word of caution is needed here: since we are using separately Calls and Puts, we end up with results of opposite signs. For one type of options, i.e., Calls for the reasons explained in the Introduction, the parameter $\hat{\alpha}$ is positive and for the other type (Puts), it is negative. As a consequence, the pricing kernel is negatively correlated with Calls and positively with Puts.

We will consider two cases for f(Grm), namely:

$$f\left(Grm_{t+1}\right) = \lambda_0 Grm_{t+1}{}^{-\lambda_1} \tag{14}$$

$$f\left(Grm_{t+1}\right) = \pi_0 Grm_{t+1} + \pi_1 Grm_{t+1}{}^2 \tag{15}$$

These are the two standard forms found in the literature. The first one (14) is the power form which directly stems from the Constant Relative Risk Aversion utility function that is often assumed for the Representative Investor. The second one is motivated by recent evidence showing that higher moments of the market portfolio are relevant for asset pricing reflecting the fact that agents care about kurtosis and skewness (Dittmar (2002)).

When the SDF is taken to be (14), then:

$$\hat{\alpha} = \lambda_1 \tag{16}$$

and, using (15), we will have:

$$\hat{\alpha}_t = -\frac{Grm_{t+1}\left(\pi_0 + \pi_1 Grm_{t+1}\right)}{\pi_0 Grm_{t+1} + \pi_1 Grm_{t+1}{}^2} \tag{17}$$

Thus the first pricing kernel is associated with a constant parameter of relative risk aversions while the second one is associated with a time varying parameter of risk aversion.

## 3. DATA

We use data from the Israeli Stock Exchange Market, the TASE. The main index on this market is the TA 25 that contains 25 stocks and was previously denominated the MAOF. Stocks included into the TA 25 Stock Index are those with the highest market capitalization and are among the 75 shares with the highest average daily turnover. The Index is updated twice a year, on January and July. The stock index is dividend protected, so that its value is obtained assuming the reinvestment of any dividend into the stock distributing the dividend. For our sample period that contains 7 years, from 1/1/96 to 31/12/02, summary statistics of the behavior of the stock index are gathered in Table I.

**Table I**

The average daily return was 0.008% (around 3% annually) but with a high standard deviation for daily returns, 119%. The range of the index daily return was between – 9.4% and 7.4 %, thus a wide range for daily returns. Given the skewness and the kurtosis of the return distribution, the index return was clearly not normally distributed and this will be reflected both in the option prices (smile of volatility) and in the dynamics of the pricing kernels.

European options on the stock index started to be traded on August 1993. Until 2000, options were traded with expiration date every two months. Since 2000, a new serie of options is introduced each month with three months to expiration so that each month one traded serie expires. The average daily number of contracts traded has known a constant growth for the past years, reaching more than 100 000 contracts per day. In terms of the underlying (weighted by the deltas), option trading represents more than 800% of the turnover in the TA 25 shares. Futures contracts on the stock index have been introduced on October 1995 with three months to maturity. There are no official market makers on the TASE: their absence could explain the lack of trading in those futures contracts[11].

Following Evans et al. (2003) and Ofek et al. (2003), we use closing prices for TA 25 options. Our sample covers 7 years of data, from January 1, 1996 to December 31, 2002. Our data set, graciously provided by the TASE, includes daily option closing price, trading volume, open interest and the number of transactions. We drop all the options for which trading volume was zero and for which the implied volatility was more than 100%. Then we group the options with respect to their moneyness (defined here as the exercise price divided by the underlying). We choose six moneyness groups: less than 90%, between 90% and 95%, between 95% and 100%, between 100% and 105%, between 105% and 110% and more than 110%. Moreover, we build two groups of options: the first one comprises short term options, which correspond to options with the shortest maturity each day, while the second one comprises long term options, which correspond to options with the second maturity each day. For each day and for each moneyness-maturity couple, we only have one option return which corresponds to the average return of all the options in this group that day.

Summary statistics for our data are in the following Table II.

**Table II**

Short term options, both Calls and Puts, have an average maturity of one month while the corresponding long term options have an average maturity of two months. The data about the implied volatilities feature the standard volatility smile which is present in most options markets around the world. The average implied volatility in short term

[11] On June 17, 2003, the TASE announced the introduction of market makers on both derivatives markets and primitive asset markets. Since February 2004, market makers started trading on the Euro/NIS exchange rate currency options market.

options is higher that the average implied volatility in long term options, and this feature holds both for Calls and Puts. Another interesting feature of the data is that Put implied volatilities are above the corresponding Call implied volatilities meaning that Puts sell more expensive than Calls. Such a feature is a standing feature of option markets as shown by Bondarenko (2003) for example. Finally, we find that around the money options concentrate most of the trading activity in the market and that Calls and Puts trading volume are similar. This means that no option was more traded more than the other. Finally, sample sizes are reasonable and allow a reliable statistical investigation.

The behavior during our sample period of the options returns is gathered in the following Table III.

**Table III**

The daily average return for short term in-the-money Calls was positive while the daily average return for short term out-of-the-money Calls was negative. These returns were highly volatile and skewed. Therefore, it cannot be concluded that it was worth buying in-the-money Calls and selling out-of-the-money Calls without further investigation. As for Long term Calls, in-the-money-options still have an average positive daily return while deep-out-of-the-money options had a negative average return. Still, the relatively high level of volatility of the return cannot be used to conclude that there was some beneficial trading strategy. When comparing short term Calls to long term Calls, it turns out that the former had a higher average return but also a higher volatility level. Puts have similar characteristics.

Thus we are left with a group of 24 assets that allow us to estimate the Euler equation (9) which we do in the next section.

## 4. EMPIRICAL FINDINGS

Our strategy for assessing the level of risk aversion in the Economy is by estimating the Euler equation (9) for each of the 24 assets and inferring using (10) the parameter of risk aversion. While some Authors derive the parameter of risk aversion for different maturities and others for different moneyness, we derive both so as to highlight some horizon effects in the behavior of this important parameter.

We first start with the findings related to the power pricing kernel (14). The results are in the following Table.

**Table IV**

In Table IV, we report both the estimates of the intertemporal marginal rate of substitution curvature and the estimate of the relative risk aversion parameter. It turns out that these two parameters are significantly different. This shows the impact of heterogeneity on option prices and suggests that markets are incomplete. The estimate of $\hat{\alpha}$ when Calls were used ranges from 6.39 to 24.11 and was always positive, while it ranges from 6.50 to 24.73 for Puts in absolute terms and was always negative. If the obtained estimates were interpreted as if they were those of the relative risk aversion parameter, it might have been concluded to the existence of an Equity Premium Puzzle in the option markets like in the equity market. However, correcting for incompleteness, and thus for heterogeneity among traders, yields more reasonable estimates of the true parameter of relative risk aversion. The values obtained when using Calls vary from 1.69 to 3.17, while for Puts they vary from 1.62 to 3.20. These values are clearly reasonable and encouraging results.

**Figure I**

As it appears in Figure I, short term options always yield an implied parameter of relative risk aversion that is higher than long term options. Moreover, this parameter features an inverted smile with respect to option moneyness. The results obtained here are similar to the best results obtained elsewhere in the literature. However, it cannot be concluded that asset prices are cheaper than they would have been had markets been complete, or alternatively, that the risk premium requested under incomplete markets are always higher than the corresponding risk premia under complete markets. The main reason stems from the fact the subjective discount factor in (9) may attenuate the effect of a higher curvature. Indeed, parts of these subjective discount factors were higher than one.

An important implication of the previous findings is worth stressing. Since the estimated parameter has reasonable values for all the moneyness and maturity couples, it means in particular that once market incompleteness is accounted for, deep out-of-the-money Puts are not overpriced. This finding is clearly different from the one found by Bondarenko (2003) who showed that no available model could explain observed Put prices.

What happens to such features if one allows for time variation in this parameter? The results which are obtained are very similar to those obtained using a constant RRA as reported in Table V.

**Table V**

It turns out that the value of the parameters ranges from 1.69 to 3.08 for Calls while short term Calls feature higher parameters than long term Calls. The same holds for Puts. The most interesting feature of these data is the relatively low volatility of the parameters. It must also be noted that a very low percentage of negative values for the time-varying parameter is obtained. Time varying risk aversion allows us to deepen the analysis in different directions.

First and most importantly, we can look at the behavior of the parameter of risk aversion with respect to wealth or in our case, with respect to the market gross return which is a proxy for wealth changes. A standing feature of existing studies is the procyclicality of this relation for reasonable ranges of the market return. This means that is the parameter tends to be high when the market return is high and vice versa. This has been interpreted as not being sustainable from an economic point of view. In our setting, the obtained distributions are in the following Figures.

**Figure II**

**Figure III**

**Figure IV**

**Figure V**

It turns out that when Calls are used, the parameter of risk aversion is a convex increasing function of the market return and the inverse is true for Puts. The same holds whether short term or long term options are considered. It should however be concluded that Puts yield better results than Calls. As explained in the Introduction, such a finding is a direct consequence of the methodology followed here and in most papers dealing with this issue: the factor representation of the pricing kernel (or the Stochastic Discount Factor) takes into account the market index which is also the underlying of the options. As a consequence, and given that option returns amplify the changes in the underlying, we are bound to obtain a positive increasing function for Calls and a negative one for Puts. Similar findings hold for the behavior of the pricing kernels.

## 5. CONCLUDING REMARKS

Market incompleteness has been seen as a potential improvement of the standard complete market setting that may help solve the Equity Premium Puzzle. The results which have been obtained so far from theoretical analyses and simulations (Heaton and Lucas (1996) and Levine and Zame (2002)) have been disappointing. Our findings show that the model suggested by Constandinides and Duffie (1996) is a promising path for improving our understanding of the behavior of asset prices.

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**Table I**

Summary Statistics for the TA 25 (Market Index) daily changes for the period starting from 1/1/96 to 31/12/02. This Table reports the characteristics of the daily changes expressed in percentage.

| | |
|---|---|
| Mean | 0.008 |
| Median | -.004 |
| Standard Deviation | 1.197 |
| Kurtosis | 5.101 |
| Skewness | -.419 |
| Minimum | -.412 |
| Maximum | 7.402 |
| Observations | 1587 |

**Table II**

Summary Statistics of Option Characteristics. For each day, we grouped the options in five groups of moneyness: smaller or equal to 90%, between 90% and 95%, 95% and 100%, 100% and 105%, 105% to 110% and more than 110%. In days where there were more than one option for a group, we averaged the daily return of all the options pertaining to the same group such as we were left with one observation for each day and for each group. We retained each day and for each group, options with the lowest maturity (we call these options short term options) and options with the next maturity (we call them long term options).

Calls

| | ≤ 0.90 | ]0.90;0.95] | ]0.95;1.00] | ]1.00;1.05] | ]1.05;1.10] | > 1.10 |
|---|---|---|---|---|---|---|
| **Short Term** | | | | | | |
| Average Moneyness (%) | 87 | 93 | 98 | 102 | 108 | 115 |
| Average Time To Maturity (Days) | 28 | 27 | 26 | 25 | 25 | 25 |
| Average Trading Volume (Units) | 77 | 524 | 3563 | 7081 | 2753 | 625 |
| Average Turnover (NIS) | 37,258,808 | 144,871,203 | 467,468,723 | 380,926,519 | 60,600,313 | 7,860,068 |
| Average Implied Volatility (%) | 34.23 | 26.65 | 22.63 | 21.47 | 23.63 | 29.78 |
| Observations | 810 | 1248 | 1541 | 1650 | 1557 | 1146 |
| **Long Term** | | | | | | |
| Average Moneyness (%) | 87 | 93 | 98 | 103 | 108 | 114 |
| Average Time To Maturity (Days) | 61 | 66 | 69 | 72 | 71 | 66 |
| Average Trading Volume (Units) | 37 | 104 | 348 | 842 | 796 | 329 |
| Average Turnover (NIS) | 20,786,223 | 33,081,719 | 68,612,156 | 84,256,755 | 39,463,498 | 9,314,046 |
| Average Implied Volatility (%) | 25.33 | 23.01 | 22.78 | 21.92 | 21.13 | 21.93 |
| Observations | 574 | 1175 | 1442 | 1576 | 1465 | 967 |

**Table II (Continued)**

Puts

| | ≤ 0.90 | ]0.90;0.95] | ]0.95;1.00] | ]1.00;1.05] | ]1.05;1.10] | > 1.10 |
|---|---|---|---|---|---|---|
| **Short Term** | | | | | | |
| Average Moneyness (%) | 86 | 93 | 98 | 103 | 108 | 115 |
| Average Time To Maturity (Days) | 27 | 25 | 25 | 25 | 27 | 27 |
| Average Trading Volume (Units) | 541 | 2513 | 5845 | 3181 | 513 | 145 |
| Average Turnover (NIS) | 6,826,315 | 64,314,028 | 303,474,871 | 362,542,071 | 117,588,759 | 64,988,700 |
| Average Implied Volatility (%) | 36.39 | 29.29 | 25.20 | 23.92 | 27.83 | 36.62 |
| Observations | 1559 | 1637 | 1651 | 1608 | 1358 | 858 |
| **Long Term** | | | | | | |
| Average Moneyness (%) | 87 | 93 | 98 | 103 | 108 | 115 |
| Average Time To Maturity (Days) | 70 | 72 | 72 | 72 | 70 | 67 |
| Average Trading Volume (Units) | 199 | 557 | 783 | 418 | 173 | 98 |
| Average Turnover (NIS) | 5,428,784 | 24,683,988 | 65,241,386 | 58,837,584 | 38,748,714 | 44,312,963 |
| Average Implied Volatility (%) | 28.96 | 26.81 | 25.56 | 24.24 | 23.31 | 24.09 |
| Observations | 1333 | 1575 | 1599 | 1582 | 1421 | 819 |

**Table III**

Summary Statistics of Option Returns. For each day, we grouped the options in five groups of moneyness: smaller or equal to 90%, between 90% and 95%, 95% and 100%, 100% and 105%, 105% to 110% and more than 110%. In days where there were more than one option for a group, we averaged the daily return of all the options pertaining to the same group such as we were left with one observation for each day and for each group.

Calls

| | ≤ 0.90 | ]0.90;0.95] | ]0.95;1.00] | ]1.00;1.05] | ]1.05;1.10] | > 1.10 |
|---|---|---|---|---|---|---|
| **Short Term** | | | | | | |
| Mean | 1.017 | 1.020 | 1.043 | 0.996 | 0.933 | 0.997 |
| Maximum | 1.859 | 2.142 | 15.849 | 13.563 | 9.000 | 8.185 |
| Minimum | 0.699 | 0.532 | 0.398 | 0.025 | 0.105 | 0.022 |
| Std. Dev. | 0.116 | 0.165 | 0.499 | 0.567 | 0.469 | 0.659 |
| Skewness | 0.963 | 0.847 | 18.212 | 8.369 | 5.181 | 4.961 |
| Kurtosis | 8.298 | 5.392 | 512.278 | 159.227 | 68.707 | 38.312 |
| Observations | 809 | 1247 | 1540 | 1649 | 1556 | 1146 |
| **Long Term** | | | | | | |
| Mean | 1.019 | 1.014 | 1.009 | 1.013 | 0.994 | 0.948 |
| Maximum | 1.524 | 2.102 | 2.828 | 5.418 | 5.383 | 4.083 |
| Minimum | 0.570 | 0.561 | 0.424 | 0.412 | 0.407 | 0.240 |
| Std. Dev. | 0.112 | 0.132 | 0.159 | 0.232 | 0.278 | 0.294 |
| Skewness | 0.262 | 1.051 | 1.579 | 5.067 | 3.715 | 3.034 |
| Kurtosis | 4.914 | 8.558 | 16.142 | 86.382 | 47.631 | 26.255 |
| Observations | 573 | 1174 | 1441 | 1575 | 1464 | 967 |

**Table III (Continued)**

Puts

| | ≤ 0.90 | ]0.90;0.95] | ]0.95;1.00] | ]1.00;1.05] | ]1.05;1.10] | > 1.10 |
|---|---|---|---|---|---|---|
| **Short Term** | | | | | | |
| Mean | 0.986 | 0.942 | 1.010 | 1.043 | 1.032 | 1.033 |
| Maximum | 21.250 | 15.216 | 17.017 | 9.898 | 5.026 | 2.879 |
| Minimum | 0.091 | 0.023 | 0.039 | 0.355 | 0.499 | 0.641 |
| Std. Dev. | 0.943 | 0.548 | 0.676 | 0.434 | 0.231 | 0.134 |
| Skewness | 15.248 | 11.952 | 10.925 | 8.186 | 5.760 | 3.689 |
| Kurtosis | 294.362 | 288.198 | 217.903 | 133.385 | 82.585 | 46.327 |
| Observations | 1558 | 1636 | 1650 | 1607 | 1357 | 858 |
| **Long Term** | | | | | | |
| Mean | 0.996 | 1.007 | 1.024 | 1.017 | 1.021 | 1.054 |
| Maximum | 9.885 | 3.636 | 8.462 | 6.279 | 4.326 | 12.112 |
| Minimum | 0.403 | 0.365 | 0.407 | 0.452 | 0.570 | 0.726 |
| Std. Dev. | 0.421 | 0.252 | 0.341 | 0.242 | 0.184 | 0.415 |
| Skewness | 11.248 | 2.396 | 12.338 | 9.057 | 5.696 | 23.275 |
| Kurtosis | 201.257 | 18.301 | 252.702 | 175.553 | 88.157 | 616.670 |
| Observations | 1332 | 1574 | 1598 | 1581 | 1420 | 819 |

**Table IV**

This Table reports the results of the GMM estimation of the Euler equation: $E\left[R_{j,t+1}e^{-\hat{\rho}_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\hat{\alpha}}\middle|\phi_t\right]=1$

where $e^{-\hat{\rho}_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\hat{\alpha}}$ is approximated by $\lambda_0 Grm_{t+1}^{-\lambda_1}$. Therefore, the estimate for the estimate for $\hat{\alpha}$ is indeed the estimate of $\lambda_1$. The parameter $\alpha$ is obtained by using the following relationship: $\hat{\alpha}=\alpha\left(1+\frac{\alpha(\alpha+1)}{2}\right)$

| **Moneyness** | $\leq$ 0.90 | ]0.90;0.95] | ]0.95;1.00] | ]1.00;1.05] | ]1.05;1.10] | > 1.10 |
|---|---|---|---|---|---|---|
| **<u>Calls</u>** | | | | | | |
| **Short Term** | | | | | | |
| $\hat{\alpha}$ | 6.39 | 10.4 | 17.38 | 24.11 | 21.78 | 14.04 |
| *t statistic* | (29.88) | (29.35) | (16.19) | (19.75) | (24.3) | (10.47) |
| $\alpha$ | 1.81 | 2.24 | 2.78 | 3.17 | 3.04 | 2.54 |
| **Long Term** | | | | | | |
| $\hat{\alpha}$ | 5.54 | 7.79 | 9.53 | 12.55 | 14.78 | 11.99 |
| *t statistic* | (15.7) | (35.75) | (36.31) | (24.95) | (21.6) | (15.24) |
| $\alpha$ | 1.69 | 1.98 | 2.16 | 2.43 | 2.60 | 2.38 |
| **<u>Puts</u>** | | | | | | |
| **Short Term** | | | | | | |
| $\hat{\alpha}$ | 18.65 | 23.48 | 24.73 | 18.89 | 11.96 | 6.50 |
| *t statistic* | (12.17) | (24.72) | (31.91) | (41.13) | (41.00) | (20.40) |
| $\alpha$ | 2.86 | 3.14 | 3.20 | 2.88 | 2.38 | 1.82 |
| **Long Term** | | | | | | |
| $\hat{\alpha}$ | 14.52 | 14.60 | 13.77 | 11.41 | 9.13 | 5.09 |
| *t statistic* | (20.82) | (36.37) | (24.05) | (27.36) | (29.51) | (4.30) |
| $\alpha$ | 2.58 | 2.59 | 2.53 | 2.33 | 2.12 | 1.62 |

**Table V**

This Table reports the results of the GMM estimation of the Euler equation: $E\left[R_{j,t+1}e^{-\hat{\rho}_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\hat{\alpha}}\middle|\phi_t\right]=1$ where $e^{-\hat{\rho}_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\hat{\alpha}}$ is approximated by $\pi_0 Grm_{t+1}+\pi_1 Grm_{t+1}{}^2$. Therefore, the estimate for the estimate for $\hat{\alpha}$ is given by $-\frac{Grm_{t+1}(\pi_0+\pi_1 Grm_{t+1})}{\pi_0 Grm_{t+1}+\pi_1 Grm_{t+1}{}^2}$. The parameter $\alpha$ is obtained by using the following relationship: $\hat{\alpha}=\alpha\left(1+\frac{\alpha(\alpha+1)}{2}\right)$

Calls

| | ≤ 0.90 | ]0.90;0.95] | ]0.95;1.00] | ]1.00;1.05] | ]1.05;1.10] | > 1.10 |
|---|---|---|---|---|---|---|
| **Short Term** | | | | | | |
| Mean | 1.78 | 2.23 | 2.72 | 3.08 | 2.99 | 2.53 |
| Maximum | 2.63 | 4.57 | 15.29 | 8.87 | 7.24 | 3.92 |
| Minimum | 1.51 | 1.55 | -5.62 | -6.18 | -6.64 | -7.15 |
| Std. Dev. | 0.11 | 0.19 | 0.51 | 0.64 | 0.60 | 0.39 |
| Skewness | 1.04 | 2.08 | 7.69 | -2.76 | -4.97 | -12.51 |
| Kurtosis | 7.83 | 22.05 | 292.58 | 69.39 | 96.13 | 316.97 |
| Observations | 809 | 1247 | 1540 | 1649 | 1556 | 1146 |
| % RRA < 0 | | | 0.06% | 0.18% | 0.19% | 0.09% |
| | | | | | | |
| **Long Term** | | | | | | |
| Mean | 1.69 | 1.95 | 2.15 | 2.42 | 2.58 | 2.40 |
| Maximum | 2.00 | 3.10 | 4.00 | 8.78 | 5.55 | 9.38 |
| Minimum | 1.45 | 1.59 | 1.48 | 1.50 | -6.63 | 1.50 |
| Std. Dev. | 0.09 | 0.13 | 0.17 | 0.28 | 0.37 | 0.33 |
| Skewness | 0.23 | 1.16 | 1.57 | 8.12 | -9.85 | 10.47 |
| Kurtosis | 3.19 | 8.91 | 14.68 | 172.97 | 269.31 | 219.78 |
| Observations | 573 | 1174 | 1441 | 1575 | 1464 | 967 |
| % RRA < 0 | | | | | 0.07% | |

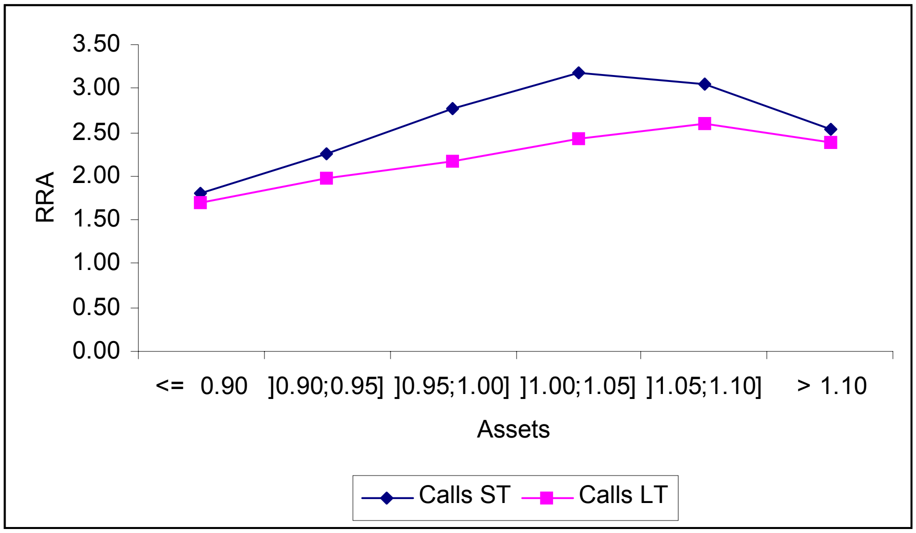
**Table V (Continued)**

Puts

| | ≤ 0.90 | ]0.90;0.95] | ]0.95;1.00] | ]1.00;1.05] | ]1.05;1.10] | > 1.10 |
|---|---|---|---|---|---|---|
| **Short Term** | | | | | | |
| Mean | 2.86 | 3.05 | 2.97 | 2.68 | 2.32 | 1.80 |
| Maximum | 4.62 | 6.18 | 13.00 | 3.86 | 7.00 | 2.27 |
| Minimum | 2.12 | -4.42 | -4.65 | -6.99 | 1.91 | 1.63 |
| Std. Dev. | 0.26 | 0.41 | 0.51 | 0.42 | 0.18 | 0.05 |
| Skewness | 1.38 | -1.94 | 0.08 | -15.33 | 14.07 | 1.73 |
| Kurtosis | 7.33 | 75.47 | 163.34 | 324.91 | 350.38 | 15.88 |
| Observations | 1558 | 1636 | 1650 | 1607 | 1357 | 858 |
| % RRA < 0 | | 0.6% | 0.18% | 0.19% | | |
| **Long Term** | | | | | | |
| Mean | 2.58 | 2.56 | 2.40 | 2.27 | 2.08 | 1.61 |
| Maximum | 3.38 | 8.31 | 5.29 | 5.14 | 3.26 | 1.85 |
| Minimum | 2.10 | 2.02 | -7.70 | 1.88 | 1.79 | 1.50 |
| Std. Dev. | 0.17 | 0.22 | 0.30 | 0.13 | 0.09 | 0.03 |
| Skewness | 0.87 | 11.44 | -23.12 | 7.74 | 3.10 | 1.38 |
| Kurtosis | 4.79 | 288.31 | 815.10 | 146.52 | 35.81 | 11.89 |
| Observations | 1332 | 1574 | 1598 | 1581 | 1420 | 819 |
| % RRA < 0 | | | 0.6% | | | |

## Figure I

Estimates of the parameter of relative risk aversion for a power pricing kernel implying a constant relative risk aversion (RRA) parameter.



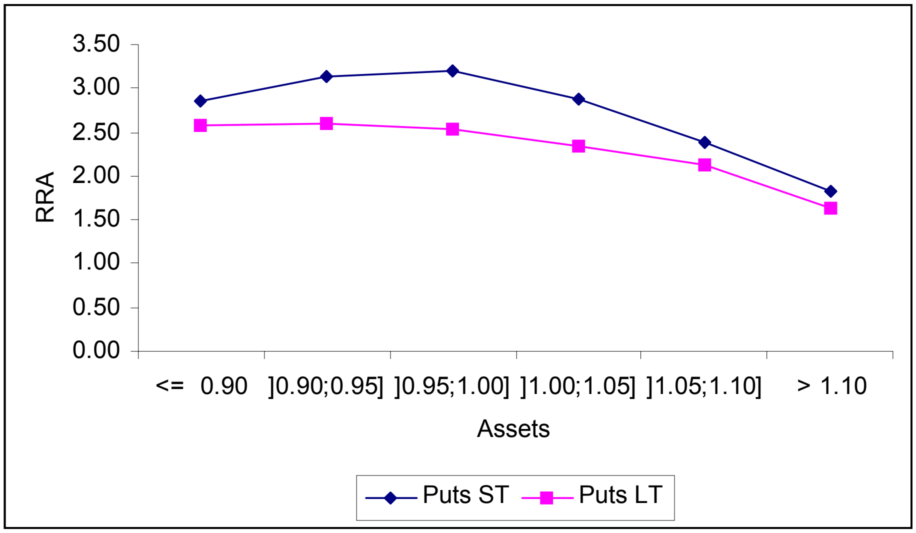



**Figure II**

Option Returns, Pricing Kernels and Parameter of Relative Risk Aversion (RRA) as functions of the Market Gross Return for Short Term Calls.

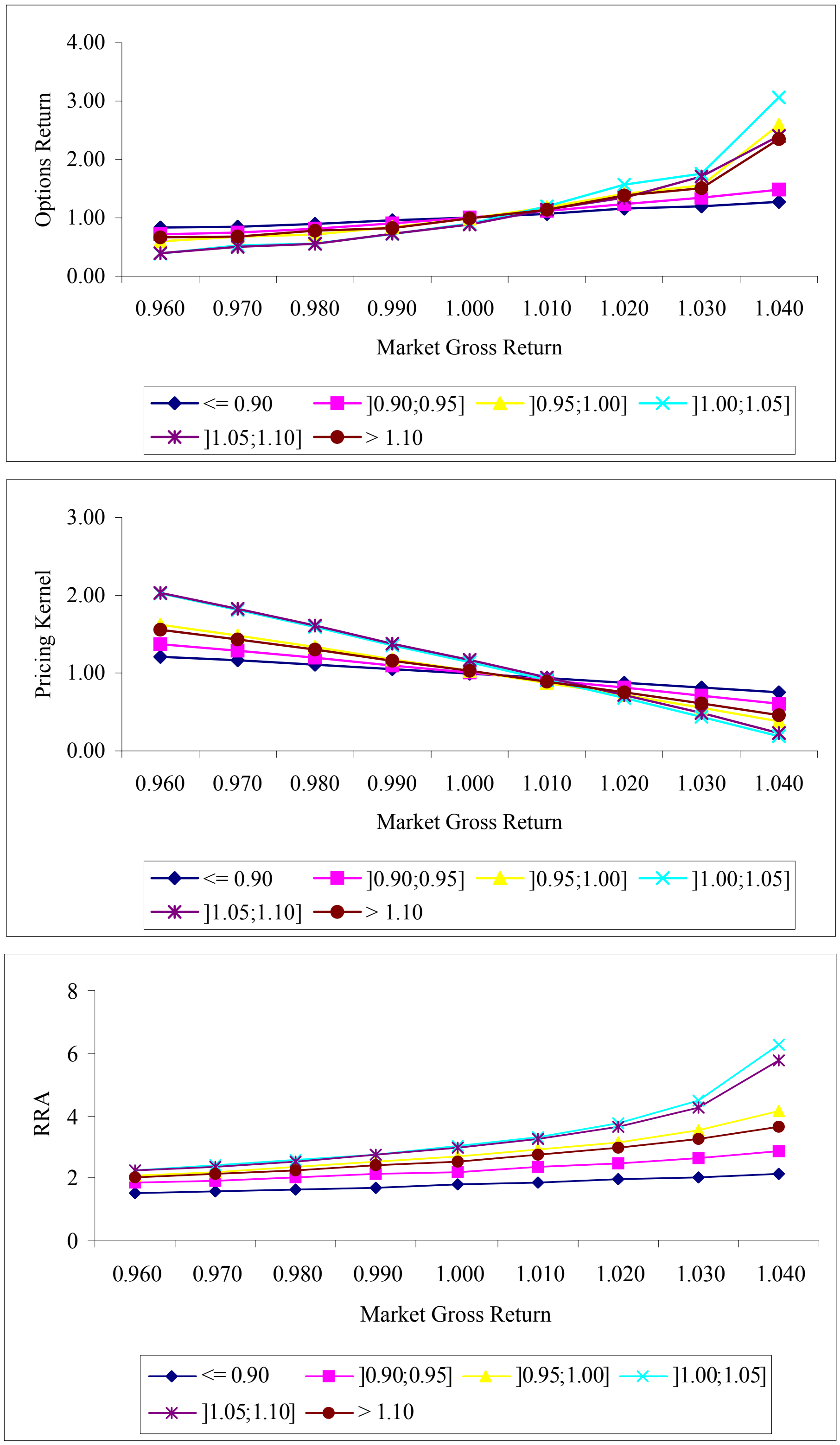
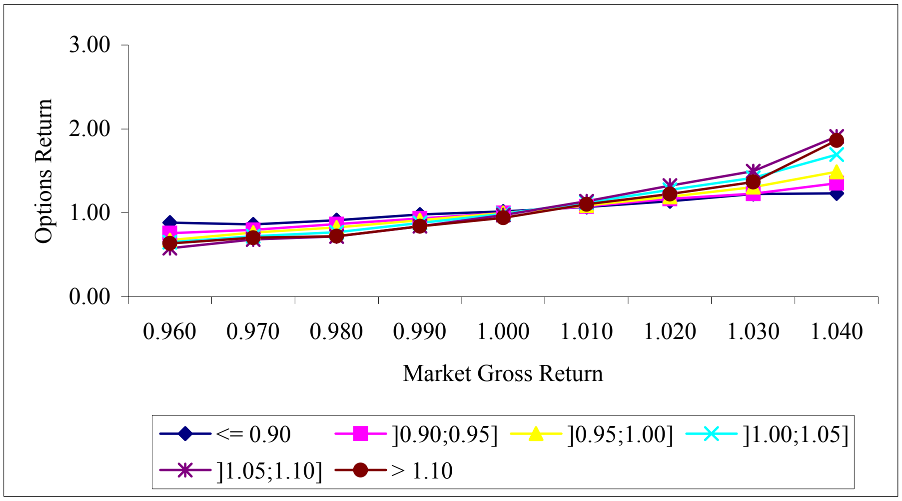
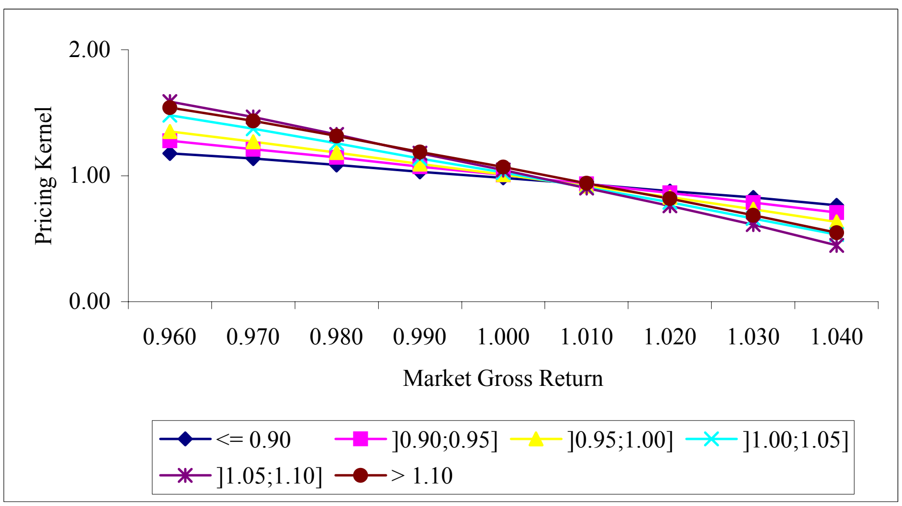


**Figure III**

Option Returns, Pricing Kernels and Parameter of Relative Risk Aversion (RRA) as functions of the Market Gross Return for Long Term Calls.



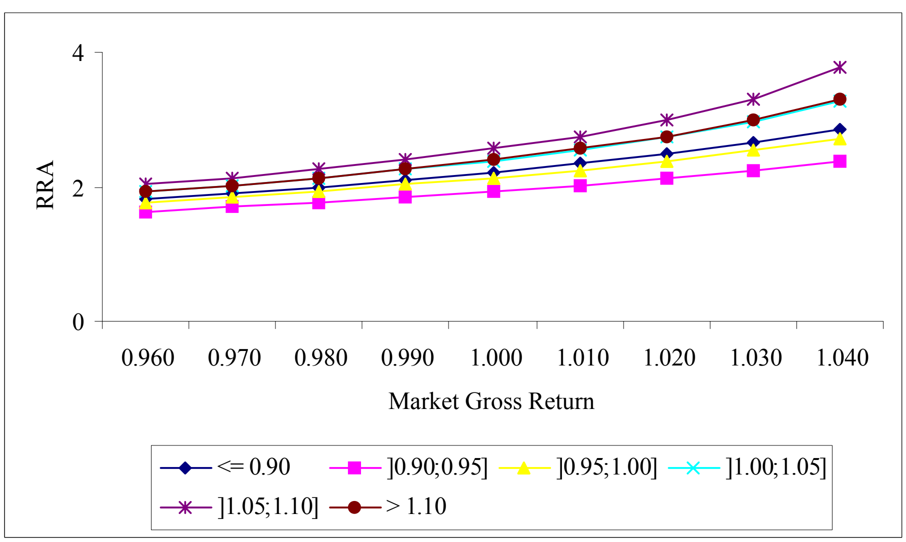
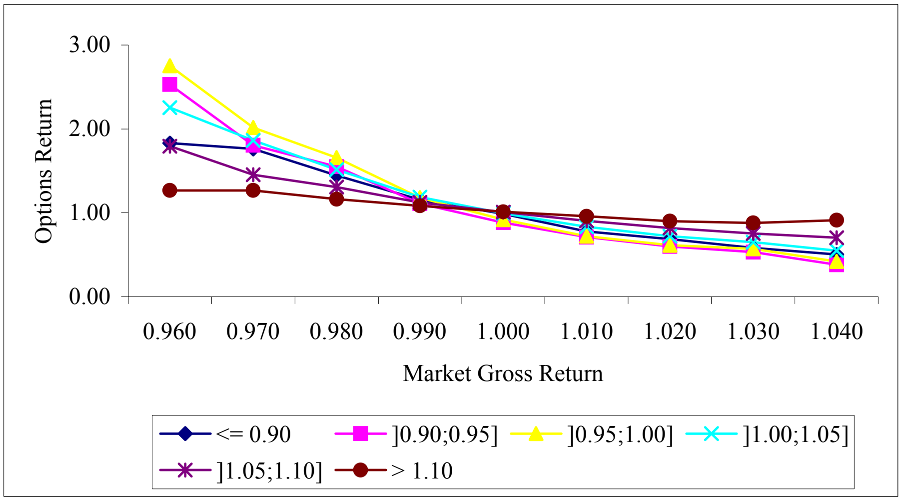
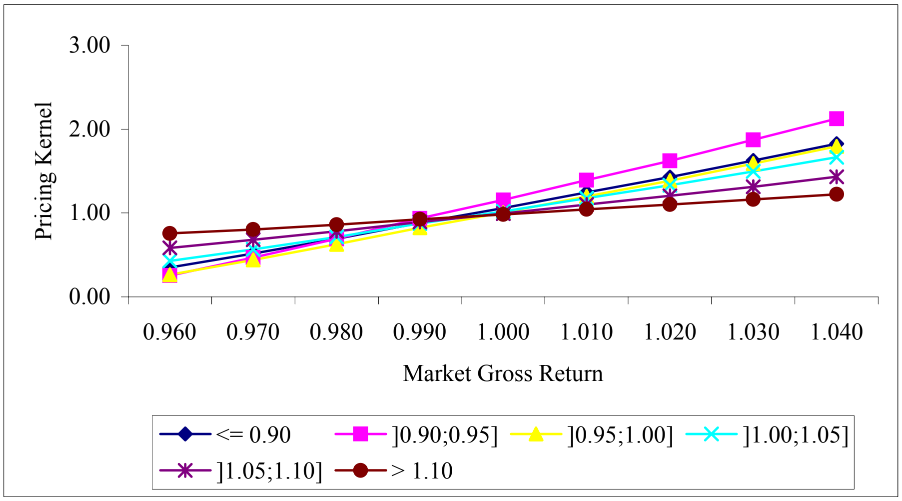
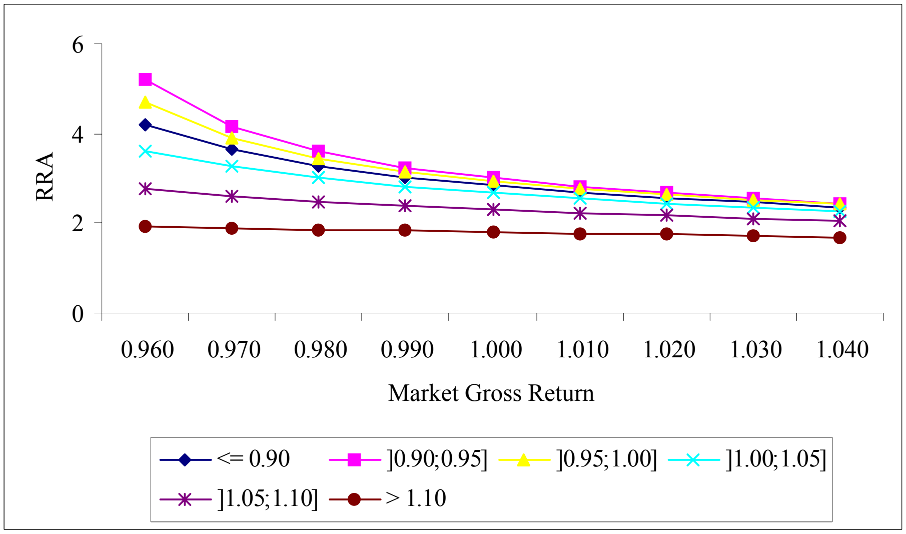



**Figure IV**

Option Returns, Pricing Kernels and Parameter of Relative Risk Aversion (RRA) as functions of the Market Gross Return for Short Term Puts.



## Figure V

Option Returns, Pricing Kernels and Parameter of Relative Risk Aversion (RRA) as functions of the Market Gross Return for Long Term Puts.