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Review Article

Review of Anisotropic Flow Correlations in Ultrarelativistic Heavy-Ion Collisions

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Anisotropic flow phenomena are a key probe of the existence of Quark-Gluon Plasma. Several new observables associated with correlations between anisotropic flow harmonics are developed, which are expected to be sensitive to the initial fluctuations and transport properties of the created matter in heavy-ion collisions. I review recent developments of correlations of anisotropic flow harmonics. The experimental measurements, together with the comparisons to theoretical model calculations, open up new opportunities of exploring novel QCD dynamics in heavy-ion collisions.

1. Introduction

One of the fundamental questions in the phenomenology of Quantum Chromo Dynamics (QCD) is, what are the properties of matter at extreme densities and temperatures where quarks and gluons are in a new state of matter, the so-called Quark-Gluon Plasma (QGP)? [1, 2]. Collisions of high-energy heavy ions, at the Brookhaven Relativistic Heavy Ion Collider (RHIC) and the CERN Large Hadron Collider (LHC), allow us to create and study the properties of the QGP matter in the laboratory. This matter expands under large pressure gradients, which transfer the inhomogeneous initial conditions into azimuthal anisotropy of produced particles in momentum space. This anisotropy of produced particles is one of the probes of the properties of the QGP [3, 4]. It can be characterized by an expansion of the single-particle azimuthal distribution $P(\varphi)$:

$$P(\varphi) = \frac{1}{2\pi} \sum_{n=-\infty}^{+\infty} \overrightarrow{V}_n e^{-in\varphi}, \qquad (1)$$

where φ is the azimuthal angle of emitted particles, $\overrightarrow{V}_n$ is the nth order flow vector defined as $\overrightarrow{V}_n = v_n e^{in\Psi_n}$, its magnitude v_n is the nth order anisotropic flow harmonic, and its orientation is symmetry plane (participant plane) angle Ψ_n. Alternatively, this anisotropy can be generally given by the joint probability density function (PDF) in terms of v_n and Ψ_n as

$$P(v_m, v_n, \dots, \Psi_m, \Psi_n, \dots) = \frac{1}{N_{\text{event}}} \frac{dN_{\text{event}}}{v_m v_n \cdots dv_m dv_n \cdots d\Psi_m d\Psi_n \cdots}. \qquad (2)$$

In the last decade, the experimental measurements of anisotropic flow v_n [5–55], combined with theoretical advances from calculations made in a variety of frameworks [56–62], have led to broad and deep knowledge of initial conditions and properties of the created hot/dense QCD matter. In particular, the precision anisotropic flow measurements based on the huge data collected at the LHC experiments and the successful description from hydrodynamic calculations demonstrate that the QGP created in heavy-ion collisions behaves like a strongly coupled liquid with a very small specific shear viscosity η/s [63–68], which is close to a quantum limit $1/4\pi$ [69].

It has been investigated into great details of event-by-event fluctuations of single flow harmonic. Based on the measurements of higher-order cumulants of anisotropic flow [43, 48, 51, 74, 75] and the event-by-event v_n distributions [40], it was realized that the newly proposed Elliptic-Power function [76–78] gives the best description of underlying

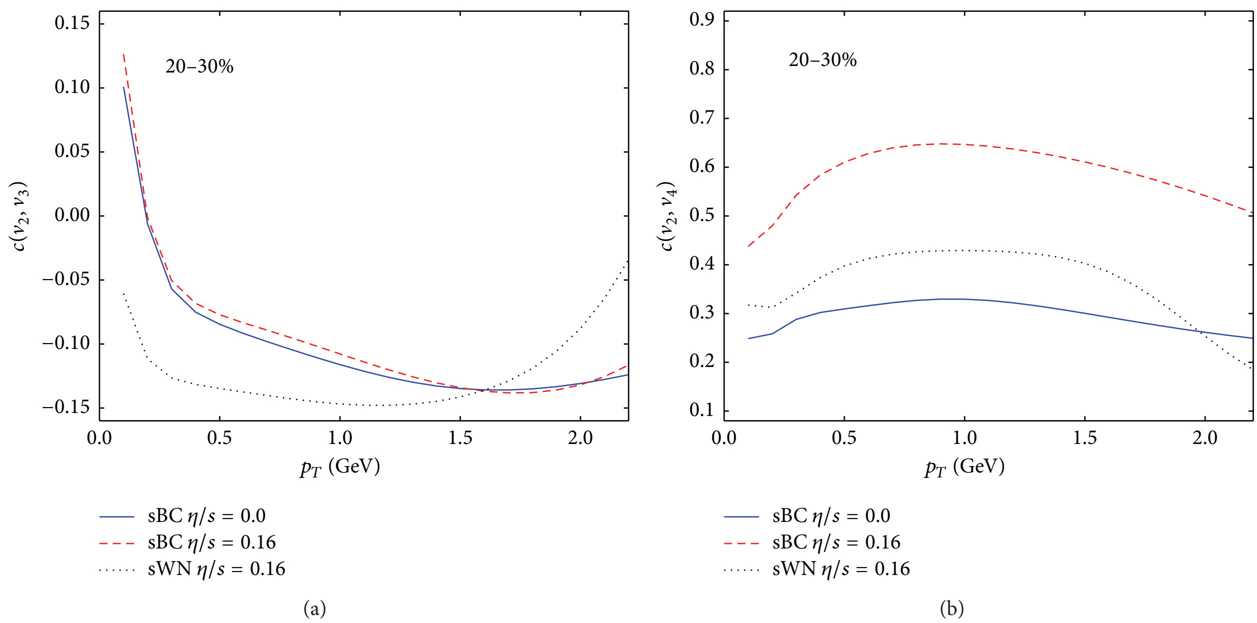

FIGURE 1: p_T dependence of $c(v_2, v_3)$ (a) and $c(v_2, v_4)$ (b) in centrality 20–30% in Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. Figures are taken from [83].

PDF of single harmonic v_n distributions [72, 79, 80]. On the other hand, it has been known for a while that both the flow harmonic (magnitude) v_n and its symmetry plane (orientation) Ψ_n of the flow vector $\overrightarrow{V_n}$ fluctuate event-by-event [81–83], but only recently p_T and η dependent flow angle (Ψ_n) and magnitude (v_n) were predicted by hydrodynamic calculations [84, 85]. Many indications were quickly obtained in experiments by looking at the deviations from unity of $v_n[2]/v_n\{2\}$ [86] and factorization ratio r_n [52, 55, 86]. These measurements were nicely predicted or reproduced by hydrodynamic calculations and are found to be sensitive to the initial state density fluctuations and/or the shear viscosity of the expanding fireball medium [84, 85, 87]. Most of these above-mentioned studies are focused on the fluctuations of single flow harmonics and their corresponding symmetry planes, as a function of collisions centrality, transverse momentum p_T, and pseudorapidity η. Results of correlations between symmetry planes [28, 41] reveal a new type of correlations between different order flow vectors, which was investigated in the observable of v_{2n/Ψ_n} before [88–90]. In particular, some of the symmetry planes correlations show quite different centrality dependence from the initial state and final state, and this characteristic sign change during system evolution is correctly reproduced by theoretical calculations [62, 82, 91], thus confirming the validity of hydrodynamic framework in heavy-ion collisions and further yielding valuable additional insights into the fluctuating initial conditions and hydrodynamic response [62, 82, 92].

In addition to all these observables, the (anti)correlations between anisotropic flow harmonics v_m and v_n are found to be extremely interesting [45, 62, 70, 71, 93]. A completely new set of information on the joint probability density function (PDF) can be obtained from the rich correlation pattern observed in experiments. On the other hand, no existent theoretical calculations [62, 70, 71, 93] could provide quantitative descriptions of data [36]. Thus, it is crucial to investigate in depth the relationship between different flow harmonics: whether they are correlated, anticorrelated, or not correlated from both experimental and theoretical points of view.

2. Correlations of v_n and v_m Fluctuations

It is found recently that the relationship between different order flow harmonics can be used to probe the initial state conditions and the hydrodynamic response of the QGP [36, 71, 93–95]. In order to better understand the event-by-event $P(\varphi)$ distribution, it is critical to investigate the relationship between v_m and v_n. Considering the naive ellipsoidal shape of the overlap region in noncentral heavy-ion collisions generating nonvanishing even flow harmonics v_{2n}, the correlations between the even flow harmonics are expected. However, it is not straightforward to use geometrical argument to explain the relationship between even flow harmonics for central collisions, where all the harmonics are driven by fluctuations instead of geometry, and to explain the relationship between even and odd flow harmonics for central and noncentral collisions [80]. A linear correlation function $c(v_m, v_n)$ was proposed to study the relationship between v_m and v_n [83]. It is defined as

$$c\left(v_m, v_n\right) = \left\langle \frac{\left(v_m - \langle v_m \rangle_{ev}\right)\left(v_n - \langle v_n \rangle_{ev}\right)}{\sigma_{v_m}\sigma_{v_n}} \right\rangle_{ev}, \quad (3)$$

where σ_{v_m} is the standard deviation of the quantity v_m; $c(v_m, v_n)$ is 1 (or −1) if v_m and v_n are linearly (antilinearly) correlated and is 0 if they are not correlated. It was shown in Figure 1 that there is an anticorrelation between v_2 and

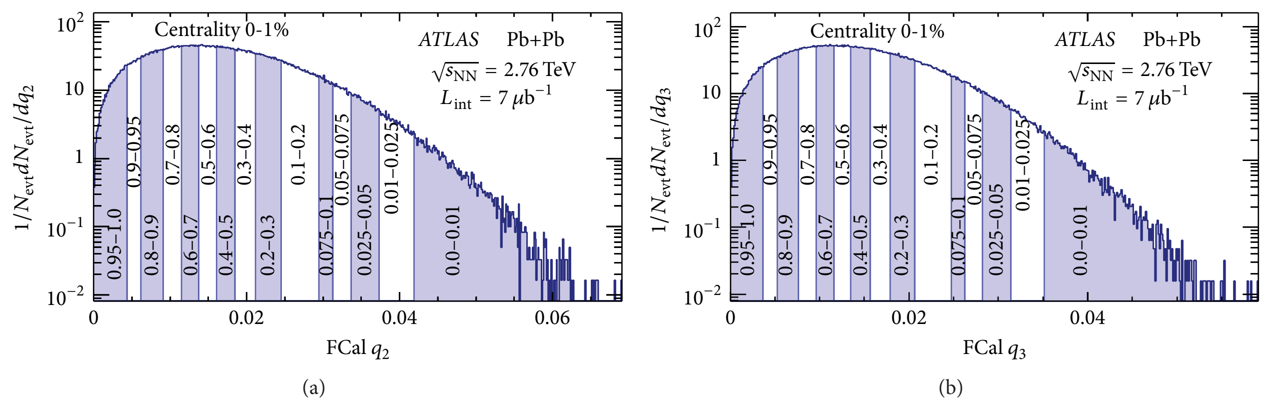

Figure 2: Distributions of V_2 (a) and V_3 (b) calculated with ATLAS forward calorimeter for centrality interval 0-1%. Figures are taken from [45].

v_3, while a correlation was observed between v_2 and v_4. In addition, it was demonstrated that $c(v_2, v_4)$ depends on both the initial conditions and η/s, while $c(v_2, v_3)$ is only sensitive to η/s [83]. Nevertheless, it cannot be accessible easily in experimental measurements, which rely on two-particle and multiparticle correlations techniques. Thus, it is critical to find an observable which studies the relationship between flow harmonics without contributions from symmetry plane correlations and can be accessed with observable techniques from experiments. Two different approaches, named *Event Shape Engineering* and *Symmetric Cumulant*, are discussed in the following section.

2.1. Event Shape Engineering (ESE). The first experimental attempt was made by ATLAS Collaboration [45], using the Event Shape Engineering (ESE) [96]. This is a technique to select events according to the magnitude of reduced flow vector $\overrightarrow{V_n}$. Figure 2 shows the performance of event shape selection on V_2 (a) and V_3 (b) in ATLAS detector. For each centrality the data sample is divided into several event classes according to V_2 or V_3 distributions. Then v_2 and v_3 relationship was investigated by measurements of v_2 and v_3 in each event class from ESE selection. Without using ESE selection, a boomerang-like pattern was observed for the centrality dependence of v_2-v_3 correlation. This is mainly due to the fact that v_3 has weaker centrality dependence than v_2. By using ESE, it was observed in Figure 3(b) that, for event class with the same centrality (shown as the same color), v_3 decreases as v_2 increases. It suggests that v_2 is anticorrelated with v_3. Considering the linear hydrodynamic response of v_2 and v_3 from eccentricity ε_2 and triangularity ε_3, the anticorrelation between v_2 and v_3 might reveal the anticorrelation between ε_2 and ε_3 of the initial geometry. This indication of initial anticorrelations between ε_2 and ε_3 is observed in model calculations [96, 97].

Figure 4 shows the investigation of relationship between v_2 and v_4. A boomerang-like pattern, although weaker than that for v_2-v_3 relationship shown in Figure 3(a), is observed in Figure 4(a), prior to the ESE selection. After the ESE selection, it is found in Figure 4(b) that v_4 increases with increasing v_2. This suggests a correlation between the two harmonics and it can be understood by the interplay between linear and nonlinear collective dynamics in the system evolution [45]. This nonlinear contribution of v_4 from v_2 is further investigated by fitting the correlation pattern using $v_4 = \sqrt{c_0^2 + (c_1 v_2^2)^2}$, where c_0 and c_1 denote the linear and nonlinear components. It is found that the linear component has weak centrality dependence, while the nonlinear component, increasing dramatically with collision centrality, becomes the dominant contribution in the most peripheral collisions [45].

These (anti)correlation patterns between v_m and v_n observed in experiments open a new window to the understanding of the collectivity phenomena in heavy-ion collisions. However, it was also noticed that these measurements were based on 2-particle correlations, which might be suffered by nonflow effects, and they require subdividing such calculations and modeling resolutions associated with ESE due to finite event-wise multiplicities. Considering the computational constraints, this approach cannot be performed easily in hydrodynamic calculations which usually are based on limited statistics compared to experimental data.

2.2. Symmetric Cumulants (SC). A new type of observables for the analyses of flow harmonic correlations, *symmetric cumulants* (originally named *Standard Candles* (SC) in [93]), was proposed as $\mathrm{SC}(m,n) = \langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4)\rangle\rangle_c$. If $m \neq n$, the isotropic part of the corresponding four-particle cumulant is given by

$$\begin{aligned} &\langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4)\rangle\rangle_c \\ &\quad = \langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4)\rangle\rangle \\ &\qquad - \langle\langle \cos[m(\varphi_1 - \varphi_2)]\rangle\rangle \langle\langle \cos[n(\varphi_1 - \varphi_2)]\rangle\rangle \\ &\quad = \langle v_m^2 v_n^2\rangle - \langle v_m^2\rangle \langle v_n^2\rangle . \end{aligned} \tag{4}$$

For a detector with uniform acceptance in azimuthal direction, the asymmetric terms, for example, $\langle\langle \cos(m\varphi_1 - n\varphi_2)\rangle\rangle$,

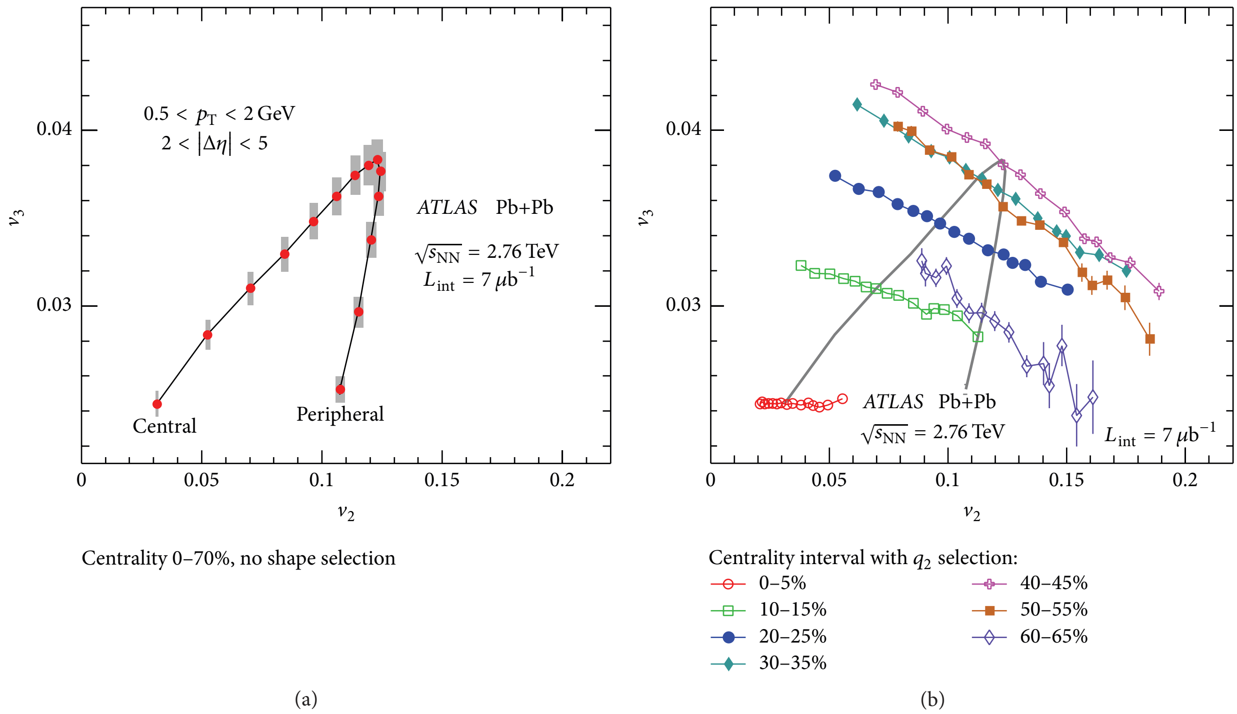

FIGURE 3: The correlation of v_2 (x-axis) with v_3 (y-axis) measured in $0.5 < p_T < 2$ GeV/c. (a) shows v_2 and v_3 values for fourteen 5% centrality intervals over the centrality range of 0–70% without event shape selection. (b) shows v_2 and v_3 values in 15 q_2 intervals in seven centrality ranges (markers) with larger v_2 value corresponding to larger q_2 value. Figures are taken from [45].

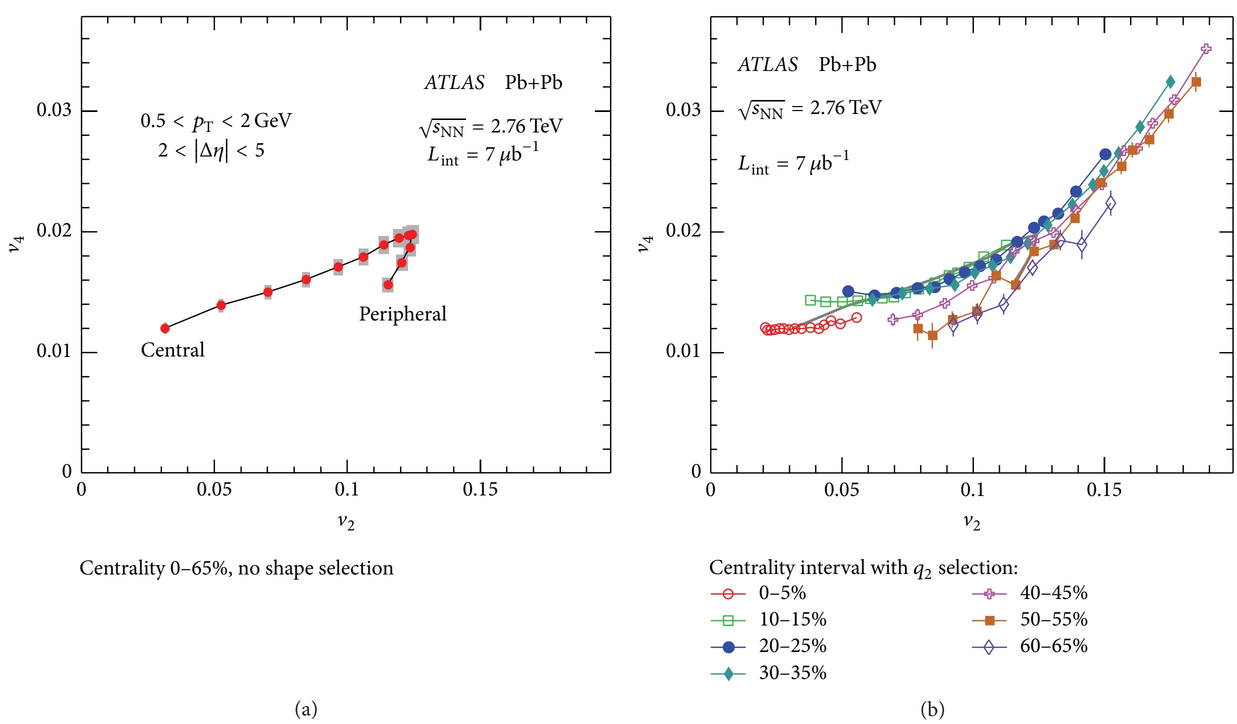

FIGURE 4: The correlation of v_2 (x-axis) with v_4 (y-axis) measured in $0.5 < p_T < 2$ GeV/c. (a) shows v_2 and v_3 values for fourteen 5% centrality intervals over the centrality range of 0–70% without event shape selection. (b) shows v_2 and v_4 values in 15 q_2 intervals in seven centrality ranges (markers) with larger v_2 value corresponding to larger q_2 value. Figures are taken from [45].

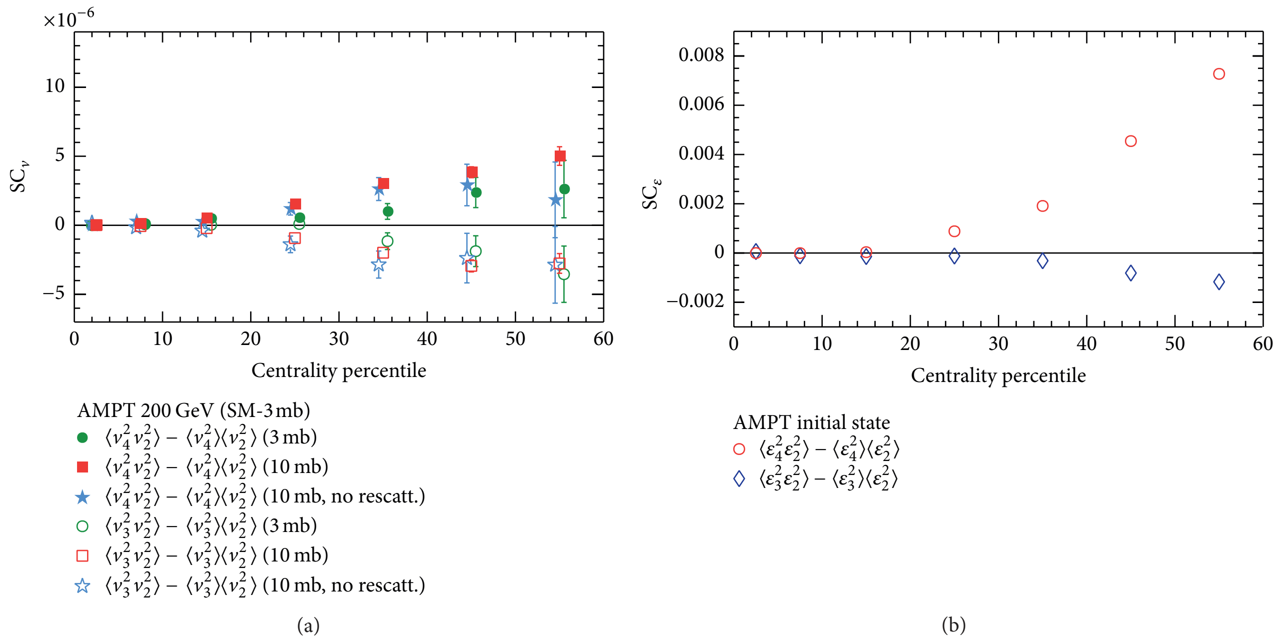

Figure 5: The centrality dependence of symmetric cumulants SC(4, 2) and SC(3, 2) at $\sqrt{s_{NN}}$. Figures are taken from [72, 93].

are averaged to zero. The single event 4-particle correlation $\langle\langle\cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4)\rangle\rangle$ could be calculated as

$$\begin{aligned}\langle\cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4)\rangle &= \frac{1}{M(M-1)(M-2)(M-3)}\left[|V_m|^2|V_n|^2\right.\\ &- 2\Re\mathfrak{e}\left[V_{m+n}V_m^*V_n^*\right] - 2\Re\mathfrak{e}\left[V_mV_{m-n}^*V_n^*\right] + |V_{m+n}|^2\\ &\left.+ |V_{m-n}|^2 - (M-4)\left(|V_m|^2 + |V_n|^2\right) + M(M-6)\right].\end{aligned} \quad (5)$$

And the single event 2-particle correlation $\langle\langle\cos[m(\varphi_1 - \varphi_2)]\rangle\rangle$ could be obtained as

$$\langle\cos[m(\varphi_1 - \varphi_2)]\rangle = \frac{1}{M(M-1)}\left[|V_m|^2 - M\right]. \quad (6)$$

Then, the weights of $M(M-1)$ and $M(M-1)(M-2)(M-3)$ are used to get the event-averaged 2-particle and 4-particle correlations, as introduced in [93]. Due to the definition, this new type of 4-particle cumulants SC(m,n) is independent of the symmetry planes Ψ_m and Ψ_n and is expected to be less sensitive to nonflow correlations, which should be strongly suppressed in 4-particle cumulants. This was confirmed by SC(m,n) calculation using HIJING model [98, 99] which does not include anisotropic collectivity but, for example, azimuthal correlations due to jet production. It is observed that both $\langle\langle\cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4)\rangle\rangle$ and $\langle\langle\cos[m(\varphi_1 - \varphi_2)]\rangle\rangle\langle\langle\cos[n(\varphi_1 - \varphi_2)]\rangle\rangle$ are nonzero, while SC(m,n) are compatible with zero in HIJING simulations [36]. This confirms that SC(m,n) measurements are nearly insensitive to nonflow correlations. Therefore, it is believed that SC(m,n) is nonzero if there is (anti)correlations of v_n and v_m. The investigation of SC(m,n) will allow us to know whether finding v_m larger than $\langle v_m\rangle$ in an event will enhance or reduce the probability of finding v_n larger than $\langle v_n\rangle$ in that event, which provides unique information for the event-by-event simulations of anisotropic flow harmonics.

Figure 5 shows the first calculation of SC(4, 2) (solid markers) and SC(3, 2) (open markers) as a function of centrality from AMPT model [93]. Nonzero values for both SC(4, 2) and SC(3, 2) are observed. Positive SC(4, 2) suggests a correlation between the event-by-event fluctuations of v_2 and v_4, which indicates that finding v_2 larger than $\langle v_2\rangle$ in an event enhances the probability of finding v_4 larger than $\langle v_4\rangle$ in that event. On the other hand, the negative results of SC(3, 2) imply that finding v_2 larger than $\langle v_2\rangle$ enhances the probability of finding v_3 smaller than $\langle v_3\rangle$ [93].

Several configurations of the AMPT model have been investigated to better understand the results based on AMPT simulations [93]. Partonic interactions can be tweaked by changing the partonic cross section: the default value is 10 mb, while using 3 mb generates weaker partonic interactions in ZPC [100, 101]. One can also change the hadronic interactions by controlling the termination time in ART. Setting NTMAX = 3, where NTMAX is a parameter which controls the number of time steps in ART (rescattering time), will effectively turn off the hadronic interactions [100, 101]. For SC(4, 2) and SC(3, 2) calculations for three different scenarios, (a) 3 mb, (b) 10 mb, and (c) 10 mb, no rescattering is presented in Figure 5. It is found that when the partonic cross section is decreasing from 10 mb (lower shear viscosity) to 3 mb (higher shear viscosity), the strength of SC(4, 2) decreases. Additionally, the "10 mb, no rescattering" setup seems to give slightly smaller magnitudes of SC(4, 2) and SC(3, 2).

Further studies have been performed in AMPT initial conditions, based on the observable of SC$(m,n)_\varepsilon$ which is defined as $\langle\varepsilon_m^2\varepsilon_n^2\rangle - \langle\varepsilon_m^2\rangle\langle\varepsilon_n^2\rangle$ [72]. The centrality dependence

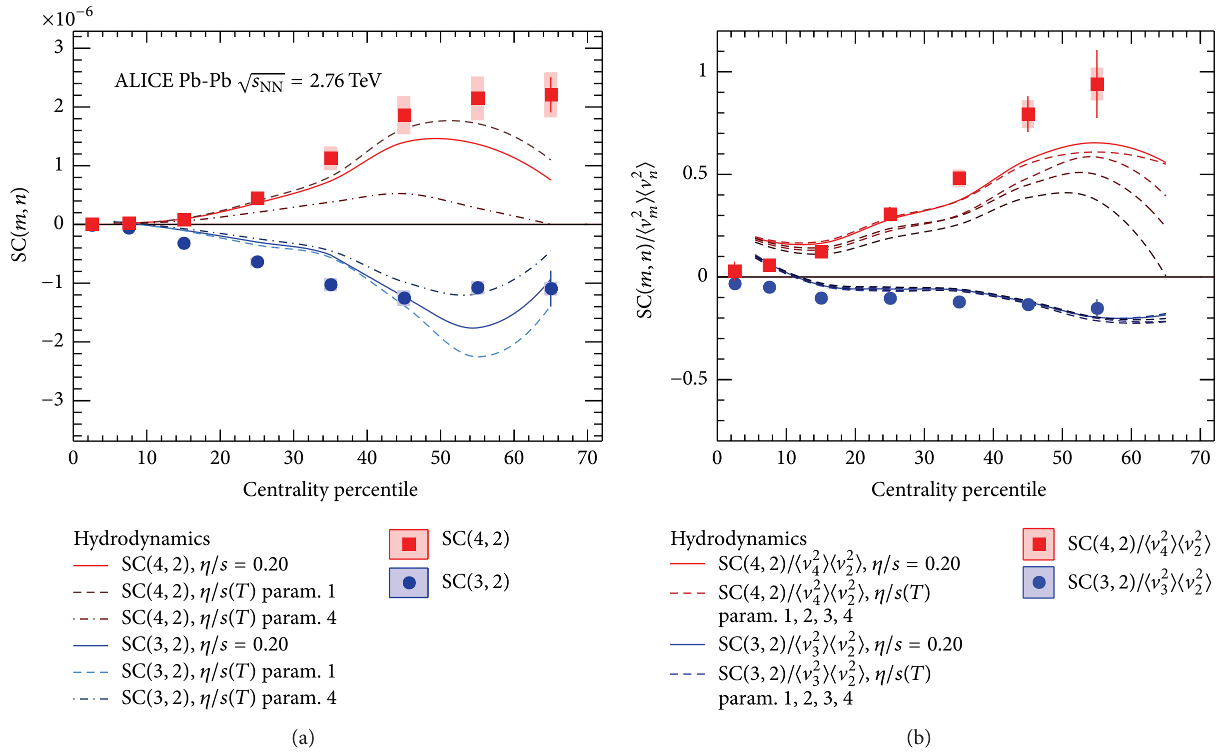

Figure 6: The centrality dependence of symmetric cumulants SC(4, 2) (red markers) and SC(3, 2) (blue markers) at $\sqrt{s_{NN}}$ = 2.76 TeV Pb–Pb collisions. Figures are taken from [36].

of $SC(4,2)_\varepsilon$ and $SC(3,2)_\varepsilon$ is presented as red circles and blue diamonds in Figure 5(b). Positive and increasing trend from central to peripheral collisions has been observed for $SC(4,2)_\varepsilon$. In contrast, negative and decreasing trend was observed for $SC(3,2)_\varepsilon$ in the AMPT initial conditions. This shows that finding ε_2 larger than $\langle\varepsilon_2\rangle$ in an event enhances the probability of finding ε_4 larger than $\langle\varepsilon_4\rangle$, while in parallel enhancing the probability of finding ε_3 smaller than $\langle\varepsilon_3\rangle$ in that event. Same conclusions were obtained using MC-Glauber initial conditions [75].

Based on AMPT calculations, it seems that the signs of $SC(m,n)_v$ (for $m,n = 2,3,4$) in the final state are determined by the correlations of $SC(m,n)_\varepsilon$ in the initial state, while its magnitude also depends on the properties of the created system. This clearly suggests that $SC(m,n)_v$ is a new promising observable to constrain the initial conditions and the transport properties of the system.

The first experimental measurements of centrality dependence of SC(4, 2) (red squares) and SC(3, 2) (blue circles) are presented in Figure 6(a). Positive values of SC(4, 2) are observed for all cases of centrality. This confirms a correlation between the event-by-event fluctuations of v_2 and v_4. On the other hand, the measured negative results of SC(3, 2) show the anticorrelation between v_2 and v_3 magnitudes. The same measurements are performed using the like-sign technique, which is another powerful approach to estimate nonflow effects [27]. It was found that the difference between correlations for like-sign and all charged combinations, which might be mainly due to nonflow effects, is much smaller compared to the magnitudes of $SC(m,n)$ itself. This further proves that nonzero values of $SC(m,n)$ measured in experiments cannot be explained by nonflow effects solely.

In addition, the comparison between experimental data and the event-by-event perturbative-QCD+saturation+hydro ("EKRT") calculations [62], which incorporate both initial conditions and hydrodynamic evolution, is shown in Figure 6. It was shown that this model can capture quantitatively the centrality dependence of individual v_2, v_3, and v_4 harmonics in central and mid-central collisions [62]. However, it can only qualitatively but not quantitatively predict $SC(m,n)$ measurements by ALICE. For given $\eta/s(T)$ parameterization tuned by individual flow harmonic, the calculation cannot describe SC(4, 2) and SC(3, 2) simultaneously for any single centrality. Experimental measurements are also compared to the VISH2+1 model calculations (see Figure 7), using various combinations of initial conditions (IC) from (a) MC-Glb, (b) MC-KLN, and (c) MC-AMPT with $\eta/s = 0.08$ and 0.20. It is noticed that the one with MC-Glb IC and $\eta/s = 0.08$ is compatible with SC(4, 2) measurement and the calculation with MC-AMPT IC and $\eta/s = 0.08$ can describe SC(3, 2) measurement [70]. However, just like EKRT calculations, none of these combinations is able to describe SC(4, 2) and SC(3, 2) simultaneously. Thus, it is concluded that the new $SC(m,n)$ observables provide better handle on the initial conditions and $\eta/s(T)$ than each of the individual harmonic measurements alone.

After being presented for the first time at Quark Matter 2015 conference, preliminary results of SC(4, 2) and SC(3, 2)

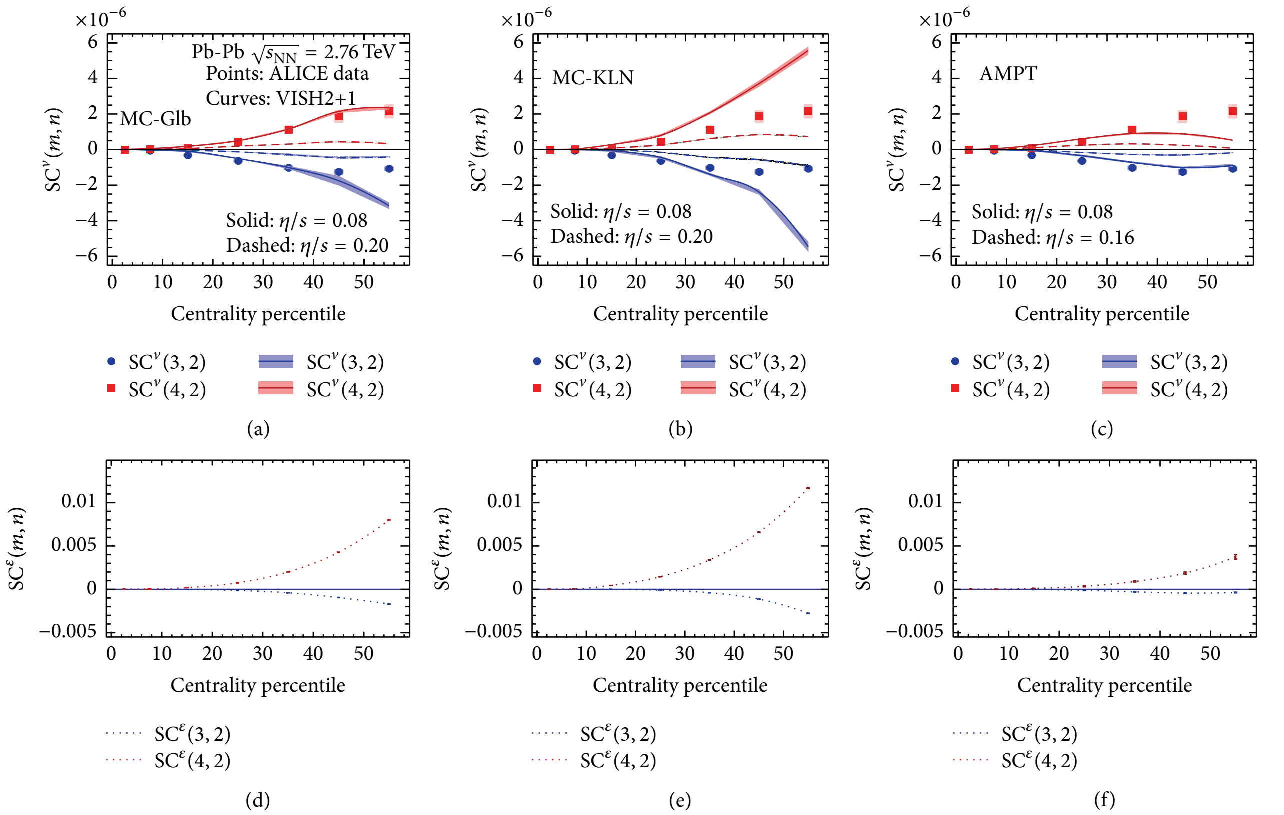

FIGURE 7: The centrality dependence of symmetric cumulants SC(4, 2) (red markers) and SC(3, 2) (blue markers) at $\sqrt{s_{NN}}$ = 2.76 TeV Pb–Pb collisions by VISH2+1 simulations. Figures are taken from [70].

gained a lot of attention [102]. One of the key suggestions was to normalize $SC(m,n)$ by dividing with the products $\langle v_m^2\rangle\langle v_n^2\rangle$ in order to get rid of influences from individual flow harmonics. The results are shown in Figure 6(b), with normalized SC(3, 2) and SC(4, 2) observables by dividing with the products $\langle v_3^2\rangle\langle v_2^2\rangle$ and $\langle v_4^2\rangle\langle v_2^2\rangle$, respectively [36]. The 2-particle correlations $\langle v_m^2\rangle$ and $\langle v_n^2\rangle$ are obtained with a pseudorapidity gap of $|\Delta\eta| > 1.0$ to suppress contributions from nonflow effects. It was shown in Figure 8(a) that the normalized SC(4, 2) observable exhibits clear sensitivity to different η/s parameterizations and the initial conditions, which provides a unique opportunity to discriminate between various possibilities of the detailed setting of $\eta/s(T)$ of the produced QGP and the initial conditions used in hydrodynamic calculations. On the other hand, normalized SC(3, 2) is independent of the setting of $\eta/s(T)$. In addition, it was demonstrated in Figure 9 that the normalized SC(3, 2), also named $NSC^v(3,2)$ in the following text, is compatible with its corresponding observable $SC^\varepsilon(3,2)$ in the initial state. Thus, $NSC^v(3,2)$ could be taken as golden observable to directly constrain initial conditions without demands for precise knowledge of transport properties of the system [70]. Furthermore, none of existing theoretical calculations can reproduce the data; there is still a long way to go for the development of hydrodynamic calculations.

Predictions of relationship between other harmonics are provided in [70] and shown in Figure 8. Besides different sensitivities to IC and η/s as seen above, the centrality dependence of the relationship between flow harmonics seems quite different. For instance, despite the differences in the initial conditions, a maximum value of SC(5, 3) is observed in central collision using $\eta/s = 0.20$, while the maximum value is seen in more peripheral collision if $\eta/s = 0.08$ is used.

Compared to the previous measurements of relationship between flow harmonics investigated using the ESE technique, $SC(m,n)$ observable provides a quantitative measure of these correlation strengths. Further investigations on relationship between flow harmonics using list of observables in Table 1 could be performed as a function of centrality, transverse momentum, and pseudorapidity et al., which is clearly nontrivial. Although one did not use the information of symmetry planes in both ESE and SC studies, recent study just reveals that flow harmonic correlations might not be completely independent on symmetry plane correlations [73]. The proportionality relations between symmetric cumulants involving higher harmonics v_4 or v_5 and symmetry plane correlations are derived, which seem to build the bridge between flow harmonic correlations and flow angle correlations (symmetry plane correlations). This might point out to a new direction of investigations of correlations between flow vectors and will shed a new light on the nature of fluctuating initial conditions and η/s of the created QGP in heavy-ion collisions.

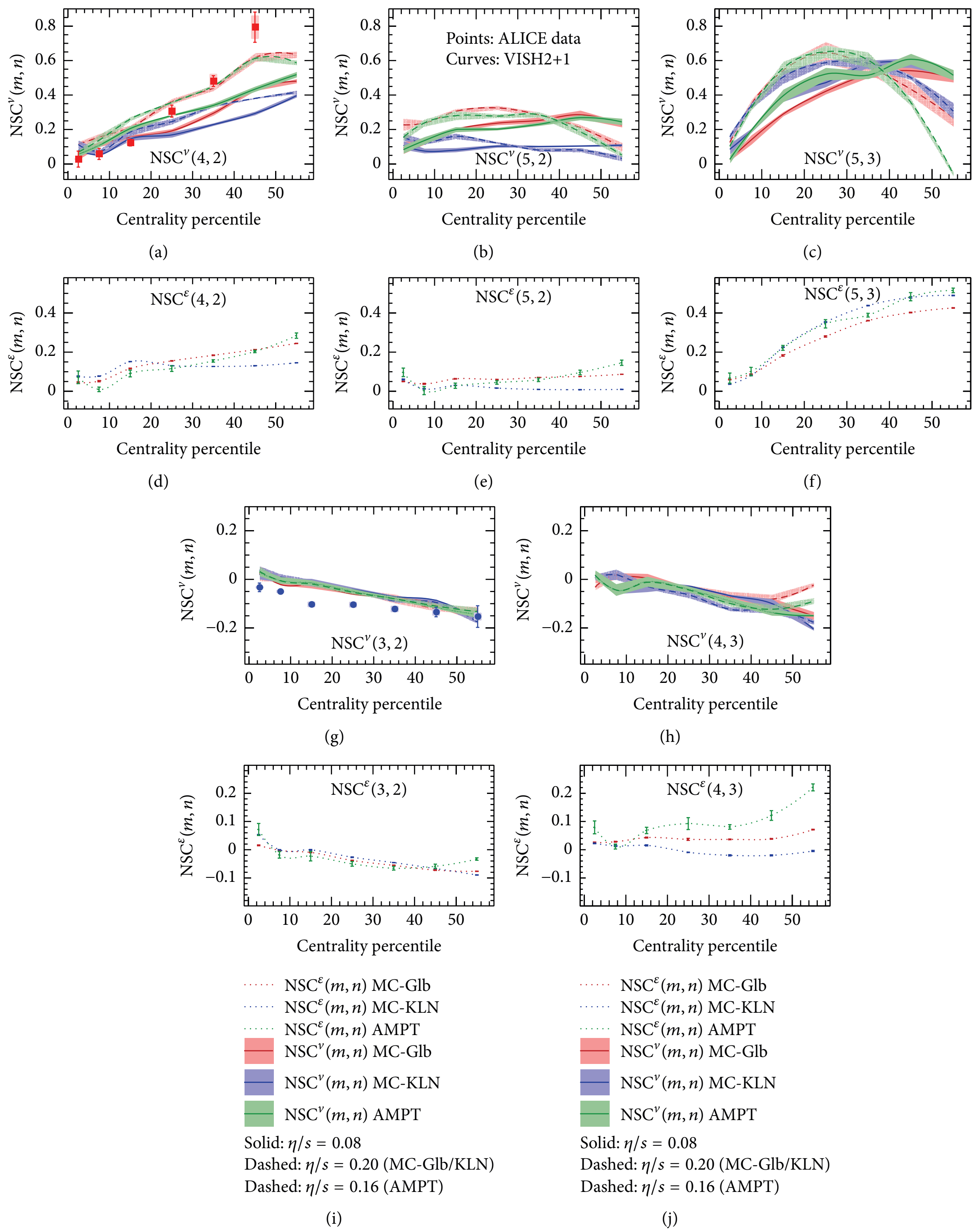

FIGURE 8: The centrality dependence of normalized symmetric cumulants NSC(m, n) at $\sqrt{s_{\mathrm{NN}}}$ = 2.76 TeV Pb–Pb collisions by VISH2+1 simulations. Figures are taken from [70].

3. Summary

In the past two decades, the underlying PDF of each single harmonic $P(v_n)$ was investigated in great detail. However, at the moment, how the joint underlying PDF, including different order symmetry planes and harmonics, is described is an open question, especially if these correlations between different flow harmonics modify the single harmonics $P(v_n)$. New observables discussed here begin to answer these open questions. Nevertheless, many more investigations between

Table 1: List of observables for correlations of flow harmonics, including all combinations of symmetric 2-harmonic 4-particle cumulants (up to v_6).

Observables	Equations	Number of particles	Exp.	Th.
$\langle\langle \cos(2\varphi_1 + 3\varphi_2 - 2\varphi_3 - 3\varphi_4)\rangle\rangle_c$	$\langle v_2^2 v_3^2\rangle - \langle v_2^2\rangle\langle v_3^2\rangle$	4	[36]	[70–72]
$\langle\langle \cos(2\varphi_1 + 4\varphi_2 - 2\varphi_3 - 4\varphi_4)\rangle\rangle_c$	$\langle v_2^2 v_4^2\rangle - \langle v_2^2\rangle\langle v_4^2\rangle$	4	[36]	[70–73]
$\langle\langle \cos(2\varphi_1 + 5\varphi_2 - 2\varphi_3 - 5\varphi_4)\rangle\rangle_c$	$\langle v_2^2 v_5^2\rangle - \langle v_2^2\rangle\langle v_5^2\rangle$	4		[70, 71, 73]
$\langle\langle \cos(2\varphi_1 + 6\varphi_2 - 2\varphi_3 - 6\varphi_4)\rangle\rangle_c$	$\langle v_2^2 v_6^2\rangle - \langle v_2^2\rangle\langle v_6^2\rangle$	4		
$\langle\langle \cos(3\varphi_1 + 4\varphi_2 - 3\varphi_3 - 4\varphi_4)\rangle\rangle_c$	$\langle v_3^2 v_4^2\rangle - \langle v_3^2\rangle\langle v_4^2\rangle$	4		[70]
$\langle\langle \cos(3\varphi_1 + 5\varphi_2 - 3\varphi_3 - 5\varphi_4)\rangle\rangle_c$	$\langle v_3^2 v_5^2\rangle - \langle v_3^2\rangle\langle v_5^2\rangle$	4		[70, 71, 73]
$\langle\langle \cos(3\varphi_1 + 6\varphi_2 - 3\varphi_3 - 6\varphi_4)\rangle\rangle_c$	$\langle v_3^2 v_6^2\rangle - \langle v_3^2\rangle\langle v_6^2\rangle$	4		
$\langle\langle \cos(4\varphi_1 + 5\varphi_2 - 4\varphi_3 - 5\varphi_4)\rangle\rangle_c$	$\langle v_4^2 v_5^2\rangle - \langle v_4^2\rangle\langle v_5^2\rangle$	4		
$\langle\langle \cos(4\varphi_1 + 6\varphi_2 - 4\varphi_3 - 6\varphi_4)\rangle\rangle_c$	$\langle v_4^2 v_6^2\rangle - \langle v_4^2\rangle\langle v_6^2\rangle$	4		
$\langle\langle \cos(5\varphi_1 + 6\varphi_2 - 5\varphi_3 - 6\varphi_4)\rangle\rangle_c$	$\langle v_5^2 v_6^2\rangle - \langle v_5^2\rangle\langle v_6^2\rangle$	4		
...	...	6		

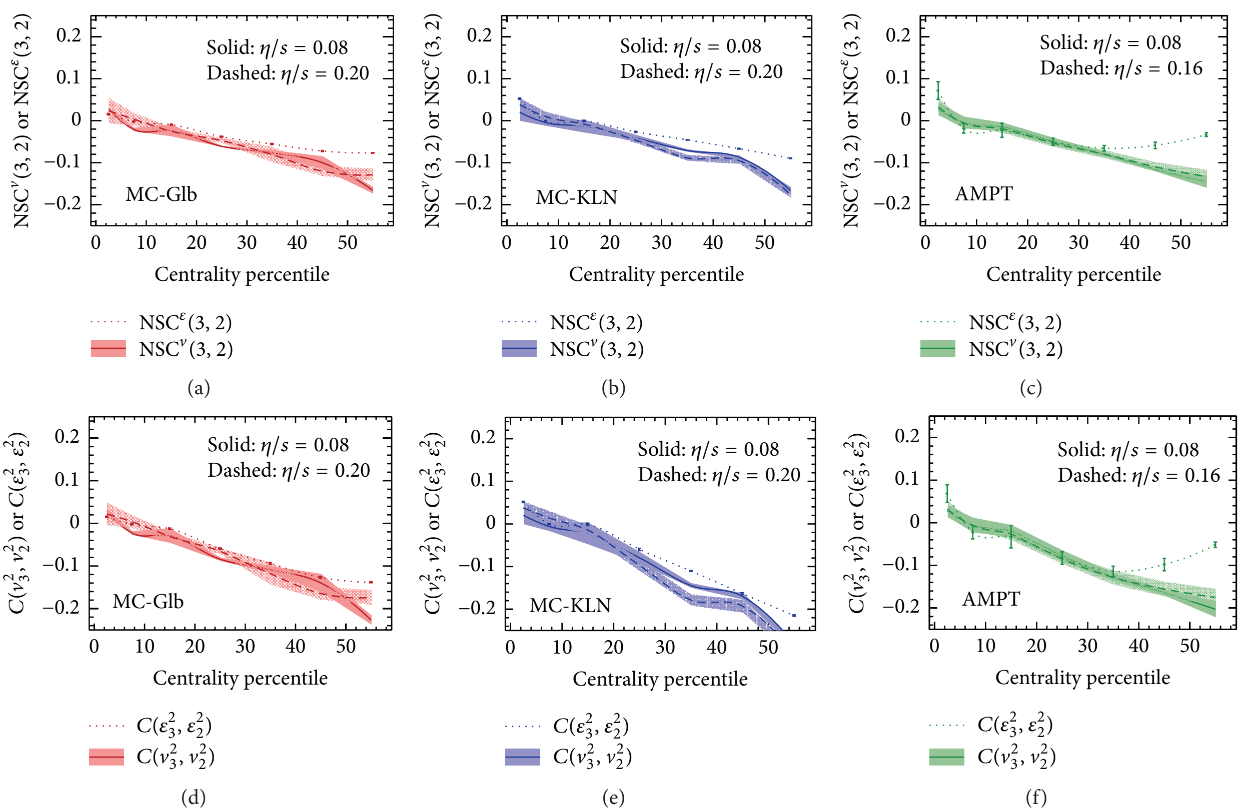

Figure 9: The centrality dependence of NSC(3, 2) (a, b, and c) and $C(v_3^2, v_2^2)$ (d, e, and f) and the corresponding observables in the initial conditions at $\sqrt{s_{NN}}$ = 2.76 TeV Pb–Pb collisions from VISH2+1. Figures are taken from [70].

different flow harmonics, including higher-order cumulants and higher harmonics, are necessary to reasonably constrain the joint PDF and ultimately lead to new insights into the nature of fluctuation of the created matter in heavy-ion collisions. How to turn the multitude of measured and possibly measurable in future relationships between anisotropic flow harmonics into a focused search for correct initial conditions and detailed setting of η/s is an exciting challenge for the theory community.

Competing Interests

The author declares that there are no competing interests regarding the publication of this paper.

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