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REVIEW OF CRITICAL FLOW RATE,  
PROPAGATION OF PRESSURE PULSE,  
AND SONIC VELOCITY IN TWO-PHASE MEDIA

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# REVIEW OF CRITICAL FLOW RATE, PROPAGATION OF PRESSURE

## PULSE, AND SONIC VELOCITY IN TWO-PHASE MEDIA

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### SUMMARY

For single-phase media, the critical discharge velocity, the sonic velocity, and the pressure pulse propagation velocity can be expressed in the same form by assuming isentropic, equilibria processes. In two-phase mixtures, the same concept is not valid due to the existence of interfacial transports of momentum, heat, and mass. Thus, the three velocities should be treated differently and separately for each particular condition, taking into account the various transport processes involved under that condition. This report reviews various attempts to predict the critical discharge rate or the propagation velocities by considering slip ratio (momentum change), evaporation (mass and heat transport), flow pattern, etc.. Experimental data were compared with predictions based on various theorems. The importance is stressed of the time required to achieve equilibrium as compared with the time available during the process, for example, of passing a pressure pulse.

### INTRODUCTION

In a system involving two-phase mixtures, such as a space fuel tank, a nuclear reactor, or a cryogenic storage tank, it is necessary to know the rate of discharge in case there is accidental leakage, the rate of pressure propagation in case there is a pressure surge, or the sonic velocity to determine the onset of flow instability. All of these propagation velocities, namely, the critical discharge velocity  $U_c$ , the propagation rate of a small pressure pulse  $a$ , and the sonic velocity  $c$  (which is the propagation rate of a continuous train of acoustic waves of small amplitude), are closely related. In fact, for single-phase flow, they are almost synonymous. However, in a two-phase flow system, nonequilibrium, nonhomogeneity, and other complicating factors are present. Under such conditions, these three terms cannot be assumed mutually interchangeable. The

objective of this report is to review the experimental and analytical studies on these propagation rates in a two-phase medium and to discuss the relation and differences of these three rates. In the following sections, the theories and experimental findings reported in the literature for each type of propagation velocity are examined, and the controlling parameters are discussed. Emphasis is placed on the effect of the flow pattern in determining the propagation rates. We first discuss the problem associated with critical flow, then those problems associated with the other two phenomena, and finally the relation between the three rates.

## SYMBOLS

A	attenuation coefficient
a	pressure propagation velocity
a, b, c, d, e	coefficients in eqs. (25) to (30)
B	dispersion coefficient
C	specific heat
$C_M$	coefficient related to virtual mass
c	sonic or acoustic velocity
$c_m$	acoustic velocity of the mixture
$c_o$	sonic velocity of gas
$c_p$	specific heat of gas at constant pressure
$c'_p$	specific heat of suspension at constant pressure
$c_v$	specific heat of gas at constant volume
D	diameter
$F_1, F_2$	coefficients in eq. (72)
$F_p$	force on a particle
G	flow rate, $\rho U$
h	enthalpy
$h_o$	stagnation enthalpy
K	wave number
$K_1, K_2$	real and imaginary parts of wave number, $K = K_1 + iK_2$
k	slip ratio

L	length
N	coefficient in eq. (36)
n	coefficient in eq. (52)
$n_b$	bubbles per unit volume
P	pressure
Pr	Prandtl number
$Q_p$	heat-transfer rate in particle
R	radius, also gas constant
$R_d$	drop radius
$R_o$	radius of an unperturbed bubble
S	entropy
T	temperature
t	time
U	velocity
$U_c$	critical velocity
v	specific volume
x	quality of vapor
$x_s$	mass fraction of suspension
Z	mass of air to mass of water ratio
z	distance in longitudinal direction
$\alpha$	void fraction
$\beta$	dispersion coefficient
$\gamma$	ratio of specific heats
$\lambda$	thermal conductivity
$\mu$	viscosity
$\nu$	kinematic viscosity
$\nu_t$	thermal diffusivity
$\rho$	density
$\tau$	relaxation time
$\Omega$	dimensionless frequency

$\omega$	frequency
Subscripts:	
c	critical
D	drag
d	displacement or drag
E	equilibrium
e	exit
fg	liquid-vapor transition
g	gas
H	homogeneous
HT	homogeneous, isothermal
L	liquid
m	maximum
mom	momentum
o	stagnation
s	suspension
t	thermal
w	wave
z	at z-location
Superscript:	
'	$\partial/\partial P$

## PROPAGATION AND DISCHARGE RATES IN SINGLE-PHASE MEDIA AND THEIR RELATION TO TWO-PHASE FLOW

First we consider the propagation of an infinitesimal pressure pulse in a single-phase medium. Following the approach by Shapiro (ref. 1, p. 46), consider a control volume around the wave front traveling at velocity  $c$  in a channel of constant cross section  $A$  (fig. 1(a)). From the Lagrangian point of view, an observer traveling with the wave front would see the surroundings in the way described in figure 1(b). Neglecting the



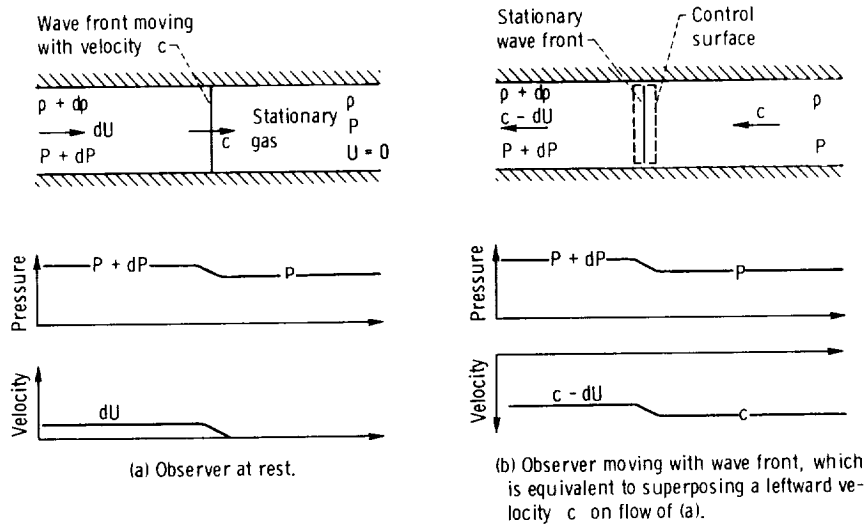


Figure 1. - Propagation of an infinitesimal pressure pulse (ref. 1).

viscous shear and the head effects and assuming one-dimensional flow yield the momentum equation

$$A[P - (P + dP)] = w[(c - dU) - c]$$

or

$$A dP = \rho c A dU$$

The continuity equation is

$$\rho c = (\rho + d\rho)(c - dU)$$

or

$$\frac{dP}{\rho} = \frac{dU}{c} \quad (2)$$

Combining equations (1) and (2) yields

$$c = \sqrt{\left(\frac{dP}{d\rho}\right)_s} \quad (3)$$

The subscript *s* denotes that the processes are isentropic since the pressure and temperature changes are vanishingly small and thus are nearly reversible.

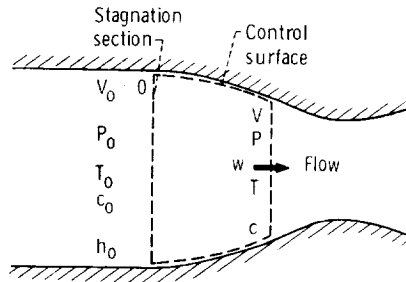


Figure 2. - Flow between stagnation section and any other section (ref. 1).

Now consider the isentropic flow through a passage of varying cross section as shown in figure 2. Again following Shapiro (ref. 1, p. 75) gives the enthalpy equation as

$$h_0 = h + \frac{U^2}{2}$$

or

$$dh = -d\left(\frac{U^2}{2}\right) \quad (4)$$

From the isentropic condition

$$T dS = 0 = dh - \frac{dP}{\rho}$$

Thus,

$$dh = -\left(\frac{dP}{\rho}\right)_s \quad (5)$$

The continuity equation is

$$\rho AU = GA = \text{constant}$$

or

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dU}{U} = 0 \quad (6)$$

Combining equations (4), (5), and (6) gives

$$\frac{dA}{A} = \frac{dP}{\rho} \left[ \frac{1}{U^2} - \left( \frac{d\rho}{dP} \right)_s \right] \quad (7)$$

or, when expressed in terms of the mass flux  $G$ ,

$$-\frac{dG}{G} = \rho dP \left[ \frac{1}{G^2} - \frac{1}{\rho^2} \left( \frac{d\rho}{dP} \right)_s \right] \quad (8)$$

From equation (8) it is obvious that the maximum mass flux occurs when  $dG/dP$  is zero or when

$$G_{\max} = \rho \sqrt{\frac{dP}{d\rho}} = \sqrt{-\frac{dP}{dv}} \quad (9)$$

This maximum flow rate occurs at the throat of a nozzle where  $dA/A = 0$ . The  $G_{\max}$  is commonly called the maximum discharge rate or critical flow rate. A comparison of equations (2) and (9) shows that for single-phase flow critical flow velocity and the sonic velocity are given by the same expression. Furthermore, since in the sonic velocity analysis the pressure pulse is assumed to be infinitesimally small and the process is very close to equilibrium and reversible, it does not matter whether there is a single pressure pulse or a train of small waves.

## SPECIAL PROBLEMS PECULIAR TO PROPAGATION AND DISCHARGE RATES IN TWO-PHASE FLOW

As mentioned in the last section, the analyses on critical flow rate or sonic velocity  $c$  are based on the assumption that the flow is one-dimensional, single-phase (hence homogeneous), in equilibrium, and isentropic. In the two-phase flow, all these assumptions are subject to violation. The coexistence of two phases in various flow patterns makes homogeneous flow a near approximation only under the special condition of highly dispersed flow. When two phases are separated, such as in annular or slug flow, the homogeneous assumption is no longer true. Also, when there is strong slip between phases, the homogeneous assumption may not be true even in the case of bubbly or mist flow. When the flow is undergoing strong interfacial mass transfer due to evaporation,

or interfacial momentum transfer due to flow acceleration, the equilibrium and isentropic concepts are violated. Here we note that the concept of equilibrium covers both thermodynamic and hydrodynamic (in terms of shear stress) equilibria. If the flow channel has a very wide angle of divergence, the one-dimensional assumption is in danger. Furthermore, since the interfacial transports are involved, a relaxation time is required for the flow to adjust to a finite disturbance. Thus, a single pulse or a series of pulses would impose different effects on the two-phase flow. Therefore, in two-phase flow, all those factors that are in disagreement with these assumptions used for the idealized single-phase approach necessitate the reexamination of each assumption for each specific case.

## TWO-PHASE CRITICAL FLOW

### General

Since the derivation for critical flow as shown in the last section is for isentropic flow, it is no longer applicable for two-phase flow when strong interfacial transports are involved and the processes are highly nonequilibrium. Furthermore, the concept of density is totally different for a flowing two-phase mixture. Thus, an alternative derivation should be used, so that the isentropic restriction can be removed and a new compressibility concept can be used.

Starting with the momentum equation, neglecting the hydrostatic head and viscous dissipation term, and assuming one-dimensional steady state, we have

$$\rho U \frac{dU}{dz} = - \frac{dP}{dz} \quad (10)$$

Combining this equation with equation (6) again gives

$$\frac{dA}{A} = \frac{dP}{\rho} \left( \frac{1}{U^2} - \frac{d\rho}{dP} \right) \quad (11)$$

except that  $d\rho/dP$  is no longer held at constant entropy as used in equation (7). The critical flow rate at the throat is again

$$G_c = \rho \sqrt{\frac{dP}{d\rho}}$$

which is the general form for homogeneous flow. Now, if the slip ratio of the two phases is not unity, the momentum equation should be changed to (ref. 2)

$$G \frac{d}{dz} \left[ xU_g + (1 - x)U_L \right] = - \frac{dP}{dz} \quad (12)$$

Since for a critical flow the criterion is

$$\frac{\partial G}{\partial P} = 0 \quad (13)$$

we can combine the last two equations into

$$-1 = G \frac{\partial}{\partial P} \left[ xU_g + (1 - x)U_L \right] \quad (14)$$

at a given  $z$ . The liquid velocity and vapor velocity are related by the slip ratio  $k$ , which is defined by

$$U_g = kU_L \quad (15)$$

The mass flow rates can be expressed as

$$(1 - x)G = \frac{(1 - \alpha)U_L}{v_L} \quad (16a)$$

$$xG = \frac{\alpha U_g}{v_g} = \frac{\alpha U_L k}{v_g} \quad (16b)$$

From these equations, when  $G$  is eliminated,

$$\alpha = \frac{xv_g}{k(1 - x)v_L + xv_g} \quad (17)$$

Combining equations (16b) and (17) gives  $G$  as

$$G = \left[ \frac{k}{k(1 - x)v_L + xv_g} \right] U_L \quad (18)$$

Eliminating  $U_L$  between equations (18) and (14) gives

$$G_c = \frac{1}{\frac{\partial}{\partial P_e} \left\{ \frac{[k(1-x)v_L + xv_g][xk + (1-x)]}{k} \right\}_z} \quad (19)$$

which is the general form of the critical flow equation. The single-phase critical flow equation (eq. (9)) is different from the two-phase critical flow equation (eq. (19)) in the number of variables. In equation (9), only the variation of  $v_g$  with  $P$  needs to be known. In two-phase flow, the variables  $v_g$ ,  $x$ , and  $k$  can be functions of  $P$  and the problem is to describe the relations. Fauske (ref. 3) pointed out in his discussion on pressure pulse propagation that there are three interfacial transport processes to be considered in a two-phase flow situation. This concept can also be applied to the critical flow. The three interfacial transport processes considered by Fauske are the following:

- (1) Interfacial heat transfer. The heat-transfer rate between the gas phase and the surrounding liquid or solid phase determines the term  $\partial v_g / \partial P$  in equation (19).
- (2) Interfacial momentum transfer. This transfer determines how fast each phase is accelerated, thus controlling the term  $\partial k / \partial P$  in equation (19).
- (3) Interfacial mass transfer. This transfer determines the rate of evaporation or condensation, that is, the term  $\partial x / \partial P$  in equation (19).

In summary, the discussion on critical flow shows the following: for a single-phase flow, only the volume change  $\partial v_g / \partial P$  is of concern; for a two-component, two-phase flow, both momentum and heat transfer or  $\partial k / \partial P$  and  $\partial v_g / \partial P$  should be considered; for a one-component, two-phase flow, all three processes or  $\partial k / \partial P$ ,  $\partial v_g / \partial P$ , and  $\partial x / \partial P$  should be considered. The whole effort in this field of two-phase critical flow can be viewed as an attempt to describe these three transport processes in light of specific interfacial configurations (flow pattern) and the time available for reaching the equilibrium of each or all of these processes. Comprehensive reviews on this subject can be found in reports by Fauske (ref. 4) and Henry (ref. 2).

## Theories on Critical Flow

Many theories on critical flow have been proposed over the years, and some of them are quite similar. We will only cover a few which have been reported more recently. In the following sections, we discuss three models which propose various ways of determining slip ratio and change of equilibrium quality and one model which, in addition to slip ratio consideration, tried to include the nonequilibrium effect in the analysis.

Fauske's theory (ref. 4). - Fauske suggested that in two-phase flow the maximum discharge rate may not necessarily be accompanied by a shock front. He proposed that at the critical flow condition the absolute value of the pressure gradient at a given location is maximum but finite for a given flow rate and quality, or mathematically

$$\left| \left( \frac{\partial P}{\partial z} \right)_{G, x} \right| = \text{maximum, finite} \quad (20)$$

In two-phase critical flow,  $G$  is fixed when  $x$ ,  $P$ , and  $k$  are fixed. Since in computing  $\partial P / \partial z$  both  $x$  and  $G$  are fixed,  $P$  is a function of  $k$  only. Therefore, Fauske says that  $(\partial P / \partial z)_{\max}$  should occur when

$$\frac{\partial}{\partial k} \left( \frac{\partial P}{\partial z} \right) = 0$$

It was shown by Fauske that to maximize  $\partial P / \partial z$  the conditions to be satisfied are

$$\frac{\partial f}{\partial k} = 0 \quad (21)$$

and

$$\frac{\partial v_{\text{mom}}}{\partial k} = 0 \quad (22)$$

where  $f$  is the friction factor and  $v_{\text{mom}}$ , the term in braces in equation (19), is called by Fauske "the momentum specific volume" to represent the volume weighed by momentum of each species:

$$v_{\text{mom}} = \frac{[k(1-x)v_L + xv_g][xk + (1-x)]}{k} \quad (23)$$

Using equations (22) and (23) shows that at critical flow

$$k = \sqrt{\frac{v_g}{v_L}} \quad (24)$$

Fauske further assumed that the flow is isenthalpic and follows the saturation line. From this assumption, together with equation (24), one can obtain the terms  $\partial x/\partial P$ ,  $\partial k/\partial P$ , and  $\partial v_g/\partial P$  as follows:

$$\frac{\partial x}{\partial P} = - \frac{1}{h_{fg}} \left( \frac{\partial h_f}{\partial P} + x \frac{\partial h_{fg}}{\partial P} \right)_{\text{sat}} \quad (25a)$$

$$\frac{\partial v_g}{\partial P} = \left. \frac{dv_g}{dP} \right|_{\text{saturation}} \quad (25b)$$

$$\frac{\partial v_L}{\partial P} = 0 \quad (25c)$$

$$\frac{\partial k}{\partial P} = \frac{1}{2 \sqrt{v_L v_g}} \frac{\partial v_g}{\partial P} \quad (25d)$$

From equations (20), (24), and (25) the  $G_c$  can be calculated. The theoretical prediction of the critical flow rate as a function of quality and pressure for a stream-water system is shown in figure 3. (Note that in fig. 3 two sets of plots using quality and pressure as abscissa are superimposed and the reader can use either of them.)

Fauske used annular flow as an illustration. His analysis is not limited to annular flow only and has been compared favorably to experimental data over a wide range of quality ( $0.01 < x < 1.0$ ) (see ref. 5), which certainly covers a wide variety of flow patterns.

Fauske's theory predicts the experimental flow rate data of Falleti (ref. 6), Moy (ref. 7), and DaCruz (ref. 8) successfully for the range  $0.1 < x < 1.0$ . However, Fauske's theory consistently over predicted the slip ratio  $k$  when compared with more recent data of Klingebiel (ref. 9), Fauske (ref. 10), and Henry (ref. 2). Since Fauske's flow rate equation made use of his expression for slip ratio, it may indicate that the flow rate prediction is not very sensitive to the value of slip ratio.

Moody's model (ref. 11). - Moody's analysis is based on an annular flow model with uniform axial velocities of each phase and equilibrium between two phases. The major difference of Moody's approach from that of Fauske's is in the expression for the slip ratio  $k$ . While Fauske obtained  $k$  by minimizing the momentum volume  $\partial v_{\text{mom}}/\partial k$ , Moody maximized the flow rate with respect to both  $k$  and  $P$ :



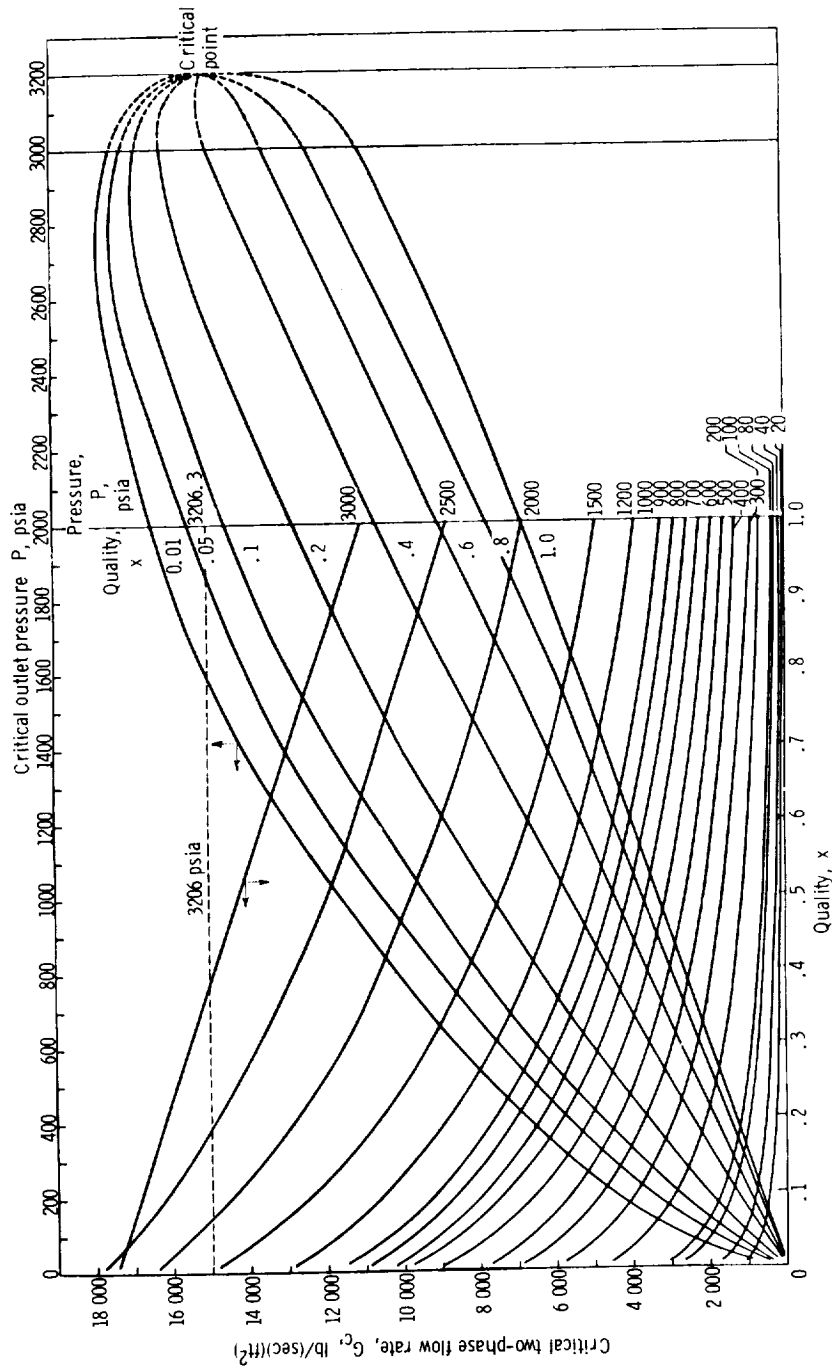


Figure 3. - Theoretical predictions of critical flow rates by Fauski's model (ref. 4).

$$\left. \frac{\partial G}{\partial P} \right|_k = 0 \quad (26)$$

$$\left. \frac{\partial G}{\partial k} \right|_P = 0 \quad (27)$$

The resulting expression for  $k$  is

$$k_c = \left( \frac{v_g}{v_L} \right)^{1/3} \quad (28)$$

The use of this expression for  $k$  leads to the critical flow rate equation

$$G_c = \sqrt{\frac{-2g_c(v_L + xv_{fg})}{a(ad + 2be)}} \quad (29)$$

where

$$a = kv_L + x(v_g - kv_L) \quad (30a)$$

$$b = \frac{1}{k^2} + x \left( 1 - \frac{1}{k^2} \right) \quad (30b)$$

$$d = \left[ \frac{S'_g}{k^2 S_{fg}} - \frac{S'}{S_{fg}} - \frac{(S_{fg} k^2)'}{k^4 S_{fg}} \right] + x \left[ \frac{(S_{fg} k^2)'}{k^4 S_{fg}} - \frac{S'_{fg}}{S_{fg}} \right] \quad (30c)$$

$$e = \left[ S_{fg} \left( \frac{kv_L}{S_{fg}} \right)' + \left( \frac{kv_L}{S_{fg}} \right) S'_g - \left( \frac{v_L}{S_{fg}} \right) S'_L \right] + x \left[ S_{fg} \left( \frac{v_g}{S_{fg}} \right)' - S_{fg} \left( \frac{kv_L}{S_{fg}} \right)' \right] \quad (30d)$$

and the superscript ' denotes  $\partial/\partial P$ . All properties are those at local static pressure at the maximum flow rate condition.

Moody's equation was compared with experimental results of Faletti (ref. 6), Moy (ref. 7), Fauske (ref. 4), and Zaloudek (ref. 12). It appears that his equation slightly overpredicts the maximum discharge rate in the low quality range ( $x < 0.1$ ), predicts

well in the moderate quality range ( $0.2 < x < 0.6$ ), and underpredicts in the higher quality range ( $x > 0.60$ ) (see fig. 4).

Levy's model (ref. 5). - Most assumptions used by Levy are the same as those postulate by Moody - namely, thermal equilibrium, separated phases with each represented by a uniform velocity, and the neglect of frictional and hydrostatic head. Levy departed from the two previous theories in his method of calculating the slip ratio  $k$ . In the previous two analyses, the  $k$  was determined by some maximization principle, while Levy used his method of momentum exchange which was originally developed to predict void fraction (ref. 13). In this approach, the momentum of each phase was considered separately, and the phase pressure drops were equated. From this method the quality  $x$  can be expressed as a function of the void fraction and phase densities:

$$x = \frac{\alpha(1 - 2\alpha) + \alpha \sqrt{(1 - 2\alpha)^2 + \alpha \left[ \frac{2\rho_L}{\rho_g} (1 - \alpha)^2 + \alpha(1 - 2\alpha) \right]}}{\left( \frac{2\rho_L}{\rho_g} \right) (1 - \alpha^2) + \alpha(1 - 2\alpha)} \quad (31)$$

The momentum specific volume  $v_{\text{mom}}$  in equation (23) can be expressed as a function of quality  $x$  and pressure  $P$ . Consequently,

$$dv_{\text{mom}} = \left. \frac{\partial v_{\text{mom}}}{\partial P} \right|_x dP + \left. \frac{\partial v_{\text{mom}}}{\partial x} \right|_P dx \quad (32)$$

Since for isentropic flow the entropy change  $dS$  is zero,

$$dS = \left. \frac{\partial S}{\partial P} \right|_x dP + \left. \frac{\partial S}{\partial x} \right|_P dx = 0 \quad (33)$$

$$\left. \frac{\partial x}{\partial P} \right|_S = - \frac{\left[ x \frac{\partial S_g}{\partial P} + (1 - x) \frac{\partial S_L}{\partial P} \right]}{(S_g - S_L)} \quad (34)$$

Combining equations (32), (33), and (34) gives

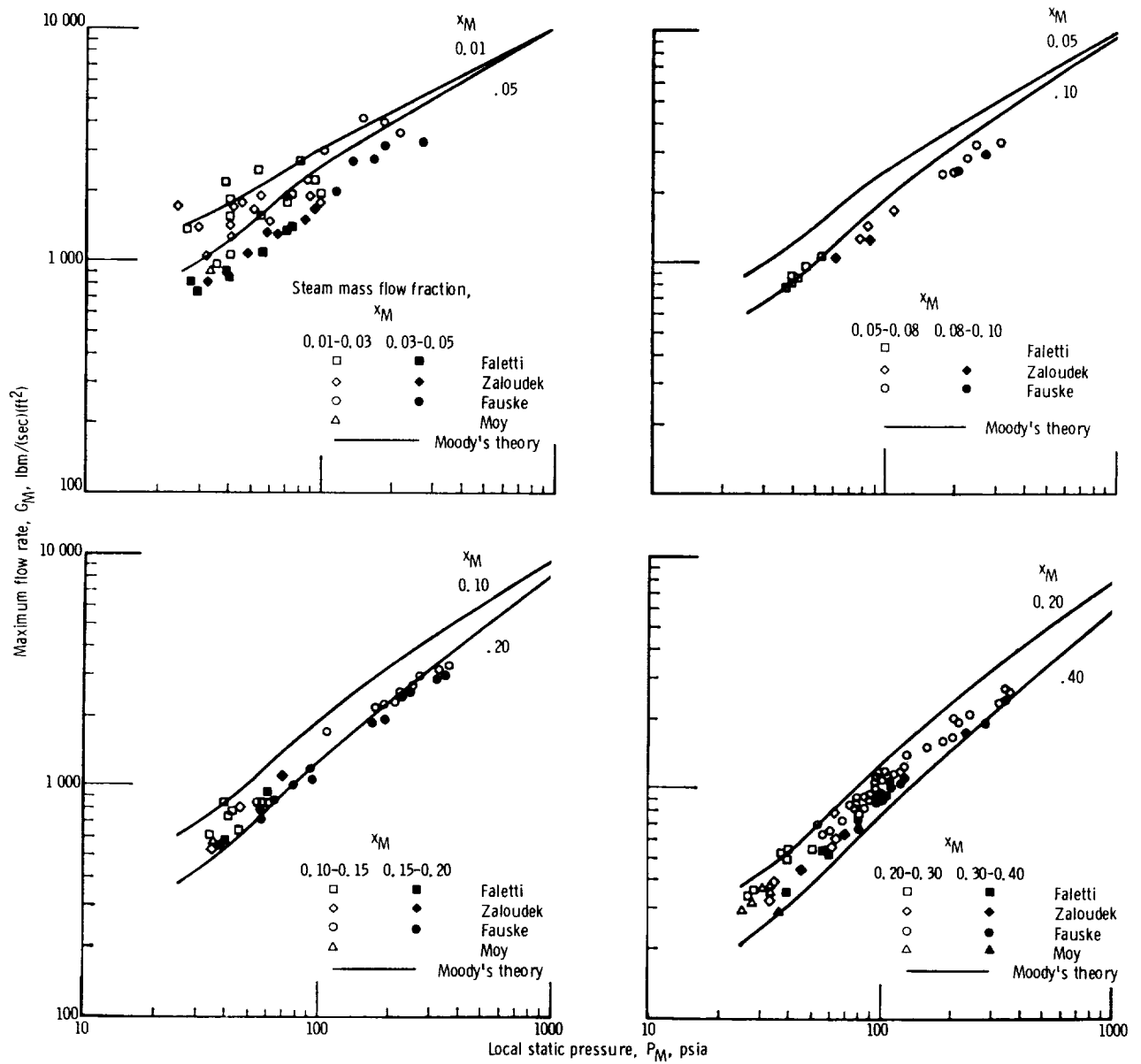


Figure 4. - Comparison of Moody's calculated maximum steam-water flow rates with data (ref. 11).

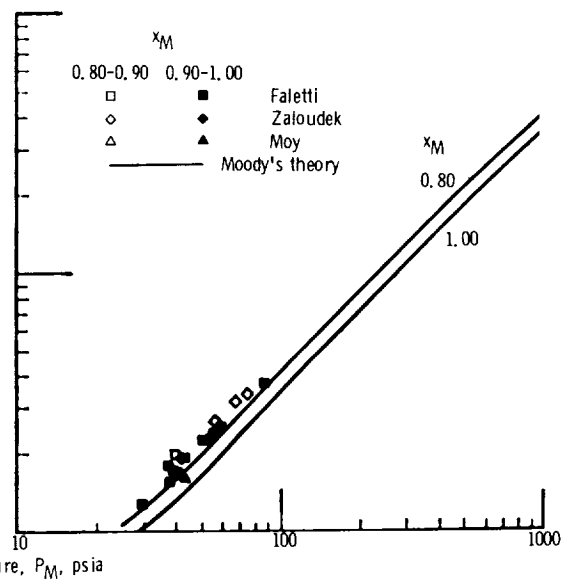
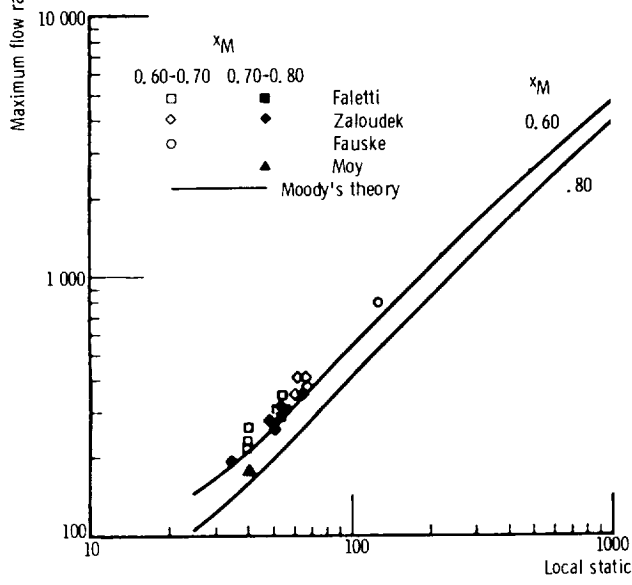
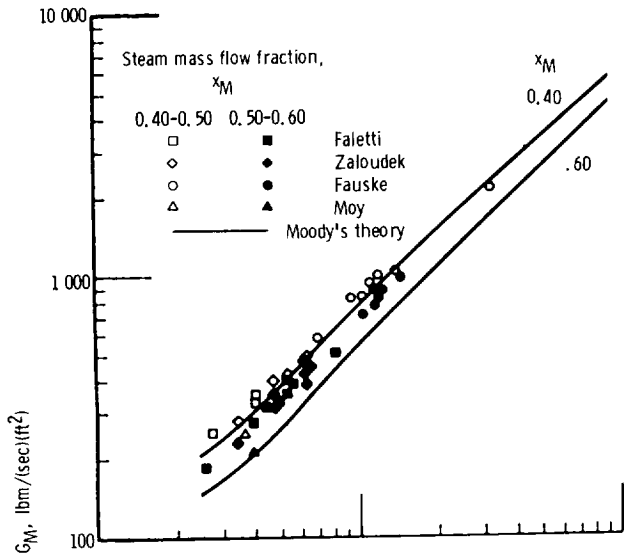


Figure 4. - Concluded.

$$\left. \frac{\partial v_{\text{mom}}}{\partial P} \right|_S = \left( \frac{\partial v_{\text{mom}}}{\partial P} \right)_x - \left( \frac{\partial v_{\text{mom}}}{\partial x} \right)_P \frac{\left[ x \frac{\partial S_g}{\partial P} + (1-x) \left( \frac{\partial S_L}{\partial P} \right) \right]}{(S_g - S_L)} \quad (35)$$

The terms  $\partial v_{\text{mom}}/\partial P|_x$  and  $\partial v_{\text{mom}}/\partial x|_P$  can be obtained from equations (23) and (31). Levy calculated critical flow rates for isentropic and isenthalpic processes and showed that there is not much difference between these two processes, as shown by figure 5 where the critical flow rate for both processes is plotted as a function of quality at various pressures. However, there are two significant differences as compared with Fauske's model:

(1) Fauske's slip ratio  $k = \sqrt{v_g/v_L}$  is independent of quality while Levy's slip ratio increases with quality.

(2) Fauske's critical flow rate decreases monotonically with quality while Levy's exhibits a slight maximum for the low pressure steam-water system with a maximum located around a quality fraction of 0.01.

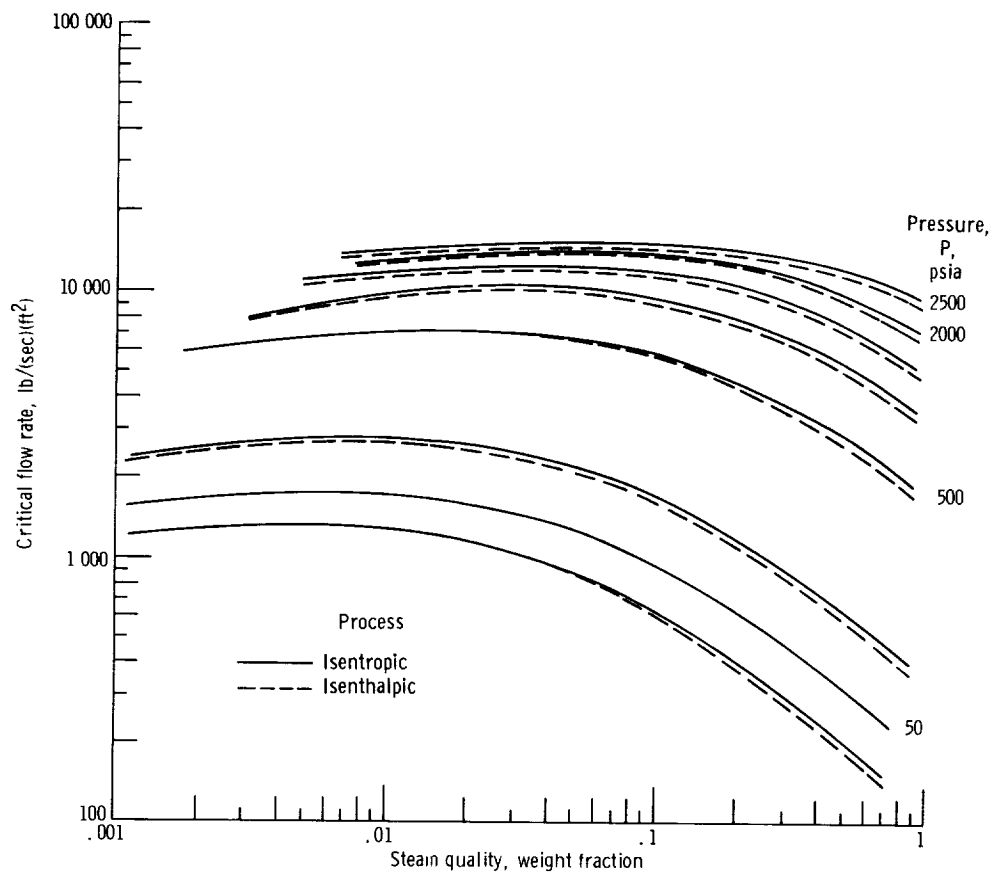


Figure 5. - Critical flow rate production for steam-water mixture from Levy's model (ref. 5).

In general, however, Levy's and Fauske's predicted critical flow rates are quite close except in the high pressure regime.

At this point one should pause to note that the previous three models assume separated flow in thermal equilibrium. The only difference is in their way of determining slip ratio. As has been noted previously, the predicted critical flow results are not very different. Among them Moody's is more interesting to practicing engineers since in his paper the critical flow rate was also presented as a function of the stagnation condition (see fig. 6). This figure is convenient for direct use without knowing the local static condition. However, it was pointed out by Neusen in his discussion of Moody's paper

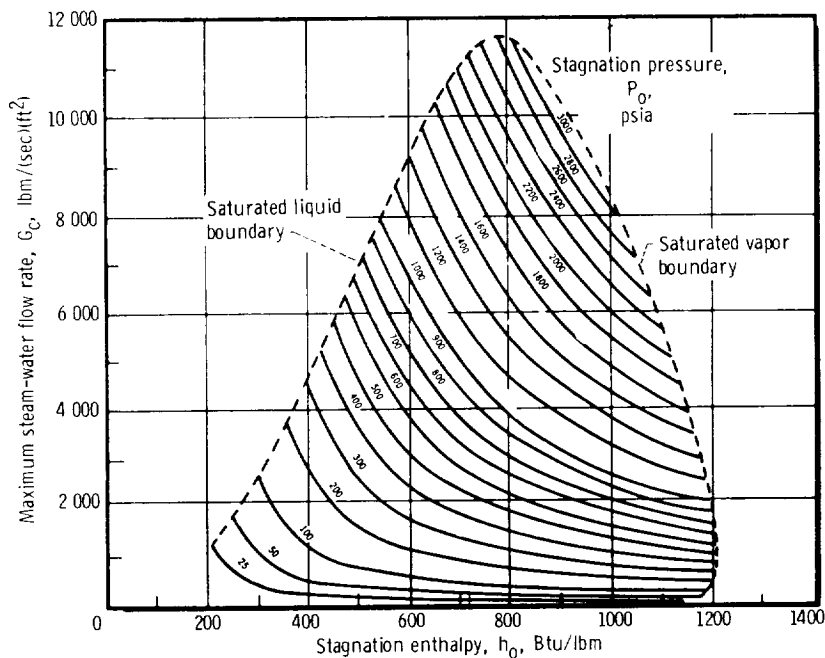


Figure 6. - Maximum steam-water flow rate and local stagnation properties according to Moody's model (ref. 11).

(ref. 11) that when the stagnation condition was used for calculation, Moody's analysis overpredicted the experimental results for steam-water in nozzle flow. In any case, since in all three models the flow was assumed to be in equilibrium, the departure from equilibrium, which has been recognized in many experiments, has not been accounted for. To this end, we introduce a later model by Henry which attempted to account for this nonequilibrium effect empirically.

Henry's model (ref. 2) for low quality, high pressure cases. - To take the nonequilibrium effect into consideration, Henry assumed that the real quality is related to the equilibrium quality  $x_E$  by the following equation in which he inserted an unknown empirical parameter  $N$ :

$$x = Nkx_E \quad (36)$$

Combining this equation with equation (19), neglecting the smaller terms, and using a simplifying assumption, we have

$$G_c^2 = \frac{-1}{Nkx_E \left( \frac{\partial v_g}{\partial P} \right)_{h_0} + v_g N \left( \frac{\partial x_E}{\partial P} \right)_{h_0} + v_g x_e \left( \frac{\partial N}{\partial P} \right)_{h_0}} \quad (37)$$

where  $h_0$  refers to constant stagnation enthalpy.

Henry further argued that from experimental data the  $N$  term varies much slower than the pressure variation with distance near the throat; thus,  $\partial N / \partial P = 0$ . Furthermore, when the perfect gas law is assumed for the isothermal case for simplicity, equation (37) can be reduced to

$$G_c = \frac{-1}{\left\{ N \left[ \frac{x_E v_g}{P} + v_g \left( \frac{\partial x_E}{\partial P} \right)_{h_0} \right] \right\}_e} \quad (38)$$

But since  $(\partial x_E / \partial P)_{h_0}$  is not convenient to use, Henry used  $(\partial x_E / \partial P)_S$  for computation purposes.

Since the expression in the bracket is nothing other than  $1/G_{c,H,E}^2$  (i. e., for the case of homogeneous, equilibrium flow (eq. (19)), we have

$$\frac{G_c}{G_{c,H,E}} = \sqrt{\frac{1}{N}} \quad (39)$$

For low qualities where  $1 - x \approx 1$ , the relation between quality and void (eq. (17)) becomes

$$x = k \frac{\alpha v_L}{1 - \alpha v_g} \quad (40)$$

Henry combined equation (36) with equation (40) and obtained



$$N = \frac{\alpha v_L}{x_E(1 - \alpha)v_g} \quad (41)$$

From equation (41), the empirical value of  $N$  could be obtained from void data. The  $N_{\text{emp}}$  can be correlated to the equilibrium quality by

$$N = 20x_E \quad (42)$$

But because of the assumptions used to neglect some of the terms, Henry's analysis is restricted to low void flows; thus, it should be valid only for flow with low quality or under high pressure.

Later Henry (ref. 14) further elaborated on his analysis so that for a pipe flow

$$\left. \begin{aligned} N &= 20x_E & \text{for } x < 0.05 \\ N &= 1 & \text{for } x \geq 0.05 \end{aligned} \right\} \quad (42)$$

(i. e., in higher void, thermodynamic equilibrium is achieved), and since he observed a strong  $L/D$  effect (to be discussed later), he restricted the previous analysis to the case where  $L/C > 12$ .

For nozzle flow, where the flow pattern and pressure gradients are different from that of the pipe flow, Henry and Fauske (ref. 15) proposed that the real quality be determined by using  $N$  as

$$\left. \begin{aligned} N &= \frac{x_E}{0.14} & \text{for } x_E < 0.14 \\ N &= 1 & \text{for } x_E > 0.14 \end{aligned} \right\} \quad (43)$$

The critical flow rate determined from this nonequilibrium model is compared with those based on the equilibrium and frozen model (where quality does not change with distance) shown in figure 7. In the same reference, Henry and Fauske (ref. 15) also proposed corrections for subcooled conditions.

The application of Henry's analysis for low quality flow (ref. 2) has been tested by Bergles and Kelly (ref. 16) for two-phase flow under the heating condition. They found that for equilibrium quality no less than 0.01 the critical flow rate for the heating case is comparable to that of Henry's for the adiabatic case. However, for very low quality ( $x < 0.01$ ), the adiabatic correlation underpredicts the data for  $0.003 < x < 0.025$  and

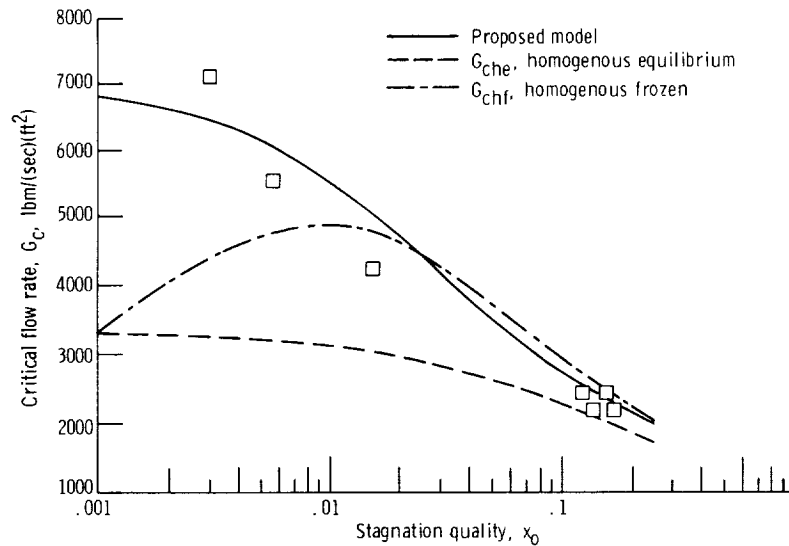


Figure 7. - Comparison of critical flow predictions and experimental data of reference 15. Pressure,  $P_0 = 500$  psia.

overpredicts for  $x < 0.003$ . Apparently, under the heating condition, the wall heat flux, as well as depressurization, cause flow quality to deviate from the equilibrium value.

It should be noted that in Henry's method the critical flow rate can be determined from a stagnation condition; thus, it is convenient to use. However, the empiricism they used to determine the quality coefficient  $N$  restricts their analysis to the range of conditions from which the data were drawn. More general theory to verify this empiricism is yet to be developed.

## Experimental Observations

In addition to comparing experimental results to the theoretical predictions given in the last sections, there are several experimental observations that should be discussed in more detail.

Comparison of experimental results with simplified solutions. - Smith (ref. 17) presented various highly simplified solutions and compared their results with experimental data for hydrogen, nitrogen, and oxygen. The purpose was to discern the limit of applicability of these simplified solutions. Basically, the fluid can be in equilibrium or non-equilibrium (frozen), the flow can be homogeneous or separated, and the process can be isentropic or isenthalpic. The three pertinent equations are the following:

Homogeneous equilibrium model:

$$G_c = \left\{ \frac{\frac{h_{fg}}{v_{fg}}}{\frac{d \left( h_L - h_{fg} \frac{v_L}{v_{fg}} \right)}{dP} + [(1-x)v_L + xv_g] \left[ \frac{d \left( \frac{h_{fg}}{v_{fg}} \right)}{dP} - 1 \right]} \right\}^{1/2} \quad (44a)$$

Homogeneous frozen model:

$$G_c = \left[ \frac{\left( \frac{x}{1-x} \right) C_{Pg} + C_L}{\left( \frac{x}{1-x} \right) C_{vg} + C_L} \cdot \frac{P}{xv_g} \right]^{1/2} \quad (44b)$$

Vapor choking model (i. e., flow controlled by vapor phase only):

$$G_c = \frac{\alpha \rho_g}{x} a_g \quad (44c)$$

where  $\alpha$  is the void fraction, which can be determined from the Martinelli-Nelson correlation.

The comparison of theoretical prediction with experimental result indicates the following area of application:

(1) Low quality, adiabatic flow in a short tube. The data are bracketed by the homogeneous equilibrium and homogeneous frozen solutions.

(2) Adiabatic, high quality flow. The data can be predicted by the separated vapor choking model; that is, the flow acts like an all-vapor flow. A similar dependence of critical flow rate on quality was reported by Brennan, Edmond, and Smith (ref. 18).

Slip ratio as function of quality. - In both Moody's and Fauske's analyses the slip ratio is independent of quality. In Levy's model, the ratio increases with quality. It was shown by Henry (ref. 2) that for a 50-psia steam-water system the experimental slip ratios based on measured void and equilibrium quality  $k_E$  are of the order of 1 to 10, which is not near any one of those analytical models. Furthermore, the theories did not even predict the trend of the variation (fig. 8). Thus, the question of slip ratio is still

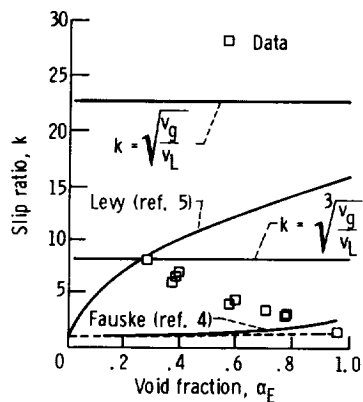


Figure 8. - Comparison between experimental equilibrium velocity ratios and theoretical models for exit pressure  $P_e = 50$  psia (ref. 2).

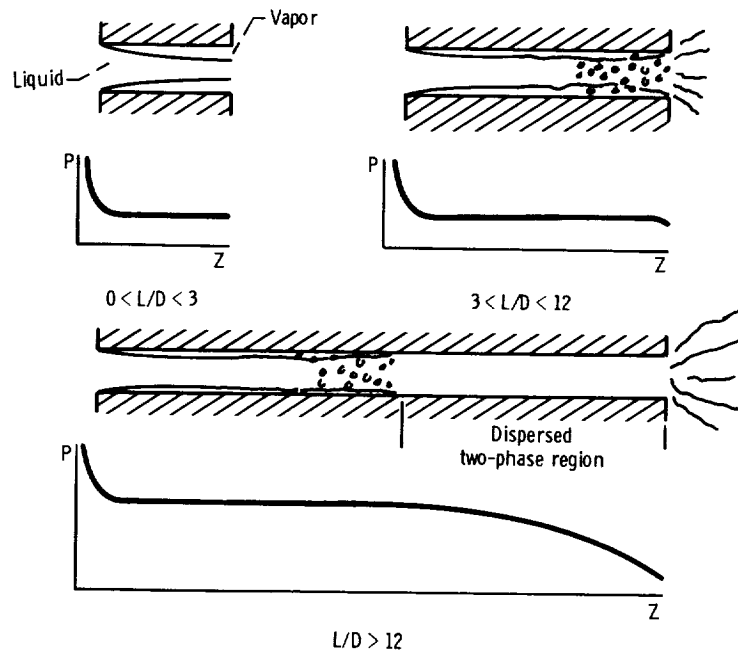


Figure 9. - Flow patterns for critical discharge of saturated or subcooled liquids through short tubes (ref. 14).

unanswered. Further research is needed. However, as discussed previously, the critical flow rate apparently can be predicted without much knowledge of slip ratio.

**Effect of geometry.** - A two-phase flow critical discharge rate is affected by the geometry of the passage, including the length to diameter ratio, the entrance effect, and the shape of the nozzle:

(1)  $L/D$  effect. Depending on the length of the tube or nozzle, the flow could behave differently. According to Henry (ref. 14) the following phenomena have been observed for a sharp-edged entrance with subcooled or saturated one-phase flow at the entrance (fig. 9):

- $0 < L/D < 3$       Liquid flow is in the form of free streamline jet.
- $3 < L/D < 12$     Pressure is essentially constant. The jet begins to breakup. However, the interface mass transfer is low.
- $L/D = 12$         Breakup of jet is completed. The flow ceases to be annular.
- $L/D > 12$         Pressure begins to drop drastically.

A similar phenomenon was observed by Fauske (ref. 19) who showed that as  $L/D$  is increased the flow rate approaches Fauske's model of equilibrium flow with slip. The length effect was also observed by Edmonds and Smith (ref. 20). They showed that the flow rate for a short nozzle is higher than that for a long nozzle.

(2) Entrance effect. The previous description of separated flow as varying with  $L/D$  is restricted to subcooled or saturated one-phase flow entering the tube through a sharp-edged entrance. For a dispersed saturated two-phase flow entering an orifice, the flow will stay dispersed. In this case, Henry and Fauske (ref. 15) proposed the use of the empirical relation

$$\text{Real flow} = 0.84 \text{ (critical flow in ideal nozzle)}$$

as obtained for the case of single-phase compressible flow in an orifice.

(3) Two-dimensional effect. Henry (ref. 2) found that flow passing through a  $7^\circ$  divergent nozzle behaves differently from flow through a  $120^\circ$  divergence angle nozzle. The axial pressure variation near the exit for the  $7^\circ$  nozzle appears to be more like those for single-phase flow, while in the  $120^\circ$  nozzle a back pressure increase causes upstream pressure and exit pressure to rise. Henry contends that for the  $120^\circ$  nozzle, the flow in the divergent section is undergoing a strong radial expansion and there is a pressure gradient in the radial direction. Hence, if one measures the throat pressure in a nozzle with a large divergent angle, the radial pressure gradient will cause error in the pressure reading.

## PROPAGATION OF PRESSURE PULSES AND WAVES

The topics of propagation of a pressure pulse and pressure waves have been important in the study of acoustics. To two-phase flow engineers, the interest has been centered on the effect of the propagation rate on critical discharge and on the onset of flow instability. Many papers have been published on this subject, some dating back to the 1930's. Good reviews can be found in the paper by Gouse and Evans (ref. 21) and by Moody (ref. 22). We will only touch on more recent contributions.

In the earlier years, it appeared that two-phase flow pressure pulse sonic velocity and critical discharge velocity were considered to be the same entity, since this was a valid approach in single-phase flow. However, as shown in the previous sections, the critical flow rates for two-phase mixtures should be treated differently from those for the single phase due to the various interfacial transport processes involved. Similarly, the propagation velocities for single-phase mixtures should not be applicable to two-phase mixtures, since in the ideal single-phase problem, the perturbations were assumed small and equilibrium was assumed to be maintained all the time. Furthermore, in two-phase flow, one should not confuse pressure pulse propagation velocity with the sonic velocity. The basic differences between these two propagation rates lie in the different forms of perturbation and the different time scale as compared with the relaxation time of the fluid to reach equilibrium state. For the propagation of a single pressure pulse, the wave front is usually steep. The fluid subjected to such a moving front does not have time to

equilibrate to the rapid change in state, and thus it usually can be considered to be approximated by the frozen state. Therefore, for the propagation rate of a single pulse, one only worries about the state of change while the pulse is passing and no concern is given to states after the pulse has passed. On the other hand, sound is propagated through continuous waves of small amplitude. The fluid is subject to a periodical change of pressure and has to respond to such periodical changes. The extent of nonequilibrium is dependent on the period of the wave. If the frequency is low or the amplitude is infinitesimal, an equilibrium state can be approached. Conversely, for a wave of high frequency or high amplitude, fluid response will be lagging.

In order to facilitate discussion, we first address ourselves to the subject or propagation of a single pressure pulse and then take up the sonic velocity.

### Pressure Pulse Propagation

Before going into a detailed discussion of each kind of propagation velocity, it is instructive to derive a general form of the propagation equation. If we travel with the wave front and establish a control volume around the wave front, as we did with equation (3), the equations for two-phase mixture (corresponding to eqs. (1) and (2) for single phase) are

$$a [\alpha \rho_g dU_g + (1 - \alpha) \rho_L dU_L] = dP \quad (45)$$

and

$$ad [\alpha \rho_g + (1 - \alpha) \rho_L] = \alpha \rho_g dU_g + (1 - \alpha) \rho_L dU_L \quad (46)$$

Combining equations (45) and (46) results in

$$a^2 = \frac{dP}{d[\alpha \rho_g + (1 - \alpha) \rho_L]} \quad (47)$$

Since  $\alpha$  is function of quality  $x$  and the slip ratio  $k$  (eq. (17)), equation (47) can be expanded to read

$$a^2 = \frac{[(1 - x)\rho_g + x\rho_L]^2}{x\rho_L^2 \left(\frac{\partial \rho_g}{\partial P}\right) + (1 - x)\rho_g^2 \left(\frac{\partial \rho_L}{\partial P}\right) - (\rho_L - \rho_g) \left(\frac{\partial x}{\partial P}\right) + x(1 - x)(\rho_L - \rho_g)\rho_g \rho_L \left(\frac{\partial k}{\partial P}\right)} \quad (48)$$

The terms  $\partial\rho_g/\partial P$ ,  $\partial\rho_L/\partial P$ ,  $\partial x/\partial P$ , and  $\partial k/\partial P$  are determined by interfacial heat transfer, mass transfer, and momentum transfer, respectively, just as discussed in the section on critical flow.

The propagation characteristics of compression and rarefaction pressure pulses were studied experimentally by Grolmes and Fauske (ref. 23) and by Barclay, Ledwidge, and Cornfield (ref. 24). In reference 24, it was found that compression waves travelled faster than the rarefaction wave. The wave shapes were studied in both references 23 and 24. The wave shapes from reference 23 are shown in figure 10. From these wave

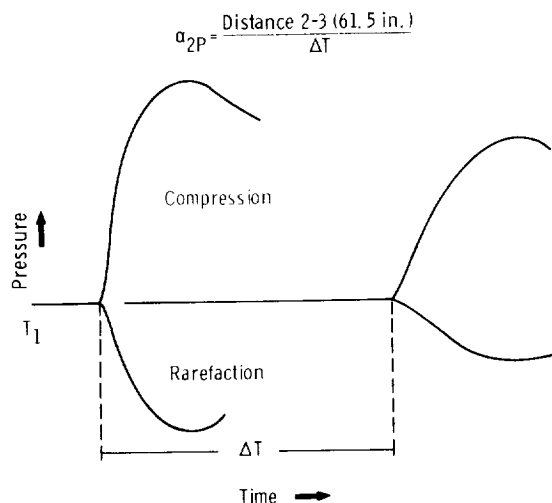


Figure 10. - Superposition of representative oscilloscope pressure traces at locations 2 and 3 for compression and rarefaction pressure pulses in low void fraction steam-water mixtures (ref. 23).

profiles Grolmes and Fauske concluded that the mass transfer, or  $\partial x/\partial P$  term of equation (48), can be neglected; that is, the frozen state can be assumed. But it is important to note that the pressure wave propagation in a flow system is different from the propagation in a stagnant two-phase medium (ref. 24). This difference is due to the fact that the term  $\partial k/\partial P$  is very much a function of flow pattern. Different expressions for  $\partial k/\partial P$  can be derived depending on whether the flow pattern is stratified ( $\partial k/\partial P \neq 0$ ) or dispersed flow ( $\partial k/\partial P = 0$ ). This effect, which was first recognized by Henry, Grolmes, and Fauske (ref. 25), is illustrated in figure 11. Later, Henry (refs. 26 and 27) proposed a series of refined models for propagation velocity for various flow patterns to take into account the virtual mass effect on  $\partial k/\partial P$  as well as heat-transfer effect on  $\partial v_g/\partial P$ . When momentum transfer is considered, Henry included the virtual mass which is the inertial effect of the accelerating gas and its surrounding liquid. The problem for each flow pattern is now discussed.

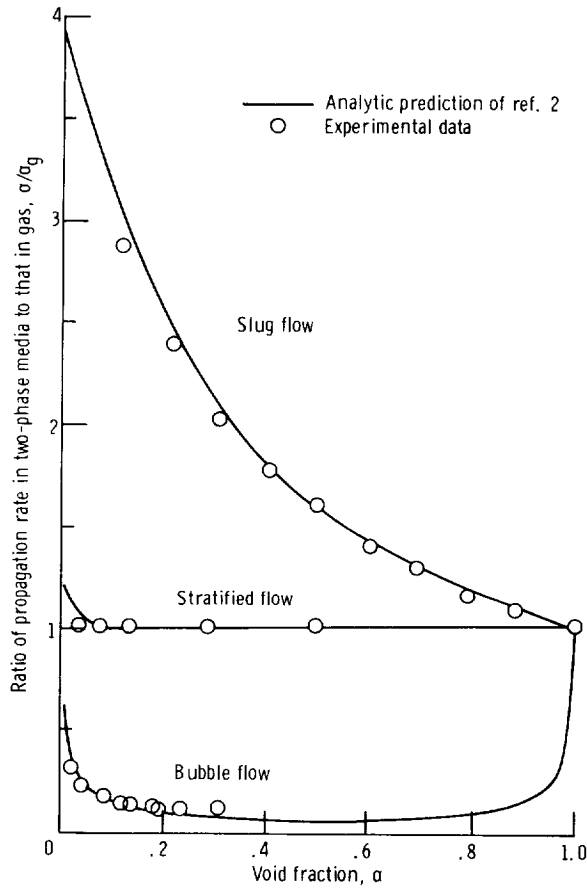


Figure 11. - Measured and calculated pressure wave propagation velocity in air-water mixtures at 25 psia (ref. 25).

**Bubbly flow.** - In his earlier model, Henry assumed that  $\partial k/\partial P = 0$ . However, he later proposed that the virtual mass term of the gaseous volumes should be included:

$$-\frac{\partial P}{\partial z} = \rho_g U_g \frac{\partial U_g}{\partial z} + \frac{\rho_L}{C_M} \left( U_g \frac{\partial U_g}{\partial z} - U_L \frac{\partial U_L}{\partial z} \right) \quad (50)$$

where the last term is the virtual mass term and can be related to the change of slip ratio  $k$ . The constant  $C_M$  varies depending on the geometry of the gas phase. Thus, he assumed that  $C_M$  is dependent on the void fraction. He further assumed that the gas compressibility term is also a function of void. Then using data from the air-water system with  $P = 25$  psia and void fractions up to 0.5, he empirically correlated the correction



factor which includes the effects of interfacial momentum transfer  $C_M$  and interfacial heat transfer  $\partial v_g / \partial P$ :

$$\frac{a}{a_{HT}} = 1.035 + 1.671\alpha \quad (51)$$

where  $a_{HT}$ , the value for homogeneous, isothermal conditions, is

$$a_{HT}^2 = \left\{ \alpha^2 + \alpha(1 - \alpha) \frac{\rho_L}{\rho_g} + \left[ (1 - \alpha)^2 + \alpha(1 - \alpha) \frac{\rho_g}{\rho_L} \right] \frac{nP}{\rho_g a_L^2} \right\}^{-1} \frac{nP}{\rho_g} \quad (52)$$

with

$$n = \frac{(1 - x)C_L + xC_{pg}}{(1 - x)C_L + xC_{vg}}$$

The empirical equation was checked against data for the 10 to 285 psia pressure range and found to be successful.

Annular flow, smooth interface (ref. 25). - Since the interface is relatively small compared to dispersed flow and assumed to be smooth, there is no significant momentum transfer or mass transfer between phases. Each phase is accelerated separately. Under such conditions the change of slip ratio with pressure is

$$\frac{\partial k}{\partial P} = - \frac{1}{a^2} \left( \frac{1}{\rho_g} - \frac{1}{\rho_L} \right) \quad (53)$$

The resulting propagation equation is

$$a^2 = \frac{\left\{ \left[ (1 - x)\rho_g + \rho_L x \right]^2 + x(1 - x)(\rho_L - \rho_g)^2 \right\}}{\left[ \frac{x\rho_L^2}{a_g^2} + \frac{(1 - x)\rho_g^2}{a_L^2} \right]} \quad (54)$$

or

$$\left(\frac{a}{a_g}\right) = \sqrt{1 + \frac{(1-x)(\rho_g)^2}{x(\rho_L)^2}} \quad (55)$$

Annular flow, wave interface (ref. 27). - Under this condition the virtual mass of gas flowing over wavy surfaces is approximated by flow over a surface made of continuous rows of half cylinders:

$$\frac{dP}{dz} = \rho_L U_L \frac{dU_L}{dz} - \rho_g \left( U_g \frac{dU_g}{dz} - U_L \frac{dU_L}{dz} \right) \quad (56)$$

where the last term on the right side is the virtual mass of the liquid filament in the accelerating gas or vapor stream. The resulting propagation is

$$\frac{a}{a_g} = \sqrt{\alpha} \quad (57)$$

Mist flow, two components. - The momentum equation is

$$\frac{dP}{dz} = \rho_L U_L \frac{dU_L}{dz} - \frac{\rho_g}{2} \left( U_g \frac{dU_g}{dz} - U_L \frac{dU_L}{dz} \right) \quad (58)$$

assuming spherical drops. The resulting equation for propagation is

$$\frac{a}{a_g} = \left[ \frac{1 + \frac{2\alpha^2(1-\alpha)\rho_L}{(1+\alpha)\rho_g}}{\alpha^2 + \alpha(1-\alpha)\frac{\rho_L}{\rho_g}} \right]^{1/2} \approx \sqrt{\frac{2\alpha}{1+\alpha}} \quad (59)$$

for  $\alpha > 0.5$ .

Mist flow, one component. - In a one-component system, with drops finely dispersed, the mass transfer between phases over such large interfacial area has to be considered. Henry (ref. 27) argued that for the compression wave the frozen state can be assumed since subcooled liquid and superheated vapor conditions generated by the wave are fairly stable. Thus, the expressions for the two-component system are valid (eq. (40)). However, for a rarefaction wave, the vapor becomes subcooled and the liquid

becomes superheated. When the wave front passes the liquid phase is assumed to adjust from the metastable state at an equilibrium rate. If isentropic processes are assumed, the mass-transfer rate can be shown as

$$\frac{\partial x}{\partial P} = - \frac{(1-x)}{S_{fg}} \frac{\partial S_L}{\partial P} \quad (60)$$

Inclusion of this mass-transfer term results in the propagation velocity

$$a^2 = \frac{1 + \frac{2\alpha^2(1-\alpha)\rho_L}{(1+\alpha)\rho_g}}{\frac{\left[ \alpha^2 + \alpha(1-\alpha) \frac{\rho_L}{\rho_g} \right]}{a_g^2} + \rho_L \frac{\alpha(1-\alpha)}{xS_{fg}} \frac{\partial S_L}{\partial P}} \quad (61)$$

The need for different expressions for compression and rarefaction waves is consistent with the experimental observation of Barclay et al. (ref. 24) that the compression wave travels faster than the rarefaction wave.

Slug flow. - For the slug flow, the time required for a pressure pulse to sweep across the length of a slug is

$$t = \frac{L_L}{a_L} + \frac{L_g}{a_g}$$

and the void fraction is

$$\alpha = \frac{L_g}{L_g + L_L}$$

Thus, the propagation velocity is

$$\frac{a}{a_g} = \frac{a_L}{\alpha a_L + (1-\alpha)a_g} \quad (62)$$

A similar approach has been taken by Bowles and Manion (ref. 28) in dealing with the problem of propagation of acoustic waves. This is discussed in the next section. A sum-

mary of the models proposed by Henry et al. has recently been published in a single report (ref. 29).

## Sonic Wave Propagation

As mentioned previously, sonic waves differ from pressure pulses in two respects:

(1) The pressure fluctuation is small. (Theoretically, sonic waves ought to be limited to very small fluctuations, but in reality the amplitude has to be large enough to avoid being completely attenuated within short distances.)

(2) The waves are continuous. The propagation, attenuation, and dispersion of acoustic waves in a two-phase system has been an important subject of research in the field of acoustics. A good survey can be found in the paper by Gouse and Evans (ref. 21) and Gouse and Brown (ref. 30). However, most of the research was performed in a two-phase medium with no flow conditions. As Gouse and Evans pointed out, very little research was done in two-phase flow systems. There is one of the few exceptions. Although the flow condition may be expected to bring in new variables, such as slip ratio, it is reasonable to assume that the basic phenomena observed in the nonflow condition also occur in the flow condition. We will thus proceed to examine the various important parameters according to two basic two-phase configurations - namely, droplet suspension and bubbly mixture.

Droplet suspensions (gas-liquid system). - Since the inertia of a liquid suspended in the gas phase is higher than the inertia of the gas, the time for the displacement of liquid under the pressure waves should be considered.

A model was proposed by Temkin (ref. 31) to account for the response of suspension with pressure and temperature changes. In this analysis, the suspensions are considered to move with the pressure waves according to the Stokes law

$$F_p = 6\pi\mu R(U_p - U) \quad (63)$$

The heat is transferred by conduction between the gas and the suspension. The equation for the heat transfer is that of conduction from an infinite body to an immersed sphere:

$$Q_p = 4\pi R\lambda(T_p - T) \quad (64)$$

The use of equations (63) and (64) is the approximation of oscillatory state equation by steady-state equation with the oscillatory terms neglected. This approximation is valid

if the relaxation time to the wave period ratio is small or

$$0 < \tau_D \omega \approx 1, \quad \tau_t \omega = 0 \quad (65)$$

where

$$\tau_D = \frac{2R^2 \rho_L}{9 \mu_g} \text{ (drag relaxation time)} \quad (66)$$

$$\tau_t = \frac{3}{2} \left( \frac{c_{PL}}{c_{Pg}} \right) \text{Pr}_g \tau_D \text{ (thermal relaxation time)} \quad (67)$$

In Temkin's analysis, mass transfer is assumed to be absent. Continuity equations, energy equations, and momentum equations are then written for both the gas phase and the dispersed phase with the interfacial transport expressed as equations (63) and (64). The responses of temperature, velocity, and densities to a small perturbation of pressure are then determined.

Under this system, the perturbation changes with time and distance according to

$$e^{i(K_1 z - \omega t)} e^{-K_2 t}$$

Thus, a positive  $K_1$  means that the amplitude of the perturbation attenuates with the distance  $z$ . Temkin found that an attenuation coefficient

$$A = \frac{2c_o K_2}{x_s \omega}$$

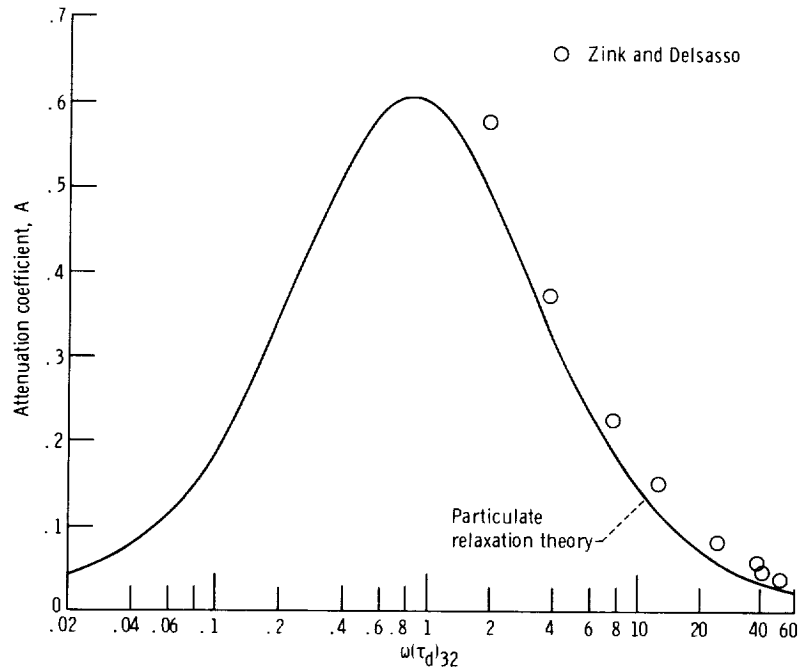
is function of the relaxation times  $\tau_D$  and  $\tau_t$  and the mass fraction of suspension  $x_s$ :

$$A = \frac{2c_o K_2}{x_s \omega} = \frac{\omega \tau_D}{1 + \omega^2 \tau_D^2} + (\gamma - 1) \left( \frac{c_{PL}}{c_{Pg}} \right) \left( \frac{\omega \tau_t}{1 + \omega^2 \tau_t^2} \right) \quad (68)$$

Another term of interest is the dispersion coefficient

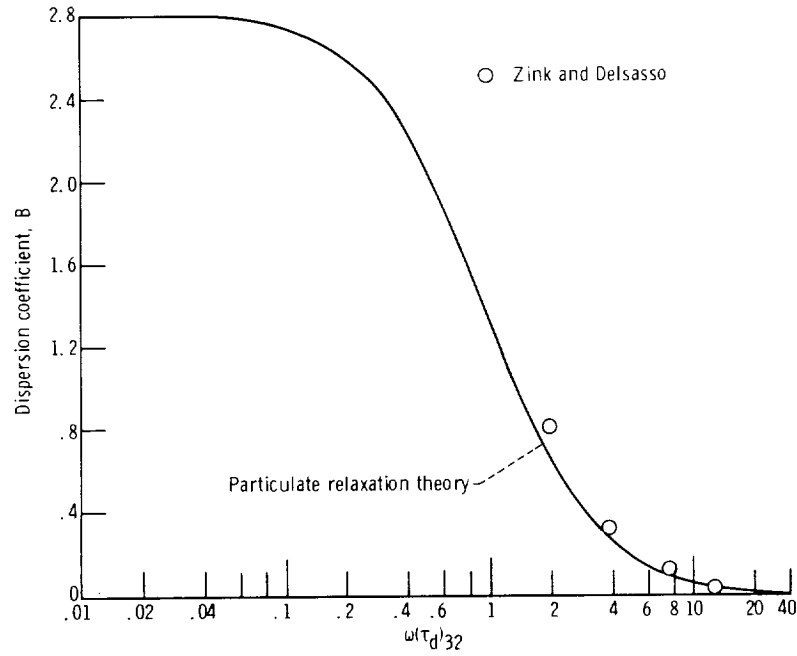
$$B = \frac{\left(\frac{c_0}{c} - 1\right)}{x_s} = \frac{1}{1 + \omega^2 \tau_D^2} + \frac{(\gamma - 1) \left(\frac{c_{PL}}{c_{Pg}}\right) \omega \tau_t}{1 + \omega^2 \tau_t^2} \quad (69)$$

This term expresses the difference of sonic velocity  $c$  in a particle suspension from that in the air  $c_0$  as a function of the mass fraction of suspension  $x_s$ , the relaxation times  $\tau$ 's, and the frequency. These equations show that the acoustic velocity in a suspension is a strong function of frequency and drop size. Experimental data for A and B for a suspension of alumina particles in air were shown to compare well with the prediction (fig. 12). Later, Temkin and Dobbins (ref. 32) applied this theory to a nitrogen-oleic acid system and discussed the effects of the wall influence, gravitational settling and particle size distribution. The last two effects were found to be small.



(a) Attenuation by alumina particles in air.

Figure 12. - Attenuation and dispersion of sound waves in air with suspended particles (ref. 31). Ratio of specific heat at constant pressure and velocity for air,  $\gamma = C_p/C_v = 1.4$ ; specific heat ratio of solid to gas,  $c'_p/c_p = 1.0$ ; Prandtl number,  $Pr = 0.71$ .



(b) Dispersion by alumina particles in air.

Figure 12. - Concluded.

**Bubbly mixture (gas-liquid two-component system).** - Most of the earlier work on acoustic velocity was based on the assumptions of (1) homogeneous distribution of small bubbles, (2) the velocity being primarily a function of void fraction, and (3) the acoustic velocities being the single-phase values of the liquid and vapor phases  $C_L$  and  $C_g$ . One example is that of Hsieh and Plesset (ref. 33). In their analysis, the two-phase homogeneous mixture is assumed to be represented as a uniform medium with physical properties synthesized from the constituent phases and weighted according to void fraction and quality. This analysis is limited to the small bubble sizes such that the bubble radius is much smaller than the wavelength and the frequency is well below the bubble resonance. Using such a model they were able to show that the gas compression is essentially isothermal and the acoustic velocity can be approximated as

$$\frac{1}{C^2} = \frac{1}{\frac{\rho_g(1-x)}{\rho_L} \left[ 1 + \frac{x\rho_L}{(1-x)\rho_g} \right]^2} \left( \frac{x}{1-x} \frac{1-\alpha}{\alpha} \frac{1}{C_L^2} + \frac{\alpha}{1-\alpha} \frac{1}{C_g^2} \right) \quad (70)$$

where

$$C_g^2 = \frac{\partial P}{\partial \rho_g} = \frac{P}{\rho_g}$$

since the process is isothermal. In this equation the acoustic velocity is independent of wave frequency. A similar analysis was performed by McWilliam and Duggins (ref. 34) to show the effects of pressure and bubble size on sonic velocity in bubbly mixture. They showed that sonic velocity decrease with increasing bubble size and decreasing pressure. In reference 35 Van Wijngaarden derived equations to show there is a dispersion of the acoustic wave; that is

$$\omega = \frac{K}{(1 + K^2)^{1/2}} \quad (71)$$

where  $\omega$  and  $K$  are frequency and wave number, respectively. In reference 36 Van Wijngaarden showed further that the wave propagation in two-phase bubble mixture can be treated as an analogy to water waves. He showed that for high gas content ( $R_o n_b^{1/3} \approx 1$ ) the dispersion is not small, where  $R_o n_b^{1/3}$  is the ratio between bubble radius and bubble distance. Thus, the acoustic velocity is not only a function of void but also of frequency and bubble size. Karplus (ref. 37) made experimental measurements of sound velocity in a gas-liquid mixture for the void range of 1 to 67 percent, with and without detergent. He found that the velocity is strongly dependent on frequency, void fraction, and pressure, as shown in figures 13(a) and (b). It was also found that there is a  $C_{min}$  with respect to

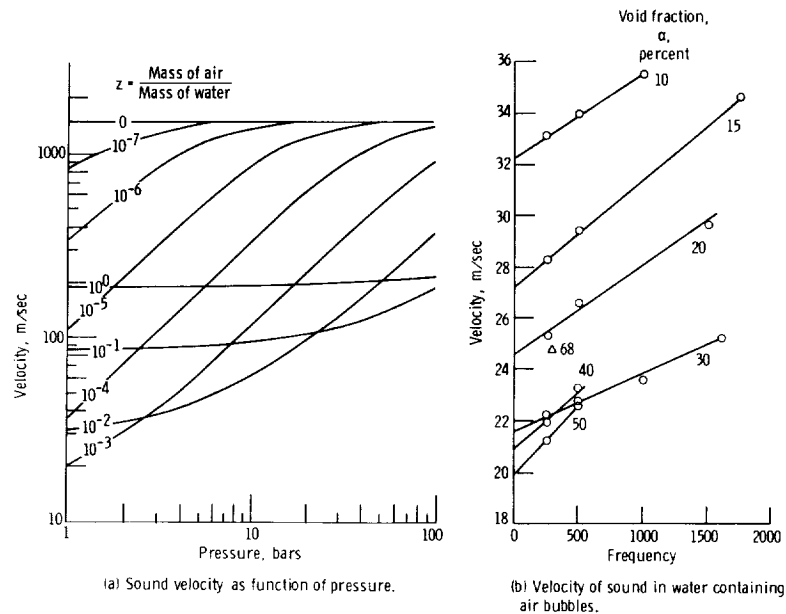


Figure 13. - Sound velocity as affected by pressure and frequency (ref. 37).



the void fraction when  $\alpha = 0.5$ . The effect of the detergent appears to be inconclusive.

The attenuation of sonic waves in bubble mixtures of carbon dioxide and water was experimentally shown by Bowles and Manion (ref. 28). They found that in a freshly poured club soda or fresh seltzer water the sonic waves are greatly reduced as compared with an old solution which has already lost some of the bubbles.

Vapor-liquid system. - In the last two sections we discussed the studies of acoustic velocity in gas-liquid mixtures. In the boiling system where liquid and vapor coexist, the change of pressure causes condensation or evaporation. Thus, the mass transfer is the additional complexity to be considered. There are far fewer studies on vapor-liquid acoustic velocity than their counterparts in the gas-liquid system. Karplus (ref. 38) made computations of sound velocity in mixtures of water and steam, assuming thermodynamic equilibrium. Acoustic velocity is determined from  $\partial P/\partial \rho$  along constant entropy lines. The calculation showed that the velocity is lower than in single-phase water or steam, and there is a discontinuity in velocity at both the saturated liquid and saturated vapor lines. A small discontinuity in velocity exists when the dry steam is transitioned to wet steam with just a trace of liquid. At the other end of the range, the discontinuity is very drastic, and the velocity in a low void mixture is two orders of magnitude lower than that of liquid with any void. This is especially true near the triple point. It may be noted that the experimental values of  $C$  are higher than the analytical prediction based on the equilibrium assumption. Clinch and Karplus (ref. 39) analyzed the propagation of pressure waves in a hydrogen liquid-vapor system (of droplet suspension). In this study, they took into account the effects of viscosity and thermal diffusivity on sonic velocity. They concluded that for the mass transfer the relaxation time ratios  $\omega R_d^2/\nu_t$  and  $\omega R_d^2/\nu$  should be considered. For very low frequency, equilibrium can be assumed and the existing equation for gas-liquid can be used (i. e.,  $C = f(\text{void})$ ). For very high frequency, where there is no time for mass and heat transfer, the acoustic velocity approaches that of the dominating phase. But for intermediate frequency, the acoustic velocity is dependent on quality, the relaxation time ratio, and the spacing between them. An example is shown in figure 14.

The importance of relaxation time on mass transfer has also been shown by Hsu and Watts (ref. 40). In their study of the compressibility of stationary vapor bubbles under pressure pulsation, they found both analytically and experimentally that the controlling factor for compressibility is  $\omega R^2/\nu_t$ . The bubble radius change in response to a pressure pulse is

$$\frac{\frac{\Delta R}{R}}{\frac{\Delta P}{P}} = \left\{ \frac{2\sigma}{RP} - \frac{3P_g}{P_b} \left(1 + \frac{2\sigma}{RP}\right) + \frac{\rho_L \nu_t}{PR^2} \Omega^2 - \left(\frac{h_{fg}}{\nu_{fg} P_b}\right) \left[ \frac{iF_2 \Omega \left(1 + \frac{2\sigma}{PR}\right)}{1 + iF_1 \Omega + \sqrt{i\Omega}} \right] \right\}^{-1} \quad (72)$$

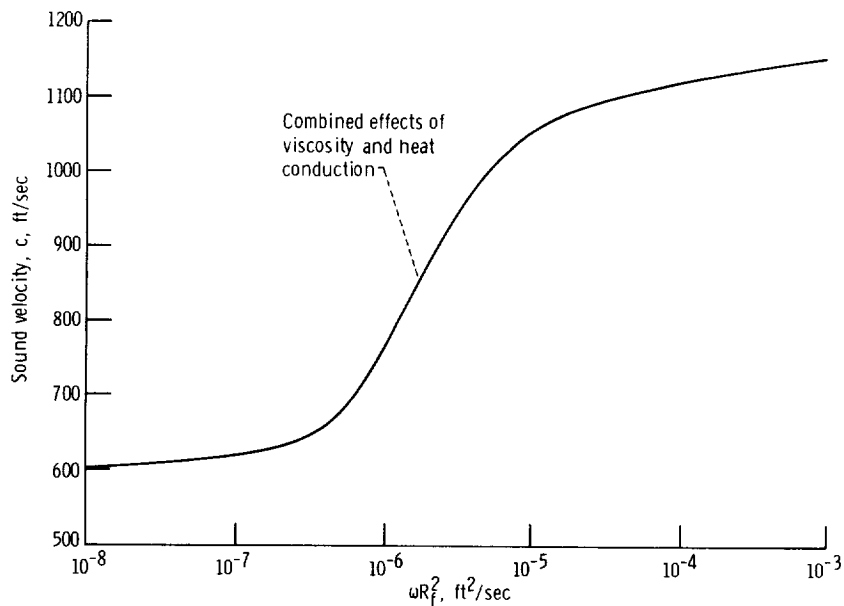


Figure 14. - Sound velocity in hydrogen mixture against  $\omega R_f^2$  at atmospheric pressure (ref. 39). Critical quality,  $x_c = 0.375$ .

In this equation  $F_1$ ,  $F_2$  are groupings of properties,  $\Omega = \omega R^2 / \nu_t$ , and  $P_g$  is the partial pressure of the gas. In this study, it was found that a trace of noncondensable gas in the vapor phase can significantly reduce the compressibility. Thus, for a vapor-liquid system, insufficient time for reaching equilibrium (i. e., high frequency) and a trace of gas can both cause an increase in the acoustic velocity. A subsequent unpublished work by Watts and Hsu shows that for a vapor bubble slipping with respect to the liquid, the compressibility is even higher. Thus for a two-phase mixture in flowing condition with slip, the sonic velocity should be even lower than that in stationary two-phase medium.

## RELATION BETWEEN CRITICAL DISCHARGE RATE, PRESSURE PROPAGATION RATE, AND SONIC VELOCITY

### General Comments

In the previous sections we have discussed the various approaches used to treat the three propagation or discharge rates ( $G_c$ ,  $C$ , and  $a$ ) in two-phase mixtures. In this section, we attempt to discuss the relation between these three terms.

In the INTRODUCTION we mentioned that in single phase the three terms  $G_c$ ,  $C$ , and  $a$  are closely related to the isentropic equilibrium compressibility of the fluid. In two-phase flow or two-phase mixture, these three terms are still shown to be determined

by the compressibility of the mixture, with the important difference from the single-phase case being that the fluid is no longer under equilibrium and homogeneous conditions. Much of the effort reviewed in this report was directed to determining how much two-phase fluids deviated from the equilibrium and homogeneous states. The nonhomogeneity is determined from knowledge of the flow pattern and the extent of nonequilibrium is determined by knowing the momentum, energy, and mass transport processes. In general, with a larger interfacial area (which is a function of flow pattern) and a longer time available in comparison with relaxation time, an equilibrium state is more closely approached. But for each propagation rate or velocity, the process has to be examined separately.

## Relation Between Critical Discharge and Pressure Propagation

The critical discharge rate and the pressure propagation rate should be related if one considers the existence of a stationary wave front in a discharging flow. At the wave front, the pressure propagation rate moving upstream is balanced by the outflow of the discharging fluid. However, it was shown by Fauske (ref. 4) that the critical discharge rate could not be equated to the single-phase sonic velocity of either the liquid or vapor. Isbin et al. (ref. 41) suggested that the pressure was propagated through a liquid film. Moody (ref. 22) postulated that during critical discharge of a separated flow (i. e., annular flow) the pressure pulse travels at supersonic speed in vapor and subsonic speed in liquid. One source of confusion is the misuse of all the concepts for single-phase on the two-phase flow system. In the real situation, there is a great deal of interfacial transport which affects slip ratio, quality, etc., and thus the two phases should not be considered separately. These interfacial transports tend to adjust the system, at least partially, to accommodate the change in pressures. The time available for this accommodation should be associated with the characteristic time  $L/U$ . Thus a longer channel should be closer to equilibrium situation than a short channel. This is demonstrated in figure 15. Furthermore, it should be remembered that during the discharging process, the flow acceleration makes an adjustment to equilibrium even more difficult. Considering all these factors, it is difficult to assign a right set of local properties and parameters such as local quality, local slip ratio, etc. at the wave front to make the critical discharge flow rate equal to the local pressure propagation rate but of opposite direction. Furthermore, for a two-phase flow, the choke condition may not always be met at the throat. For example, some unpublished data obtained from Gutierrez of Lewis Research Center has shown that through a transparent divergent nozzle the wave front for a two-phase flow is a little downstream of the throat.

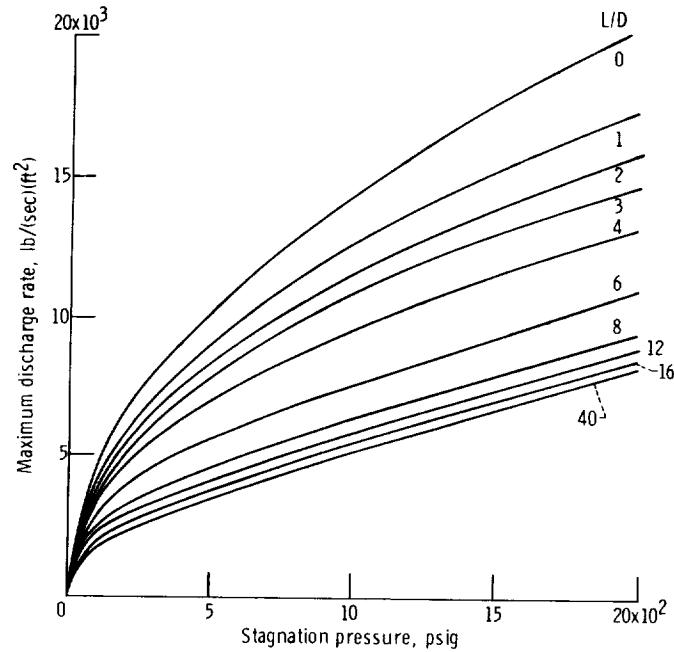


Figure 15. - Maximum discharge rates of saturated water for 0.25-inch inside diameter tube (ref. 19).

## Relation Between Sonic Velocity and Pressure Pulse Propagation

As to the relation between the sonic velocity and the propagation velocity of the single pressure pulse, it was mentioned before that the imposed pressure disturbances are different - one being a continuous wave and another being a single impulse. The difference is analogous to the steady periodic heating of a block by a cyclical change in surface temperature as compared with momentarily changing the surface temperature to a new level. The responses to these two kinds of disturbance are certainly different, both in depth of penetration and in lagging time. For two-phase flow, it involves the time required for the interfacial transport to respond to the changing boundary condition. It is easier to define the relaxation time for a sonic wave propagation, such as shown in equations (66) and (67). When the single pressure pulse is imposed, the shape of the pulse (i. e., the steepness and magnitude of the pulse) should really be included in considering the extent of nonequilibrium involved.

## CONCLUSIONS

The discussions in previous sections can be summarized as follows:

1. In the single-phase system, critical discharge rate and the pressure pulse propagation rate (and sonic velocity) were of the same expression even though they were derived from different approaches. This is due to the common basic assumptions of the equilibrium, isentropic process. In two-phase systems this basic assumption of the isentropic process under equilibrium condition is no longer valid. Consequently, the three propagation or discharge rates are no longer expressed in similar ways. Any temptation to treat them as interchangeable items should be strongly cautioned against.

2. The compressibility associated with various propagation or discharge processes is strongly affected by the nonequilibrium and nonhomogeneous state usually existing in two-phase flow. The extent of nonhomogeneity is determined by the flow pattern while nonequilibrium is determined by the interfacial transport processes and the time available to relax toward equilibrium.

3. In the existing analyses, the effect of the flow pattern has been recognized and considered. The time factor is still not fully accounted for. For sonic velocity, the characteristic time is the period of the wave; for pressure propagation it is the duration of the wave front, and for critical discharge it is the time it takes the fluid to travel through the discharge channel. A study of the effect of these time factors on the extent of nonequilibrium could be fruitful.

Lewis Research Center,  
National Aeronautics and Space Administration,  
Cleveland, Ohio, March 28, 1972,  
113-31.

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