# Review of efficiency measurement methodologies to inform hospital resource allocation decisions in NSW: a rapid review

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An **Evidence Check** review brokered by the Sax Institute

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This rapid review was brokered by the Sax Institute.

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## Contents

## EXECUTIVE SUMMARY

The Centre for Efficiency and Productivity Analysis conducted this review to identify efficiency modelling methodologies and data considerations relevant to Australia and of use to NSW Health and the Independent Hospital Pricing Authority in driving decisions about hospital resource allocation. Key findings include:

- Measures of efficiency relevant to health funding and price benchmarking decisions include cost efficiency, input-oriented technical efficiency and cost-allocative efficiency. Estimating these measures of efficiency involves identifying the input-output combinations that are technically feasible, ie the production technology.
- There are two main efficiency modelling techniques to estimate the production technology: stochastic frontier analysis and data envelopment analysis. The standard stochastic frontier analysis model allows for statistical noise and can be used to compute measures of reliability (eg standard errors) for efficiency estimates. Data envelopment analysis can be used to identify efficient firms (peers), which inefficient firms should study in their efforts to become more efficient. A weakness of the standard data envelopment analysis model is that it does not allow for statistical noise.
- There were more than 100 applications of stochastic frontier analysis and data envelopment analysis modelling techniques to hospital data. The most comprehensive and relevant applications were conducted by the Productivity Commission in 2009 and 2010. The Commission found that, on average, Australian hospitals can potentially reduce inputs by 10% and still produce the same quantities and types of outputs.
- The Commission identified a number of data problems that will limit the use of efficiency modelling techniques (and any other technique) in informing hospital funding and price benchmarking decisions. These include: a lack of consistent data on capital costs (especially for public hospitals); the medical costs of doctors exercising their rights of practice in public hospitals; staffed beds in public and private hospitals; and measures of quality (including rates of hospital-acquired infections).
- Few studies have used efficiency measurement methods to inform health funding decisions. One study used data envelopment analysis to estimate efficient budgets for New Zealand hospitals in a way that accounts for variables such as ethnicity, rurality, clinical complexity and out-of-catchment tertiary care.
- Data envelopment analysis and stochastic frontier analysis can be used to estimate the minimum cost of providing a bundle of hospital services in ways that account for variations in output quantities and types, input prices, technical change, input and output quality, the production environment, and inefficiency.
- Stochastic frontier analysis can also be used to estimate the marginal cost of providing individual hospital services. If the efficient price of a bundle of hospital services is the minimum cost of providing those services, then the efficient price of a particular hospital service is its marginal cost.

## Search strategy

The Authors used a two-stage search strategy:

1. **Preliminary search:**
   Relevant studies were identified in an electronic search of the main economic research database (ECONLIT), web of science (WOS) and PubMed. Keywords included 'efficiency', 'productivity', 'hospital', 'health care', 'health centre', 'data envelopment analysis', 'stochastic frontier', 'production frontier' and 'cost frontier'. Each relevant paper identified was examined for references to other studies that might have been missed (if they were not in any of these three electronic databases). Additional papers were then obtained from the respective journals or via standard web search engines (eg Google). This search resulted in more than 250 studies. The majority were published journal articles, book chapters or technical reports. Some studies were working papers. More than 100 papers examining excessively narrow or broad types of health care services (eg physicians, health districts) were removed from the list.

2. **Systematic review**:
   Individual studies were critically assessed to determine the validity and applicability of efficiency modelling techniques. Each study was carefully reviewed to determine: the research hypotheses; country/region/jurisdiction; numbers of firms and periods; analytical methods used; model specifications; analytical results; validity and robustness of the techniques used; and findings and policy implications. Papers that referred to health funding and price benchmarking were further assessed to determine if they were relevant to the Australian context.

## What efficiency modelling techniques can be used to inform health funding and price benchmarking decisions?

To obtain certain measures of firm performance we must be aware of the production possibilities available to firms, ie the production technology. We also need to know how decisions are made concerning the amounts of inputs firms use and the outputs they produce. In this section we first describe: alternative representations of the production technology; common assumptions concerning the optimising behaviour of firms; and production, distance and cost functions. Second, some of these functions are used to define various economic measures of efficiency that are relevant to health funding and price benchmarking decisions. These include measures of input-oriented technical efficiency (a measure of distance to a production frontier) and cost efficiency (a measure of the reduction in costs that may result from input substitution). Third, we outline the two main modelling techniques available for estimating levels of efficiency: stochastic frontier analysis (SFA) and data envelopment analysis (DEA). SFA and DEA can be viewed as different methods for estimating the production frontier. They are known as efficiency modelling techniques because if the production frontier can be estimated then it is possible to estimate various measures of efficiency. Last, we present examples of the application of these techniques to health data.

### *Production technology*

The term production technology refers to the set of input-output combinations that are technically feasible. The set of combinations that are technically feasible in period *t* is known as the period-*t* production possibilities set:

(1) $$T^t = \{(x_{it}, q_{it}) : x_{it} \text{ can produce } q_{it}\}$$

where $x_{it} = (x_{1it}, \ldots x_{Kit})'$ is a $K \times 1$ vector of inputs (eg surgeons, nurses, capital, equipment) and $q_{it} = (q_{1it}, \ldots q_{Jit})'$ is a $J \times 1$ vector of outputs (eg outpatients, inpatients) for firm *i* in period *t*. It is common for economists to assume that $T^t$ is nonempty, closed, convex and bounded. These properties are known as regularity properties and ensure, for example, that any linear combination of two feasible production points is also feasible. A production possibilities set for a single-input single-output firm is shown in Figure 1.

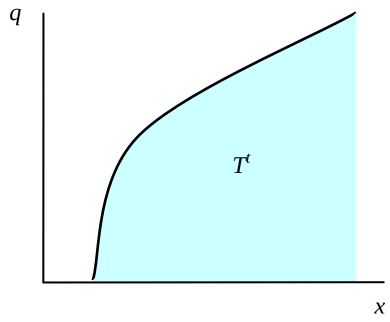


**Figure 1: Single-input single-output production possibilities set**

Production technologies can also be represented using many other types of sets. For example, an input set is the set of all input combinations that are capable of producing a given output vector. Mathematically:

(2) $$L(q_{it}) = \{x_{it} : (x_{it}, q_{it}) \in T^t\}.$$

An input set for a two-input multiple-output firm is presented in Figure 2. The input set inherits its regularity properties from the production possibilities set.

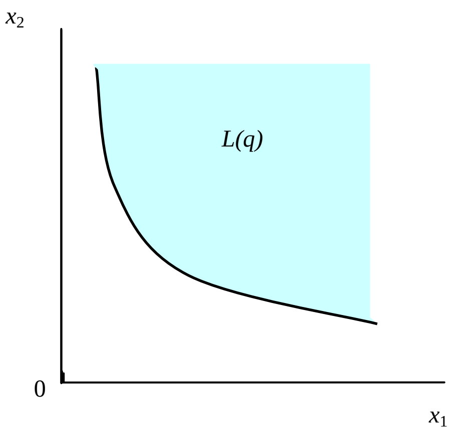


**Figure 2: Input set for a two-input multiple-output firm**

Set representations of technologies are not convenient for econometric analysis. Alternative and more convenient representations of technologies include production functions, distance functions, profit functions and cost functions. For example, the production function defines the boundary of the production possibilities set of a single output firm. Mathematically:

(3) $$q_{it} = f(x_{it})$$

where $q_{it}$ is a scalar. A simple example of a production function is the boundary of the production possibilities set shown in Figure 1. The production function inherits its regularity properties (including monotonicity and concavity) from the regularity properties of the production possibilities set.

A particularly useful representation of a multiple-input multiple-output technology is the input distance function. This function gives the maximum factor by which a firm can radially contract its input vector and still produce the same output vector. Mathematically:

(4) $$D_I^t(x_{it}, q_{it}) = \max_{\rho}\{\rho > 0 : (x_{it}/\rho, q_{it}) \in T^t\}.$$

For example, if half of the inputs are capable of producing the same outputs, the input distance is $\rho$ = 2. Sometimes we are interested in the minimum input level that can produce the output vector while holding the input mix fixed. By definition, this input level is $x_{it} / D_I^t(x_{it}, q_{it})$. The distance function inherits a number of regularity properties from those of the production possibilities set.

The production function and input distance functions are defined over input and output quantities, not prices. Such functions are sometimes called primal representations of the production technology. Other functional representations of the technology are defined over prices and are known as dual representations. Dual representations include cost and profit functions. For example, the cost function gives the minimum cost of producing a given output vector at a given set of input prices. Mathematically:

(5) $$c(w_{it}, q_{it}) = \min_x \left\{ w_{it}'x : (x, q_{it}) \in T^t \right\}$$

where $w_{it} = (w_{1it}, ... w_{Kit})'$ denotes the vector of input prices paid by firm *i* in period *t*. Again, the cost function inherits a number of regularity properties (including monotonicity and concavity in prices) from the regularity properties of the production possibilities set.

## Measures of efficiency

There are many measures of efficiency defined in the economics literature. Measures of efficiency that are most relevant to health funding and price benchmarking decisions include:

- **Input-oriented technical efficiency (ITE):** a measure of the degree to which a firm can radially contract its inputs while holding outputs fixed. Mathematically, ITE is the inverse of the input distance function defined in the previous section: $ITE_{it} = D_I^t(x_{it}, q_{it})^{-1}$. ITE can also be viewed as a measure of the reduction in total cost that is possible while holding outputs and the input mix fixed.
- **Cost efficiency (CE):** a measure of the reduction in total cost that is possible while holding outputs fixed (ie no restrictions on input mix or levels). CE can be expressed in terms of the cost function introduced in the previous section: $CE_{it} = c(w_{it}, q_{it}) / w_{it}'x_{it} =$ (minimum cost)/(observed cost).
- **Cost-allocative efficiency (CAE)**: a measure of the reduction in total cost that is possible for a technically efficient firm while holding outputs fixed (ie no restrictions on input mix or levels). Mathematically: $CAE_{it} = CE_{it} / ITE_{it}$. It follows that CE is the product of technical efficiency (movements towards the frontier) and CAE (movements around the frontier to find a less expensive input mix): $CE_{it} = CAE_{it} ITE_{it}$.

The next two sections describe the computation and interpretation of these efficiency measures. Other measures of efficiency are available and can be viewed as measures of the improvements in productivity, cost, revenue or profit that are possible while restricting (or not restricting) input and output choices in different ways. More detail on these other efficiency measures is provided elsewhere.[1,2]

## Stochastic frontier analysis

SFA involves the use of econometric methods to estimate either primal or dual representations of the production technology. The choice of functional representation is often determined by data availability. For example: if we only have data on input and output quantities then we can only estimate production frontiers and distance functions; if we only have data on input prices and output quantities then we can only estimate cost frontiers. SFA also involves assumptions about the regularity properties of the frontier (eg monotonicity, concavity), the functional form of the frontier (eg linear, translog) and the distributions of error terms representing inefficiency and statistical noise (eg means, variances). The unknown parameters of these functions and error distributions are usually estimated using the method of maximum likelihood (ML). This section outlines the main features of the SFA approach.

**Technical efficiency**
Technical efficiency refers to the ability of the firm to transform inputs into outputs. Consider a firm that uses $K$ inputs to produce a single output. If we have data on input and output quantities then the production technology might be represented using a production frontier having a Cobb-Douglas functional form:

(6) $$\ln q_{it} = \beta_0 + \sum_{k=1}^{K} \beta_k \ln x_{kit} + v_{it} - u_{it}$$

where $v_{it}$ is a symmetric random error term representing approximation errors and other sources of statistical noise, and $u_{it}$ is a non-negative random variable representing technical inefficiency. The choice of functional form is partly driven by the need to minimise the distance between the assumed function and the true unknown function, ie the need to minimise errors of approximation. SFA involves estimating the unknown parameters $\beta_k$ ($k$ = 0, ..., $K$) along with the following output-oriented measure of technical efficiency:

(7) $$OTE_{it} = \exp(-u_{it}).$$

Several methods can be used to estimate the unknown parameters in (6), and the best method depends on the assumptions made about the distributions of the random error terms $v_{it}$ and $u_{it}$. It is common to assume the errors are independently and identically distributed random variables that are uncorrelated with the explanatory variables. It is also common to assume:

| | | |
|---|---|---|
| (8) | $E(v_{it}) = 0$ | (zero mean) |
| (9) | $E(v_{it}^2) = \sigma_v^2$ | (homoskedastic) |
| (10) | $E(v_{it} v_{js}) = 0$ | (uncorrelated) |
| (11) | $E(u_{it}^2) = \text{constant}$ | (homoskedastic) |
| (12) | $E(u_{it} u_{js}) = 0$ | (uncorrelated) |

Under these assumptions it is possible to obtain consistent estimates of the slope parameters $\beta_k$ ($k$ = 1, ..., $K$) using ordinary least squares (OLS). However, the OLS estimator of the intercept parameter $\beta_0$ is biased downwards. One solution is to use a corrected ordinary least squares estimator (COLS) proposed by Winston.[3] A more common solution is to make some stronger distributional assumptions concerning the error terms and estimate the parameters using the method of ML. ML estimators are generally preferred to COLS estimators because they have many desirable statistical properties.

Aigner *et al.*[4] developed ML methods for estimating the SFA model where the symmetric and one-sided errors are assumed to be normal and half-normal, respectively:

| | | |
|---|---|---|
| (13) | $v_{it} \sim N(0, \sigma_v^2)$ | (normal) |
| (14) | $u_{it} \sim N^+(0, \sigma_u^2)$ | (half-normal) |

ML estimation involves using the probability density functions (pdfs) of these random variables to derive a joint pdf for the data, expressed as a function of the unknown parameters. This joint pdf is known as a likelihood function. ML estimation involves choosing the values of the unknown parameters to maximise the likelihood function. ML estimation of stochastic frontier models is straightforward using built-in commands in well-known econometrics software packages such as LIMDEP and Stata and R.

Many other SFA models and methods are used in empirical studies. A variant of the model given by equations (6) to (14) is a constant returns to scale (CRS) model obtained by imposing the restriction:

(15) $$\sum_{k=1}^{K} \beta_k = 1.$$

Under this restriction, output and input-oriented measures of efficiency are identical:

(16) $$ITE_{it} = OTE_{it} = \exp(-u_{it}) = D_I^t(x_{it}, q_{it})^{-1}. \qquad \text{(under CRS)}$$

Consider a group of four hypothetical hospitals producing one output using two inputs, ie $K = 2$. The output is the number of treated cases per month, and the two inputs are labour (measured by the number of medical staff) and capital (measured by the number of beds). The hypothetical output and inputs for the four hospitals in a particular month are given in Table 1.

**Table 1: Hospital inputs and outputs**

| Hospital $i$ | Month $t$ | Treated cases $q_{it}$ | Staff $x_{1it}$ | Beds $x_{2it}$ | Staff per treated case $x_{1it}/q_{it}$ | Beds per treated case $x_{2it}/q_{it}$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 200 | 200 | 600 | 1 | 3 |
| 2 | 1 | 300 | 600 | 1200 | 2 | 4 |
| 3 | 1 | 200 | 600 | 300 | 3 | 1.5 |
| 4 | 1 | 100 | 500 | 200 | 5 | 2 |

ML estimates of the parameters of the CRS model given by equations (6) to (15) are presented in Table 2. The estimated slope parameters ($\beta_1$ and $\beta_2$) are positive as required by economic theory (see the regularity conditions discussed in the section *Production technology*) and statistically significant at usual levels of significance.

**Table 2: Parameter estimates**

| Parameter | Variable | Estimate | Asymptotic st. error |
|---|---|---|---|
| $\beta_0$ | Constant | -0.832 | 1.667 |
| $\beta_1$ | Staff | 0.362 | 0.125 |
| $\beta_2$ | Beds | 0.638 | 0.125 |

Estimated technical efficiency scores are presented in Table 3 and indicate that all hospitals are approximately 95% efficient. This means it is technically possible for them to produce the same outputs using approximately 5% fewer inputs.

**Table 3: Technical efficiency estimates**

| Hospital $i$ | Month $t$ | Technical efficiency $ITE_{it}$ | 95% CI lower bound | 95% CI upper bound |
|---|---|---|---|---|
| 1 | 1 | 0.960 | 0.886 | 0.998 |
| 2 | 1 | 0.947 | 0.865 | 0.997 |
| 3 | 1 | 0.961 | 0.888 | 0.999 |
| 4 | 1 | 0.951 | 0.871 | 0.998 |

Note: CI = Confidence Interval

Figure 3 plots the input per unit of output data presented in Table 1, to provide additional insight. The curved line in Figure 3 is the ML estimate of the production frontier. This is an estimate of an isoquant of the type shown in Figure 2. Note that the distance from each data point to the frontier includes both noise and inefficiency components, eg the distance from point 2 to point F is noise, the distance from point F to the frontier is inefficiency, and points 1 and 3 are below the estimated frontier due to noise. All hospitals are estimated to be about 95% efficient (Table 3). One of the strengths of SFA is that it is straightforward to obtain measures of reliability for technical efficiency estimates. For example, Table 3 reports 95% confidence interval limits for the technical efficiency scores and shows that we can be 95% confident that hospital 2 is between 87% and 100% technically efficient.

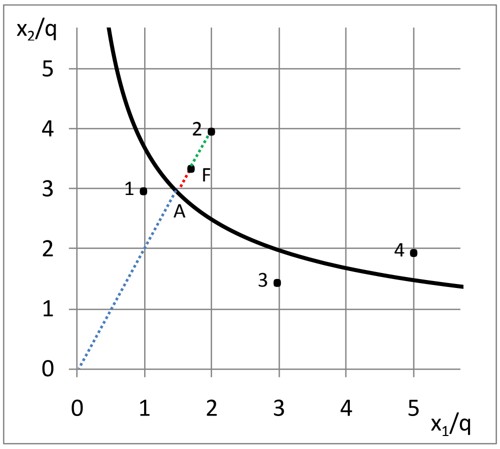


**Figure 3: Estimated frontier using stochastic frontier analysis**

**Cost efficiency**

If we have information on input prices, SFA can also be used to measure other types of efficiency, including CE. Recall that CE is a measure of the firm's ability to produce its outputs at minimum cost.

Consider a firm that uses $K$ inputs to produce a single output. If we have data on costs, input prices and output quantities, then the production technology of the firm might be represented using a cost frontier with a Cobb-Douglas functional form:

$$(17) \qquad \ln c_{it} = \alpha_0 + \sum_{k=1}^{K} \alpha_k \ln w_{kit} + \gamma \ln q_{it} + v_{it} + u_{it}$$

where $c_{it}$ denotes the total cost of production of firm $i$ in period $t$, and $v_{it}$ and $u_{it}$ are still symmetric and non-negative random variables representing noise and inefficiency.

SFA involves estimating the unknown parameters $\alpha_k$ ($k$ = 0, ..., $K$) and $\gamma$ along with the following measure of CE:

$$(18) \qquad CE_{it} = \exp(-u_{it})$$

Again, appropriate assumptions regarding the error terms allow the model to be estimated by ML.

Suppose the input prices in the hospital problem are $w_1 = \$50$ and $w_2 = \$100$. It is possible to estimate the parameters of equation (17) by the method of ML, but this is unnecessary because we have already estimated the parameters of the technology by way of equation (6). Table 4 reports the estimates for CE, technical efficiency and CAE calculated using the parameter estimates in Table 2. The efficiency estimates show that hospital 3 is the most cost efficient hospital, but could nevertheless reduce its costs by 7.5% through efficiency improvements: 3.9% by moving closer to the frontier; and a further 3.8% by moving around the frontier to find a better input mix.

**Table 4: Stochastic frontier analysis efficiency estimates**

| Hospital | Month | Cost efficiency | Technical efficiency | Cost-allocative efficiency |
|---|---|---|---|---|
| *i* | *t* | $CE_{it}$ | $ITE_{it}$ | $CAE_{it}$ |
| 1 | 1 | 0.826 | 0.960 | 0.861 |
| 2 | 1 | 0.882 | 0.947 | 0.931 |
| 3 | 1 | 0.925 | 0.961 | 0.962 |
| 4 | 1 | 0.882 | 0.951 | 0.928 |

Figure 4 shows the hospital cost minimisation problem. The line *WW'* in Figure 4 is an isocost line with slope $-w_1/w_2$ that maps out the set of all input combinations that have the same cost. The technically efficient point that minimises the cost of producing one unit of output is the point of tangency between the isocost line *WW'* and the isoquant. Thus, in Figure 4 the optimal, ie least-cost technically-efficient, point of production is point H, which has co-ordinates $(x_{1it}/q_{it}, x_{2it}/q_{it}) =$ (2.507, 2.187). The cost of production at this point is $344.

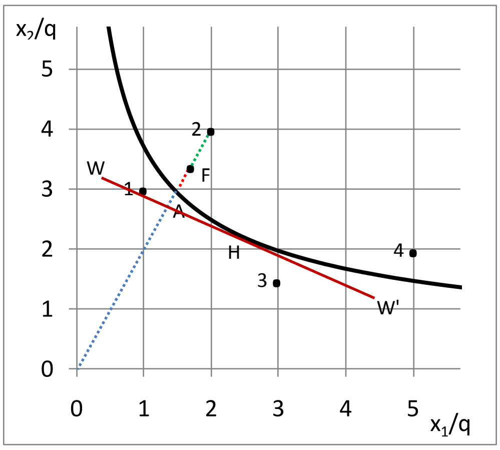


**Figure 4: Cost minimisation using stochastic frontier analysis**

## Data envelopment analysis

DEA uses linear programming methods to estimate the production technology. Primal and dual representations of the technology can be estimated using this approach. DEA requires assumptions concerning the regularity properties of the production frontier, eg if the production possibilities set is not convex then the DEA model is known as a Free Disposal Hull model. The functional form assumption underpinning DEA is that the production or cost frontier is locally linear. DEA is often described as a non-parametric approach because it does not involve any error terms, so does not involve any assumptions about the parameters (means, variances) of the distributions of those error terms. This section describes the main features of the DEA approach.

**Technical Efficiency**

Consider a firm that uses *K* inputs to produce a single output. Local linearity means that for any input vectors in the neighbourhood of $x_{it} = (x_{1it}, x_{2it}, ..., x_{Kit})'$ the production frontier takes the linear form:

(19) $$\mu q_{it} = \alpha + \upsilon' x_{it}$$

where $\mu$ and $\upsilon$ are non-negative and $\alpha$ is unsigned to allow for variable returns to scale.

Equation (19) is the DEA analogue of equation (6). An alternative representation of this local linear technology is the input distance function:

(20) $$D_I^t(x_{it}, q_{it}) = \frac{\upsilon' x_{it}}{\mu q_{it} - \alpha} \geq 1.$$

As we have seen, the input distance is the inverse of the measure of ITE: $ITE_{it} = D_I^t(x_{it}, q_{it})^{-1}$. DEA involves selecting values of the unknown parameters in (20) to minimise the value of the input distance function (or maximise its inverse).

Aside from the non-negativity constraints on $\mu$ and $\upsilon$, the only constraints on the parameters are that they must satisfy $D_I^t(x_{it}, q_{it}) \geq 1$ for all observations in the data set. This optimisation problem has an infinite number of solutions, and it is common to identify a unique solution by setting $\upsilon' x_{it} = 1$. The resulting problem is the input-oriented DEA linear program (LP):

(21) $$\begin{aligned} D_I^t(x_{it}, q_{it})^{-1} = \max_{\alpha,\mu,\upsilon} \quad & \mu q_{it} - \alpha \\ \text{s.t.} \quad & \mu q_{nr} - \upsilon' x_{nr} - \alpha \leq 0 \quad \text{for } n = 1,\ldots,N \text{ and } r = 1,\ldots,t \\ & \upsilon' x_{it} = 1 \\ & \mu, \upsilon \geq 0. \end{aligned}$$

This form of the DEA problem is sometimes called the primal DEA problem. The terms primal and dual are being used here in a different sense to the way they were used in the section *Production technology*. Every normal primal LP has a dual form with the following property: if the primal and the dual LPs both have feasible solutions then the optimised values of the two objective functions are equal. The dual form of the normal maximisation LP (21) is:

(22) $$\begin{aligned} D_I^t(x_{it}, q_{it})^{-1} = \min_{\rho,\lambda} \quad & \rho \\ \text{s.t.} \quad & \sum_{n=1}^{N}\sum_{r=1}^{t} \lambda_{nr} q_{nr} \geq q_{it} \\ & \rho x_{it} - \sum_{n=1}^{N}\sum_{r=1}^{t} \lambda_{nr} x_{nr} \geq 0 \\ & \sum_{n=1}^{N}\sum_{r=1}^{t} \lambda_{nr} = 1 \\ & \rho, \lambda_{nr} \geq 0 \text{ for } n = 1,\ldots,N \text{ and } r = 1,\ldots,t. \end{aligned}$$

Consider the hypothetical hospital data presented in Table 1, still assuming the technology exhibits CRS (set $\alpha = 0$). Solving the DEA LP (22) for each of the four hospitals yields the efficiency scores in the last column of Table 5.
In Table 5, the $\lambda$ values are weights that are computed as part of the DEA solution algorithm. One of the strengths of DEA is that these weights can be used to identify efficiency targets and peers.

**Table 5: Hospital efficiency scores**

| Hospital | Month | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $ITE_{it}$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 0 | 1 |
| 2 | 1 | 0.818 | 0 | 0.182 | 0 | 0.682 |
| 3 | 1 | 0 | 0 | 1 | 0 | 1 |
| 4 | 1 | 0 | 0 | 1 | 0 | 0.75 |

It is useful to graph the DEA problem to explain what these terms mean (Figure 5). Figure 5 is the DEA analogue of Figure 3. Here, the hospitals closest to the origin and the two axes (hospitals 1 and 3) tend to use the smallest amounts of inputs per unit of output, so they are the most efficient. A line has been drawn from hospital 1 to hospital 3 to represent the 'efficient frontier'. Observe that this frontier has been extended above hospital 1 and to the right of hospital 3 and parallel to the respective axes. This kinked frontier envelops all the data points (hence the term 'data envelopment analysis') and is a nonparametric estimate of the isoquant shown in Figure 2. Hospitals 1 and 3 are fully efficient (they lie on the frontier) and hospitals 2 and 4 are inefficient (they lie to the north-east of the frontier).

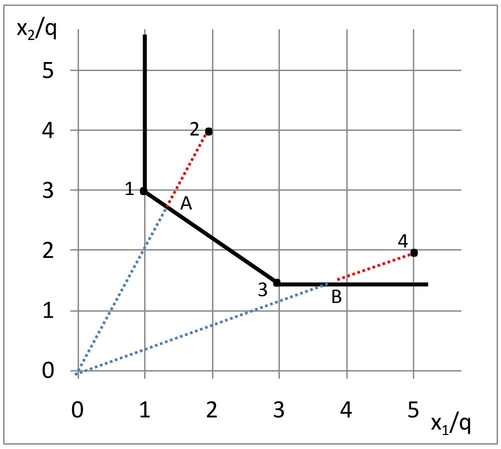


**Figure 5: Estimated frontier using data envelopment analysis**

It appears that hospital 2 could reduce its use of both inputs by about 30% before it would reach the efficient frontier at point A. Its technical efficiency score can be calculated as:

(23) $$TE_{21} = \frac{\text{distance from the origin to the frontier}}{\text{distance from the origin to hospital 2}} = \frac{\|0A\|}{\|02\|} = 0.682$$

which is the value reported in Table 2. Likewise, the efficiency score of hospital 4 can be calculated as:

(24) $$TE_{41} = \frac{\text{distance from the origin to the frontier}}{\text{distance from the origin to hospital 4}} = \frac{\|0B\|}{\|04\|} = 0.75$$

An efficiency score of 0.682 for hospital 2 means it is technically possible for it to produce the same level of output using no more than 68.2% of inputs used. A reduction in input use of this order implies hospital 2 will use (0.682)(2) = 1.364 staff per treated case and (0.682)(4) = 2.728 beds per treated case. This represents a radial contraction towards the origin, from point 2 to point A (Figure 5). Thus, point A, which has co-ordinates (1.364, 2.728), can be regarded as a target for hospital 2.

An efficiency score of 0.75 for hospital 4 means it is technically possible for it to produce the same level of output using no more than 75% of inputs used. This represents a radial contraction towards the origin from point 4 to point B. Point B, which has co-ordinates (3.75, 1.5), can be regarded as a target for hospital 4. However, Figure 1 shows that point B lies on the segment of the efficient frontier which lies parallel to the horizontal axis. Thus, even at point B, hospital 4 is still not fully efficient because it is technically possible to produce the same output with even fewer inputs. Specifically, at point B it is technically possible to treat the same number of cases using the same number of beds, but with a reduced number of staff. This extra reduction in inputs is known in the DEA literature as an input 'slack', representing a move from point B to point 3 in Figure 5.

Calculating input and output targets may be complicated by the presence of input slacks. Fortunately, input and output targets can be calculated without having to explicitly calculate these slacks. Specifically, we can calculate the input and output targets for firm *i* by taking a weighted average of the output and input vectors of all firms, using the solution values from the DEA problem for firm *i* as weights. For example, the targets for hospital 2 are computed by multiplying the outputs and inputs of hospitals 1 to 4 by $\lambda_1 = 0.818$, $\lambda_2 = 0$, $\lambda_3 = 0.182$ and $\lambda_4 = 0$:

Output target = (0.818 × 200) + (0 × 300) + (0.182 × 200) + (0 × 100) = 200 treated cases
Input 1 target = (0.818 × 200) + (0 × 600) + (0.182 × 600) + (0 × 500) = 272.8 staff
Input 2 target = (0.818 × 600) + (0 × 1200) + (0.182 × 300) + (0 × 200) = 545.4 beds

or if we wish to express these targets on an input per unit of output basis:

Input 1 target = 272.8/200 = 1.364 staff per treated case
Input 2 target = 545.4/200 = 2.728 beds per treated case

Targets for all four hospitals can be calculated this way. Not only do the $\lambda$ values allow us to calculate output and input targets for firm *i*, they can be used to identify other firms that should be studied by firm *i* in its efforts to improve its efficiency levels (these are known as peers). Recall when we calculated the input and output targets for hospital 2, the inputs and outputs of hospital 1 were given a weight of $\lambda_1 = 0.818$ while the inputs and outputs of hospital 3 were given a weight of $\lambda_3 = 0.182$. The inputs and outputs of the remaining hospitals were given zero weight. Because hospitals 1 and 3 were given non-zero weights, they are the peers for hospital 2. Because the inputs and outputs of hospital 1 were given more weight than the inputs and outputs of hospital 3, hospital 1 is regarded as a more important peer than hospital 3. Refer to Figure 5, where point A is closer to point 1 than point 3 and divides the interval between the two points in the ratio 0.182:0.818. The peers for all hospitals can be identified this way, and are reported in Table 6.

**Table 6: Hospital peers and peer weights**

| Hospital | Peers | Peer Weights |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 1, 3 | 0.818, 0.182 |
| 3 | 3 | 1 |
| 4 | 3 | 1 |

**Cost Efficiency**

If we have information on input prices we can also use DEA to estimate CE. Let $x_1^*,\ldots,x_K^*$ denote the input levels that minimise the cost of producing the output levels that are produced by firm *i* in period *t*. We can find these $x_k^*$ values (and the minimum cost) by solving:

$$\text{(25)} \qquad c(w_{it}, q_{it}) = \min_{\lambda, x_k^*} \sum_{k=1}^{K} w_{kit} x_k^*$$

$$\text{s.t.} \quad \sum_{n=1}^{N}\sum_{r=1}^{t} \lambda_{nr} q_{nr} \geq q_{it}$$

$$x^* - \sum_{n=1}^{N}\sum_{r=1}^{t} \lambda_{nr} x_{nr} \geq 0$$

$$\sum_{n=1}^{N}\sum_{r=1}^{t} \lambda_{nr} = 1$$

$$\lambda_{nr} \geq 0 \text{ for } n = 1,\ldots,N \text{ and } r = 1,\ldots,t.$$

The constraints in this problem are identical to the constraints in (22) except that $\rho x_{it}$ has been replaced with $x^*$. The LP (25) relaxes the constraint on input mix and chooses a set of input quantities that minimises the cost of placing the firm on the frontier. Having solved this LP, the CE of the firm is calculated as:

$$(26) \qquad CE_{it} = \frac{\text{minimum cost of producing the outputs of the firm}}{\text{observed cost of producing the outputs of the firm}} = \frac{\sum_{k=1}^{K} w_{kit} x_k^*}{\sum_{k=1}^{K} w_{kit} x_{kit}}$$

Reconsider the hospital problem, where input prices are $w_1 = \$50$ and $w_2 = \$100$. The cost minimisation LP for hospital 2 has solution $(x_1^*, x_2^*, c)$ = (3, 1.5, 300). Thus, the minimum cost of producing one unit of output is \$300. The CE of hospital 2 is:

$$(27) \qquad CE_{21} = \frac{300}{50(2)+100(4)} = \frac{300}{500} = 0.6$$

This means hospital 2 can reduce its costs by 40% and still produce the same number of treated cases. Refer to the frontier shown in Figure 5, and reproduced in Figure 6 for further insight. The line *WW'* is the same isocost line as shown in Figure 4 (the SFA analogue). The technically efficient point that minimises the cost of producing one unit of output is the point of tangency between the isocost line *WW'* and the frontier. Thus, in Figure 6 the optimal point of production is point 3, where hospital 3 is operating. This is the point identified in the solution to the DEA cost minimisation problem.

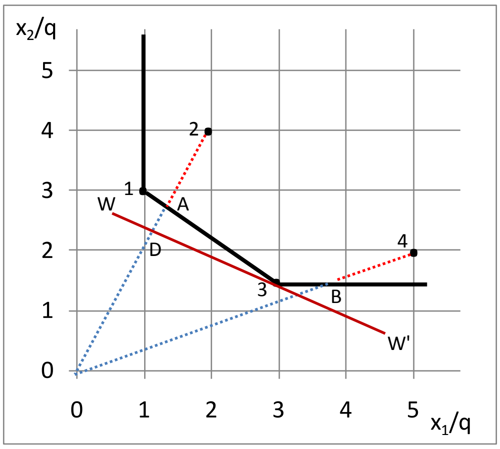


**Figure 6: Cost minimisation using data envelopment analysis**

Note that the cost of inputs used at point 3 is the same as the cost of inputs used at point D. This cost is less than the cost of inputs used at point 2 (where hospital 2 is operating). This suggests we could measure the CE of hospital 2 as:

$$(28) \qquad CE_{21} = \frac{\text{distance from the origin to point } D}{\text{distance from the origin to hospital 2}} = \frac{\|0D\|}{\|02\|} = 0.6$$

which is the CE score calculated using equation (27). The CE scores for all hospitals can be calculated the same way and are reported in Table 7.

Table 7: Data envelopment analysis efficiency estimates

| Hospital<br>*i* | Month<br>*t* | Cost Efficiency<br>$CE_{it}$ | Technical Efficiency<br>$ITE_{it}$ | Cost-Allocative Efficiency<br>$CAE_{it}$ |
|---|---|---|---|---|
| 1 | 1 | 0.857 | 1 | 0.857 |
| 2 | 1 | 0.600 | 0.682 | 0.880 |
| 3 | 1 | 1 | 1 | 1 |
| 4 | 1 | 0.667 | 0.750 | 0.889 |

## *Other methodologies*

Deterministic frontier analysis (DFA) and index number methods are other methodologies that can be used to inform health funding and price benchmarking decisions.

DFA models are special cases of SFA models that do not allow for statistical noise ($v_{it}=0$). The parameters of DFA models are estimated in different ways depending on the assumptions made about the error term $u_{it}$. A linear programming method pioneered by Aigner and Chu is commonly used.[5] Grosskopf *et al.* provide an example in a health funding context.[6]

Index number methodology is not generally regarded as an efficiency modelling technique and has not been used for this purpose. However, if the sample size is large and all firms produce the same output and face the same input prices, the minimum observed cost in the sample can be regarded as an estimate of minimum cost. The index number that compares the cost of a particular firm with the cost of the cost-minimising firm is a measure of CE. Unfortunately, the conditions under which this measure is valid are unlikely to be met in practice.

## *Applications to health data*

Examples of the application of SFA and DEA modelling techniques to health data are in Appendix 1 of this report [and Appendix E of *Public and Private Hospitals, Productivity Commission Research* Report[7]]. Detail on how these techniques can be used to inform health funding and price benchmarking decisions is discussed in the last section of this report. The most comprehensive applications are those by the Productivity Commission in 2009 and 2010.[7,8] The centrepiece of the Commission's work is a translog distance function estimated using SFA. The Commission found that the average level of technical efficiency was 90%, using data on nine outputs (four admitted patient separations + five non-admitted services) and five inputs (two categories of staff + drugs + supplies + other inputs) from more than 500 public and private acute hospitals during several years. This indicates that, on average, Australian hospitals can potentially reduce inputs by 10% and still produce the same outputs.[8]

## What efficiency modelling techniques have been used to inform health funding and price benchmarking decisions in other jurisdictions (national and overseas)?

Australian hospitals are funded using one of three funding schemes[7]:

- "Public hospitals are funded with capped budgets, at least when treating public patients" (Productivity Commission 2009;196).
- "In all states and territories except for the ACT funding for acute inpatient services is distributed at least partly on the basis of a casemix scheme." (Productivity Commission, 2009;22) Under this scheme hospitals are paid a fixed price per unit of output [diagnosis-related group (DRG)]. The Australian Refined Diagnosis Related Groups (AR-DRG) classification system is used to define the casemix (current version is AR-DRG 6.0) in all states except WA.
- "Across most jurisdictions grant (or per day) funding is used for certain types of acute care …" (Productivity Commission, 2009;22). Under this scheme, payments are made for each patient according to length of stay. Private insurers have traditionally funded private hospitals this way.

Several studies have used SFA and DEA or free disposal hull (FDH) to inform health funding and price benchmarking decisions by conducting ex post assessments of alternative funding mechanisms. A smaller number of studies have exploited efficiency modelling techniques to set health care funding levels or set prices for hospital or health care outputs.

### *Using data envelopment analysis and stochastic frontier analysis for ex post assessment*

Appendix 2 provides examples of efficiency modelling techniques used in ex post assessment of health funding arrangements. These studies generally do not control for all variables that are normally expected to affect levels of hospital efficiency and cost.

The findings of these studies cannot be used to reliably inform health funding and price benchmarking decisions in Australia. The findings include:

- United States hospitals funded under casemix had lower costs than those funded using other schemes.[9]
- Norwegian hospitals funded under casemix were more efficient than those funded by global budgets.[10]
- Introducing casemix in Taiwan led to improvements in productivity and quality.[11]
- Swedish hospitals funded with global budget caps were more efficient than those funded on a per output basis.[12]

### *Using data envelopment analysis and stochastic frontier analysis to set funding levels*

There are few papers in this category. The two main papers are:

- **Rouse and Swales (2006)[13] (New Zealand):** Separate DEA models are used to estimate levels of CE in five service categories: medical/surgery; pregnancy/childbirth; community health; disability support; and mental health. Each DEA model defines two or more outputs in the relevant service category and one input (expenditure). The medical/surgery and pregnancy/childbirth service categories are assumed to exhibit CRS; all other service categories are assumed to exhibit variable returns to scale. An aggregate historical base funding level (the efficient budget) for each service category is obtained by calculating $C_{it}^{*} = CE_{it} \times w_{it}' x_{it}$ for each hospital and then averaging. A base price per equivalent unit is then obtained as $C^{*}/Q$ where $C^{*}$ is average base funding and $Q$ is weighted volumes of output. Adjustments are then made for variables such as ethnicity, rurality, clinical complexity and out-of-catchment tertiary care. Data is from the most recently available year (usually two years preceding the target funding year).

- **Kuntz *et al.* (2007)[14] (Germany):** A two-step method is used to determine reallocation of hospital beds, ie not funding levels. Step 1 involves calculating efficiency scores using a modified DEA model (weight restrictions are used for hospitals with low capacity utilisation). Step 2 involves calculating optimal bed reallocations using a linear optimisation model (beds are reallocated from hospitals that were identified as inefficient in step 1 to hospitals that were identified as efficient). Although this method does not inform funding decisions directly, it determines hospital capacity which feeds into the hospital budgeting process.

Another study on assessing economies of scale for activity-based hospital funding provides some evidence of economies of scale as a source of higher than average cost of service provision.[15] However, it does not use efficiency methodology. This study uses an average cost function to estimate a scale coefficient which is then fed into the funding formula as a loading factor. Refer to the section *Efficient pricing using data envelopment analysis* for detail on the relationship between average cost functions and cost frontiers.

## What are the strengths and weaknesses of different modelling techniques of relevance to Australia?

Different modelling techniques are known to have different strengths and weaknesses in any empirical context, not just health. In this section and in Appendix 3 we summarise these strengths and weaknesses. Both modelling techniques may be unreliable if sample sizes are small. We also describe other data limitations that may affect the reliability of these techniques.

### *Strengths and weaknesses of stochastic frontier analysis*

Main strengths:

- Allows for measurement errors, omitted exogenous variables and other sources of statistical noise
- Can be used to construct confidence intervals and conduct conventional tests of hypotheses easily (see Table 3 for an example).

Main weaknesses:

- May be unreliable when sample sizes are small
- Need to make assumptions concerning functional form, eg linear, quadratic, translog
- Need to make assumptions concerning the distributions of the error terms, eg half-normal.

### *Strengths and weaknesses of data envelopment analysis*

Main strengths:

- Only requires weak assumptions concerning functional form (local linearity)
- Needs no assumptions concerning error terms
- Fast DEA packages are available for computing targets, peers and various measures of efficiency.

Main weaknesses:

- Does not allow for noise, so cannot distinguish inefficiency from noise
- Practically impossible to compute economic quantities that involve partial derivatives of the production frontier, eg elasticities of output response with respect to inputs, marginal costs
- Computationally difficult to obtain measures of reliability for efficiency scores
- Results may be sensitive to outliers
- Efficiency estimates are upwardly biased in small samples.

### *Data limitations*

If efficiency modelling techniques (or any other techniques) are to be useful in an Australian hospital funding context, detailed and consistent reporting is required of: variable input prices (eg wage rates); fixed input quantities (eg capital assets); output quantities (eg separations); and costs. The Productivity Commission identifies a number of data issues that will limit the ability of Australian policy makers to make informed decisions about hospital funding[7]:

- Inconsistent methods for collecting and reporting data, eg differences in the way states and territories assign clinical urgency categories in emergency departments, differences in the way patient-costed sites and cost-modelled sites measure costs for the National Hospital Cost Data Collection, differences in the way staffed beds are reported for public and private hospitals.

- Missing data on values, eg asset value data required to calculate a user cost of capital, head office overheads, the cost of medicines prescribed to hospital patients, the medical costs of doctors exercising their rights of practice in public hospitals.
- Missing data on health outcomes, eg data on mortality rates and life expectancy requires tracking of patient health after discharge.
- Missing data on measures of quality, eg no nationally consistent data on hospital-acquired infections.
- Measurement errors, eg surgery waiting list times tend to underestimate the actual wait for surgery, C* = C x CE pricing provides incentives to overstate C.
- Limited data access, eg because of legal requirements to maintain confidentiality and privacy.

The Productivity Commission observes that hospital outputs (eg number of patients treated, number of procedures performed) tend to be easier to measure than outcomes (eg changes in mortality rates and life expectancy), because the latter requires tracking of patient health after hospital discharge.[7] Until these data issues are resolved, SFA should be regarded as a more reliable modelling technique than DEA.

## What modelling techniques can be used by NSW and the Independent Hospital Pricing Authority to inform setting national efficient public hospital prices and loadings that account for hospital type, size and location, patient complexity and indigenous status?

SFA and DEA models are commonly used to estimate primal and dual representations of multiple-input multiple-output production technologies under the assumption of variable returns to scale. Thus, all of these models automatically account for hospital size. In this section we first describe ways in which SFA and DEA models can also be used to account for variations in input and output quality (eg patient complexity and indigenous status) and characteristics of the hospital operating environment (eg hospital location and type). Last, we describe how DEA can be used to set efficient prices for bundles of hospital outputs and how SFA can be used to set efficient prices for individual and bundles of outputs.

### *Accounting for quality*

Like any firm, a hospital can always use the same input(s) to produce more output(s) of lower quality. Equivalently, a hospital with access to higher-quality input(s) will have a capacity to produce more (or better quality) output(s). Failure to allow for variations in input and output quality can lead to biased estimates of efficiency and associated measures of hospital performance. Two methods are commonly used to account for variations in quality: disaggregation and cross-classification; and quality adjustment.

Disaggregation and cross-classification involves disaggregating inputs and outputs into finer (and therefore more homogeneous) categories. For example, inpatient separations can be disaggregated into acute care, newborn, rehabilitation and palliative care separations, or into medical, surgical and other separations, or by DRG (of which there are more than 600). Accident and emergency separations can be disaggregated into resuscitation, emergency, urgent, semi-urgent and non-urgent separations. Full time equivalent (FTE) staff can be disaggregated into medical officers, nurses, administrative staff and domestic staff. Beds can be disaggregated into beds in specialist care units and beds in general wards. Appendix 1 provides more examples of output and input variables.

For quality adjustment, indicators of input and output quality are often observed, and it is common to implicitly combine these indicators with observed input and output variables to obtain quality-adjusted inputs and outputs. Missing indicators of quality can be regarded as sources of statistical noise. Let $z_{kit}$ be an (observed) indicator of the quality of the (observed) input $x_{kit}$ and assume that our aim is to estimate a relationship of the form:

(29) $$\ln q_{it} = \beta_0 + \sum_{k=1}^{K} \beta_k \ln x_{kit}^{\#} + v_{it} - u_{it}$$

where $x_{kit}^{\#}$ denotes an unobserved quality-adjusted input. Estimating (29) is problematic because the quality-adjusted inputs are unobserved. One simple way forward is to assume a relationship between observed inputs and unobserved quality-adjusted inputs of the form:

(30) $$x_{kit}^{\#} = z_{kit}^{\delta_k} x_{kit}$$

Substituting (30) into (29) yields an SFA model defined over observed inputs and quality indicators:

$$\ln q_{it} = \beta_0 + \sum_{k=1}^{K} \phi_k \ln z_{kit} + \sum_{k=1}^{K} \beta_k \ln x_{kit} + v_{it} - u_{it} \quad (31)$$

where $\phi_k = \beta_k \delta_k$. Quality indicators can be incorporated into DEA models in a similar way (recall from the section *Data envelopment analysis* that the DEA model is also underpinned by a functional form assumption). Examples of output quality indicators used in a hospital context include:

- Waiting times for elective surgery and emergency department services; bed occupancy rates, which can be viewed as a measure of timely access to hospital care[7]
- Number of adverse events in acute and sub-acute care settings, including adverse drug events, rates of hospital-acquired infections, pressure ulcers, falls resulting in patient harm, and intentional self-harm
- Average length of stay[7]
- Number of unplanned or unexpected readmissions within 28 days of surgical admission
- Rates of return to operating theatres or intensive-care units
- Mortality rates (especially for people diagnosed with cancer)
- Measures of patient satisfaction around key aspects of care they received.

Examples of input quality indicators include:

- Patient characteristics including gender, age, indigenous status and ethnicity
- Socio-economic status
- Charlson co-morbidity scores
- Evans-Walker indexes of patient complexity.[7]

Differences in input quality can also be viewed as differences in the production environment. For example, hospitals working with rudimentary medical equipment are clearly working in a different environment from hospitals working with the latest medical technology. Methods for dealing with these types of quality variations are, for all intents and purposes, equivalent to methods for accounting for production environment changes.

### *Accounting for the environment*

An environmental variable is any variable that affects the ability to transform inputs into outputs and which has not already been included in the analysis. Time trends and variables indicating location are common examples of environmental variables. Time trends allow for changes in the stock of knowledge and other factors, eg levels of resistance to antibiotics that cause the production relationship to vary in time. Location generally does not affect the position of the production frontier and should not usually be included as an environmental variable when estimating primal representations of the production technology (eg distance functions). Exceptions may include hospitals in tropical locations where high humidity levels may hamper the successful treatment of infections. However, as the Productivity Commission observes, location may have a bearing on: the prices paid for inputs (eg salaried medical officers); input choices (eg special care units, number of beds); and output levels and mix (ie number and type of patients).[7] Thus, it may be appropriate for location to be included as an environmental variable when estimating dual representations of production technologies, eg cost functions.

Note that some inputs and outputs are outside the control of hospital managers. However, this fact is generally irrelevant to the problem of measuring different types of efficiency. It does not affect the production possibilities set or any of the functions that might be used to represent it, eg production frontiers and cost frontiers. Knowing that some inputs are outside the control of the hospital is important for understanding why hospitals may be located in different areas, and why different hospitals might be more or less technically, scale or allocatively efficient.

From a measurement viewpoint, environmental variables cause shifts in the production frontier and can be introduced as separate variables just like any other inputs and outputs. The distinction between environmental variables and conventional inputs and outputs is sometimes blurred. For example, if the number of untreated patients is not regarded as a hospital input then it might be viewed as an environmental variable and included anyway. If data on untreated patients were unavailable then it might be reasonable to include some of the determinants of the number of untreated patients (eg population density, location) in the form of environmental variables.

Often environmental variables are accounted for in SFA models by:

- Including a time trend
- Including dummy variables that allow selected coefficients to vary for selected firms
- Estimating group-specific frontiers, eg within a meta-frontier framework.[16]

Often environmental variables are accounted for in DEA models by estimating:

- Separate frontiers using data from different time periods
- Group-specific frontiers (eg within a meta-frontier framework).

### *Efficient pricing using data envelopment analysis*

The federal government will fund 60% of the national efficient price of public hospital services delivered to public patients.[17] The national efficient price will be: "an independent and objectively determined calculation of the cost of providing public hospital services" (Department of Health and Ageing, 2010:17). The federal government will ensure that the efficient price continues to reflect the actual cost of providing hospital services, and developments in best practice.[18]

The section *Data envelopment analysis* shows the way in which DEA can be used to estimate the minimum cost of providing a bundle of hospital services (in the illustrative example, which involved only one output, the minimum cost of producing one unit of output was $300). Hospitals will only be able to operate at this minimum cost level (ie will be cost efficient) if they operate on the estimated frontier (ie are technically efficient) and choose inputs in combinations that minimise cost (ie are allocatively efficient). Different DEA models can be used to estimate the minimum cost and, by implication, set a national efficient price. In practice, the DEA models used should be chosen considering these facts:

- Some inputs and outputs are outside the control of hospital managers (see the previous section). Coelli *et al*. (2005) describes DEA models for handling non-discretionary inputs and outputs.[1]
- Managers may rationally choose variable input levels that cause them to operate inside the production frontier, eg when the cost of acquiring the latest medical technology is high relative to the benefits it may provide and when certain levels of capacity need to be reserved for unforeseen emergencies. O'Donnell *et al.* describe DEA models for handling uncertainty and risk.[19]
- Changes in characteristics of the production environment occur, including technical change. The previous section describes DEA models for handling these changes.

Some of the weaknesses of DEA we have previously identified are particularly salient if DEA is used to inform health funding decisions:

- Estimates of minimum cost may be sensitive to measurement errors and other sources of statistical noise.
- It is computationally difficult to obtain measures of reliability for efficiency scores (and, therefore, estimates of minimum cost).
- It is practically impossible to estimate marginal costs. Estimating marginal costs is important because if the efficient price of a *bundle* of hospital services is the minimum cost of providing those services then the efficient price of a *particular* hospital service is its marginal cost.

These weaknesses can be largely overcome using SFA.

### *Efficient pricing using stochastic frontier analysis*

The federal government will be moving from payment for public hospital services based on recurrent expenditure to payment based on a national efficient price for each hospital service.[18] Like DEA, SFA can be used to estimate the minimum cost of providing a bundle of hospital services. Unlike DEA, SFA can also be used to estimate the marginal cost of providing each service.

Consider the following special cases, and generalisations of, the SFA cost frontier model (discussed in the section *Stochastic frontier analysis,* equation 17), which place SFA efficient pricing in a broader context:

(32) $\ln c_{it} = \alpha_0 + v_{it}$ (geometric average)

(33) $\ln c_{it} = \alpha_0 + \sum_{j=1}^{J} \gamma_j \ln q_{jit} + v_{it}$ (+ control for output quantities)

(34) $\ln c_{it} = \alpha_0 + \sum_{k=1}^{K} \alpha_k \ln w_{kit} + \sum_{j=1}^{J} \gamma_j \ln q_{jit} + v_{it}$ (+ control for input prices)

(35) $\ln c_{it} = \alpha_0 + \gamma t + \sum_{k=1}^{K} \alpha_k \ln w_{kit} + \sum_{j=1}^{J} \gamma_j \ln q_{jit} + v_{it}$ (+ control for technical change)

(36) $\ln c_{it} = \alpha_0 + \gamma t + \sum_{g=1}^{G} \phi_g \ln z_{git} + \sum_{k=1}^{K} \alpha_k \ln w_{kit} + \sum_{j=1}^{J} \gamma_j \ln q_{jit} + v_{it}$ (+ control for environment/quality)

(37) $\ln c_{it} = \alpha_0 + \gamma t + \sum_{g=1}^{G} \phi_g \ln z_{git} + \sum_{k=1}^{K} \alpha_k \ln w_{kit} + \sum_{j=1}^{J} \gamma_j \ln q_{jit} + v_{it}$ (+ allow for inefficiency)

A crude estimate of the minimum cost of producing the output bundle $q_{it}$ (ie an efficient price for $q_{it}$) can be obtained by taking the antilogarithm of the predictions obtained from model (32). This estimate is crude as it takes no account of the fact that hospitals providing few services (in both quantity and type) will generally cost less to run than hospitals providing many services. The model given by equation (33) accounts for variations in the quantity and type of output but is still somewhat crude in that it fails to account for variations in input prices. The model given by equation (34) accounts for input prices but fails to account for technical change. The model given by equation (37) is a fully specified SFA model that can be used to estimate minimum cost in a way that accounts for variations in output quantities and types, input prices, technical change, input and output quality, the production environment, and inefficiency.

Estimating minimum cost using the model given by (37) has two advantages over the DEA methodology discussed in the previous section. First, it is straightforward to compute standard errors and confidence intervals for estimates of minimum cost. Second, for the model given by

(37), an estimate of the marginal cost of providing each service (ie an efficient price for each service) can be computed as:

$$mc_{jit} = \frac{\partial c(z_{it}, w_{it}, q_{it}, t)}{\partial q_{jit}} = \frac{\gamma_j c_{it}}{q_{jit}} \qquad \text{for } j = 1,\ldots, J. \tag{38}$$

Computing standard errors and confidence intervals for these prices is straightforward. Computing prices at the level of a hospital, region, state or nation by simply evaluating (38) at relevant (estimated) average costs and quantities is also straightforward.

If input prices are unavailable and it is impossible to estimate the models given by (34) to (37) then other SFA methods can be used to estimate efficient prices. For example, Grosskopf *et al.* exploit the relationship between the output distance function and the revenue function to estimate normalised relative shadow prices.[6] The shadow price of an output can be viewed as the marginal cost (or opportunity cost) of producing an additional unit of that output. The shadow price of output *j* relative to the shadow price of output *k* is given by:

$$\frac{p^*_{jit}}{p^*_{kit}} = \left(\frac{\partial D^t_O(x_{it}, q_{it})}{\partial q_{jit}}\right)\left(\frac{\partial D^t_O(x_{it}, q_{it})}{\partial q_{kit}}\right)^{-1} = \frac{\partial q_{kit}}{\partial q_{jit}} \tag{39}$$

where $D^t_O(x_{it}, q_{it}) = \min_{\lambda}\{\lambda > 0 : (x_{it}, q_{it}/\lambda) \in T^t\}$ is the output distance function giving the (inverse of the) maximum factor by which a firm can radially expand its output vector while holding the input vector fixed. Grosskopf *et al.* estimated a translog output distance function and computed normalised relative shadow prices for the outputs of non profit hospitals in California and New York.[6] Normalising (39) by the output ratio gives the elasticity of substitution between outputs: $(p^*_{jit}/p^*_{kit})(q_{jit}/q_{kit}) = (\partial q_{kit}/\partial q_{jit})(q_{jit}/q_{kit}) = \partial \ln q_{kit}/\partial \ln q_{jit}$. Recently, Morrison Paul estimated an output distance function and computed normalised relative shadow prices for the outputs of NSW hospitals.[20] Unfortunately, estimating distance functions is not as straightforward as these authors make it appear. In particular, an endogeneity issue has been raised (and not satisfactorily resolved).[21,22] Refer to Coelli *et al.* (2005;265) for more details.[1]

The SFA models and methods that are used will depend on the data available. The choice of model should also be made regarding the same matters raised for DEA in the previous section.

## Appendix 1: Applications of stochastic frontier analysis and data envelopment analysis modelling techniques to hospital data

| Authors (year) | Country | Hospitals; number of observations | Efficiency Measures | Technique & Function | Outputs | Inputs | Other Variables |
|---|---|---|---|---|---|---|---|
| Al-Shammari (1999)[23] | Jordan | Ministry of Health hospitals; 15 | OTE | DEA | Patient days; minor surgical & major surgical operations | Bed days; FTE physicians; health staff | |
| Athanassopoulos & Gounaris (2001)[24] | Greece | Public hospitals; 98 | ITE | DEA | Medical & surgical patients; medical examinations; lab tests | Medical services, administrative & nursing staff; operating, pharmaceutical, medical supply & other supply costs; beds | |
| Athanassopoulos *et al.* (1999)[25] | Greece | Hospitals; 98 | OTE | DEA | Patients general medicine; patients surgical; lab tests; clinical examinations | Doctors in general medicine, surgical & labs; management & nursing staff; hospital beds | |
| Bates *et al.* (2006)[26] | US | Hospital industries (by metropolitan areas); 306 | ITE | DEA | Inpatient days; emergency room & non emergency room OPV; surgeries; births | FTE registered nurses, licensed practical nurses & other salaried staff; beds; expenditures on materials & supplies; active physicians | Hospitals per capita in metropolitan area; state HMO penetration rate; fraction of hospital in state with HMO contract; concentration ratio of three largest health insurers in the state; Medicare enrolees as % of metropolitan population; existence of a state certificate of Need law; Log of per capita income in metropolitan area; existence of teaching hospital; mortality rate; log of population; regional & state dummy |

| Authors (year) | Country | Hospitals; number of observations | Efficiency Measures | Technique & Function | Outputs | Inputs | Other Variables |
|---|---|---|---|---|---|---|---|
| Berta *et al.* (2010)[27] | Italy | General hospitals; 134 | OTE | SFA - CD & TL | Casemix discharges | Beds; physicians; nurses; administrative staff | Emergency department; concentration of services; teaching; hospital group; up coding; cream skimming; readmission |
| Bilodeau *et al.* (2004)[28] | Canada | Short term hospitals; 121 | ITE | DEA | Inpatient days; OPV; lab exams performed for pay; laundry & cafeteria services; teaching | Hours & expenses on labour; expenditure on supplies, food & meals; total expenditure on drugs, energy & other categories; equipment; building; physicians | Quebec financial DRG; inpatient diversity index; inpatient complexity index; outpatient diversity index; outpatient complexity index; % of patients >65 years; increase in patients >65 years since 1981; density |
| Bilodeau *et al.* (2009)[29] | Canada | Short term hospitals; 121 | ITE | SFA - TL | Inpatient days; OPV; lab exams; laundry & cafeteria services; teaching | Hours & expenses of labour; expenditures on supplies, food, meals, drugs, energy & others | |
| Biorn *et al.* (2002)[10] | Norway | Hospitals; 48 | ITE | SFA | Inpatient services; outpatient services | FTE physicians & other labours; medical & total running expenses | Total revenue per bed; % of total revenue that was outpatient revenue; dummy if hospital was activity based reimbursed; % of total in hospital days that were irregularly long length of stay; beds; dummy if hospital was a university clinic or a central hospital |
| Blank & Valdmanis (2010)[30] | Netherlands | General hospitals; 69 | ITE | DEA with bootstrap | Discharges groups 1, 2,3,4, first time visits | Administrative, nursing, paramedical & other staff; material supplies; variable cost | Part time factor staff; seniority staff; composition of capital; physicians' intensity |

| Authors (year) | Country | Hospitals; number of observations | Efficiency Measures | Technique & Function | Outputs | Inputs | Other Variables |
|---|---|---|---|---|---|---|---|
| Brown (2003)[31] | US | Hospitals; 613 | ITE | SFA - Log linear cost function | All cases; <1 DRG cases; 1 to 2 DRG cases; >2 DRG cases | Beds; capital expenses; FTE employees | Mean DRG weight; residents; COTH; public, for profit; metropolitan statistical area 4 firm concentration; year; HMO; PPO |
| Burgess & Wilson (1998)[32] | US | Non-psychiatric hospital in VA system & non VA system; 1545 | ITE, OTE | DEA | Acute care inpatient days; casemix weighted acute care inpatient discharges; long term care inpatient days; OPV; ambulatory & inpatient surgical procedures | Acute care hospital beds weighted by scope of service index; long term hospital beds; FTE registered nurses, licensed practical nurses, other clinical labour, nonclinical labour & long term care labour | State/local government; non profit; for profit; VA; member of the Council on teaching hospitals; HHI; average LOS; medical wage index; % of registered nurses; ratio of FTE clinical & non clinical staff; administrative cost per bed day of care |
| Butler & Li (2005)[33] | US | Rural hospitals; 57 | OTE | DEA | Inpatient days; inpatient & outpatient surgical operations; emergency room & OPV | Non-salary expenses; beds; employees; services offered (to measure the complexity of services) | |
| Carey (2003)[34] | US | Hospitals; 1209 | ITE | SFA - TL cost function | Adjusted admissions; adjusted patient days | Expenditure; beds; average annual salary | Casemix index; HMO penetration rate; HHI; system (% affiliated etc); non profit; government; teaching |
| Chang & Troyer (2009)[35] | US | Acute hospitals; 27 | ITE | SFA - TL | Inpatient admission; OPV | Total cost; price of capital | Mortality rate; % OPV that were emergency visits; personal income of the county; population of the county; time; HHI for county for profit motivation |
| Chang (1998)[36] | Taiwan | Hospitals; 6 | ITE | DEA | Clinic visits (including regular & emergency); weighted patient days | FTE physicians, nurses & medical supporting staff; FTE general & administrative staff | Scope of service; occupancy rate; % patients retired veterans; year |

| Authors (year) | Country | Hospitals; number of observations | Efficiency Measures | Technique & Function | Outputs | Inputs | Other Variables |
|---|---|---|---|---|---|---|---|
| Chang *et al.* (2004)[37] | Taiwan | Regional & district hospitals; 483 | OTE | DEA | Patient days; clinic or OPV; surgical patients | Beds; physicians; nurses; supporting medical staff (including ancillary service staff) | |
| Chen (2006)[11] | Taiwan | Public hospitals; unknown | OTE | MPI | OPV; intensive care patients; inpatient discharges; surgeries performed; quality attribute | Doctors, nurses & other medical staff; beds | Quality attribute; National Health Insurance; public hospital; bed utilisation rate; average LOS; severity of illness; log bed; HHI index |
| Chen *et al.* (2005)[38] | US | Hospitals; 89 | ITE | DEA | Routine care bed days; special care bed days | General service, routine & special case costs; cumulative capital investment; ancillary service cost | For profit; church; government; teaching; beds; outpatient revenue/total revenue; debt equity ratio; % of Medicare bed days; doctors/1000 population; beds in a county; hospitals in county; median income |
| Chern & Wan (2000)[39] | US | Hospitals; 80 | ITE | DEA | Casemix adjusted discharges; OPV | Beds; service complexity; FTE non-physicians; operating expenses (not including payroll, capital or depreciation) | |
| Chirikos (1998)[40] | US | Hospitals; 186 | ITE | SFA - TL cost function | Casemix adjusted admission; post-admission patient days corresponding to three different payer groups; two outpatient indices | Physicians; beds; cost | Ownership (dummies); proprietary (investor-owned hospital or not); voluntary (non profit); government (public vs private); market share; population (of the area where hospital locates); teaching |

| Authors (year) | Country | Hospitals; number of observations | Efficiency Measures | Technique & Function | Outputs | Inputs | Other Variables |
|---|---|---|---|---|---|---|---|
| Cremieux & Ouellette (2001)[41] | Canada | Hospitals; 1383 | ITE | SFA - TL cost function | Inpatient days; OPV; lab & physiological exams; laundry & cafeteria; residents | Cost share of labour, drugs, supplies, energy, food laundry & other variable inputs; buildings; equipment; physicians | Inpatient complexity index; inpatient diversity index; outpatient complexity index; outpatient diversity index |
| Daidone & D'Amico (2009)[42] | Italy | Private & public hospitals; 108 | OTE | SFA - TL | Weighted acute patients; general medicine; general surgery | Beds; gini (hospital specialisation index); nurses | Hospital casemix; ownership; non profit; geographical dummy; time |
| Dalmau-Matarrodona & Puig-Junoy (1998)[43] | Spain | General acute care hospitals; 94 | ITE | DEA | Casemix adjusted discharged patients; inpatient days in acute, sub acute, intensive, long term & other services; surgical interventions; hospital day care services; ambulatory visits; resident physicians | FTE physicians (including residents), nursing & equivalents, & other non sanitary staff; inpatient beds | Non profit, for profit; public; HHI; competitors in local market; % hospital revenue from NHS; >1 hour surgical interventions/100 patients; teaching status; % recovered discharged patients; beds; squared beds |
| Deily & McKay (2006)[44] | US | Acute care hospitals; 139 | ITE | SFA - TL cost function | Admissions; OPV | Total expenses | Intensive patient days/total patient days; emergency visits/OPV; Medicaid admission/total admission; teaching; non profit, for profit; government; central region, north, panhandle, south, west; risk adjusted predicted mortality rate; board certified active medical staff/bed; FTE resident/bed; open heart transplant/total admission; dummy for transplant program |

| Authors (year) | Country | Hospitals; number of observations | Efficiency Measures | Technique & Function | Outputs | Inputs | Other Variables |
|---|---|---|---|---|---|---|---|
| Ferrari (2006)[45] | Scotland | Acute hospitals; 52 | OTE | SFA - TL output distance function | Inpatient & outpatient index | Capital; FTE medical, nursing & other staff; beds | Year, teaching |
| Ferrari (2006)[46] | Scotland | Acute hospitals; 53 | OTE | DEA | Inpatient surgery, medical & other; outpatient day cases & day patients | Total capital charges; FTE medical, nursing & other staff: beds | |
| Ferrier & Valdmanis (1996)[47] | US | Rural hospitals; 360 | ITE | DEA | Acute days; sub-acute days; intensive days; surgeries performed; discharges; outpatients | Staff; beds | Quality measured as the ratio of hospital risk adjusted predicted mortality rate to its actual mortality rate; quality; total patient days; occupancy rate; % patients treated as outpatients; intensity of care; public or not; three states (as dummies) |
| Folland & Hofler (2001)[48] | US | Hospitals; 791 | ITE | SFA - Homotheti c cost function | General medical surgical; paediatrics; obstetrics/gynaecolog y; all other inpatient (all measured by annual inpatient days) OPV | Total cost | % board certified; reservation quality |
| Frech & Mobley (2000)[49] | US | Short term general hospitals; 378 | ITE | DEA; CD cost function | Total inpatient discharges in each of 6 payoff categories; OPV; FTE interns & residents/staff bed (teaching output) | Net plant property & equipment at beginning of period (measured by depreciation & amortisation); licensed physicians with admitted privileges | Infant mortality index; 5 casemix indices; proportions of OPV that were non-surgical; sub acute; newborns; medical surgical acute care; intensive care; expenditure on charity care & donation; scope of service index; worker age index; income per capita in the hospital city; medical doctors per capita in area; rural dummy |

| Authors (year) | Country | Hospitals; number of observations | Efficiency Measures | Technique & Function | Outputs | Inputs | Other Variables |
|---|---|---|---|---|---|---|---|
| Fujii (2001)[50] | Japan | Municipal hospitals; 954 | ITE | SFA - TL cost function | Inpatients & outpatients/day; clinical examinations/100 patients | Total cost, beds, emergency beds | Dummy for teaching, general, nursing standard, meal standard, bed standard, inverse of bed occupancy rate, subsidised & urban |
| Fujii & Ohta (1999)[51] | Japan | Municipal hospitals; 927 | ITE | DEA – Log linear cost function | Inpatients & outpatients/day | Total cost | Ratio of inpatient/outpatient; (depreciation & interest)/book value; examinations/100 patients; dummies for emergency hospital, general hospital, nursing standard |
| Gerdtham *et al.* (1999)[52] | Sweden | County Council hospitals; 12 | ITE | DEA & OLS | Actual cost; beds | Internal market; Solidity (=equity/total assets); political majority in county councils (conservative/liberal); % population >70 years; bed days | Discharges for surgical & short term internal medicine; surgical operations in short term care; physician visits in short term care & internal medicine |
| Goncalves *et al.* (2007)[53] | Brazil | Public hospitals; 27 | ITE | DEA | % of admission relating to three chapters of ICD with the greatest mortality rate; mean value paid through the hospital admission authority | Mortality rate; mean LOS in hospital | |
| Griffin & Steel (2004)[54] | US | Non teaching hospitals; 382 | ITE | SFA Bayes - TL cost function | Discharges; inpatient days; beds; OPV; casemix index | Total cost; capital stock | Time trend |
| Grosskopf *et al.* (2001)[55] | US | Hospitals; 792 | ITE | DEA | Patients; inpatient surgical; outpatient surgical; ER visits; OPV | Beds; medical staff, residents & interns; registered & licensed practical nurses; FTE other labours | |

| Authors (year) | Country | Hospitals; number of observations | Efficiency Measures | Technique & Function | Outputs | Inputs | Other Variables |
|---|---|---|---|---|---|---|---|
| Grosskopf *et al.* (2004)[56] | US | Teaching hospitals; 254 | ITE | DFA TL output distance function | Inpatients; inpatient & outpatient surgeries; OPV | Fully licensed & staffed beds; FTE physicians, registered nurses, licensed practical nurses, medical residents & other staff | Public, non profit, for profit; HMO & PPO contracts with each hospital, patients covered by either PPO or HMO; residents/physicians; residents/beds; member of COTH; affiliated with a medical school; accredited by JCAHCO |
| Gruca & Nath (2001)[57] | US | Community general hospitals; 168 | ITE | DEA | RIW weighted inpatient care; weighted OPV; long-term care days | FTE nursing, ancillary services, administrative staff; services & supplies (including drug & medical surgical supplies); beds | |
| Hajialiafzali *et al.* (2007)[58] | Iran | Hospitals; 53 | ITE | DEA | OPV; emergency visits; medical interventions; ratio of major surgeries to total surgeries (for complexity) | FTE medical doctors, nurses & other staff; staffed beds | |
| Harris *et al.* (2000)[59] | US | Hospitals; 20 | ITE | DEA | Adjusted discharges; OPV | Service mix; size; employees; operational expenses | |
| Herr (2008)[60] | Germany | Public & private hospitals; 1500 | ITE, OTE | SFA – CD; TL | Weighted cases | Doctors & other staff; beds; prices of doctors, other staff & beds; total cost | Non profit, private, public; >75 ratio; surgery ratio; female ratio; east; death rate; year dummy |
| Hofmarcher *et al.* (2002)[61] | Austria | Hospital wards; 31 | ITE | DEA | Patient days; discharges; LDF point (payment system) | Medical, paramedical & administrative staff; beds | |

| Authors (year) | Country | Hospitals; number of observations | Efficiency Measures | Technique & Function | Outputs | Inputs | Other Variables |
|---|---|---|---|---|---|---|---|
| Hollingsworth & Parkin (1995)[62] | Scotland | Acute hospitals; 75 | ITE | DEA | Acute inpatient days (medical & surgical); A&E & outpatient attendances; obstetrics & gynaecology inpatient days; other specialty inpatient days | Average number of staffed beds; trained, learning & other nurses; professional, technical administrative & clerical staff; total junior & senior non nursing medical & dental staff; drug supply; hospital capital charge | |
| Kibambe & Koch (2007)[63] | South Africa | Public hospitals; 42 | ITE | DEA | OPV; total admissions; inpatient days; theatre cases/surgeries | Active beds; medical doctors & specialists; nurses | |
| Kirigia *et al.* (2004)[64] | Kenya | Public health centre; 32 | ITE | DEA | Three groups of diseases treated & general OPV | Clinical officers & nurses; physiotherapists & others; lab technicians; administrative staff; non-wage expenditure; beds | |
| Kontodimopoulos *et al.* (2006)[65] | Greece | Small-scale hospitals in remote areas; 17 | ITE | DEA | Patient admissions; outpatients; preventive medicine services | Doctors; nurses; beds | |
| Koop *et al.* (1997)[66] | US | Hospitals; unknown | ITE | SFA - TL cost function | Discharges; inpatient days; beds; OPV; casemix index | Total cost; capital stock | Time trend; dummy for non profit & profit hospitals; dummy if ratio of clinical workers to average daily census was above average |

| Authors (year) | Country | Hospitals; number of observations | Efficiency Measures | Technique & Function | Outputs | Inputs | Other Variables |
|---|---|---|---|---|---|---|---|
| Li & Rosenman (2001)[67] | US | Hospitals; 90 | ITE | SFA | Patient days; OPV | Beds; total costs | Casemix index; % medical outpatient visit; % patient days Medicare; western region of Washington; public hospital; urban; for profit |
| Linna (1998)[68] | Finland | Acute hospitals; 43 | ITE | DEA & SFA - Box Cox transformation cost function | Emergency, scheduled & follow up visits; DRG weighted admissions; bed days; residents receiving 1 year of training; impact-weighted scientific publications; on the job training weeks of nurses | Net operating cost; beds | Teaching dummy; readmission rate; year of observation |
| Linna *et al.* (2006)[69] | Norway, Finland | Public hospitals; 98 | ITE | DEA | DRG weighted discharges; weighted day cares; outlier days; weighted OPV | Cost (adjusted) | |
| Lothgren (2000)[12] | Sweden | County Councils health care (aggregated); 26 | OTE | SFA - TL cost function | Operations; admissions; OPV; output vector index; 2 transformation indices | Total cost; beds | year dummy |
| Magnussen (1996)[70] | Norway | Acute care non teaching hospitals; 46 | ITE | DEA | Medical, surgical, simple & complex patient days; medical & surgical patients; long term care days; OPV | Physicians, nurses & other staff; beds | |

| Authors (year) | Country | Hospitals; number of observations | Efficiency Measures | Technique & Function | Outputs | Inputs | Other Variables |
|---|---|---|---|---|---|---|---|
| Maniadakis *et al.* (1999)[71] | Scotland | Acute hospitals; 75 | ITE | DEA | A&E attendances; adjusted inpatients, outpatients & day cases; standardised survivals after admission for stroke; fractured neck of femur; myocardial infarction | Doctor, nurse & other staff; beds; cubic meter; admission for stroke; fractured neck of femur; myocardial infarction | Standardised survivals after admission for stroke, fractured neck of femur, myocardial infarction |
| Maniadakis & Thanassoulis (2000)[72] | UK | Acute hospitals; 75 | ITE | DEA | A&E attendances; adjusted inpatients & outpatients; adjusted day stays | Doctors, nurses & other staff; beds; cubic meters/100 | |
| Masiye (2007)[73] | Zambia | Hospitals; 30 | ITE | DEA | Ambulatory care; maternal & child health inpatients (deliveries); lab tests; x-ray & theatre operations | Non-labour cost; medical doctors, nurses, administrative & other staff | |
| McCallion *et al.* (2000)[74] | Northern Ireland | Hospitals; 23 | ITE | DEA | Discharges for general surgeries, general medical, maternity, A&E (both inpatient & outpatient) | FTE nursing, administrative & ancillary staff; specialists; bed complement | |

| Authors (year) | Country | Hospitals; number of observations | Efficiency Measures | Technique & Function | Outputs | Inputs | Other Variables |
|---|---|---|---|---|---|---|---|
| McKay *et al.* (2002)[75] | US | Hospitals; 4075 | ITE | SFA - TL cost function | Admissions; inpatient days; OPV | Total cost/bed | Dummy for accredited hospitals; FTE residents/bed; % intensive care beds; inpatient surgical operations/admission; % outpatients that were surgical & emergency; high technology services; for profit; government; Medicare; Medicaid; hospitals; market share; area occupancy; population; income; rural; teaching; beds; system; management; HMO; PPO |
| McKillop *et al.* (1999)[76] | Northern Ireland | Acute hospitals; 23 | OTE | DEA | Surgical, medical, obstetrics & gynaecology; A&E | Nursing, consultant, administrative & ancillary staff; beds | Control variables were not specified |
| Mobley & Magnussen (1998)[77] | Norway, US | Urban hospitals; 228 | ITE | DEA | Patient days in 3 age groups; OPV; casemix index for patient >65 years | FTE physicians, residents & other labours; beds | Hospital types |
| Morey & Dittman (1996)[78] | US | North Carolina hospitals; 105 | ITE | DEA | Patient days for patients <14 years, 14-65 years & >65 years | Nursing, ancillary, administrative & general services; intensive care, acute care & other beds; % of patient days requiring intensive care; % of patient days as either intensive or acute; estimated capital involved with hospital | |

| Authors (year) | Country | Hospitals; number of observations | Efficiency Measures | Technique & Function | Outputs | Inputs | Other Variables |
|---|---|---|---|---|---|---|---|
| Morrison Paul (2002)[20] | Australia | NSW public hospitals; 223 | OTE | SFA – TL output distance function | Public & private acute inpatient separations; sub-acute & non-acute inpatient bed days; non-admitted patient occasions of service; teaching & research outputs (proxy by number of junior medical officers) | Salaries; superannuation; visiting medical officers; goods & services; repairs & maintenance (labour, materials, capital, research & other) | Social indicator of education & occupation; rurality; standardised mortality ratio; diagnoses |
| O'Neil (1998)[79] | US | Urban hospitals; 27 | OTE | DEA | Adjusted inpatient medical & surgical; adjusted outpatient; residents trained | Technological services; beds; FTEs; supply (operational expenses excluding payroll, capital & depreciation) | |
| Parkin & Hollingsworth (1997)[80] | Scotland | Acute hospitals; 75 | OTE | DEA | Medical & surgical acute discharges; A&E & outpatient attendances; obstetrics & gynaecology & other specialty discharges | Average staffed beds; trained, learning & other nurses; professional, technical, administrative & clerical staff; junior & senior non nursing medical & dental staff; drug supply; hospital's capital charge | |

| Authors (year) | Country | Hospitals; number of observations | Efficiency Measures | Technique & Function | Outputs | Inputs | Other Variables |
|---|---|---|---|---|---|---|---|
| Pilyavsky *et al.* (2006)[81] | Ukraine | Community hospitals; 61 | OTE | DEA | Medical and surgical admissions | Beds; physicians; nurses | Location (west, east); provincial budget allocated to health; physician density; outpatient utilisation/population base; distance to major urban area; % population that were elderly; % surgical in hospitals; average salary in hospital |
| Productivity Commission (2009)[7] | Australia | Public & private acute hospitals; 508 | OTE | SFA ML - CD & TL output distance functions | Inpatient casemix adjusted separations; non-admitted occasions of service | beds; nursing, diagnosis & other staff; drugs, supplies; clerical | Hospital type (ICU, teaching); mortality rate; patient characteristics; socioeconomic status; Charlson co-morbidity scores; Evans-Walker index of patient risk/complexity |
| Productivity Commission (2010)[8] | Australia | Public & private acute hospitals; 1806 | ITE, OTE | SFA TL output distance function & cost function | Inpatient casemix adjusted separations; non-admitted occasions of service | beds; nursing, diagnosis & other staff; drugs, supplies; clerical; total cost | Hospital type (ICU, teaching); mortality rate; patient characteristics; socioeconomic status; Charlson co-morbidity scores; Evans-Walker index of patient risk/complexity |
| Puig-Junoy (2000)[82] | Spain | Acute care hospitals; 94 | ITE | DEA | Casemix adjusted discharged patients; inpatient days in acute & sub acute services, intensive care, long term care & other services; surgical interventions; ambulatory visits; resident physicians | FTE physicians, nurses & equivalents, & other non-salary staff; inpatient beds | Non profit, for profit; public; HHI; competitors in the local market; % hospital revenue from NHS; >1 hour surgical interventions/100 patients; teaching status; % recovered discharged patients; beds; squared beds |
| Ramanathan (2005)[83] | Oman | Hospitals; 20 | ITE | DEA | OPV; inpatients; major & minor surgical procedures | Beds; doctors & others | |

| Authors (year) | Country | Hospitals; number of observations | Efficiency Measures | Technique & Function | Outputs | Inputs | Other Variables |
|---|---|---|---|---|---|---|---|
| Rebba & Rizzi (2007)[84] | Italy | Hospitals; 85 | ITE | DEA | DRG weighted inpatient cases; treatment days; emergency service cases | Physicians, nurses & other employees; hospital beds; acute care admissions (proxy for hospital demands) | Three dummies for hospital types (public, hospital trust, non profit, for profit); beds; casemix index; rotation index (ratio of total discharges & number of beds) |
| Renner *et al.* (2005)[85] | Sierra Leone | Peripheral health units; 37 | OTE | DEA | Antenatal & post natal care; babies delivered; nutrition/growth monitoring visits; family planning visits; children <5 years & pregnant women immunised; health education | Technical staff & sub-ordinate technical staff; materials & supplies; capital inputs | |
| Rodriguez-Alvarez & Lovell (2004)[86] | Spain | Public hospitals; 67 | ITE | SFA – TL input distance | Discharges in medicine, surgery, obstetrics, paediatrics & intensive care; ambulatory visits | Care graduates, care technicians & other staff; supplies; beds | Teaching; year |
| Rosko (1999)[87] | US | Short-term community hospitals; 3262 | ITE | SFA - TL cost function | OPV; inpatient discharges; post admission days | Total expenses | Casemix index; Emergency visit/total outpatients; dummy for hospitals that were members of teaching hospitals; dummy for teaching hospitals that were not a COTH member; HHI; dummy for profit hospital; % unemployed of labour force; % Medicare HMO enrolment/Medicare beneficiaries; % Medicare discharges/total discharges; % Medicaid discharges/total discharges |

| Authors (year) | Country | Hospitals; number of observations | Efficiency Measures | Technique & Function | Outputs | Inputs | Other Variables |
|---|---|---|---|---|---|---|---|
| Rosko (2001)[88] | US | Urban hospitals; 1557 | ITE | SFA - TL cost function | DRG weighted inpatient discharges; OPV | Total expenses | Dummy for being member of COTH; dummy for teaching hospitals not being a member of COTH; emergency/OPV; Outpatient surgeries/OPV; HMO enrolment/population; Medicare discharges/total discharges; Medicaid discharges/total discharges; dummy for investor owned hospital; HHI for concentration of hospital admissions |
| Rosko (2001)[89] | US | Short-term community hospitals; 1996 | ITE | SFA - TL cost function | Inpatient discharges; OPV; days in long term units | Total expenses | Dummy for being member of COTH; dummy for teaching hospitals not being a member of COTH; Medicare patient casemix index; emergency/OPV; Outpatient surgeries/OPV; HMO enrolment/population; Medicare discharges/total discharges; Medicaid discharges/total discharges; dummy for non profit; multihospital system membership; unemployment rate; dummy for hospitals in areas of low HHI |
| Rosko & Mutter (2008)[90] | US | Hospitals; 2218 | ITE | SFA - CD TL; DEA | Inpatient admission; OPV; days in other inpatient units; outpatient surgeries | Total cost; price of capital | Teaching hospital; Emergency department; ownership; HMO; Medicare; Medicaid |

| Authors (year) | Country | Hospitals; number of observations | Efficiency Measures | Technique & Function | Outputs | Inputs | Other Variables |
|---|---|---|---|---|---|---|---|
| Rosko & Proenca (2005)[91] | US | Private short-term general hospitals; 1368 | ITE | SFA | Adjusted inpatient discharges; OPV; dates in long term visits | Total expenses | Emergency/outpatient; outpatient surgeries/OPV; COTH member; teaching hospital; low/medium/high network; low/medium/high system; alliance member; HMO penetration; Medicare share; Medicaid share; highly competitive market; for profit; unemployment rate |
| Rouse & Swales (2006)[13] | New Zealand | Public hospitals; unknown | OTE | DEA | Medical/surgical model: discharges, non-DRG volume, specialist treatments; Pregnancy/childbirth model: discharges, outpatient attendances; Community health model: nursing & home visits, dental treatments, % population served; Mental Health model: bed days, contacts; Disability Support model: bed days, assessments | Total expenditure | |
| Sahin & Ozcan (2000)[92] | Turkey | Public hospitals of 80 provinces; 80 | ITE | DEA | OPV; discharged patients; hospital mortality rate | Beds; specialists, general practitioner, nurses & other allied professionals; revolving funds expenditure | |

| Authors (year) | Country | Hospitals; number of observations | Efficiency Measures | Technique & Function | Outputs | Inputs | Other Variables |
|---|---|---|---|---|---|---|---|
| Smet (2007)[93] | Belgium | General care hospitals; 187 | ITE | SFA - TL cost function | Admissions; patient days for 7 categories | Total operating cost; beds | Arrivals/day; service time; occupancy rate; queuing indicator; 2 region dummies; university affiliation; dummy for private or public |
| Sommersguter-Reichmann (2003)[94] | Austria | Hospitals; 22 | ITE | DEA | Outpatient; credit points multiplied by a steering factor | FTE labour; beds; expenses for external medical services | |
| Staat (2006)[95] | Germany | Local & regional hospitals; 160 | ITE | DEA | Cases; reciprocal LOS; casemix for medicine; surgery & fields of specialisations | Per diem; beds | |
| Street (2003)[96] | UK | Public hospitals; 226 | ITE | SFA & OLS - CD cost function | Casemix adjusted inpatients; first outpatient attendances weighted by specialty; emergency attendances; transfers into & out of hospital per spell; emergency admissions per spell; finished consultant episode inter specialty transfers per spell; episodes per spell | Total cost | Non primary outpatient attendances/inpatient spell; standardised index of unexpected emergency admission/total emergency admissions; % patients <15 years, >60 years, female; student whole time teaching equivalent/spell; % of total revenue spent on research; market forces factor |

| Authors (year) | Country | Hospitals; number of observations | Efficiency Measures | Technique & Function | Outputs | Inputs | Other Variables |
|---|---|---|---|---|---|---|---|
| Street & Jacobs (2002)[97] | UK | Acute hospitals; 217 | ITE | SFA - CD cost function | Transfers into & out of hospital per spell; emergency admissions per spell; finished consultation episode inter-specialty transfers per spell; episodes per spell | Casemix cost index | Non primary outpatient attendances/inpatient spell; standardised index of unexpected emergency admission/total emergency admissions; HRG weight (casemix index); % patients <15 years, >60 years, female; student whole time teaching equivalent/spell; % revenue spent on research; market forces factor (weighted average of staff, land, building etc) |
| Wagstaff & Lopez (1996)[98] | Spain | Hospitals; 43 | ITE | SFA - Cost function | Ambulatory visits; emergency visits; inpatient discharges | Cost | % ambulatory visits that were not first time; casemix index for inpatient cases; teaching status; presence of CAT, patients undergoing rehabilitation program, day hospital, & oncology department in hospital; operating theatres |
| Yaisawarng & Burgess (2006)[99] | US | US Department of Veteran Affairs hospitals; 131 | ITE | SFA - TL cost function | Basic 1 & 2; complex; non vest 1 & 2 | Two categories of total cost applied for 2 veteran types; beds; ICU | Quality of care for all beds except psychiatry beds; delight index; outpatient delight index; outpatient overall quality of care index; readmission rate; average LOS in days for the readmissions; average days to readmit; in hospital death rate; inpatient quality of care index; outpatient quality of care index |

| Authors (year) | Country | Hospitals; number of observations | Efficiency Measures | Technique & Function | Outputs | Inputs | Other Variables |
|---|---|---|---|---|---|---|---|
| Yong & Harris (1999)[100] | Australia | Public hospitals; 35 | ITE | SFA - CD cost function | Weighted inliers equivalent separation (casemix adjusted); on campus medical clinical occasion of services; emergency/casualty occasion of services | Admitted inpatient expenditure; total operating expenditure | Teaching; A1 hospital; occupancy rate; size; input use (medical staff/weighted-inlier equivalent separations) |
| Zere *et al.* (2001)[101] | South Africa | Hospitals; 86 | OTE | DEA | OPV; inpatient days | Beds; recurrent expenditure | Occupancy rate; average LOS; OPV as % inpatient days; local dummies |

**Efficiency Measures:** ITE = input-oriented technical efficiency; OTE = output-oriented technical efficiency; **Technique:** DEA = data envelopment analysis; DFA = deterministic frontier analysis; ML = maximum likelihood; MPI = Malmquist productivity index; OLS = ordinary least squares; SFA = stochastic frontier analysis; **Function:** CD = Cobb-Douglas; TL = translog; **Other:** CAT = computed tomography (CT or CAT Scan); COTH = council of teaching hospitals; DRG = diagnosis-related group; FTE= full time equivalent; HHI = Herfindahl–Hirschman index; HMO = health maintenance organisation; HRG = health care resource group; ICD = International Classification of Diseases; ICU = intensive care unit; JCAHCO = Joint Commission on Accreditation of Healthcare Organisations; LDF = price for points (Leistungsorientierte Diagnose-Fallgruppen); LOS = length of stay; NHS = National Health Service; OPV = outpatient visits; PPO = preferred provider organisation; RIW = resource intensity weight; VA = veterans’ affairs

## Appendix 2: Applications of stochastic frontier analysis and data envelopment analysis modelling techniques to inform hospital funding and price benchmarking decisions

| Authors (year) | Country | Firms | Efficiency Measures | Technique | Purpose |
|---|---|---|---|---|---|
| Biorn *et al.* (2002)[10] | Norway | Hospitals | ITE | SFA | Introduction of an activity based contract has improved efficiency when measured as technical efficiency. Empirical analysis supports the prediction of the theoretical model that increases in cost efficiency are lower than technical efficiency, & even negative |
| Brown (2003)[31] | US | Hospitals | ITE | SFA | |
| Chern & Wan (2000)[39] | US | Hospitals | ITE | DEA | |
| Chirikos (1998)[40] | US | Hospitals | ITE | SFA | Caution should be taken when using an efficiency estimate for policy formulation because efficiency scores & ranking can change when using different models |
| Gerdtham *et al.* (1999)[52] | Sweden | County Council hospitals | ITE | DEA & OLS | A change in resource allocation (funding) method significantly changes the efficiency score |
| Grosskopf *et al.* (2004)[56] | US | Teaching hospitals | ITE | DEA | Competition in the form of managed care reduces resource use, ie increases efficiency |
| Gruca & Nath (2001)[57] | US | Community general hospitals | ITE | DEA | Under the single payer system, there were no significant differences in efficiency across ownership types for non profit hospitals |
| McKillop *et al.* (1999)[76] | Northern Ireland | Acute hospitals | OTE | DEA | Benchmarking performance is useful |
| Morey & Dittman (1996)[78] | US | North Carolina Hospitals | ITE | DEA | Recommend a reward/punishment scheme based on the efficiency measurement, ie cut down the reimbursement based on the level of inefficiency of the previous year |
| Rosko (2001)[88] | US | Urban hospitals | ITE | SFA | Advocate market oriented approaches for hospital cost containment |
| Rouse & Swales (2006)[13] | New Zealand | Public hospitals | OTE | DEA | Identify efficient expenditure levels to set prices for hospital services at the DRG level. In use since 1997. |
| Sommersguter -Reichmann (2003)[94] | Austria | Hospitals | ITE | DEA | Improvement of technology has been the immediate result of the introduction of an activity based hospital financing system |

| Authors (year) | Country | Firms | Efficiency Measures | Technique | Purpose |
|---|---|---|---|---|---|
| Yaisawarng & Burgess (2006)[99] | US | US Department of Veterans Affairs hospitals | ITE | SFA | |
| Yong & Harris (1999)[100] | Australia | Public hospitals | ITE | SFA | Assessment of efficiency level when using casemix funding models |

**Technique:** DEA = data envelopment analysis; OLS = ordinary least squares; SFA = stochastic frontier analysis; **Efficiency Measures:** ITE = input-oriented technical efficiency; OTE = output-oriented technical efficiency; **Other:** DRG = diagnosis-related group

## Appendix 3: Strengths and weaknesses of different efficiency modelling techniques

| Technique | Strengths | Weaknesses | Misconceptions clarified |
|---|---|---|---|
| Stochastic frontier analysis (SFA) | • Allows for measurement errors, omitted exogenous variables & other sources of statistical noise<br>• Can be used to conduct conventional tests of hypotheses (easily) | • Sampling theory estimates may be unreliable if sample sizes are small (Bayesian methodology can be used to make valid finite sample inferences)<br>• Need to make assumptions concerning the distribution of the error terms<br>• Need to make assumptions concerning functional form | • SFA can be used with time-series data (Coelli *et al.*, 2005;312)[1] |
| Data envelopment analysis (DEA) & free disposal hull (FDH) | • Does not require (additional) assumptions concerning functional form – see last column<br>• Does not require assumptions concerning the distribution of error terms<br>• Fast DEA software packages are available for computing targets, peers & various measures of technical, scale & mix efficiency | • Does not allow for noise, so cannot distinguish inefficiency from noise<br>• Practically impossible to establish relationships between individual inputs & outputs, eg elasticities of output response<br>• Computationally difficult to obtain measures of reliability (standard errors) for efficiency scores (requires bootstrapping)<br>• Results may be sensitive to outliers<br>• Results may be unreliable if sample sizes are small | • DEA/FDH can account for environmental factors. Different frontiers can be estimated for groups of firms classified according to the values of environmental variables<br>• DEA/FDH can account for variations in output & input quality. Indicators of quality can be included in both DEA & SFA models just like any other input or output<br>• DEA/FDH involves a functional form assumption. DEA is underpinned by the assumption that the production frontier is linear in the neighbourhood of each data point, ie is locally linear<br>• DEA can be used with time-series data (Coelli *et al.*, 2005;312)[1] |
| Index numbers | • Does not require any assumptions about functional form or the distributions of error terms | • If index number methods are to be used for efficiency analysis then we must have measures of relative value of inputs & outputs, eg prices<br>• Computing total factor productivity efficiency involves identifying the point of maximum productivity, which may be unreliable if sample sizes are small | |

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