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ScienceDirect
Renewable and Sustainable Energy Reviews
12 (2008) 1822–1842
RENEWABLE & SUSTAINABLE ENERGY REVIEWS
www.elsevier.com/locate/rser

Review of Ranque–Hilsch effects in vortex tubes

Smith Eiamsa-ard[a,1], Pongjet Promvonge[b,*]

[a]*Department of Mechanical Engineering, Faculty of Engineering, Mahanakorn University of Technology, Bangkok 10530, Thailand*

[b]*Department of Mechanical Engineering, Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang, Bangkok 10520, Thailand*

Received 2 February 2007; received in revised form 2 February 2007; accepted 22 March 2007

Abstract

The vortex tube or Ranque–Hilsch vortex tube is a device that enables the separation of hot and cold air as compressed air flows tangentially into the vortex chamber through inlet nozzles. Separating cold and hot airs by using the principles of the vortex tube can be applied to industrial applications such as cooling equipment in CNC machines, refrigerators, cooling suits, heating processes, etc. The vortex tube is well-suited for these applications because it is simple, compact, light, quiet, and does not use Freon or other refrigerants (CFCs/HCFCs). It has no moving parts and does not break or wear and therefore requires little maintenance. Thus, this paper presents an overview of the phenomena occurring inside the vortex tube during the temperature/energy separation on both the counter flow and parallel flow types. The paper also reviews the experiments and the calculations presented in previous studies on temperature separation in the vortex tube. The experiment consisted of two important parameters, the first is the geometrical characteristics of the vortex tube (for example, the diameter and length of the hot and cold tubes, the diameter of the cold orifice, shape of the hot (divergent) tube, number of inlet nozzles, shape of the inlet nozzles, and shape of the cone valve. The second is focused on the thermo-physical parameters such as inlet gas pressure, cold mass fraction, moisture of inlet gas, and type of gas (air, oxygen, helium, and methane). For each parameter, the temperature separation mechanism and the flow-field inside the vortex tubes is explored by measuring the pressure, velocity, and temperature fields.

The computation review is concentrated on the quantitative, theoretical, analytical, and numerical (finite volume method) aspects of the study. Although many experimental and numerical studies on the vortex tubes have been made, the physical behaviour of the flow is not fully understood due to its complexity and the lack of consistency in the experimental findings. Furthermore, several different

*Corresponding author. Tel.: +662 3264197; fax: +662 3264198.
E-mail addresses: smith@mut.ac.th (S. Eiamsa-ard), kppongje@kmitl.ac.th (P. Promvonge).
[1]Tel./fax: +662 9883666x241.

doi:10.1016/j.rser.2007.03.006

hypotheses based on experimental, analytical, and numerical studies have been put forward to describe the thermal separation phenomenon.

Keywords: Vortex tube; Ranque–Hilsch vortex tube; Temperature separation; Ranque–Hilsch effects

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Contents

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1. Introduction of engineering background of vortex tube

The vortex tube (also called the *Ranque–Hilsch vortex tube*) is a mechanical device operating as a refrigerating machine without any moving parts, by separating a compressed gas stream into a low total temperature region and a high one. Such a separation of the flow into regions of low and high total temperature is referred to as the temperature (or energy) separation effect. The vortex tube was first discovered by Ranque [1,2], a metallurgist and physicist who was granted a French patent for the device in 1932, and a United States patent in 1934. The initial reaction of the scientific and engineering communities to his invention was disbelief and apathy. Since the vortex tube was thermodynamically highly inefficient, it was abandoned for several years. Interest in the device was revived by Hilsch [3], a German engineer, who reported an account of his own comprehensive experimental and theoretical studies aimed at improving the efficiency of the vortex tube. He systematically examined the effect of the inlet pressure and the geometrical parameters of the vortex tube on its performance and presented a possible explanation of the energy separation process. After World War II, Hilsch's tubes and documents were uncovered, which were later studied extensively. Indicative of early interest in the vortex tube is the comprehensive survey by Westley [4] which included over 100 references. Other literature surveys such as Curley and McGree [5], Kalvinskas [6],

Nomenclature

COP	coefficient of performance
C_p	specific heat at constant pressure (kJ/kg K)
d	cold orifice diameter (m)
D	vortex tube diameter (m)
M	mass flow rate (kg/s)
P	pressure (Pa)
Q_c	cooling rate (kJ)
R	gas constant (kJ/kg K)
T	temperature (K)
ΔT	temperature drop (K)
w	mechanical energy (kJ)

Greek letters

β	cold orifice diameter ratio
η	efficiency (%)
γ	specific heat ratio
μ_c	cold mass fraction

Subscripts

a	atmosphere
c	cold air
h	hot air
i	inlet air
is	isentropic

Dobratz [7] and Nash [8] provided extensive reviews of vortex tube applications and enhancements. Since vortex flow phenomenon taking place in a vortex tube is compressible and complex, the simulation and solution of turbulent vortex flows is a difficult and challenging task. Vortex tubes have been used commercially for low-temperature applications, such as to cool parts of machines, set solders, dehumidify gas samples, cool electric or electronic control cabinets, chill environmental chambers, cool food, and test temperature sensors [9–11]. Other practical applications include quick start-up of steam power generation, liquefaction of natural gas [12], cooling of equipments in laboratories dealing with explosive chemicals [13,14], temperature control of divers' air suppliers [15], manned underwater habitats [16], hyperbaric chambers [17], separating particles in the waste gas industry [18], cooling for low-temperature magic angle spinning nuclear magnetic resonance (NMR) [19], nuclear reactors, and cooling of firemen's suits [20], etc. In general, the vortex tube has been known by different names. The most well-known names are: vortex tube, Ranque vortex tube (first discoverer), Hilsch vortex tube or Ranque–Hilsch (who improved the performance of the vortex tubes after Ranque), and Maxwell–Demon vortex tube (derived from the name of Maxwell and Demon group who together studied the molecule of hot air moving within the tube). Although there are various names, only "vortex tube" will be used in this report.

The purpose of this article is to present an overview of the past investigations of the mean flow and temperature behaviours in a turbulent vortex tube in order to understand the nature of the temperature separation or Ranque–Hilsch effect, which is the total temperature difference between the temperature in the tube and the inlet temperature. This report is separated into six sections. Section 2 presents some details of the important parameter definitions. Section 3 describes the type of the vortex tubes. Section 4 describes a parametric study of the geometry of the vortex tube. Section 5 presents the survey of the past research on both experimental and computational works. Observation results are summarized in the final section.

2. Important definitions

In this section, a few important terms commonly used in vortex tube work are defined.

2.1. Cold mass fraction

The cold mass fraction is the most important parameter indicating the vortex tube performance and the temperature/energy separation inside the vortex tube. Cold mass fraction is defined as the ratio of cold air mass flow rate to inlet air mass flow rate. The cold mass fraction can be controlled by the cone valve, which is placed at the hot tube end. This can be expressed as follows:

$$\mu_{\mathrm{c}} = \frac{M_{\mathrm{c}}}{M_{\mathrm{i}}}, \tag{1}$$

where M_c is the mass flow rate of cold air and M_i is the mass flow rate of the entry air.

2.2. Cold air temperature drop

Cold air temperature drop or temperature reduction is defined as the difference in temperature between entry air temperature and cold air temperature:

$$\Delta T_{\mathrm{c}} = T_{\mathrm{i}} - T_{\mathrm{c}} \tag{2}$$

in which T_i is the entry air temperature and T_c is the cold air temperature.

2.3. Cold orifice diameter

Cold orifice diameter ratio (β) is defined as the ratio of cold orifice diameter (d) to vortex tube diameter (D):

$$\beta = d/D. \tag{3}$$

2.4. Isentropic efficiency

To calculate the cooling efficiency of the vortex tube, the principle of adiabatic expansion of ideal gas is used. As the air flows into the vortex tube, the expansion in isentropic process occurs. This can be written as follows:

$$\eta_{is} = \frac{T_i - T_c}{T_i(1 - (P_a/P_i)^{(\gamma-1/\gamma)})}, \tag{4}$$

where η_{is}, P_i, P_a and γ are the isentropic efficiency, inlet air pressure, atmosphere pressure and specific heat ratio, respectively.

2.5. Coefficient of performance

To find the coefficient of performance (COP) defined as a ratio of cooling rate to energy used in cooling, the same principle of isentropic expansion of ideal gas is employed and the equation becomes

$$\mathrm{COP} = \frac{Q_c}{w} \tag{5}$$

and

$$\mathrm{COP} = \frac{\mu_c C_p(T_i - T_c)}{(\gamma/\gamma - 1)RT_i[(P_i/P_c)^{\gamma-1/\gamma} - 1]} \tag{6}$$

in which Q_c is cooling rate per unit of air in the inlet vortex tube, and w is mechanical energy used in cooling per unit of air inlet.

3. Classifications of the vortex tube

Generally, the vortex tube can be classified into two types. One is the counter-flow type (often referred to as the standard type) and the other the parallel or uni-flow type, as shown in Figs. 1a and b, respectively.

3.1. Counter-flow vortex tube

The counter-flow vortex tube, as shown in Fig. 1a, consists of an entrance block of nozzle connections with a central orifice, a vortex tube (or hot tube) and a cone-shaped valve. A source of compressed gas (e.g. air) at high pressure enters the vortex tube tangentially through one or more inlet nozzles at a high velocity. The expanding air inside the tube then creates a rapidly spinning vortex. The air flows through the tube rather than pass through the central orifice located next to the nozzles because the orifice is of much smaller diameter than the tube. The length of the tube is typically between 30 and 50 tube diameters, and no optimum value has been determined between these limits. As the air expands down the tube, the pressure drops sharply to a value slightly above atmospheric pressure, and the air velocity can approach the speed of sound. Centrifugal action will keep this constrained vortex close to the inner surface of the tube.

The air that escapes at the other end of the tube can be varied by a flow-control valve, usually shaped as a cone. The amount of air released is between 30% and 70% of the total airflow in the tube. The remainder of the air is returned through the centre of the tube, along its axis as a counter-flowing stream. Once a vortex is set up in the tube, the air near the axis cools down while the air at periphery heats up in comparison with the inlet temperature. This phenomenon is known as temperature separation effect (also called the

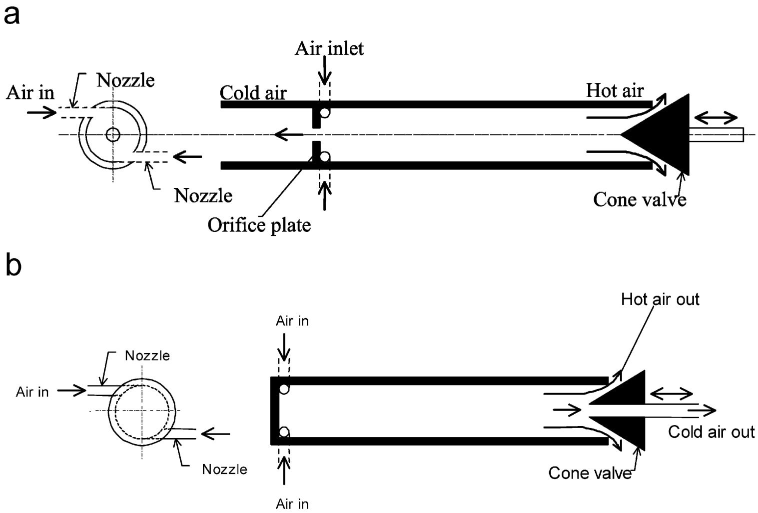

Fig. 1. Basic operation of vortex tubes: (a) the Ranque–Hilsch standard vortex tube or counter-flow vortex tube and (b) the uni-flow or parallel flow vortex tube.

Ranque–Hilsch effect). As a result, the gas escaping through the orifice is cold and the hot gas flows out in the other direction. A remarkable feature of this device is the absence of moving parts and simplicity of operation.

3.2. *Uni-flow vortex tube*

The uni-flow vortex tube (Fig. 1b) comprises an entrance block of inlet nozzles, a vortex tube and a cone-shaped valve with a central orifice. Unlike the more popular counter-flow version, the cold air exit is located concentrically with the annular exit for the hot air. The operation of the uni-flow vortex tube is similar to the operation of the counter-flow one. The temperatures of the air leaving the hot and cold ends can differ by as much as 140–160 °C, but extremes of up to 230 °C have been measured by Comassar [21]. In general, the practical low-temperature limit for the cold air stream is −40 °C, although temperatures as low as −50 °C have been obtained with research equipment. The practical limit for the high temperature is 190 °C, but temperatures in excess of 225 °C have been observed by Bruno [13,14]. The main applications of the vortex tube are in those areas where compactness, reliability, and low equipment costs are the major factors and the operating efficiency is of no consequence. Some typical applications are cooling devices for airplanes, space suits and mines; instrument cooling; and industrial process coolers.

4. Parametric study of the vortex tube

The analysis in the past investigated had showed that the vortex thermal separation phenomenon comes mainly from the diffusion process of mean kinetic energy. Low temperatures (or large temperature separation), both total and static, are found near the tube axis, becoming lower towards the orifice or the cold exit of the standard vortex tube. One might want to know how the diffusion process of mean kinetic energy affects the design of vortex tube. In general, a vortex tube is designed to obtain either (i) the maximum temperature separation or (ii) the maximum efficiency. At a given supply pressure, however, many vortex tubes with different design parameters can yield the same temperature separation [22]. This is not in doubt if the separation phenomenon in the tube is understood clearly. If any design parameter of a particular vortex tube affects the flow field, it would certainly affect the performance of the tube.

In the design of a standard vortex tube, there are several tube parameters to be considered, such as (1) tube diameter, (2) cold orifice diameter, (3) number, size and location of the inlet nozzles, (4) tube length and (5) hot valve shape. There are no critical dimensions of these parameters that would result in a unique value of maximum temperature separation. Knowledge of the temperature separation phenomenon suggests a relative design procedure for a vortex tube with the physical realities of its operation. For fixed inlet conditions (supply pressure) a very small diameter vortex tube would offer considerably higher back pressures and, therefore, the tangential velocities between the periphery and the core would not differ substantially due to the lower specific volume of air (still high density) while the axial velocities in the core region are high. This would lead to low diffusion of kinetic energy which also means low temperature separation. On the other hand, a very large tube diameter would result in lower overall tangential velocities both in the core and in the periphery region that would produce low diffusion of mean kinetic energy and also low temperature separation.

A very small cold orifice would give higher back pressure in the vortex tube, resulting, as discussed above, in low temperature separation. On the other hand, a very large cold orifice would tend to draw air directly from the inlet and yield weaker tangential velocities near the inlet region, resulting in low temperature separation. Similarly, a very small inlet nozzle would give rise to considerable pressure drop in the nozzle itself, leading to low tangential velocities and hence low temperature separation. A very large inlet nozzle would fail to establish proper vortex flow resulting again in low diffusion of kinetic energy and therefore low temperature separation. The inlet nozzle location should be as close as possible to the orifice to yield high tangential velocities near the orifice. A nozzle location away from the orifice would lead to low tangential velocities near the orifice and hence low temperature separation.

5. Review of the vortex tube

Vortex flows or swirl flows have been of considerable interest over the past decades because of their use in industrial applications, such as furnaces, gas-turbine combustors and dust collectors. Vortex (or high swirl) can also produce a hot and a cold stream via a vortex tube. The vortex tube has been used in industrial applications for cooling and heating processes because they are simple, compact, light and quiet (in operation) devices [9–20]. Several researchers put a lot of efforts to explain for the phenomena occurring during the energy separation inside the vortex tube. Research studies about these phenomena were formed mainly into two groups. The first one performed the experimental work (geometrical and thermo-physical parameters) and then through the value of their results attempted to explain the phenomena. The second performed the studies in qualitative, analytical and numerical ways in order to help in the analysis of the mechanisms present in the vortex tube.

5.1. Experimental work

The vortex tube was first discovered by Ranque [1,2], a metallurgist and physicist who was granted a French patent for the device in 1932, and a United States patent in 1934. The initial reaction of the scientific and engineering communities to his invention was disbelief and apathy. Since the vortex tube was thermodynamically highly inefficient, it was abandoned for several years. Interest in the device was revived by Hilsch [3], a German engineer, who reported an account of his own comprehensive experimental and theoretical studies aimed at improving the efficiency of the vortex tube. He systematically examined the effect of the inlet pressure and the geometrical parameters of the vortex tube on its performance and presented a possible explanation of the energy separation process. After Hilsch [3], an experimental study was made by Scheper [23] who measured the velocity, pressure, and total and static temperature gradients in a Ranque–Hilsch vortex tube, using probes and visualization techniques. He concluded that the axial and radial velocity components were much smaller than the tangential velocity. His measurements indicated that the static temperature decreased in a radially outward direction. This result was contrary to most other observations that were made later. Martynovskii and Alekseev [24] studied experimentally the effect of various design parameters of vortex tubes.

Hartnett and Eckert [25,26] measured the velocity, total temperature, and total and static pressure distributions inside a uni-flow vortex tube. They used the experimental values of static temperature and pressure to estimate the values of density and hence, the mass and energy flow at different cross sections in the tube. The results agreed fairly well with the overall mass and energy flow in the tube. Scheller and Brown [27] presented measurements of the pressure, temperature, and velocity profiles in a standard vortex tube and observed that the static temperature decreased radially outwards as in the work of Scheper [23], and hypothesized the energy separation mechanism as heat transfer by forced convection. Blatt and Trusch [28] investigated experimentally the performance of a uni-flow vortex tube and improved its performance by adding a radial diffuser to the end of the shortened tube instead of a cone valve. The geometry of the tube was optimised to maximise the temperature difference between the cold and inlet temperatures by changing the various dimensions of the tube such as the gap of the diffuser, tube length, and entrance geometry. Moreover, the effects of inlet pressure and heat fluxes were examined. Linderstrom-Lang [29] studied in detail the application of the vortex tube to gas separation, using different gas mixtures and tube geometry and found that the separation effect depended mainly on the ratio of cold and hot gas mass flow rates. The measurements of Takahama [30] in a counter-flow vortex tube provided data for the design of a standard type vortex tube with a high efficiency of energy separation. He also gave empirical formulae for the profiles of the velocity and temperature of the air flowing through the vortex tube. Takahama and Soga [31] used the same sets of the vortex tubes of Takahama [30] to study the effect of the tube geometry on the energy separation process and that of the cold air flow rate on the velocity and temperature fields for the optimum proportion ratio of the total area of nozzles to the tube area. They also reported an axisymmetric vortex flow in the tube.

Vennos [32] measured the velocity, total temperature, and total and static pressures inside a standard vortex tube and reported the existence of substantial radial velocity. Bruun [33] presented the experimental data of pressure, velocity and temperature profiles in a counter-flow vortex tube with a ratio of 0.23 for the cold to total mass flow rate and concluded that radial and axial convective terms in the equations of motion and energy were equally important. Although no measurements of radial velocities were made, his calculation, based on the equation of continuity, showed an outward directed radial velocity near the inlet nozzle and an inward radial velocity in the rest of the tube. He reported that turbulent heat transport accounted for most of the energy separation. Nash [34] used vortex expansion techniques for high temperature cryogenic cooling to apply to infrared detector applications. A summary of the design parameters of the vortex cooler was reported by Nash [35]. Marshall [36] used several different gas mixtures in a variety of sizes of vortex tubes and confirmed the effect of the gas separation reported by Linderstrom-Lang [29]. A critical inlet Reynolds number was identified at which the separation was a maximum. Takahama et al. [37] investigated experimentally the energy separation performance of a steam-operated standard vortex tube and reported that the performance worsened with wetness of steam at the nozzle outlet because of the effect of evaporation. Energy separation was absent with the dryness fraction less than around 0.98. The measurements of Collins and Lovelace [38] with a two-phase, liquid–vapour mixture, propane in a standard counter-flow vortex tube showed that for an inlet pressure of 0.791 MPa, the separation remained significant for a dryness fraction above 80% at the inlet. With a dryness fraction below 80%, the temperature separation became insignificant. But the discharge enthalpies showed considerable differences indicating that the Ranque–Hilsch process is still in effect.

Takahama and Yokosawa [39] examined the possibility of shortening the chamber length of a standard vortex tube by using divergent tubes for the vortex chamber. Earlier researchers such as Parulekar [40], Otten [41], and Raiskii and Tunkel [42] also employed divergent tubes for all or part of the vortex chamber in attempts to shorten the chamber and improve energy separation performance, but their emphasis was on the maximum and minimum temperatures in the outflowing streams. Therefore, Takahama and Yokosawa [39] compared their results with those from the straight vortex chambers. They found that the uses of a divergent tube with a small angle of divergence led to an improvement in temperature separation and enable the shortening of the chamber. Kurosaka et al. [43] carried out an experiment to study the total temperature separation mechanism in a uni-flow vortex tube to support their analysis and concluded that the mechanism of energy separation in the tube is due to acoustic streaming induced by the vortex whistle. Schlenz [44] investigated experimentally the flow field and the energy separation in a uni-flow vortex tube with an orifice rather than a conical valve to control the flow. The velocity profiles were measured by using laser-Doppler velocimetry (LDA), supported by flow visualization. Experimental studies of a large counter-flow vortex tube with short length by Amitani et al. [45] indicated that the shortened vortex tube of 6 tube diameters length had the same efficiency as a longer and smaller vortex tube when perforated plates are equipped to stop the rotation of the stream in the tube. Stephan et al. [46] measured temperatures in the standard vortex tube with air as a working medium in order to support a similarity relation of the cold gas exit temperature with the cold gas mass ratio, established using dimensional analysis. Negm et al. [47,48] studied experimentally the process of energy separation in the standard vortex tubes to support their correlation obtained using dimensional analysis and in a double stage vortex tube which found that the performance of the first stage is always higher than that of the second stage tube. Lin et al. [49] made an experimental investigation to study the heat transfer behaviour of a water-cooled vortex tube with air.

Ahlborn et al. [50] carried out measurements in standard vortex tubes to support their models for calculating limits of temperature separation. They also attributed the heating to the conversion of kinetic energy into heat and the cooling to the reverse process. Ahlborn et al. [51] studied the temperature separation in a low-pressure vortex tube. Based on their recent model calculation [50], they concluded that the effect depends on the normalized pressure ratio ($\mu_c = (P_i - P_c)/P_c$) rather than on the absolute values of the entrance pressure, P_i and exhaust pressure, P_c. In 1997, Ahlborn and Groves [52] measured axial and azimuthal velocities by using a small pitot probe and found that the existence of secondary air outward flow in the vortex tube. Ahlborn et al. [53] identified the temperature splitting phenomenon of a Ranque–Hilsch vortex tube in which a stream of gas divides itself into a hot and a cold flow as a natural heat pump mechanism, which is enabled by secondary circulation. Ahlborn and Gordon [54] considered the vortex tube mass a refrigeration device which could be analysed as a classical thermodynamic cycle, replete with significant temperature splitting, refrigerant, and coolant loops, expansion and compression branches, and natural (or built-in) heat exchangers.

Arbuzov et al. [55] concluded that the most likely physical mechanism (the Ranque effect) was viscous heating of the gas in a thin boundary layer at the walls of the vortex chamber and the adiabatic cooling of the gas at the centre on account of the formation of an intense vortex braid near the axis. Gutsol [56] explained that the centrifugal separation of ''stagnant'' elements and their adiabatic expansion causes the energy separation in the vortex tube system. Piralishvili and Polyaev [57] made experimental investigations on this effect in so-called double-circuit vortex tubes. The possibility of constructing a double-circuit vortex tube refrigeration machine as efficient as a gas expansion system was demonstrated. Lewins and Bejan [58] have suggested that angular velocity gradients in the radial direction give rise to frictional coupling between different layers of the rotating flow resulting in a migration of energy via shear work from the inner layers to the outer layers. Trofimov [59] verified that the dynamics of internal angular momentum leads to this effect. Guillaume and Jolly [60] demonstrated that two vortex tubes placed in a charged configuration or placed in series by connecting the cold discharge of one stage into the inlet of the following stage. From their results, it was found that for similar inlet temperatures, a two-stage vortex tube could be produced a higher temperature reduction than one of the vortex tubes operating independently. Manohar and Chetan [61] used a vortex tube for separating methane and nitrogen from a mixture and found that there was partial gas separation leading to a higher concentration of methane at one exit in comparison to the inlet and a lower concentration at the other exit.

Saidi and Valipour [62] presented on the classification of the parameters affecting vortex tube operation. In their work, the thermo-physical parameters such as inlet gas pressure, type of gas and cold gas mass ratio, moisture of inlet gas, and the geometry parameters, i.e., diameter and length of main tube diameter of outlet orifice, shape of entrance nozzle were designated and studied. Singh et al. [63] reported the effect of various parameters such as cold mass fraction, nozzle, cold orifice diameter, hot end area of the tube, and L/D ratio on the performance of the vortex tube. They observed that the effect of nozzle design was more important than the cold orifice design in getting higher temperature separations and found that the length of the tube had no effect on the performance of the vortex tube in the range 45–55 L/D. Riu et al. [18] investigated dust separation characteristics of a counter-flow vortex tube with lime powders whose mean particle sizes were 5 and 14.6 μm. They showed that a vortex tube can be used as an efficient pre-skimmer to separate particles from the waste gas in industry.

Promvonge and Eiamsa-ard [64] experimentally studied the energy and temperature separations in the vortex tube with a snail entrance. In their experimental results, the use of snail entrance could help to increase the cold air temperature drop and to improve the vortex tube efficiency in comparison with those of original tangential inlet nozzles. Promvonge and Eiamsa-ard [65] again reported the effects of (1) the number of inlet tangential nozzles, (2) the cold orifice diameter, and (3) tube insulations on the temperature reduction and isentropic efficiency in the vortex tube. Gao et al. [66] used a special pitot tube and thermocouple techniques to measure the pressure, velocity and temperature distribution inside the vortex tube which the pitot tube has only a diameter of 1 mm with one hole (0.1 mm diameter). In their work, the influence of different inlet conditions was studied. They found that rounding off the entrance can be enhanced and extended the secondary circulation gas flow, and improved the system's performance. Aydın and Baki [67] investigated experimentally the energy separation in a counter-flow vortex tube with various geometrical and thermo-physic parameters. The geometry of the tube was optimised to maximise the temperature difference between the cold and inlet temperatures by changing the various dimensions of the tube such as the length of the vortex tube, the diameter of the inlet nozzle, and the angle of the control valve. Moreover, the effects of various inlet pressure and different working gases (air, oxygen, and nitrogen) on temperature different in a tube were also studied.

The relevant data from the experimental work are summarized in Table 1. It is found that various tube dimensions and operating conditions are used, for example, from diameters as low as 4.6 mm and as high as 800 mm. Table 1 presents variations in the maximum temperature difference between the inlet and the hot and cold streams. In this table for the same standard tube type, Scheper [23] used an inlet pressure of 2.0 atm (abs.) and obtained a temperature difference of about 8 °C between the hot and cold streams while Vennos [32] employed inlet pressure of 5.8 atm (abs.) but obtained only a temperature difference of about 12 °C. This means that, at this point, it is nearly impossible to predict how a given tube will perform because the exact nature of flow inside the tube is in doubt. However, it can be achieved if the energy separation mechanisms are understood.

Regarding the radial static temperature gradient, Scheper [23] and Scheller and Brown [27] reported that static temperature decreased radially outward whereas other investigators reported an increase in the static temperature in the radially outward direction.

5.2. *Qualitative, analytical and numerical work*

The energy separation was first explained by Ranque in his patent in 1932. He hypothesized that the inner layers of the vortex expand and grow cold while they press upon the outer layers to heat the latter [68]. This theory, based on invicid non-conducting fluid flow was rejected by Ranque himself in 1933 when he stated that the compressed outer layers in the vortex tube have low velocities while the expanded inner layers have large velocities and hence a larger kinetic energy. This velocity distribution gives rise to considerable friction between the different layers which results in centrifugal migration of energy from the inner layers. Hilsch [3] supported the theory put forward by Ranque [1,2]

Table 1
Summary of experimental studies on vortex tubes

Year	Investigator	Dia., D (mm)	P_i, atm (abs.)	Total temperature (°C)		μ_c
				$T_h - T_i$	$T_c - T_i$	
1933	Ranque	12	7	38	−32	–
1947	Hilsch	4.6	11	140	−53	0.23
1950	Webster	8.7	–	–	–	–
1951	Scheper	38.1	2	3.9	−11.7	0.26
1956–7	Hartnett and Eckert	76.2	2.4	3.5	−40	–
1956	Martynovskii and Alekseev	4.4/28	12	–	−65	–
1957	Scheller and Brown	25.4	6.1	15.6	−23	0.506
1958	Otten	20	8	40	−50	0.43
1959	Lay	50.8	1.68	9.4	−15.5	0
1960	Suzuki	16	5	54	−30	1
1960	Takahama and Kawashima	52.8	–	–	–	–
1962	Sibulkin	44.5	–	–	–	–
1962	Reynolds	76.2	–	–	–	–
1962	Blatt and Trusch	38.1	4	–	−99	0
1965	Takahama	28/78	–	–	–	–
1966	Takahama and Soga	28/78.	–	–	–	–
1968	Vennos	41.3	5.76	−1	−13	0.35
1969	Bruun	94	2	6	−20	0.23
1973	Soni	6.4/32	1.5/3	–	–	–
1982	Schlenz	50.8	3.36	–	–	–
1983	Stephan et al.	17.6	6	78	−38	0.3
1983	Amitani et al.	800	3.06	15	−19	0.4
1988	Negm et al.	11/20	6	30	−42	0.38
1994	Ahlborn et al.	18	4	40	−30	–
1996	Ahlborn et al.	25.4	2.7	30	−27	0.4
2001	Guillaume and Jolly III	9.5	6	–	−17.37	0.4
2003	Saidi and Valipour	9	3	–	−43	0.6
2004	Promvonge and Eiamsa-ard	16	3.5	–	33	0.33
2005	Promvonge and Eiamsa-ard	16	3.5	25	30	0.38
2005	Aljuwayhel et al.	19	3	1.2	−11	0.1

Note: P_i = inlet pressure before nozzle.

stating that air in the cold stream expands from high pressure near the wall to low pressure at the core and in the process transfers a considerable part of its kinetic energy to the outer layers by internal friction. This tends to establish a constant angular velocity throughout the cross-section of the tube.

Following Hilsch, a theoretical study was made by Kassner and Knoernschild [69] who derived the laws of shear stress in circular flow and applied the results to the vortex tube. They hypothesized that initially in the vortex tube a free vortex (tangential velocity $\propto 1/r$) is formed with the corresponding pressure distribution which causes a temperature distribution corresponding to an adiabatic expansion leading to a low temperature in the region of lower pressure, near the vortex tube axis. Due to shear stresses, the nature of flow down the tube slowly changes from a free to forced vortex (tangential velocity $\propto r$). This change from a free to forced vortex starts from the boundaries, i.e., at the axis and at the walls and causes a radially outward flow of kinetic energy. In addition, turbulent transport in the presence of a strong radial pressure gradient results in a temperature profile, which almost complies with the adiabatic temperature distribution corresponding to the pressure distribution of a forced vortex. Energy transport along this temperature gradient causes even lower temperatures in the core. This is the most widely favoured explanation of the Ranque effect [70,71].

Webster [72] suggested that outward energy transfer from any given point in the swirling mass occurred in the manner of a recoil reaction to the inward expansion of the gas at that point. This view was rejected by many investigators, including Fulton [68], who presented his own version. Fulton [68] argued, like Ranque [1,2] and Hilsch [3], that the energy separation resulted from the exchange of energy between the air near the axis with a high angular velocity and the air at the periphery with a low angular velocity: the air near the axis tends to accelerate the outer air. He calculated that the ratio of centrifugal kinetic energy flux to centripetal heat flux for a free vortex was twice the turbulent Prandtl number and predicted a lower performance of the vortex tube for gases with low Prandtl numbers assuming negligible radial and axial velocity gradients. The performance of an actual vortex tube was about twice that predicted by his analysis and led Fulton to conclude that some of his simplifying assumptions were erroneous. He also suggested a shape for the flow pattern inside the tube. Scheper [23] formulated, following his measurements, a theory based on forced convection heat transfer from the core to the walls in a way similar to a double pipe heat exchanger. The heat transfer coefficient calculated on the basis of his data was 286 BTU/h ft^2 °F. The static temperature gradients necessary to transport heat were very small and not uniform at all axial stations. This theory was criticized and rejected by Fulton [73] for the lack of a proper explanation. Van Deemter [74] independently reached conclusions similar to those of Fulton [68]. He indicated that the discrepancy between the actual performance of a vortex tube and that predicted by Fulton [68] was due to incorrect estimation of the turbulent heat flux. He applied an extended Bernoulli equation to the vortex flow and predicted the temperature profiles based on various assumed velocity profiles and found some agreement with the experimental results of Hilsch [3] by introducing an additional term in the equation of energy to account for the effect of turbulent mixing.

Hartnett and Eckert [25,26] showed a simple model based on turbulent rotating flow with solid body rotation gives a temperature difference between the tube walls and the axis, which is somewhat higher but still close to their experimental values. They attributed this disagreement between the theoretical and experimental values to the axial velocities which were neglected in their simple model. They also reported that the static temperature gradient increased radially towards the walls. Deissler and Perlmutter [75,76], like other investigators, considered an axially symmetrical model in which the tangential velocity and temperature were independent of the axial position. They divided the vortex into a core and an annular region, each with a different but uniform axial mass velocity. Based on their analytical studies they concluded that the turbulent energy transfer to a fluid element is the most important factor affecting the total temperature of a fluid element. The agreement between the prediction and the experimental results of Hilsch [3] was close for overall energy separation, despite their reservation about the assumption of an axially symmetrical model. They also introduced a new parameter, the turbulent radial Reynolds number, to characterize the velocity and temperature distribution, and since it could not be estimated directly, they used instead the ratio of radial to tangential velocity at a reference radius as a parameter. It should be noted that this parameter was adjusted in order to fit experimental data and is similar to the approach of Van Deemter [74].

Lay [77,78] suggested that the assumption of constant axial velocity by Deissler and Perlmutter [60] and in his analytical model of the vortex tube was not based on any experimental data and needed verification. Lay's model consisted of a free vortex superimposed with radial sink flow and a constant axial velocity. Based on these studies, Lay presented calculations for the optimum size of the cold orifice although performance calculations for the general case were not possible. Suzuki [79] deduced the presence of large radial velocities based on his observation that the core consisted of a forced vortex and the annular region a free vortex. Sibulkin [80] replaced the steady three-dimensional flow problem by an unsteady, two-dimensional (2D) problem by replacing the axial coordinate with time. He neglected the axial and radial shear forces and his model qualitatively agreed with the experimental results of Lay [77,78] and Scheper [23]. Reynolds [81] performed numerical analysis of a vortex tube. A detailed order-of-magnitude analysis was used for the various fluxes appearing in the turbulent energy equation and the prediction was compared with his measurements. He concluded that the thermal and mechanical energy fluxes were the most significant. Lewellen [82] combined the three Navier–Stokes equations for an incompressible fluid in a strong rotating axisymmetric flow with a radial sink flow and arrived at an asymptotic series solution. Linderstrom-Lang [83] examined analytically the velocity and thermal fields in the tube. He calculated the axial and radial gradients of the tangential velocity profile from prescribed secondary flow functions on the basis of a zero-order approximation to the momentum equations developed by Lewellen [84] for an incompressible flow. The total temperature distribution in the axial and radial directions was also computed from the secondary flow functions and corresponding tangential velocity results, on the basis of an approximate turbulent energy equation. The results obtained agreed qualitatively with measurements.

Kurosaka [85] studied analytically the Ranque–Hilsch effect and demonstrated that the acoustic streaming induced by orderly disturbances with the swirling flow were an important cause of the Ranque–Hilsch effect. He showed analytically that the streaming induced by the pure tone, a spinning wave corresponding to the first tangential mode, deformed the base Rankine vortex into a forced vortex, resulting in total temperature separation in the radial direction. This was confirmed by his measurements in the uni-flow vortex tube. Schlenz [44] investigated numerically the flow field and the process of energy separation in a uni-flow vortex tube. Calculations were carried out assuming a 2D axisymmetric compressible flow and using the Galerkin's approach with a zero-equation turbulence model to solve the mass, momentum, and energy conservation equations to calculate the flow and thermal fields. The calculations failed to predict the velocity and temperature profiles in the tube but agreed qualitatively with the measurements of Lay [77,78]. A numerical study of a large counter-flow vortex tube with short length was conducted by Amitani et al. [45]. The mass, momentum and energy conservation equations in a 2D flow model with an assumption of a helical motion in the axial direction for an invicid compressible perfect fluid were solved numerically. They reported a good agreement of predictions with their measurements and concluded that in radial flow in a vortex tube compressibility is essential to temperature separation.

Stephan et al. [86] formulated a general mathematical expression for the energy separation process but this could not be solved because of the complicated system of equations. The system of equations formulated led, however, to a similarity relation for the prediction of the cold gas temperature that agreed with the similarity relation obtained by the dimensional analysis [46]. Experiments with air, helium, and oxygen as working fluid confirmed that theoretical consideration and agreed well with the similarity relation. Dimensional analysis was also used by Negm et al. [47] who found that for similarity of tube geometry, the inside tube diameter was the main parameter, and this was confirmed by their experimental measurements. The correlation obtained from the analytical and experimental results was used to predict the overall cooling performance of vortex tubes. Balmer [87] who investigated theoretically the temperature separation phenomenon in a vortex tube, used the second law of thermodynamics to show temperature separation effect with a net increase in entropy is possible when incompressible liquids are used in the tube. This was confirmed by experiments with liquid water which showed that temperature separation occurred when an inlet pressure was sufficiently high. Nash [88] analysed the thermodynamics of vortex expansion and evaluated the design limitations of vortex tubes to enhance the tube design and carried out experiments with the enhanced designs, including applications in both high and low-temperature cryogenic refrigeration systems. Borissov et al. [89] examined analytically the flow and temperature fields in a vortex tube using a model based on the analytical solution of complex spatial vortex flow in bounded regions, and based on an incompressible flow approximation to yield the three components of velocity for the complex flow structure with a helical vortex. The velocity values were introduced into the energy equation in which only the convective heat transfer due to complex topology of hydrodynamic field was considered. The predicted temperature field was in qualitative agreement with the measured.

Ahlborn et al. [50] developed a two-component model to determine the limits for the increase and the decrease in temperature within the standard vortex tube. They showed that experimental data with air as working fluid were within the calculated limits and that the flow inside the tube was always subsonic. Gutsol [56] discussed the existing theories of the Ranque effect and a new approach to the vortex effect was formulated, which provided an unfired explanation of experimental data. Gutsol and Bakken [90] studied the efficiency of thermal insulation of microwave-generated plasma using reverse vortex flow by the way of experimental and numerical simulations. They concluded that this effect would take place due to radial motion of turbulent micro-volumes with differing tangential velocities in the strong centrifugal field. Cockerill [91] studied the vortex tubes for use in gas liquefaction and mixture separation as applied to uranium enrichment in order to determine the basic performance characteristics, the relationship between cold air temperature, hot air temperature, and cold mass fraction, and the variation of the hot discharge tube wall temperature with a hot tube length. Cockerill also reported a mathematical model for the simulation of a compressible turbulence flow in a vortex tube. Frohlingsdorf and Unger [92] studied on the phenomena of velocity and energy separation inside the vortex tube through the code system CFX with the k–ε model. Promvonge [93,94] introduced a mathematical model for the simulation of a strongly swirling compressible flow in a vortex tube by using an algebraic Reynolds stress model (algebraic stress model—ASM) and the k–ε turbulence model to investigate flow characteristics and energy separation in a uni-flow vortex tube. It was found that a temperature separation in the tube exists and predictions of the flow and temperature fields agree well with measurements [25,26]. The ASM yielded more accurate prediction than the k–ε model. Behera et al. [95] investigated the effect of the different types of nozzle profiles and number of nozzles on temperature separation in the counter-flow vortex tube using the code system of Star-CD with 'Renormalization Group' (RNG) version of the k–ε model. Aljuwayhel et al. [96] reported the energy separation and flow phenomena in a counter-flow vortex tube using the commercial CFD code FLUENT and found that the RNG k–ε model predicted the velocity and temperature variations better than the standard k–ε model. This is contrary to results of Skye et al. [97] claimed that for vortex tube's performance, the standard k–ε model performs better than the RNG k–ε model despite using the same commercial CFD code FLUENT. Some of these investigators tried to employ higher-order turbulence models but they could not get converged solutions due to numerical instability in solving the strongly swirling flows.

The application of a mathematical model for the simulation of thermal separation in a Ranque–Hilsch vortex tube was reported by Eiamsa-ard and Promvonge [98,99]. The work had been carried out in order to provide an understanding of the physical behaviours of the flow, pressure, and temperature in a vortex tube. A staggered finite volume approach with standard k–ε model and an ASM with (Upwind, Hybrid, SOU, and QUICK schemes), was used to carry out all the computations. The computations showed that results predicted by both turbulence models generally are in good agreement with measurements but the ASM performs better agreement between the numerical results and experimental data. Finally, the numerical computations with selective source terms of the energy equation suppressed [99] showed that the diffusive transport of mean kinetic energy had a substantial influence on the maximum temperature separation occurring near the inlet region. In the downstream region far from the inlet, expansion effects and the stress generation with its gradient transport were also significant. Most of the computations found in the literature used simple or first-order turbulence models that are considered unsuitable for complex, compressible vortex-tube flows.

6. Observations

6.1. Experimental work

In the past experimental investigation of vortex tubes, it was divided into two main categories. The first consists of parametric studies of the effects of varying the geometry of the vortex tube components on the tube performance. The second is focused on the mechanism of energy separation and flow inside the vortex tube by measuring the pressure, velocity and temperature profiles at various stations between the inlet nozzle and the hot valve. This category mostly is concentrated on the operating condition, $\mu_c = 0.0$ by using a uni-flow vortex tube in which the tube is blocked at the cold orifice position and all the air leaves through the hot valve. The effective parameters on temperature separation in the vortex tube can be separated into two groups, the geometrical and thermo-physical parameters. The observation of both parameters can be drawn as follows:

- The increase of the number of inlet nozzles leads to higher temperature separation in the vortex tube.
- Using a small cold orifice ($d/D = 0.2$, 0.3, and 0.4) yields higher backpressure while a large cold orifice ($d/D = 0.6$, 0.7, 0.8, and 0.9) allows high tangential velocities into the cold tube, resulting in lower thermal/energy separation in the tube.

- Optimum values for the cold orifice diameter (d/D), the angle of the control valve (ϕ), the length of the vortex tube (L/D), and the diameter of the inlet nozzle (δ/D) are found to be approximately $d/D \approx 0.5$, $\phi \approx 50^\circ$, $L/D \approx 20$, and $\delta/D \approx 0.33$, respectively, which are expected to be fruitful for vortex tube designers.
- The inlet gas pressure should be 2 bar (for optimal efficiency) while the higher inlet pressure is due to high temperature separation. Inlet gas with helium gives higher temperature difference than those found from the oxygen, methane, and air.

6.2. Theoretical, analytical, and numerical work

Most of the past work efforts based on theoretical and analytical studies have been unsuccessful to explain the energy separation phenomenon in the tube. Also, a few attempts of applying numerical analysis to the vortex tube (see Table 2) have failed to predict the flow and temperature fields due to the complexity of the flow and energy separation process inside the tube. The failure of those calculations of vortex-tube flows was due to the choice of oversimplified models to describe the flow. In view of the recently computational work, the use of various turbulence models in predicting the temperature separation such as the first-order or the second-order turbulence models, leads to fairly good agreement between the predicted and the experimental results better than those found in the past decades, especially for using the second-order turbulence model.

Table 2
Summary of numerical studies on vortex tubes

Investigators	Flow considered	Model	Method or software used	Results compared with measurements
Linderstrom-Lang (1971)	Incompressible	Zero-equation	Stream-function	Poor but just trend
Schlenz (1982)	2D compressible	Zero-equation or mixing length	Galerkin's technique	Poor but qualitative trend
Amitani et al. (1983)	2D compressible	Neglected	Finite difference	Fair but assumptions in doubt
Borissov et al. (1993)	Incompressible	–	Velocity field induced by helical vortex	Qualitative agreement
Guston and Bakken (1999)	2D compressible	k–ε model	FLUENT^{TM} code	Fairly good
Frohlingsdorf and Unger (1999)	2D compressible	k–ε model	CFX code	Fairly good
Promvonge (1999)	2D compressible	ASM and k–ε model	Finite volume	Good
Behera et al. (2005)	3D compressible	k–ε and RNG k–ε models	Star-CD code	Fairly good
Aljuwayhel et al. (2005)	2D compressible	k–ε and RNG k–ε models	FLUENT^{TM} code	Fairly good
Skye et al. (2006)	2D compressible	k–ε and RNG k–ε models	FLUENT^{TM} code	Fairly good
Eiamsa-ard and Promvonge (2006)	2D compressible	ASM and k–ε model	Finite volume	Good

Note: 2D: two-dimehnsional; 3D: three-dimensional.

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