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Review of the development of Cap Models for geomaterials

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Abstract. The history of the development of Cap constitutive models is reviewed. The Cap family of models provides a powerful, yet adaptable way of representing many aspects of the dynamic stress-strain behavior of geological materials. These models have been extensively used for more than three decades to characterize the highly nonlinear behavior of soils, rocks and concrete, and are particularly well suited to the dynamic analysis arising in ground shock and seismic applications. The modern series of Cap Models is based on the adaptation of several earlier models, and was introduced in the early 1970's as a result of university and corporate R&D technology development sponsored by the United States government. Dr. Eugene Sevin played a role in these activities during his tenure at the Defense Nuclear Agency. In this paper, the basic behavior of the early models is briefly discussed and compared, and the reasons for the introduction of Cap Models are outlined. Many adaptations of the Cap Model have been developed since the first model was introduced, and the salient features of some of these model extensions are also reviewed.

Keywords: Dynamics, geological materials, constitutive models, plasticity, compaction, wave propagation, failure

1. Introduction

The modern family of Cap Models was introduced in the early 1970's as a result of government-sponsored university and corporate R&D technology development. Most of the support for this development effort was provided by the Defense Atomic Support Agency, which later became the Defense Nuclear Agency, and is now known as the Defense Threat Reduction Agency. The multi-pronged effort involved experimental work, as well as theoretical development and numerical analysis. As a consequence of one of his areas of responsibility at Defense Nuclear Agency, Dr. Eugene Sevin began to play a significant role in supporting, reviewing and evaluating this work. The objective was to arrive at a theoretically sound modeling procedure that could be used in large-scale computer analysis of ground shock. This pertained to the "free-field" ground shock emanating from detonations of nuclear or high explosive weapons, as well as to the response of structures to the ground shock loading to which they were subjected as a result of "structure-medium interaction". In later work, the Cap Model was applied to structure-medium-interaction for seismic applications.

The equations governing the dynamics of deformable media include, of course, the conservation laws of physics (conservation of mass, momentum and energy). These equations, which apply to all materials, involve more unknowns than equations, so that additional equations are needed before solution is possible. These additional equations, called constitutive equations, must describe the behavior of the material in terms of the relationship between stresses and strains at any point. The set of constitutive equations describing the behavior of any particular substance is said to be a mathematical material model for that substance, and the Cap Model is one particular way in which constitutive behavior can be represented mathematically.

The approach embodied by the Cap Model is based on the theory of plasticity, and is an outgrowth of several earlier plasticity-based models. The basic Cap Model has been extended and expanded over the years, so that it actually represents a family of models. Several examples of these model extensions and their salient features are briefly discussed below, and some of the opportunities for future extension of the models to new applications are

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indicated. Although the Cap Models have been used primarily for geological materials such as soils, rocks and concrete, as described in Sandler and Baron [10], they are also capable of representing many other types of materials. In particular, it may be possible to utilize the results of the existing body of Cap Model research for application to new composite materials that are of current commercial interest.

2. Development of the cap models

The basic Cap Model was first introduced when Drucker, Drucker et al. [4] utilized the concept of a movable cap, and added it to an otherwise ideally plastic model. The role of the cap was to enhance the model in such a way as to include a representation of the phenomenon of soil compaction (irreversible decrease in volume under pressure). In the 1960's Roscoe and his coworkers at Cambridge University in England adopted the cap feature, including it in the Critical State Model, as described by Schofield and Wroth [11]; this model is sometimes referred to as the Cam-Clay model. In addition, a group of researchers at the Massachusetts Institute of Technology also worked on a similar model, Christian [1].

The modern series of Cap Models was introduced by DiMaggio and Sandler [2] working at Columbia University and Weidlinger Associates, respectively. It should be acknowledged that several valuable contributions to the overall model development effort were made by a number of other individuals, most notably Prof. H. Bleich, also of Columbia University.

At the most fundamental level, the Cap Models were developed in order to predict the effects of nuclear and/or conventional weapons on the response of fixed hardened structures, as well as on relatively weak industrial-type structures. This effort was also aimed at determining the effectiveness of weapons in producing craters and ground motion due to ground shock. Such analysis requires advanced computational capabilities across a broad spectrum of disciplines, including dynamic analysis, structural response, constitutive modeling, numerical analysis, and coupled soil- and possibly fluid-structure interaction. Also, a considerable amount of testing, in both the laboratory and in the field, was conducted in order to evaluate, refine and validate the models and the analysis procedures utilizing them. A few years later, some of the results of the research effort were applied to earthquake-related analysis, as reported in Isenberg, Isenberg et al. [5].

Because Cap Models utilize the classical plasticity approach, the yield condition is their defining characteristic. The yield condition is an equation that sets a limit on the on the stress that can be supported by the material while it deforms elastically (i.e., reversibly). Once the yield condition is reached the material can respond in an inelastic manner under further deformation. In many cases, the yield condition can be described as a surface in a "stress space", whose coordinate axes represent components of stress; in stress space each point corresponds to a possible state of stress at a physical point in the material. Such a graphical representation of a yield condition is called a yield surface.

The yield surface used in the Cap Model is shown in Fig. 1 below. To keep this review in the simplest possible technical terms, this figure shows a two-dimensional simplified schematic representation in which the coordinates represent the values of two of the invariants of the stress tensor at any point in the material. The abscissa in the figure is the pressure p, the negative of the mean stress (which, in turn, is one-third of the first invariant of the stress tensor), while the ordinate is, roughly speaking, the "shear stress" or, more precisely, the square root of the second invariant of the deviatoric stress tensor (often denoted by the symbol J_2'). This figure may be thought of as a two-dimensional projection of a full six-dimensional "stress space", in which each point represents a possible stress state in the material.

As shown in the figure, the yield surface is composed of a failure envelope together with a movable cap that intersects the pressure axis. An associated flow rule is used, i.e., the plastic strain rate tensor is proportional through a scalar factor to the gradient (in stress space) of the function defining the yield surface. Geometrically speaking, this means that the components of plastic strain rate form a "vector" parallel to the normal to the yield surface in stress space. As in all basic plasticity formulations, the stress, as represented by a point in the figure, can never lie outside the yield surface. Therefore, in this case, three different types of behavior are possible:

Elastic, when the stress point lies within the yield surface,
Failure, when the stress point lies on the failure envelope, and
Cap, when the stress point lies on the cap.

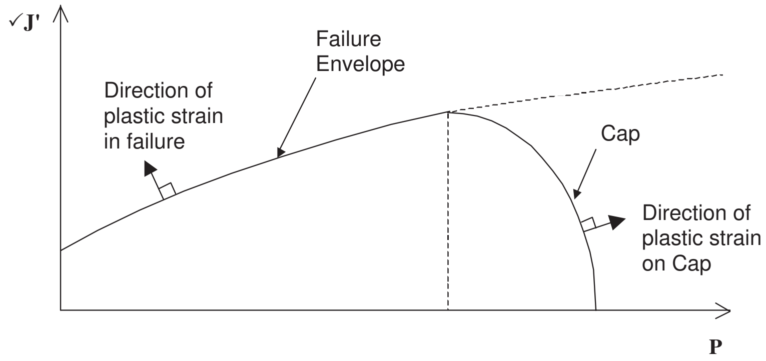

Fig. 1. Yield surface in the Cap Model.

Elastic behavior occurs whenever the stress is within the yield surface. In terms of the figure, this means that the point representing the stress state lies below the failure envelope and to the left of the cap. By definition, an elastic response always results in recoverable deformations; however, in specific applications of the model, the behavior within the yield surface has been generalized to include various forms of nonlinearly elastic behavior, and a viscoelastic representation has also been used. In some advanced versions, the elastic moduli defining this regime of behavior at each point of material are made dependent on the history of plastic deformation at the point.

During the failure mode of behavior, the stress point lies on the failure envelope. The general role of the failure envelope is to limit the level of shear stress that the material can support without failure. In the usual case, this portion of the yield surface acts ideally plastically, e.g., it is fixed in stress space. Again, however, this has been generalized over the years to introduce isotropic and/or kinematic hardening into the model when the need arose. As the figure indicates, the use of an associated flow rule, which requires that the plastic strain rate vector be directed normally to the yield surface in stress space, implies that the plastic strain be directed upward and leftward when the stress point lies on the failure envelope. Therefore, the plastic strain during failure is composed of an irreversible deviatoric (shear) component together with a volumetric (dilatant) component.

The use of the associated flow rule in this regime has often been questioned on the basis of experimental data, and various non-associated versions have also been used over the years. Also, softening (shrinking) failure envelopes have been proposed and used as well. When used in a rate-independent formulation, both of these approaches are problematic on theoretical grounds; from a purely mathematical point of view, they lead to a set of governing equations for dynamic problems that is not properly posed. Physically, this means that solutions implied by such models are (at best) unduly sensitive to small changes in initial and/or boundary conditions. (In fact, these solutions may not actually exist, and if they do, they may not be unique). Since the precise detailed physical data in any actual event is never perfectly well known, such models can not be confidently applied to any practical analysis, whether it be for design evaluation or prediction of tests.

(Nevertheless it is indeed possible to ameliorate the theoretical shortcomings of certain types of non-associated formulations through the incorporation of rate-dependent, or even non-local or so-called non-simple, behavior. Doing this successfully, however, requires a good deal of skill, and also requires the availability of appropriately relevant material property data. Additional details concerning such models are well beyond the scope of this paper).

Finally, the Cap mode of behavior occurs when the stress point lies on the movable cap and pushes it outward, i.e., to the right in the figure. The motion of the cap is related to the plastic volumetric strain through the use of a hardening rule. There is considerable leeway in the choice of the size and shape of the cap as well as the hardening rule. As the figure shows, the associated flow rule requires that the plastic strain rate vector be directed upward and rightward. Therefore, the plastic strain during this mode of behavior is composed of a shear component together with a negative volumetric component, which represents the permanent compaction often observed in geological materials.

The cap does not move during purely elastic deformation. However, when the stress point lies on the failure envelope alone, the behavior of the cap depends upon the resultant amount of dilatancy, or plastic volumetric strain (expansion). In this case the cap can move backward (leftward in the figure). This movement of the cap must stop if and when the Cap reaches the stress point (in which case the stress will lie at the corner of the yield surface, i.e., at the intersection of the Cap and the failure envelope).

As indicated earlier, many adaptations of the Cap Model have been developed for particular applications since the original model was introduced. A few specific examples of this are: a) the introduction of viscoelastic response within the yield surface, Sandler and Baron [8], b) the inclusion of hardening (either isotropic and/or kinematic) into the failure envelope, Isenberg Isenberg et al. [5], and Sandler et al. [9], c) the incorporation of the third invariant of deviatoric stress into the yield surface formulation [6], d) the use of a damage parameter to define changes in the yield surface and/or elastic moduli to represent the effects of cracked material, Mould et al. [7], and e) the utilization of rate-dependent yield surfaces, Mould et al. [7], and DiMaggio and Sandler [3]. Additional adaptations, possibly involving effects such as temperature dependence, are also possible. Given the relatively strong affect of temperature on the behavior of many of the artificial component materials used in the construction of composites, this could be an important area of future Cap Model development.

As indicated above, aside from its capacity to represent observed constitutive behavior, the Cap Model is designed to satisfy certain theoretical requirements. These requirements ensure that the initial and boundary value problems based on the constitutive model, together with the basic physical laws (e.g., conservation of mass, momentum and energy), be properly posed, i.e., that such problems have solutions that exist, are unique and depend continuously on the initial and boundary conditions. Such seemingly abstract requirements are of considerable practical importance for computational analysis. Because all numerical solutions are subject to errors, however small, any solution that is unduly sensitive to such errors is highly suspect. When properly formulated, the Cap Model satisfies the theoretical requirements to preclude such sensitivities. As such, it provides an ideal vehicle for modeling constitutive behavior in large-scale numerical analyses.

3. Conclusion

An outline has been presented of the development of the Cap family of constitutive models. These models, which were introduced in the early 1970's as a result of government-sponsored university and corporate R&D technology, provide a flexible tool for representing the many diverse aspects of the observed dynamic stress-strain behavior of geological materials. They have been successfully used for more than three decades to characterize soils and rocks, as well as concrete.

Many adaptations of the Cap Model have been developed, and the models continue to find application in several military and commercial engineering areas. Although they have been used primarily for geological materials such as soils, rocks and concrete, the Cap Models are also capable of representing may other types of materials, such as synthetics.

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