## Revised depolarization corrections for atmospheric extinction

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Fröhlich and Shaw<sup>1</sup> have estimated the Rayleigh optical depth of the atmosphere, based on laboratory measurements published up to 1968. They state that "the uncertainty of the present calculations is mainly due to the lack of accurate depolarization data." This is true, but not in the sense they intend.

First, several accurate new measurements have been published since  $1968.^{2-5}$  More importantly, both Fröhlich and Shaw<sup>1</sup> and Hoyt<sup>6</sup> err by excluding the rotational Raman contribution to the optical depth. In fact,<sup>2,3,7-9</sup> the classical formulae for Rayleigh scattering refer to the total molecular scattering. (They also remain true for any part that includes the central line.<sup>9</sup>) Obviously, the extinction optical depth must include the Raman component, as the Raman-scattered photons have come from the incident beam, just as the unshifted photons have. Excluding the Raman wings also excludes the Raman contribution to the extinction cross section.<sup>9</sup>

This matter was discussed at length by Stuart,<sup>7</sup> who points out that the integrated intensity of the rotational Raman lines can appreciably affect the total scattering by strongly anisotropic molecules, "and thereby substantially influence the observed depolarization. Hence, one could fear that all previous depolarization measurements were counterfeit and worthless," just as Hoyt<sup>6</sup> did in referring to "the failure of the earlier experimenters to isolate Rayleigh scattering from Raman scattering. The strongly depolarized Raman lines will ... give systematically high values of the depolarization factor." But Stuart<sup>7</sup> rejects this notion: "However, detailed theoretical investigation shows ... that the previously derived relation between the optical anisotropy and the degree of depolarization observed under the usual experimental conditions is still obtained when molecular rotation is considered." He then proves it with a detailed derivation, due to Cabannes and Rocard,<sup>10</sup> and concludes, "Therefore, the rotation of the molecules is without influence on the intensity and degree of depolarization of the whole scattered radiation, Rayleigh line and pure rotation lines included, in gases . . . . (emphasis added). Fabelinskii<sup>8</sup> gives a similar argument in less detail, and adds that, in the limit of an infinite moment of inertia, the rotational wings would collapse into the Rayleigh core; but in this case there would be no molecular rotation. So the formulae derived<sup>11</sup> for a nonrotating molecule are valid for real ones, when the Raman wings are included in the depolarization measurement.

It is merely an historical accident that portions of what was once called Rayleigh scattering have been parcelled out to later investigators like Raman and Brillouin, leaving Lord Rayleigh's name attached to only the central spectral component. But if one thinks simply of molecular scattering by anisotropic molecules, and asks for the molecular extinction coefficient, confusion should not arise.

The term depolarization ratio is also a possible source of confusion, as several such ratios are used. King<sup>11</sup> uses the depolarization for natural light, the source used in his day, but the laser workers<sup>2–5</sup> report depolarizations for a polarized incident beam. Table I gives various depolarizations in terms of the dimensionless anisotropy factor  $\epsilon = \delta^2/b^2$ , where b is the average polarizability and  $\delta$  is the anisotropic part.<sup>2</sup> King's correction factor<sup>11</sup> is

$$F_K = \frac{6+3\rho_0^t}{6-7\rho_0^t} = \frac{9+2\epsilon}{9},$$

in the present notation. If a measured depolarization (whether or not it includes the Raman region) is first converted to the corresponding value of  $\epsilon$ , which is then substituted into the above equation, no confusion can arise.

Table II gives  $\rho_0^t$  – values for the anisotropic atmospheric gases, according to the modern data.<sup>2–5</sup> These are accurate enough to show the dispersion of the anisotropy,<sup>4,5</sup> which Fröhlich and Shaw<sup>1</sup> neglected. Evidently the values they used are seriously in error, quite apart from the dispersion of the depolarization. The second figures given for Rowell *et al.*<sup>2</sup> are derived from their  $\rho_v^c$  values, and hence are less accurate than the first values, derived from  $\rho_v^t$ . I therefore adopt  $\rho_0^t = 0.0210$  for N<sub>2</sub>, 0.058 for O<sub>2</sub>, and 0.078 for CO<sub>2</sub>. Although CO<sub>2</sub>, which Fröhlich and Shaw<sup>1</sup> neglected, is only 1/30 as abundant as argon, which they included, it contributes nearly half a percent to the total depolarization of air because of its high refractivity and anisotropy. I omit water vapor, because good extinction data can be obtained only in very dry conditions.

The mean value for dry air is  $\rho_0^t = 0.0279$ , which is correct to a few percent. This gives a King factor of 1.0480, which is 3.1% higher Fröhlich and Shaw's value, 2.4% higher than Hoyt's, and only 1.2% lower than was used for the standard tables of Penndorf<sup>12</sup> and Elterman.<sup>13</sup> Ironically, it seems that the newer tables are worse than the older ones, precisely because of the depolarization corrections on which some claims of improved accuracy were based.

The optical depth calculations published by Fröhlich and Shaw<sup>1</sup> are in error by over four times their claimed uncertainty of 0.7%, due to the use of incorrect depolarization factors. Their results might be used, with caution, if their optical depths were first multiplied by 1.031; the accuracy will then suffer slightly outside the visible region, due to neglect of the

Table I.	Depolarization Factors in Terms of $\epsilon = (\delta/b)^2$	
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Spectral region	Incident E-vector normal to scattering plane	Incident light unpolarized
Central or Baulaish Brillouin	$a^{2} - \frac{2}{(180 \pm 4c)}$	$a^{g} = 6\epsilon/(180 + 7\epsilon)$
Total or	$p_{\rm V} = 56/(100 + 4c)$	$p_0 = 00/(100 + 10)$
Rayleigh + Raman	$\rho_{\rm v}^{\rm t}=3\epsilon/(45+4\epsilon)$	$\rho_0^{\rm t}=6\epsilon/(45+7\epsilon)$

Table II.	Depolarization Ratios (p	ι'×	100) for	Atmospheric	Gases
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Gas	Fröhlich & Shaw <sup>1</sup>	Rowell <i>et al.</i> <sup>2</sup> (488 nm)	Alms <i>et al.</i> <sup>4</sup> (515 nm)	Baas & v.d. Hout <sup>5</sup> (515 nm)
N <sub>2</sub>	$1.17 \pm 0.06$	2.14, 2.42	$2.10 \pm 0.04$	$2.00 \pm 0.06$
0,2	$1.4 \pm 0.1$	5.66, 5.87		5.8 $\pm 0.2$
$\tilde{CO}_2$	•••	7.3, 7.5	$7.97 \pm 0.10$	7.7 $\pm 0.2$

dispersion of anisotropy. However, the model should eventually be revised to include this dispersion.

Finally, if only the unshifted line is observed, as may happen in lidar work with very narrow receiver filtering, the smaller King factor of 1.0120 found from the smaller depolarization (0.00714) of the central line is appropriate. [The King correction  $(F_K^c - 1)$  for the central line is exactly one fourth of that  $(F_K^t - 1)$  for the total.] If only part of the Raman light is included, the depolarization used should include that portion of the Raman light actually used.<sup>9</sup>

The depolarization data now available are so accurate that their uncertainty affects the calculated Rayleigh optical depths by <1%. Thus, the anomalously low extinctions occasionally reported, which Fröhlich and Shaw<sup>1</sup> allude to, must be due to systematic errors of measurement rather than errors in the Rayleigh optical depths. Error sources in extinction measurements were discussed extensively a few years ago.<sup>14</sup>

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## Remote sensing optical properties of a stratified ocean: an improved interpretation

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In the passive remote sensing of ocean properties through measurement of sunlight backscattered from beneath the sea surface, it is important to know the depth from which information is obtained and the effect of stratification. This was first discussed by Gordon and McCluney,<sup>1</sup> who showed that for a homogeneous ocean, the depth above which 90% of the backscattered irradiance originated (the penetration of depth



Fig. 1. Correlation between  $R_s$  and  $\overline{x}_s$  computed using Eq. (3) with  $g(\tau) = 1$  in the lower curve and  $g(\tau)$  given by Eq. (4) in the upper curve (the values of  $\overline{x}_s$  for the upper curve have been multiplied by 10). The solid line corresponds to  $\overline{x}_s = x_h$ .

 $Z_{90}$ ) was the inverse of the downwelling irradiance attenuation coefficient K (or the depth at which the downwelling irradiance falls to 36% of its value at the surface). Gordon<sup>2</sup> then attempted to interpret the subsurface diffuse reflectance  $R_s$ of a stratified ocean in terms of the optical properties of an equivalent homogeneous ocean. Briefly (see Ref. 2 for details),  $R_s$  for a continuously stratified ocean was calculated by Monte Carlo techniques for an ocean in which the backscattering probability B was held constant<sup>3</sup> and the single scattering albedo [ $\omega_0 = b/(a + b)$ , where b and a are, respectively, the scattering and absorption coefficients of the medium] varied with optical depth<sup>4</sup>  $\tau$  according to

$$\omega_0(\tau) = \omega_\infty [1 + \zeta \tau^n \exp(-\epsilon \tau)],$$

where n can be 0 or 1. Figure 1 of Ref. 2 shows some of the variations in b/a resulting from this  $\omega_0(\tau)$ . With  $\zeta > 0$ , b/a has its maximum value at the surface for n = 0, while n = 1 leads to a subsurface maximum at  $\tau = 1/\epsilon$ . The corresponding  $\zeta < 0$  cases yield surface and subsurface minima in b/a. The  $\zeta > 0$  stratifications would be plausible near the discharge of a river carrying large quantities of sediment, while the  $\zeta < 0$  cases would be plausible if the river carried substantial quantities of absorbing material. The reflectance  $R_h$  of a homogeneous ocean can be written<sup>5,6</sup> in terms of a, b, and B (all independent of depth):

$$R_h = f(x), \tag{1}$$

where x = bB/a. The ocean constituents influence  $R_h$  through their effect on x. To interpret the reflectance  $R_s$  of a stratified ocean,  $R_h$  in Eq. 1 is replaced by  $R_s$  and solved for x, which shall be called  $x_h [=f^{-1}(R_s)]$ . It was then hypothesized that the resulting  $x_h$  was approximately the mean of  $x_s(\tau)$  over the penetration depth, i.e.,