Reweighted Random Walks for Graph Matching

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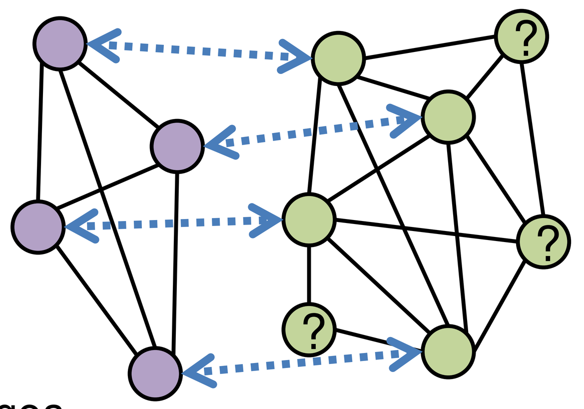
INTRODUCTION

Graph Matching Problem



- Graph Matching for object recognition: Construct a graph using features from a image as nodes, relation between features as edge attributes
- Find the correspondence or mapping between nodes of two graphs which best preserves attributes of both nodes and edges

Motivation

- Generally, numbers of nodes are different for two graphs. Some nodes could be outlier nodes
- Due to object motion or view-point change, relationships between two nodes are not exactly same

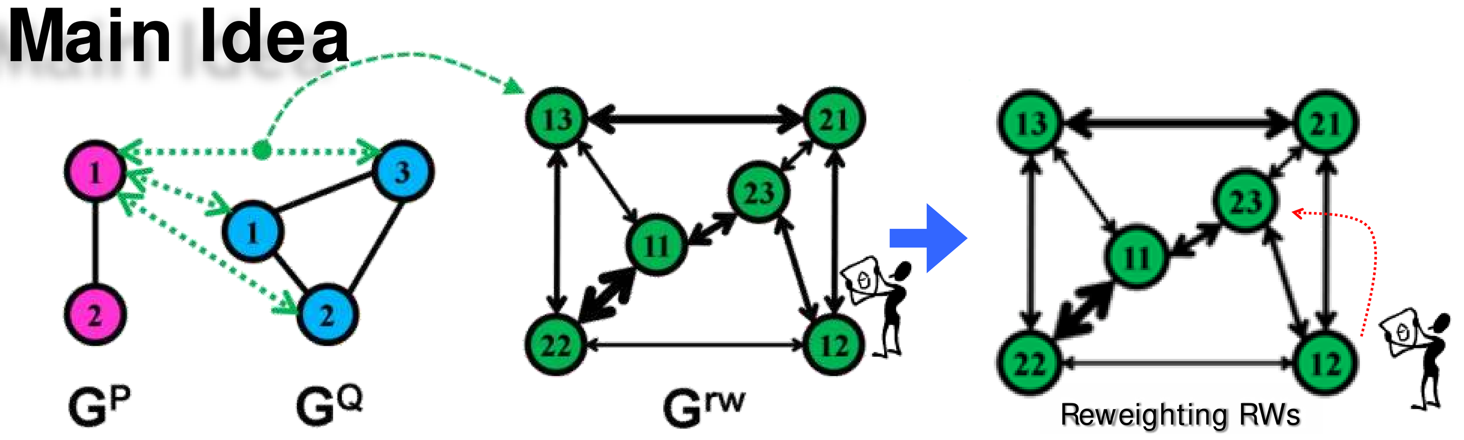
Outlier Noise → Challenging NP-hard Problem ← Deformation Noise

Contribution

- A novel random walk view for graph matching
- A state-of-the-art graph matching method robust to deform & outliers
- Extensive comparison with recent graph matching methods

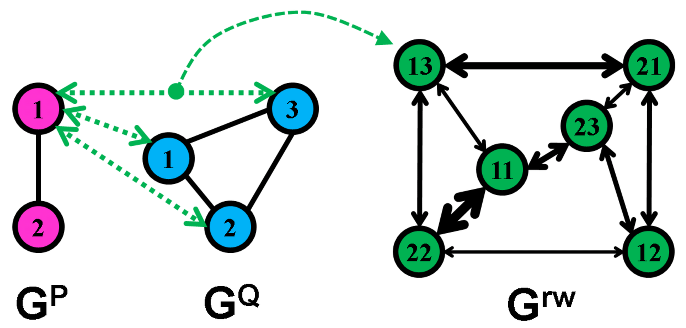
PROPOSED METHOD

Main Idea



- Random walks on an association graph using candidate matches as nodes. Rank candidate matches by stationary distribution
- Personalized jump for enforcing the matching constraints during the random walks process
- Matching constraints satisfying reweighting vector is calculated iteratively by inflation and bistochastic normalization

Association Graph



- Candidate correspondences become nodes in the association graph
- Random walker travels correspondence to correspondence in association graph

Traditional Random Walks

- Traditional random walk approaches convert the affinity matrix to the row stochastic transition matrix

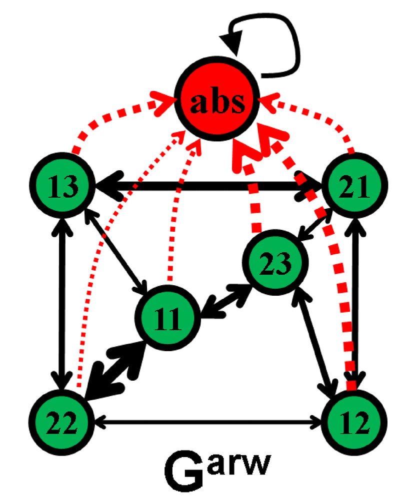
$$\mathbf{D}_{ii} = d_i = \sum_j \mathbf{W}_{ij} \qquad \mathbf{P} = \mathbf{D}^{-1}\mathbf{W} \qquad \mathbf{x}^{(n+1)T} = \mathbf{x}^{(n)T}\mathbf{P}$$

Problematic: Normalization can strengthen the adverse effect of outliers and weak correspondences

- We tested this row-Normalized Random Walk method denoted as **NRWM**

PROPOSED METHOD

Affinity-Preserving Random Walks



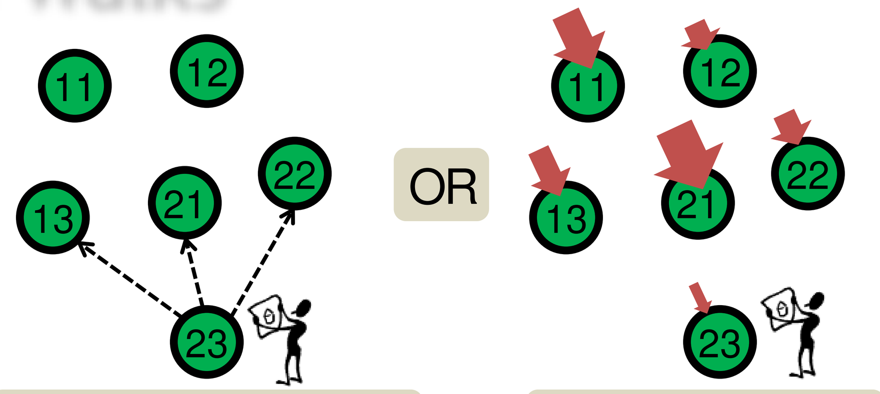
- How to preserve original affinities in the Markov chain?
- **Solution**: A new **Absorbing** node is augmented
- Absorbing node soaks affinity $d_{max} - d_i$ from the node v_i
- A candidate match with more degree has more weight than other candidates

$$\mathbf{P} = \begin{pmatrix} \mathbf{W}/d_{max} & \mathbf{1} - \mathbf{d}/d_{max} \\ \mathbf{0}^T & 1 \end{pmatrix} \qquad \begin{pmatrix} \mathbf{x}^{(n+1)T} & x_{abs}^{(n+1)} \end{pmatrix} = \begin{pmatrix} \mathbf{x}^{(n)T} & x_{abs}^{(n)} \end{pmatrix}\mathbf{P}$$

- Stationary distribution can be acquired by taking principal eigenvector of $\mathbf{W}$
- In our paper, proposed **APRW** is proven to be equivalent with **Spectral Relaxation** of **Inter Quadratic Programming** by Leordeanu & Hebert, *ICCV05*

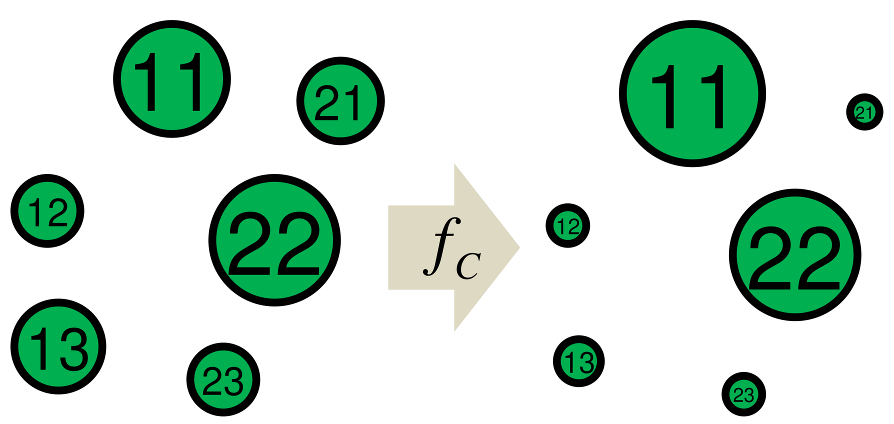
Reweighting Random Walks

- **Problem**: In affinity-preserving random walks, the matching constraints (1-to-1) are ignored
- **Solution**: **Personalized Jump** Haveliwala, Topic-sensitive pagerank, *WWW02*



$$\begin{pmatrix} \mathbf{x}^{(n+1)T} & x_{abs}^{(n+1)} \end{pmatrix} = \alpha \begin{pmatrix} \mathbf{x}^{(n)T} & x_{abs}^{(n)} \end{pmatrix}\mathbf{P} + (1-\alpha)\,\mathbf{r}^T$$

- Make reweighting vector satisfy the matching constraints using current state
- **Inflation**: Strong candidates are amplified while weak candidates are attenuated
- **Bistocastic-Norm**: Make inflated state to satisfy constraints Sinkhorn, Ann. Math. Statistics 64'



$$\begin{pmatrix} \mathbf{x}^{(n+1)T} & x_{abs}^{(n+1)} \end{pmatrix} = \alpha \begin{pmatrix} \mathbf{x}^{(n)T} & x_{abs}^{(n)} \end{pmatrix}\mathbf{P} + (1-\alpha)\begin{pmatrix} f_C(x^{(n)T}\mathbf{W})^T & 0 \end{pmatrix}$$

What f_C does: Inflation ⇄ Bistochastic Normalization

EXPERIMENTS

Project Page Open

- Full results are available: **http://cv.snu.ac.kr/research/~RRWM**
- Source code will be available soon

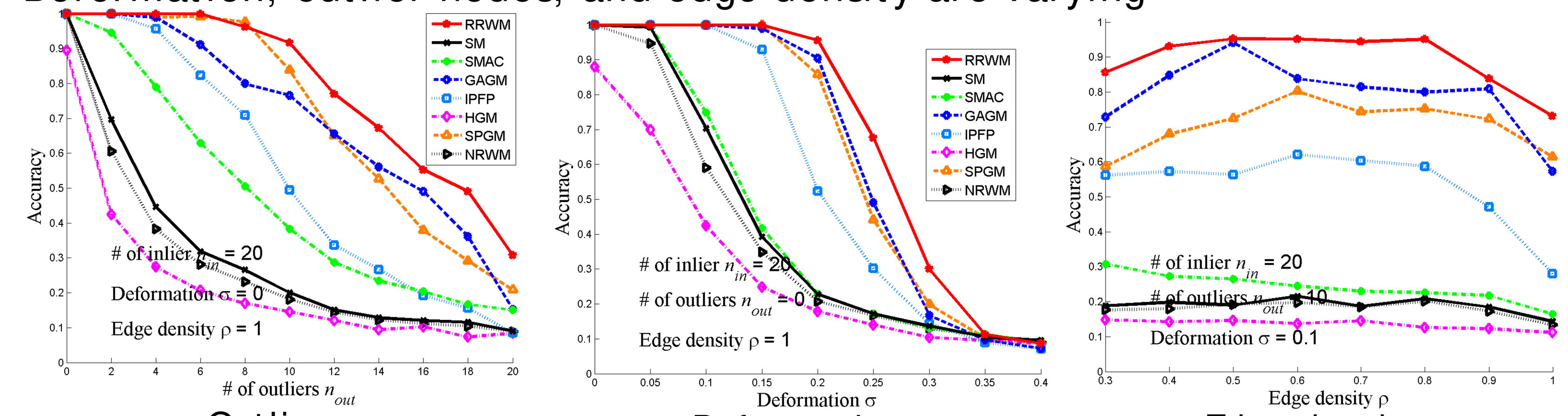
Comparing with Various Methods

- **SM**: Leordeanu & Hebert, *ICCV05*
- **IPFP**: Leordeanu & Hebert, *NIPS09*
- **SMAC**: Cour et al, *NIPS06*
- **GAGM**: Gold & Rangarajan, *PAMI96*
- **HGM**: Zass & Shashua, *CVPR08*
- **SPGM**: Wyk & Wyk, *PAMI04*
- **NRWM**: Conventional row-wise Normalized Random Walk Matching
- **RRWM**: Proposed method, Reweighted Random Walk Matching

EXPERIMENTS

Synthetic Random Graph Matching

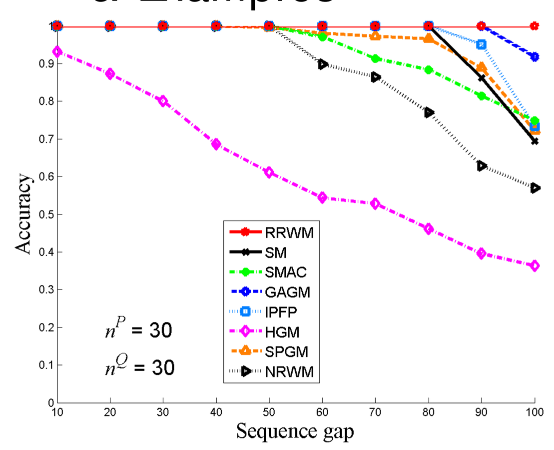
- Generate two graphs with randomly assigned edge attributes
- Pair-wise distance: difference of two edge attributes
- Deformation, outlier nodes, and edge density are varying



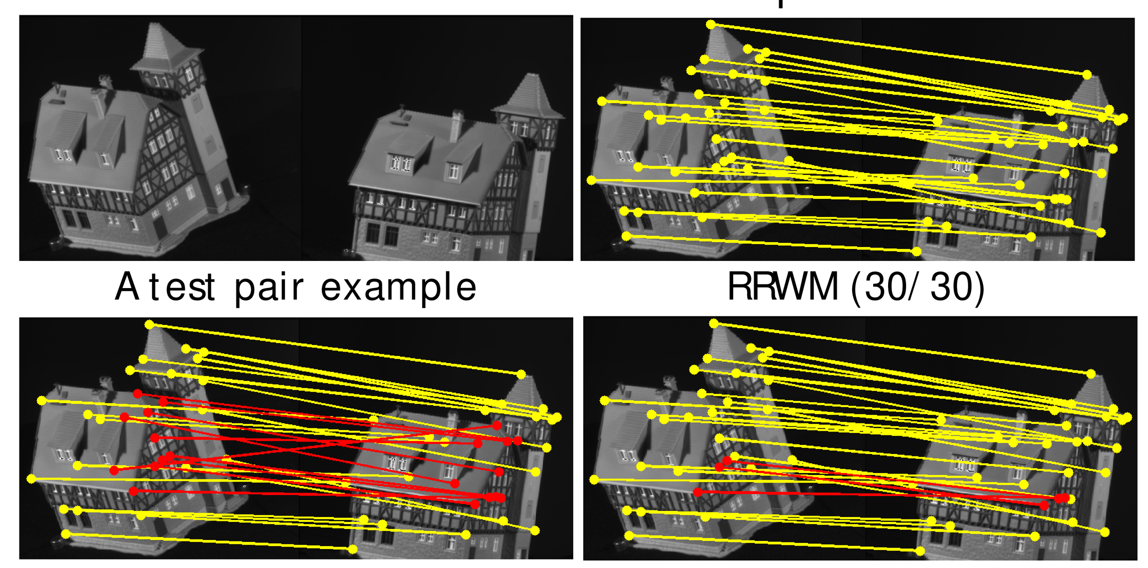
Outlier | Deformation | Edge density

Feature Point Matching across Image Sequences

- CMU house sequence
- 30 pts are manually tracked
- Pair-wise distance: difference of two distances between two points
- Matching Accuracy & Examples



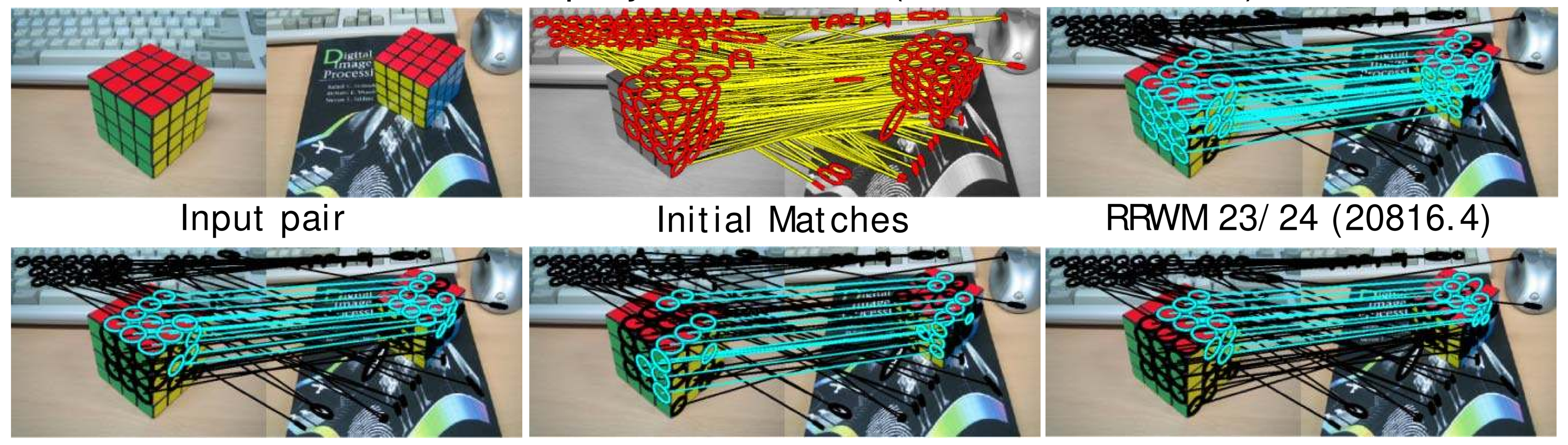
30 pts to 30 pts



A test pair example | RRWM (30/30) | SM (20/30) | GAGM (27/30)

Real Image Matching

- Caltech-101 & MSRC dataset
- MSER detector & SIFT descriptor
- Pair-wise distance: mutual projection error (Cho et al, ICCV09)



Input pair | Initial Matches | RRWM 23/24 (20816.4)
SM 12/24 (17010.9) | SMAC 10/24 (19264.6) | GAGM 10/24 (12466.3)

- Matching performance on the real image dataset (30 pairs)

Methods	RRWM	SM	SMAC	GAGM
Avg. of accuracy (%)	**64.01**	52.08	39.74	58.74
Avg. of relative score (%)	**100**	82.41	59.35	91.13

- More matching examples (Input pair / Initial Matches / Our Result)