RFQ Auctions with Supplier Qualification Screening

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Abstract

We consider a manufacturer using a Request For Quotes reverse auction in combination with supplier qualification screening to determine which qualified supplier will be awarded a contract. Supplier qualification screening is costly for the manufacturer, for example involving reference checks, financial audits, and on-site visits. The manufacturer seeks to minimize its total procurement costs, i.e. the contract payment plus qualification costs. While suppliers can be qualified prior to the auction (pre-qualification), we allow the manufacturer to delay all or part of the qualification until after the auction (post-qualification). Using an optimal mechanism analysis we analytically explore the tradeoffs between varying levels of pre- and post-qualification. While using post-qualification causes the expected contract payment to increase (bids from unqualified suppliers are discarded) we find that standard industrial practices of pre-qualification only can be improved upon by judicious use of post-qualification, particularly when supplier qualification screening is moderately expensive relative to the value of the contract to the buyer.

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1. Introduction

The average U.S. manufacturer spends 40-60% of its revenue income to purchase goods and services (U.S. Department of Commerce 2005). Vital for most companies, the procurement function must negotiate reasonable prices with suppliers and – equally important – it must make reasonable efforts to ensure that contracts are made with *qualified* suppliers who are indeed able to fulfill the contract. Contracting with unqualified suppliers can result in significant costs for the buyer. For example, in the third quarter of 1997, parts shortages contributed to Boeing's \$696M loss (Biddle 1997). In some cases, damage done to customers can be potentially irreversible, particularly in health care; for example, the production shutdown of vaccine supplier Chiron caused flu vaccine shortages in 2004 (Whalen et al. 2004). Vulnerabilities to supplier non-performance deepen for lengthly global supply chains, while price pressures and the myriad of global supply options compounds the procurement manager's challenge of sorting the able suppliers out from the charlatans.

To verify a supplier's qualification, the procurement function must spend time and money vetting suppliers with *qualification screening.* Screening often involves references checks, financial status checks, surge capacity verification, and even site visits to supplier production facilities. Typically, qualification screening precedes price negotiations with suppliers. This is particularly common when buyers use qualification screening before a Request For Quotes (RFQ) auction for a well-defined good or service, a prototypical procurement setting on which we focus this paper. (For details on procurement auctions in practice, see, for example, Jap 2003.) To convince the suppliers to bid aggressively, the buyer touts the fact that only the lowest price will win the contract when all participants in the auction are absolutely qualified to win the contract.

Yet committing to award business to the lowest bidder requires the buyer to spend significant time and resources screening *all* suppliers entering the auction. This paper analyzes how much costly supplier qualification screening should be performed before the auction. At one extreme the buyer uses pre-qualification only, in which he fully screens all suppliers entering the auction and commits to awarding the contract directly to the lowest bidder, so suppliers in an open-descending auction (were this the format used) would bid down to their true cost (but, they may not have to). At the other extreme the buyer uses post-qualification only, in which he screens suppliers only after seeing their bids in the auction; instead of screening all suppliers, the buyer homes in on the most promising bidders and screens them in sequence until finding one who is qualified. But, this comes with a tradeoff: suppliers (assumed strategic) no longer bid down to their true cost because they can potentially win the contract without being the lowest bidder – the winner will be the lowest *qualified* bidder, if any. In this paper we consider these two extremes, and mixtures of the two in which suppliers are partially screened before the auction.

Our study appears to be the first auction model in which contracting is contingent upon passing costly supplier qualification screening, a common feature of RFQs in practice. The central tradeoff of the research problem is this: while delaying some or all qualification screening until after the auction saves the buyer qualification screening costs, doing so increases contract payments and also risks non-transaction (turning to an outside option, such as internal production) if all suppliers in the auction turn out to be unqualified. By optimally balancing these tradeoffs, we find that the buyer can significantly reduce its total procurement cost (qualification cost plus contract payment) by judiciously delaying all or part of supplier qualification screening until after the auction, provided the qualification screening is not prohibitively selective (suppliers stand a reasonable chance of passing qualification) and not too costly (prompting defection to the outside option) nor too cheap (making total pre-qualification the best option). Our analysis endogenizes the level of pre- and post-qualification as well as the negotiation mechanism chosen by the buyer.

Section 2 provides a literature review, and §3 introduces the model and assumptions. Combining classical mechanism design with optimal stopping time problems, the negotiation structure (optimal reserve prices and post-qualification sequence) is derived in §4.1 and behavior of the optimal balance between pre- and post-qualification is characterized in §4.2. Cost savings are explored in §5 and §6 discusses practical considerations and extensions.

2. Literature Review

Many practical decisions faced by procurement managers have been addressed by academic research: contract type, such as fixed price versus cost plus; negotiation framework such as auction versus face-to-face; and competition type such as sole versus dual sourcing. See Elmaghraby (2000) for a detailed survey on procurement studies in economics and operations management. The present paper focuses on a procurement situation in which a buyer holds an auction to award a contract to one of several potential suppliers. There is a sizeable literature on auctions – the books by Krishna (2002) and Milgrom (2004) provide excellent treatments and detailed references – but only a handful of such studies include the processes which occur before and after an auction. Typically these processes seek to mitigate the risk of consummating a transaction in which one or more parties does not obtain what it expected, what we will call *non-performance*. Note that non-performance could describe, for example, an item falling short of the winner's expectations, or a winner failing his obligations to the auctioneer. The supplier qualification process central to this paper – and to our knowledge novel in the literature – is one such process meant to mitigate the risk of supplier non-performance in a procurement auction.

One type of non-performance in forward auctions is the winner's failure to pay. Papers dealing with this issue have looked at the ability of bidders to borrow money and the ensuing possibility of broke winners (Zheng 2001), and the use of deposits or fees forfeited to the auctioneer in the event a winning bid is reneged (Rothkopf 1991, Waehrer 1995). In a procurement auction context, surety bonds (analogous to a bid deposit) to partially offset non-performance costs are examined by Calveras et al. (2004), while Braynov and Sandholm (2003) study a buyer who is unable to directly verify the "trustworthiness" of suppliers, but knowing the form of the suppliers' cost functions can design bidding options which cause each supplier to reveal themselves as either a high or low trustworthy type, allowing the buyer to estimate the expected utility of contracting with that supplier. In contrast, we assume that the buyer verifies (at a cost) that a supplier is qualified up to some threshold prior to contracting (he will not contract with unqualified types). Practitioners we have spoken with use qualification and surety bonds in tandem, the former to proactively avoid problems (the focus and main contribution area of our paper), the latter to partially recoup costs if problems arise.

A second type of non-performance is misevaluation of the item. For example, costly bid preparation (or costly entry, or due diligence) plays a central role in forward auctions for non-standard, complex items such as an entire company or its assets. To encourage participation in auctions where bidders trade off their bid preparation costs (possibly millions of dollars) against their anticipated likelihood of winning the item, Ye (2005) suggests inviting only bidders whose bid in an initial, assumed costless round of bidding signals that they stand a good chance of winning the item. While Ye finds that screening out low value bidders can promote competition in a complex item auction by limiting bidders' unnecessary bid preparation costs, we examine screening costs borne by the auctioneer (buyer) and find that screening out unqualified suppliers promotes competition by increasing the likelihood that each bid in an RFQ auction will be qualified and therefore eligible for contracting.

In our context of relatively well-specified RFQ auctions we assume that a supplier's bid indicates the value offered to the buyer should that supplier be deemed qualified. This allows the buyer to delay all or part of the qualification process until after the auction, at which point – with bids in hand – he can home in on the suppliers offering the highest value (who may or may not turn out to be qualified). Other auction theoretic papers have focused on situations where it is costly for the auctioneer to estimate even the value offered in the suppliers' bids. In such situations, the buyer could employ a sequential search model whereby suppliers are communicated with and their bids evaluated until either finding a supplier whose cost is sufficiently low or exhausting the supply pool (McAfee and McMillan 1988). In an effort to explain unconsummated Request For Proposals auctions documented by Snir and Hitt (2003), Carr (2003) models an auction for professional services where proposals (bids) are difficult to compare; in his model, faced with high evaluation costs after the auction, the auctioneer might simply forego evaluating *any* proposals in favor of an outside option. The buyer sometimes turns to an outside option in our model, but for different reasons – either due to reserve prices, or after disqualifying all suppliers invited to the auction.

Methodologically, our study is also related to the screening literature, in particular studies such as Feinberg and Huber (1996) which assume that some form of screening can be performed cheaply (e.g., bids can be observed) relative to more costly forms of screening (e.g., qualification). In our context, partial qualification screening impacts the extent to which bidders compete in the auction, by creating randomness in the number of qualified bidders in the auction. For auctions with an uncertain number of bidders, optimal mechanisms and equilibrium bid functions have been respectively derived by McAfee and McMillan (1987) and Harstad et al. (1990), but neither studies qualification processes or the attendant possibility of having to turn to an outside option.

Our study is related in spirit to multiple dimensional auctions (Che 1993, Beil and Wein 2003, Chen 2004) in that both seek to take non-price factors into account. Although the goals of a multiple dimension auction and qualification processes are related, they are distinct: multiple dimension auctions serve as tools to better express the value of non-price abilities of suppliers, such as quality. Qualification processes seek to verify the ability of a supplier to deliver on the promises expressed by its bid, be they promises on price or any other dimensions. While we will assume a single dimension (price) auction format to keep our focus on the qualification process, a multiple dimension auction could be used at the expense of more complex analyses (discussed in §6.4).

3. Model

We model a risk-neutral cost-minimizing buyer seeking to award an indivisible contract to a *qualified* supplier. A supplier is called qualified if the buyer is willing to transact with the supplier without performing additional due diligence. Qualification thresholds vary widely in practice depending on the buyer's needs and the contract type (see Leenders and Fearon 1997 for a discussion of purchasing processes). The constituent requirements themselves exhibit varying degrees of standardization. Among the more standard requirements are the need to verify the supplier's reputability (e.g., through published ratings) and ability to ramp up production. Not all requirements are so transparent, as qualification can encompass relational aspects that are difficult to codify; e.g., a just in time manufacturer we spoke with visits with supplier management to ensure they and the supplier "see eye to eye on lean principles" before awarding contracts. These verification processes take time and can be costly, particularly if involving visits to distant supplier facilities. As is common in industry, we will refer to this verification as the *qualification process*, or the act of *qualifying* a supplier. A supplier is described as *qualified* once he successfully passes the qualification process.

To begin formalizing the model, let q_0 be the buyer's *qualification threshold*, a scalar between 0 and 1 capturing the strictness of the pre-award requirements. For each supplier i, we will define its *qualification level* q_i as the maximum qualification threshold that supplier i can pass. If $q_i \geq q_0$, the buyer's qualification process on supplier i would reveal that supplier i is qualified. The cost the buyer would incur to do so is denoted by K, the total cost to the buyer of verifying that an individual supplier meets all requirements to be deemed qualified. On the other hand, if $q_i < q_0$, supplier i

would be rejected during the buyer's qualification process after failing to meet a requirement. In this latter case, how much cost would the buyer incur? Assuming that the requirements are nested (passing threshold q implies passing threshold $q' < q$, but not vice-versa), the cost strictly increases with q_i and approaches K as $q_i \to q_0$. To streamline the exposition we further assume that the cost is linear and normalized such that a threshold of zero costs zero to verify, implying that weeding out an unqualified supplier i costs the buyer $\frac{q_i}{q_0}K$; we discuss non-linear functions in §6.2.

Each supplier i possesses two dimensions of private information. The first is a signal $s_i \in [0,1]$ about their true value of q_i. The information contained in s_i induces a conditional probability distribution over q_i given by $H(\cdot|s_i)$. (For simplicity we will assume that all distributions in this paper are continuous, increasing and differentiable over their domains.) Signals $s_1, s_2, \ldots,$ are i.i.d. random variables distributed according to G, which is common knowledge.

Supplier i's second dimension of private information is its cost to fulfill the contract, x_i, which it observes perfectly prior to the auction. Cost x_i is distributed according to a commonly known distribution F_i on domain $[0,1]$ which for simplicity is assumed statistically independent of other supplier's costs and signals, including s_i. In reality, a supplier likely to be qualified might be expected to have relatively high costs if qualification requires that costly spare capacity be kept on hand for an ability to ramp up production in the face of surge orders from the buyer; on the other hand, lower costs might prevail if qualification includes lean principles which impart efficiency. We leave such issues for future research.

The buyer completely verifies a supplier's qualification before contracting with them, so there is no adverse selection for qualification. In this paper the signals s_i serve the purpose of explicitly modelling the information over qualification, although possible extensions using these signals in the auction mechanism are discussed in §6.4. However, there is adverse selection for supplier costs. The buyer utilizes an auction to extract private cost information from the risk-neutral suppliers, who – as is standard in the auction literature – are assumed to be fully rational players following a Bayesian Nash bidding equilibrium. Bidding is assumed to be costless, as is common in the auction literature. To capture practical demands on the buyer's resources such as auction participant training and technical support, we assume the auction has a finite bidder capacity N. In §4.2 we discuss the case where n, the number of bidders invited to the auction, is chosen optimally subject to $n \leq N$. In the meantime our analysis treats $n \leq N$ as fixed.

We now describe the procurement process the buyer uses to minimize costs associated with supplier qualification screening (qualification checks) and the contract payment associated with supplier cost screening (competitive auction). The procurement process proceeds as follows.

Pre Qualification Stage. The buyer announces the qualification threshold q_0 and a *pre-qualification threshold* $q \leq q_0$. The buyer verifies requirements of supply pool members one by one until finding n suppliers who pass the pre-qualification threshold. (For simplicity we assume an infinite supplier pool; the finite case is an interesting, but complicated extension – see §6.3.)

Auction Stage. The n pre-qualified suppliers participate in a price-only auction (we discuss a potential multiple attribute extension in §6.4). To help minimize contract payment, the buyer employs optimally chosen reserve prices during the auction, as is common in the mechanism design literature (e.g., Myerson 1981). Unlike a traditional optimal auction where the low virtual cost supplier wins the contract, in our optimal auction the lowest virtual cost *qualified* supplier wins the contract (see the Post Qualification Stage description below). Virtual costs account for information rents accruing to suppliers and are defined on p14. When bidding, supplier i estimates the probability a given rival is qualified as

$$\beta = \int_0^1 \frac{1 - H(q_0|s)}{1 - H(q|s)} dG(s). \tag{1}$$

In particular, each supplier believes it faces a Binomial$(n-1, \beta)$ distributed number of qualified rivals.

Post Qualification Stage. After the auction the buyer post-qualifies suppliers up to qualification threshold q_0 in the order of increasing virtual cost until either finding a qualified supplier, quitting, or disqualifying all n suppliers (in §4.1 we prove this is optimal). In the latter two cases, the buyer turns to his outside option at a cost of C_o. For example, C_o could be the cost of in-house production.

The main tradeoffs of the model are rooted in the variable β. If $\beta = 1$, then the buyer performs all due diligence prior to the auction – the situation tacitly assumed in traditional auction theory. In such a case the buyer incurs qualification costs for *at least* n (possibly more if some are rejected en-route to being qualified) suppliers prior to the auction, and awards the contract directly to the lowest virtual cost bidder after the auction. On the other hand, $\beta = \int_{s=0}^{1}[1 - H(q_0|s)]dG(s)$ models postponement of all due diligence; after the auction, the buyer only pays qualification costs until finding the first qualified bidder or turning to his outside option. Clearly the expected pre- plus post-qualification costs are greater for the case when $\beta = 1$. But the buyer also must consider the expected costs of contracting and non-transaction, for which the cost relationship can be reversed – consider the following toy example showing why larger β (more pre-qualification) reduces the expected costs of contract payment and non-transaction in an open descending auction. Suppose supplier i faces existing bids of \$100,000 from supplier j and \$125,000 from supplier k. In order to take the lead in the auction, supplier i must enter a bid below \$100,000. However, the value of β must be considered: if $\beta = 1$, then taking the lead is a prerequisite to winning the contract, and supplier i knows he must bid below \$100,000 in order to have any hope of winning the business. Conversely, suppose $\beta << 1$; in this case, supplier i knows that rivals j and k stand a good chance of being disqualified after the auction. A bid exceeding \$100,000 – or \$125,000 – could potentially still win the contract even though it would not take the lead in the auction. Thus supplier bids are inflated in the open descending auction when $\beta < 1$, but by how much? And what about the risk that all suppliers are disqualified, forcing the buyer to use his costly outside option? As discussed in the next section, precisely capturing the effects of β when designing the buyer's optimal auction mechanism requires extension of existing (Myerson 1981, McAfee and McMillan 1987) optimal mechanism methodology.

For various pre-qualification thresholds (captured by β), the remainder of this paper specifies the buyer's optimal mechanism, quantifies the expected costs of pre-qualification $\overline{\text{PRE}}$, post-qualification $\overline{\text{POST}}$, contract payment $\overline{\text{PAY}}$, and non-transaction $\overline{\text{NT}}$, and then uses these analyses to characterize how the optimal pre-qualification threshold depends on model parameters.

4. Analysis

4.1 Expected Costs Derivations

We begin by deriving $\overline{\text{PRE}}$, the expected cost of the pre-qualification stage. We then move on to deriving $\overline{\text{PAY}}$, $\overline{\text{POST}}$, and $\overline{\text{NT}}$, the expected costs of the auction and post qualification stages.

Pre-Qualification Cost. As explained in §3 we assume an infinite supply pool from which the buyer samples until finding n suppliers passing the pre-qualification stage. The expected pre-qualification cost is comprised of n pre-qualification "successes" plus geometrically distributed numbers of pre-qualification "failures" before each success. Since a supplier with type s passes pre-qualification with probability $1 - H(q|s)$, the unconditional probability of success from the buyer's perspective equals $1 - \int_{s=0}^{1} H(q|s)dG(s)$. Because the qualification cost is linear in the amount of qualification performed, for each success the buyer pays $\frac{q}{q_0}K$ to pre-qualify a supplier with qualification $y \geq q$ up to the pre-qualification threshold q. For each failed pre-qualification on a supplier with qualification level $y < q$, the buyer pays $\frac{y}{q_0}K$. Since y is a random variable, the buyer expects to pay

$$\overline{\text{PRE}} = n\frac{q}{q_0}K + \left[\frac{n}{1 - \int_{s=0}^{1} H(q|s)dG(s)} - n\right] \int_{s=0}^{1} \int_{y=0}^{q} \frac{y}{q_0 H(q|s)} K dH(y|s)dG(s). \tag{2}$$

Costs in the Auction and Post-Qualification Stages. After pre-qualification, the n pre-qualified suppliers compete in an auction preceding post-qualification. For mechanism design purposes, the auction and post-qualification stages comprise a mechanism of awarding a contract to one of n suppliers or to the buyer's outside option. The auction mechanism with post-qualification has four components: a set of possible messages (or "bids") for each supplier; a rule that describes how the buyer sequences the suppliers during post-qualification; a rule that describes how the buyer chooses the maximum number of suppliers to post-qualify from this supplier sequence; and a payment rule mapping bids to an amount transferred between the buyer and sellers. The post-qualification sequencing rule and the max number to qualify rule are the analogues of the allocation rule in standard mechanism design problems absent post-qualification; together they capture the fact that the allocation decision itself is not entirely in the buyer's hands, as post-qualification outcomes (the realization of q_i's) play a role in determining whether or not a post-qualified supplier will actually be qualified for contract award.

All three rules (sequencing, max number to post-qualify, and payments) are functions of the messages sent by suppliers. Thanks to the revelation principle (Myerson 1981), given any mechanism and an equilibrium for that mechanism, there exists an outcome equivalent "direct" mechanism in which it is an equilibrium for each supplier to bid its true cost. This allows us to restrict our optimal mechanism search to direct mechanisms; in what follows the set of messages sent is assumed without loss of generality to be just the supplier cost vector $\vec{x} = (x_1, x_2, \ldots, x_n)$.

After the analysis which follows we find that the sequencing and max number to post-qualify rules are pure strategy (deterministic) plans the buyer announces ex ante to the suppliers in the optimal mechanism. However, for generality in the mechanism design analysis we allow these rules to be mixed strategies (probabilistic) and define them as follows. Let $\Pi : \vec{x} \to [0,1]^{n!}$ be a probability distribution over the $n!$ different sequences of suppliers, and let $\Gamma : \vec{x} \to [0,1]^{n+1}$ be a probability distribution over the $n+1$ possible different maximum number of suppliers to post-qualify (including zero). For a given vector of costs $\vec{x}$, using Π, Γ, and β we can compute Δ_i the probability that supplier i wins the contract (unconditional on s_i):

$$\Delta_i(\vec{x}) = \beta \left[\sum_{a=1}^{n!} \sum_{b=0}^{n} \Pi_a(\vec{x}) \Gamma_b(\vec{x}) (1-\beta)^{z(i,a)-1} I_{z(i,a) \leq b} \right], \tag{3}$$

where Π_a is the a^{th} element of Π, Γ_b is the b^{th} element of Γ, and $z(i,a)$ is supplier i's position in sequence a (for example, if a corresponds to sequence $(6,3,5,7,\ldots,4)$ then $z(3,a) = 2$). Here the bracketed term on the RHS of (3) is just the probability that supplier i is post-qualified by the buyer, where the leading β term is simply the probability that i is qualified (unconditional on s_i).

The expected payment rule, which we will denote as $\mathrm{M} : \vec{x} \to \mathbb{R}^n$, is the expected monetary transfer (unconditional on s_i's) from the buyer to the suppliers. Note that we have so far defined both M_i and Δ_i from the buyer's perspective, that is, they equal the expected transfer amount to, and probability of award to, supplier i based only on the vector of all costs $\vec{x}$. However, from supplier i's perspective – privileged with knowledge of s_i – these quantities are different owing to the fact that supplier i estimates its own probability of being qualified as $\frac{1-H(q_0|s_i)}{1-H(q|s_i)}$ while the buyer (and other suppliers) estimate it as β. Taking this into account, we can define the suppliers' best response functions in order to characterize the equilibrium payments of the buyer. If all suppliers $j \neq i$ report their true cost, supplier i's expected profit from reporting z_i is maximized by a truthful bid of $z_i = x_i$ if

$$x_i \quad \text{solves} \quad \max_{z_i \in [0,1]} \left\{ \frac{1 - H(q_0|s_i)}{\beta(1 - H(q|s_i))} \left[\mathrm{m}_i(z_i) - \delta_i(z_i)x_i\right] \right\} \tag{4}$$

where $\mathrm{m}_i(z_i) = \int_{\vec{x}_{-i}} \mathrm{M}_i(z_i, \vec{x}_{-i})dF_{-i}(\vec{x}_{-i})$ and $\delta_i(z_i) = \int_{\vec{x}_{-i}} \Delta_i(z_i, \vec{x}_{-i})dF_{-i}(\vec{x}_{-i})$. Here $\vec{x}_{-i}$ is $(x_1, x_2, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$ and F_{-i} its distribution. Equation (4) is known as the "incentive compatibility" constraint for supplier i, and must hold for all suppliers in a direct mechanism. Applying the envelope theorem to equation (4) implies that the derivative of supplier i's profit at $x_i \in [0, 1]$ is given by $-\delta_i(x_i)\frac{1-H(q_0|s_i)}{\beta(1-H(q|s_i))}$. Treating this as a differential equation and using the expected profit at $x_i \equiv 1$ as an integration constant yields an equation for supplier i's equilibrium expected profit:

$$\frac{1 - H(q_0|s_i)}{\beta(1 - H(q|s_i))} \left[\mathrm{m}_i(x_i) - \delta_i(x_i)x_i\right] = \frac{1 - H(q_0|s_i)}{\beta(1 - H(q|s_i))} \left[\mathrm{m}_i(1) - \delta_i(1) + \int_{x_i}^{1} \delta_i(z_i)dz_i\right]. \tag{5}$$

Solving for m_i, the buyer's expected payment to supplier i given reported cost x_i is

$$\mathrm{m}_i(x_i) = \mathrm{m}_i(1) - \delta_i(1) + \delta_i(x_i)x_i + \int_{x_i}^{1} \delta_i(z_i)dz_i. \tag{6}$$

Using a convexity argument it can be shown that a mechanism is incentive compatible (i.e., (4) holds) if and only if (5) holds and the associated $\delta_i(x_i)$ is nonincreasing in x_i for all i. We use this condition to check that our proposed mechanism is indeed incentive compatible (see Online Appendix A). Furthermore, to ensure participation of all suppliers $i = 1, \ldots, n$ we must also check that our mechanism is "individually rational," i.e., has non-negative expected profit for supplier i given any possible cost realization x_i. Equation (5) implies that individual rationality for an incentive compatible mechanism obtains if supplier i's expected profit at $x_i \equiv 1$ is non-negative.

With our direct mechanism defined and the expected payment to each supplier derived we are ready to compute all the ex ante expected costs associated with our mechanism.

Proposition 1 *Given a direct mechanism defined by* Π, Γ, *and* M, *the ex ante expected contract payment, post-qualification cost, and non-transaction cost are*

$$\overline{PAY} = \sum_{i=1}^{n}[m_i(1) - \delta_i(1)] + \sum_{i=1}^{n}\int_{\vec{x}}[\psi_i(x_i)\Delta_i(\vec{x})]\,dF(\vec{x}), \tag{7}$$

$$\overline{POST} = \int_{\vec{x}}\sum_{b=1}^{n} E[POST|b]\Gamma_b(\vec{x})dF(\vec{x}), \quad \textit{and} \quad \overline{NT} = C_o\int_{\vec{x}}\sum_{b=0}^{n}\Gamma_b(\vec{x})(1-\beta)^b dF(\vec{x}),$$

where F *is the distribution over* $\vec{x}$, f_i *is the density of* F_i, *and*

$$\psi_i(x) \triangleq x_i + \frac{F_i(x_i)}{f_i(x_i)}.$$

Furthermore, where b *is the max number of suppliers to post-qualify,* $E[POST|b]$ *equals*

$$\sum_{t=1}^{b}\left[(t-1)\overline{C}_{REJECT} + C_{ACCEPT}\right]\beta(1-\beta)^{t-1} + b(1-\beta)^b\overline{C}_{REJECT},$$

where the expected cost to post-qualify an unqualified supplier is given by

$$\overline{C}_{REJECT} \triangleq \int_{s=0}^{1}\frac{1}{H(q_0|s) - H(q|s)}\int_{y=q}^{q_0}\frac{y-q}{q_0}KdH(y|s)dG(s),$$

and the deterministic cost to post-qualify a qualified supplier is given by

$$C_{ACCEPT} \triangleq \frac{q_0 - q}{q_0}K.$$

Proof. Since the buyer's ex ante expected contract payment is simply the combined ex ante expected payments to all suppliers, we get (7) by integrating (6) over x_i and summing over $i = 1, \ldots, n$. The forms of $\overline{\text{POST}}$ and $\overline{\text{NT}}$ follow from the definition of $\Gamma_b(\vec{x})$ and β. In particular, for cost vector $\vec{x}$, with probability $\Gamma_b(\vec{x})$ the maximum number of suppliers to post-qualify is set to b. Once b is set, the number of suppliers to post-qualify depends on the number of suppliers who are rejected by post-qualification, where each rejection occurs independently with probability β; either suppliers are post-qualified until the t^{th} results in successful post-qualification, or all b are post-qualified and all are rejected. The post qualification costs $\overline{C}_{REJECT}$ and C_{ACCEPT} are derived using the fact that post-qualification of a supplier with qualification level y costs the buyer $\frac{q_0-q}{q_0}K$ if the supplier is qualified (i.e. $y \geq q_0$), $\frac{y-q}{q_0}K$ otherwise. The supplier qualification level y for an unqualified bidder is random between q and q_0 and follows conditional distribution $\frac{H(y|s)}{H(q_0|s)-H(q|s)}$ (recall suppliers with qualification level below q are prevented from entering the auction by pre-qualification), where signal s follows distribution G. ■

The derivation of costs associated with the mechanism ignores the pre-qualification cost, which is considered sunk by the auction and post-qualification mechanism. Note that if the buyer selects the pre-qualification threshold to be $q = q_0$ – i.e., he performs all qualification prior to the auction – then $\beta \equiv 1$ and the above mechanism analysis devolves into the standard mechanism design analysis of Myerson (1981). The value $\frac{F_i(x_i)}{f_i(x_i)}$ represents the informational rent accruing to supplier i's asymmetric knowledge of his cost x_i (no rent accrues to supplier i's knowledge of s_i, since there is no adverse selection for the qualification level). Following mechanism design literature tradition we will refer to $\psi_i(x_i)$ as supplier i's "virtual cost."

The Optimal Auction and Post-Qualification Mechanism. We now find a mechanism $(\Pi, \Gamma, \mathrm{M})$ that minimizes $\overline{\mathrm{PAY}} + \overline{\mathrm{POST}} + \overline{\mathrm{NT}}$ subject to the incentive compatibility and individual rationality constraints described following (6). To ensure that the optimal Π and Γ we choose will satisfy incentive compatibility, we will assume that the virtual cost $\psi_i(x_i) = x_i + \frac{F_i(x_i)}{f_i(x_i)}$ is increasing in the true cost x_i (this standard, technical condition ensures $\delta_i(x_i)$ nonincreasing in x_i for all i, and is satisfied, for example, if F_i is logconcave; see Bagnoli and Bergstrom 2005 for details about logconcave functions, which include uniform, normal, logistic and exponential distributions). For the remainder of this section we will assume without loss of generality that bidder labels are such that $\psi_1(x_1) \leq \psi_2(x_2) \leq \cdots \leq \psi_n(x_n)$.

Proposition 2 *An optimal direct, individually rational, and incentive compatible auction and post-qualification mechanism* (Π^*, Γ^*, M^*) *that minimizes* $\overline{PAY} + \overline{POST} + \overline{NT}$ *is as follows. Set* $m_i^*(x_i)$ *to the RHS of* (6) *with* $m_i^*(1) - \delta_i^*(1)$ *fixed at zero for all* i*, where* $\Pi^*(\vec{x})$ *weights with probability one the permutation* $(1, 2, 3, \cdots, n)$ *that orders suppliers by virtual cost, and* $\Gamma^*(\vec{x})$ *weights with* *probability one the value*

$$\max\{i, i = 1, \ldots, n \text{ such that } \psi_i(x_i) < C_o - C_{ACCEPT} - \frac{(1-\beta)}{\beta}\overline{C}_{REJECT}\}, \tag{8}$$

or the value zero if the above set is empty.

See the Online Appendix for proof of this proposition and all others which follow. In practice buyers often do employ a reserve price. Proposition 2 says in an optimal auction the buyer should set supplier-specific reserve prices $\{r_i,\ i = 1, \cdots, n\}$ such that $r_i = \max\{\min\{\psi_i^{-1}(C_o - C_{ACCEPT} - \frac{(1-\beta)}{\beta}\overline{C}_{REJECT}),\ 1\}, 0\}$ and promise to post-qualify, in order of ascending virtual costs, only those suppliers whose bids fall below their corresponding reserve price. When suppliers' costs are symmetrically distributed a common reserve price is optimal and the supplier with the lowest virtual cost is also the one with the lowest true cost, since the virtual cost function is assumed to be increasing. While the optimal post-qualification sequencing strategy is intuitive, the determination of the optimal reserve prices is perhaps not as straightforward. To illustrate the intuition behind the reserve prices, the remainder of this subsection provides a dynamic program interpretation.

In the post-qualification stage, when the buyer determines whether to post-qualify supplier i he has two choices. First, the buyer can post-qualify i; this incurs a cost $\psi_i(x_i) + C_{ACCEPT}$ if supplier i turns out qualified, otherwise an expected cost of $\overline{C}_{REJECT}$ if supplier i turns out to be unqualified. Second, the buyer can quit and take the outside option at a fixed cost of C_o. The buyer's problem is to determine the stopping policy, that is, when to quit or equivalently what are the reserve prices, given that suppliers are qualified with probability β and unqualified with probability $1-\beta$. To find the optimal stopping policy, consider an equivalent $n+1$-period dynamic program. Let v_i be the optimal expected cost to go given that the buyer did not transact or quit in the first $i-1$ periods, where

$$v_i = \min\{C_o,\ [\psi_i(x_i) + C_{ACCEPT}]\beta + [v_{i+1} + \overline{C}_{REJECT}](1-\beta)\} \text{ for } i = 1, \ldots, n, \tag{9}$$

$$v_{n+1} = C_o, \quad \text{and} \quad \psi_1(x_1) \le \psi_2(x_2) \le \cdots \le \psi_n(x_n). \tag{10}$$

Proposition 3 *An optimal policy for the stopping problem* (9)-(10) *is as follows: in period i, the buyer should attempt to transact if and only if* $\psi_i(x_i) < C_o - C_{ACCEPT} - \frac{(1-\beta)}{\beta}\overline{C}_{REJECT}$*, and quit* *otherwise. Furthermore, this implies that*

$$\underset{\vec{x}}{E} v_1(\vec{x}) = \overline{PAY} + \overline{POST} + \overline{NT}.$$

Proposition 3 reveals that the buyer's decision in the post-qualification stage can be interpreted as an optimal stopping problem. In particular, the ex ante expected total auction and post-qualification cost with the optimal mechanism is simply the ex ante optimal cost of a dynamic program. We will exploit this structure in the sequel.

4.2 Optimal Qualification Screening Strategy

The buyer's problem is now extended to investigate optimal selection of the pre-qualification threshold (equivalently β) and the number of suppliers invited to the auction n.

Optimal Pre-Qualification Threshold. We first extend the optimization analysis by allowing the buyer to optimally select β. The buyer faces four practical concerns: uncertain supplier costs, uncertain supplier qualification levels, costs associated to verify the qualification of a supplier, and an outside option cost. Comparisons of the latter two costs determine the buyer's optimal pre-qualification threshold, for given uncertainties over supplier costs and qualification levels. The following three propositions summarize the optimal pre-qualification characterizations for general uncertainty distributions, culminating in Proposition 6's threshold results.

Proposition 4 *Under the mechanism described in Proposition 2, the more qualification due diligence the buyer performs before the auction, the more he pays before the auction but the less he pays after the auction. That is,*

$$\overline{PRE} \textit{ increases in } \beta, \textit{ while } \overline{PAY} + \overline{POST} + \overline{NT} \textit{ decreases in } \beta.$$

Therefore, the buyer trades off the pre-qualification cost against the total post-auction cost in an optimal qualification strategy.

This proposition reflects the intuition that the more due diligence work the buyer does before the auction, the more he needs to pay before the auction, given that a higher pre-qualification requirement means not only more suppliers to pre-qualify (more rejections) but also more cost to pre-qualify each supplier. Also, the more due diligence work the buyer does before the auction the less cost he expects to incur after the auction. However, showing this requires some care because $\overline{\text{PAY}}$ and $\overline{\text{POST}}$ are not monotonic in β. The proof exploits the dynamic program structure described in Proposition 3: increasing β reduces the cost to go because it increases the likelihood of success at each stage and decreases the costs associated with acceptance or rejection.

Although Proposition 4 reveals the key tradeoff made when deciding the pre-qualification threshold, solving for the optimal β^* which minimizes $\overline{\text{PRE}} + \overline{\text{PAY}} + \overline{\text{POST}} + \overline{\text{NT}}$ for fixed K and C_o is precluded by the complexity of the total cost expression (see, for example, Proposition 1). However, exploiting the dynamic program structure of the auction and post-qualification cost helps prove monotonicities of β^*, sharpening the insight of Proposition 4.

Proposition 5 *Under the mechanism described in Proposition 2, β^* decreases with K and increases with C_o.*

Given that β^* decreases with K, it is natural that there exist thresholds characterizing the switching behavior between "pre-qualification only," "a mix of pre- and post-qualification" and "post-qualification only."

Proposition 6 *Under the mechanism described in Proposition 2, for fixed C_o, there exist positive finite thresholds $K^{pre} \leq K^{post} < K^{nt}$, all increasing in C_o, such that (i) pre-qualification only is optimal if and only if $K \leq K^{pre}$, (ii) a mix of pre- and post-qualification is optimal if and only if $K^{pre} < K < K^{post}$, (iii) post-qualification only is optimal if and only if $K^{post} \leq K < K^{nt}$, and (iv) it is optimal to forego the auction in favor of the outside option if and only if $K^{nt} \leq K$.*

Proposition 6 first shows that there exists a threshold separating the complete versus partial pre-qualification decision. Keeping the outside option cost constant, if the qualification cost is below this threshold, the buyer prefers to completely pre-qualify suppliers before the auction. Furthermore, once the buyer prefers complete pre-qualification, any increase in the outside option cost while holding the qualification cost fixed will result in the buyer still preferring complete

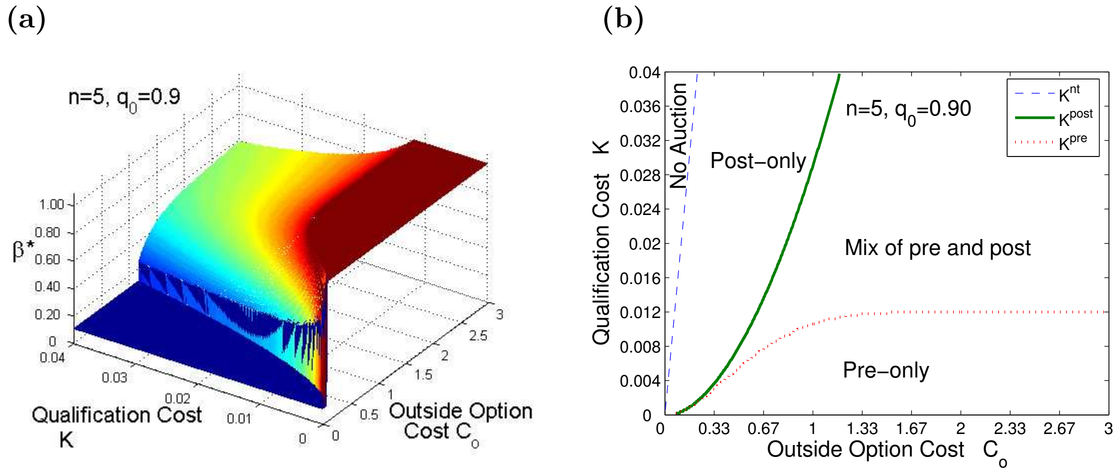

Figure 1: Optimal bidder qualification probability β^* as a function of both the per-supplier qualification cost K and the outside option cost C_o, showing (a) the values of β^* and (b) the regions of the different optimal qualification decisions. The figures are based on a numerical solution of the optimal β^* with $n = 5$ suppliers, qualification threshold $q_0 = 0.9$, and all priors over costs F_i and qualification levels H uniform over $[0, 1]$.

pre-qualification. The third part of Proposition 6 shows that a similar threshold exists between the partial pre-qualification versus the post-only decision. The first three parts of the proposition together indicate that, keeping the outside option cost fixed, as qualification cost increases from zero the decision shifts from complete pre-qualification, to a mixture of pre and post-qualification, and finally to the pure post-qualification decision. Finally, the buyer prefers to forego the auction altogether if the cost of qualification is too high, as shown in the last part of Proposition 6.

In words, Proposition 6 indicates that despite the risk of non-transaction and larger contract payment, the buyer sometimes finds it profitable to postpone some or all qualification processes until after the auction. The decision of how much qualification to postpone depends on the qualification cost, as the buyer has more incentive to risk non-transaction and higher payments with qualification postponement if doing so avoids high qualification expenses before the auction. Figure 1 illustrates the main points of this subsection so far: (a) the optimal β^* is monotone in K and C_o; and (b) the optimal policy switches between pre-only, a mixture of pre- and post-qualification, and post-only. This figure is based on uniform F_i and H, a case which we explore further in §5.

Optimal Number of Bidders. For all analyses above we fixed the number of suppliers invited to bid in the auction n. We now consider the problem of jointly optimizing β and the number of suppliers $n \leq N$, where N is the auction capacity. The complexity of the cost expressions preclude closed form expressions for the optimal number of suppliers and the optimal bidder qualification probability as functions of auction capacity, denoted $n^*(N)$ and $\beta^*(N)$, respectively. However, threshold results do obtain; the following proposition characterizes when using full auction capacity is optimal and when post-qualification only is optimal.

Proposition 7 *Under the mechanism described in Proposition 2, given any* q_0*,* $C_o > 0$ *and* $0 < K < K^{nt}$*, there exists some* $\bar{N}_1 \leq \bar{N}_2 < \infty$*, such that* $n^*(N) = N$ *for all* $N \geq \bar{N}_1$ *and* $\beta^*(N) = \int_{s=0}^{1} [1 - H(q_0|s)] dG(s)$ *(post-only) for all* $N \geq \bar{N}_2$*.*

Having a large number of bidders drives bids towards zero despite some bidders being possibly unqualified. Intuitively, inviting up to capacity ($n^* = N$) and delaying all qualification is optimal for large N because after the auction the buyer then simply locates a single qualified supplier who charges close to zero. More bidders always benefit the buyer (and at no additional cost) when he delays all qualification until after the auction, which explains why the buyer invites up to capacity when all qualification is delayed ($\bar{N}_1 \leq \bar{N}_2$). The managerial implication is that post-qualification alone can be highly attractive when the buyer has the capacity to run a very large auction, but the buyer might prefer to use some pre-qualification and might even not invite up to capacity if the auction capacity is small. The next section provides numerical insights into of the effect of optimally selecting n, and suggests that compared to setting $n \equiv N$ the profit increase is usually minor ($0-7\%$ in most cases studied), but can be significant when both the outside option and qualification costs are large: an expensive outside option scares the buyer into using a pre-qualification only strategy; with each bidder very expensive to invite, the buyer finds it profitable to reduce the auction size.

5. Cost Savings: Symmetric Uniform Priors Example

We now illustrate cost savings from optimal qualification screening strategies for a canonical case in which qualification level and cost distributions are uniform. In particular, $H(\cdot|s) \sim U[0,1]$ for all s, and normalizing to reflect dollar values, we take $F_i \sim U[\$500,000, \$1,000,000]$ for all i. This models the case in which all suppliers are equally unsure about their ability to qualify for the contract, and the supplier cost types are evenly dispersed.

Cost Savings From Optimal Pre-Qualification Threshold. We have shown that pre-only qualification is not always optimal even though it is typically considered the default method for supplier qualification in practice. If the cost gap between pre-only and the optimal qualification strategy is small, one might prefer pre-only because of its simplicity; however, we often find a significant cost gap, particularly when the supplier qualification cost K is moderate. Define the rate of savings as follows:

$$\begin{aligned}\text{rate of savings} &= \frac{\text{total cost with pre-only} \;-\; \text{total cost with optimal threshold}}{\text{total cost with pre-only}},\\ &= \frac{\overline{\text{TOTAL}}(K, C_o, \beta = 1) - \overline{\text{TOTAL}}(K, C_o, \beta^*)}{\overline{\text{TOTAL}}(K, C_o, \beta = 1)}.\end{aligned}$$

In Figure 2 we illustrate the rate of savings for our uniform priors example. The buyer's outside option cost of \$1,200,000 is 20% higher than the worst possible supplier cost (e.g., this could model procurement with low cost foreign suppliers).

Cost savings from optimally balancing the qualification process can be significant: the maximal rate of saving is around 11% in Figure 2(a) and 17% in Figure 2(b). In Figure 2(a) only ten percent of all suppliers are truly qualified ($q_0 = 0.90$), mitigating the benefits of aggressive post-qualification due to non-transaction risks. This could model the case in which foreign suppliers are extremely unlikely to meet rigorous qualification requirements set by the buyer. On the other hand, Figure 2(b) depicts a case with more lenient qualification requirements that eighty percent of suppliers would meet ($q_0 = 0.20$); in this case, costly pre-qualification reveals little new information so postponing some or all qualification can be very beneficial. For example, if supplier

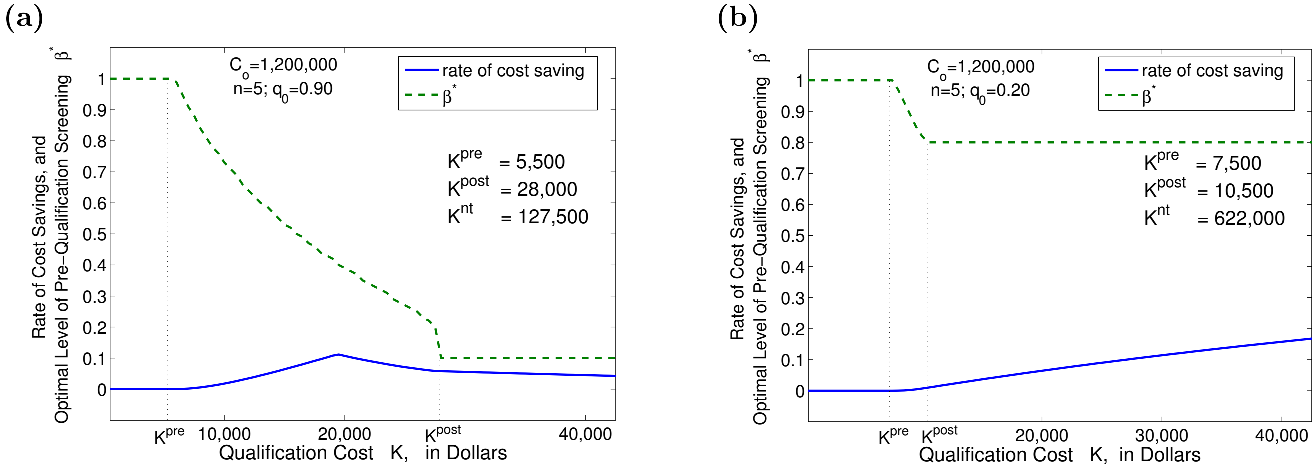

Figure 2: Percent cost savings and β^* the optimal bidder qualification probability plotted versus qualification cost K, in dollars. Pictures assume $n = 5$ suppliers, outside option cost $C_o = \$1,200,000$, supplier cost and qualification level distributions $U[\$500,000, \$1,000,000]$ and $U[0,1]$ respectively, and (a) strict ($q_0 = 0.90$) and (b) more lenient ($q_0 = 0.20$) qualification thresholds.

qualification costs \$25,000 (perhaps \$10,000 spent to purchase and test supplier products, \$10,000 to send three buyer employees to inspect supplier facilities abroad, and \$5,000 on time intensive meetings with stakeholders throughout the buyer's company), about 10% of total procurement costs are saved by postponing qualification checks. Figure 2(a) shows that as K increases the rate of saving first increases and then decreases. This is because beyond a qualification cost of \$19,500, the pre-only strategy's cost is fixed at C_o because qualification is too costly to make the auction with pre-only worthwhile. In contrast, with optimal qualification the buyer strictly prefers the auction to the outside option until the qualification cost is \$127,500 (K^{nt}). For a larger value of K than those shown, the savings curve in Figure 2(b) would eventually peak following the same logic. In summary, these results suggest the buyer should seriously consider postponing some or all of the supplier qualification process, especially when the outside option cost is high, the qualification requirement is low, and it is moderately costly to qualify a supplier.

Optimal Number of Bidders. Table 1 provides numerical results on the optimal number of bidders for our uniform priors example; its contents are as follows: the first three columns list

Table 1: Characterization of $n^*(N)$ and $\beta^*(N)$ when supplier cost and qualification level distributions are $U[\$500,000, \$1,000,000]$ and $U[0,1]$, respectively.

q_0	K (in \$000s)	C_o (in \$000s)	$\{N \mid n^*(N) < N\}$	n^*	β^*	$\bar{N}_1$	$\bar{N}_2$	Cost Advantage
0.8	5	1,200	{8,9,10}	7	1	11	30	$\leq 0.095\%$
0.8	20	1,200	{4,5,6,7}	3	1	8	12	$\leq 0.94\%$
0.8	50	1,200	ϕ	ϕ	ϕ	2	2	0
0.8	5	2,000	{8,9,10,11}	7	1	12	31	$\leq 0.097\%$
0.8	20	2,000	{4,...,9}	3	1	10	17	$\leq 1.48\%$
0.8	50	2,000	{3,...,8}	2	1	9	11	$\leq 4.71\%$
0.9	5	2,000	{6,...,23}	5	1	24	47	$\leq 1.26\%$
0.9	20	2,000	{3,...,20}	2	1	21	25	$\leq 5.93\%$
0.9	50	2,000	{3,...,13}	2	0.87	14	15	$\leq 6.51\%$
0.9	20	1,200	{3,...,10}	2	1	11	11	$\leq 2.54\%$
0.9	50	100,000	{3,...,51}	2	1	52	52	$\leq 37\%$
0.9	0.050	1,200	ϕ	ϕ	ϕ	2	419	0

parameter values; the fourth column indicates the auction capacity levels under which the buyer optimally uses only partial capacity; the fifth and sixth columns list the optimal number of bidders and the optimal β when the buyer optimally uses only partial capacity; and the seventh and eighth columns are the thresholds defined in Proposition 7. To see how much the buyer benefits from choosing an optimal number of bidders, we computed the total cost with $n = n^*(N)$ and the total cost with $n = N$ and report in the last column the upper bound of the percent cost advantage, defined to be $\frac{\text{Total cost}(n=N) - \text{Total cost}(n=n^*(N))}{\text{Total cost}(n=N)}$.

Take the first row as an example. When $q_0 = 0.8$, $K = \$5,000$, and $C_o = \$1,200,000$ it is optimal to perform pre-qualification only and invite up to N when $N = 2, ..., 7$. However, it is optimal to choose $n = 7$ and do pre-qualification only when $N = 8, 9, 10$ because the benefits from additional bidders are dominated by the costs of additional qualification work. As shown by Proposition 7, post-qualification is more attractive if there are more bidders; accordingly, when the buyer is able to invite at least 11 bidders, he would like to invite as many bidders as possible and postpone part or all of the due diligence until after the auction. Eventually – when the auction capacity is at least 30 bidders – a post-qualification only strategy dominates.

The table indicates that $n^* < N$ decreases in K and q_0, which makes sense because the higher the qualification cost or the stricter the qualification requirement, the higher the pre-qualification cost to find each bidder. Similar reasoning explains why $\bar{N}_1$ and $\bar{N}_2$ decrease in K. However, $\bar{N}_1$ and $\bar{N}_2$ increase in C_o and q_0: the higher C_o or q_0, the less attractive post-qualification becomes due to a higher cost and greater risk of non-transaction, making more bidders required (lowering the supplier bids available to the buyer) to compensate for this effect.

The cost advantages of optimizing n^* appear moderate when K and C_o are relatively small, as the qualification and non-transaction costs do not greatly impact the buyer's total cost in such cases. Row ten shows an advantage of about 2.54% over the $n = N = 5$ case depicted in Figure 2(a) at $K = \$20,000$. Since Figure 2(a) showed 10.7% savings off the pre-only strategy in this case, Table 1 shows that optimizing n as well as β saves about $1 - 0.893 * 0.9746 = 13\%$ off the pre-only case. The cost advantage becomes significant when C_o is huge (for example, C_o equal to a hundred million dollars in row eleven) and the buyer is happy finding virtually any qualified supplier. This extreme case might represent a scenario where the buyer cannot produce in-house and the contract is extremely important – for example, flu vaccine procurement by a government. In this case, even with a moderate to large auction capacity it is still optimal to perform pre-qualification only with just two bidders; numerical studies show the cost advantage over setting $n \equiv N$ could be as high as 30% when $N = 5$ or 6. In summary, simultaneously optimizing the number of bidders n as well as the probability of bidder qualification β can yield notable cost savings, especially when C_o and K are moderate to large.

6. Practical Considerations and Extensions

6.1 Value of Credible Reserve Price

Our optimal mechanism derivation in §4.1 assumed the buyer could credibly commit to not awarding the contract to any supplier bidding above their reserve price. As Milgrom (1987) points out, an auctioneer who cannot credibly commit to throwing away bids between the reserve price and the auctioneer's own valuation is disadvantaged: the auctioneer cannot achieve the optimal ex-ante expected profits because bidders will ignore the announced reserve price.

Post-qualification of bidders places additional importance on reserve price credibility. Using Proposition 2, we can characterize the optimal reserve prices by $\psi_i(r_i) = b_{max}$, where $b_{max} \triangleq C_o - C_{ACCEPT} - \frac{(1-\beta)}{\beta}\overline{C}_{REJECT}$ is the maximum bid that the buyer would find profitable to post-qualify. Since $\psi_i(x) > x$, we have $r_i < b_{max}$; i.e., in the optimal mechanism the buyer promises to ignore bids below b_{max} that he would otherwise find profitable to post-qualify.

If this promise is uncredible, the buyer is forced to set b_{max} as the auction reserve price and cannot apply an optimal mechanism (optimal reserve prices). Supposing supplier costs are normalized to interval $[0, 1]$, this is particularly problematic when $b_{max} > 1$: if all other bidders fail post-qualification, a bidder i with cost 1 will be considered by the buyer. Because all other bidders would have failed post-qualification by the time i's offer is considered, bidder i can make a take-it-or-leave-it offer of b_{max} to the buyer. Therefore a cost type 1 bidder i expects (without conditioning on s_i) to earn $(b_{max} - 1)\beta(1 - \beta)^{n-1}$ from the auction. Note that when $\beta = 1$, as is assumed in classical auction theory, cost type 1 bidders expect to earn zero profits in the auction provided the auction has at least two bidders.

Thus, two factors inflate the costs of an uncredible buyer in our setting: forgone "price discrimination" opportunities, as seen in classical auction theory; and the "being held hostage by the last remaining bidder" effect, a consequence of post-qualification which to our knowledge is new to the literature. If, for example, supplier cost distributions are symmetric (and hence one reserve price r prevails for all suppliers), when $b_{max} > r \geq 1$ only the "being held hostage" effect exists; when $1 \geq b_{max} > r$, only the "price discrimination" effect exists; and when $b_{max} > 1 > r$ both effects exist. In words, this suggest that post-qualification should be used carefully when the buyer has little or no negotiating clout with suppliers and must rely solely on competition among suppliers for price concessions. In such situations a supplier can command a very high price in a one-on-one negotiation with the buyer, a damaging scenario for the buyer that is risked by post-qualification.

6.2 General Qualification Cost Models

Our analyses assumed that qualification costs are increasing and linear in the qualification level $q \in [0, q_0]$. However, for any non-linear qualification cost function that is strictly increasing from 0 to K over $q \in [0, q_0]$, the optimal qualification strategy can be recovered by analyzing an equivalent model in which qualification costs are linear but the buyer's prior distribution over qualification levels is redefined. The buyer's decision about how much pre-qualification money to spend rests on how much screening each successive dollar accomplishes; by preserving this relationship in the redefined linear model, the linear model analysis is sufficient. Because this redefinition is straightforward per the above, we omit the formalization.

6.3 Finite Supplier Pool

Our analyses assumes a large (effectively infinite) supplier pool from which the buyer pre-qualifies suppliers. This ensures that the buyer is always able to find as many suppliers as necessary to fill the auction. However, in practice the number of suppliers who show interest in the RFQ is finite and could be small for specialized purchases. On one hand, this would not affect post-qualification only policies provided the number of suppliers to invite to the auction does not exceed the supplier pool size. On the other hand, a finite supplier pool would affect strategies with a pre-qualification stage, since the number of suppliers passing pre-qualification and entering the auction would be non-deterministic. Presumably the buyer could more actively control the pre-qualification process, for example by dynamically adjusting the pre-qualification threshold per the remaining supplier pool size. We leave the finite supplier pool case to our future work.

6.4 Inclusion of Multiple Attributes

This paper took a first step at managing supplier qualification and price negotiation processes with an analytical model. However, the auction format was assumed to be price only. One potentially appealing but complicating extension would be to allow suppliers to report their signals s_i of their qualification level to the buyer, allowing, for example, the buyer's post-qualification decisions to factor in not only price but also the supplier's indication of how likely they are to be qualified. Since the suppliers' signals are noisy, care would be required to keep the reporting meaningful and, for example, prevent suppliers from falsely claiming to have the best possible signal. This might be accomplished if the buyer can penalize a supplier for failing post-qualification, but in turn this might tempt the buyer to arbitrarily fail suppliers for the sake of recovering a penalty, especially since in practice the buyer's qualification standards can be idiosyncratic and opaque. We leave such issues to future research.

7. Conclusions

When issuing an RFQ for competitive bid, finding a supplier truly qualified to fulfill the contract is often as important as price concerns. Costly supplier qualification processes are virtually ubiquitous in industry to help buyers proactively avoid problems and expenses associated with supplier non-performance, e.g., buyer production line stoppages and product reliability issues. This paper explicitly models and suggests optimal policies for both the supplier qualification and competitive price negotiations processes together, and to our knowledge is the first study of optimal supplier qualification processes in the operations management and auction theoretic literatures. To save on total supplier qualification and contracting costs we allow the buyer to delay all or part of the qualification process until after the competitive price negotiation (an auction) and then home in on the lowest (virtual cost) bidders. While delaying qualification is not to our knowledge common practice in industry, our study provides a mathematical framework which suggest such a post-qualification stage can indeed be beneficial.

In particular, we find that pre-qualification only is optimal solely when supplier qualification is relatively cheap. Because postponing qualification means that some attractive bids in the auction may be disqualified, it makes sense to completely pre-qualify suppliers if doing so is cheap. However, for moderate sized qualification costs the buyer can do much better if some costly qualification is delayed until after the auction, because reduced qualification costs with judicious post-qualification can more than offset expected increases in the contracting costs (as determined by our auction theoretic analysis). Figure 2(b) shows total (qualification plus procurement) cost savings of around 10% for a contract worth \$1.2M to the buyer when qualifying a supplier costs $\$25,000$ and priors over supplier costs and qualification levels are uniform and symmetric. More generally, Proposition 6 partitions the two-dimensional qualification cost (K) and outside option cost (C_o) plane into regions where either traditional pre-qualification only, our novel post-qualification only, or our novel mix of the two are optimal (illustrated for a uniform case in Figure 1(b)). Cost savings are even higher (Table 1) when the buyer also optimizes the number of bidders invited to the auction.

While operations management analyses such as ours may merely galvanize a reconsideration of current procurement policies, as supply chains lengthen and supply sources become globalized and more varied, the increase in potential new suppliers and the growing number of RFQ events could make the standard pre-qualification only strategy prohibitive for resource-constrained procurement departments that cannot possibly fully pre-qualify all suppliers invited to all bidding events. Post-qualification might eventually by used out of sheer necessity to accommodate constrained qualification resources, but fortunately our study shows that post-qualification can be part of an optimally balanced supplier qualification strategy even without such resource constraints.

Our study is built on classical auction theory, shouldering the auction with a stopping problem model of pre-qualification and a dynamic program model of post-qualification. Expansions to the auction theory literature are developed in pursuit of an optimal auction and post-qualification mechanism, boiling down to Proposition 3's dynamic programming interpretation of the expanded auction model. The spirit of the results are general in the sense that they characterize the tradeoff between costly pre-qualification and increased bidding competition. Section 6 discusses several possible extensions to our model, which while fairly general does make some important assumptions in order to keep the analyses focused and tractable.

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Online Appendix for
RFQ Auctions with Supplier Qualification Screening

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A. Proof of Proposition 2

The optimal mechanism design problem can be formulated as

$$\min_{\Pi,\ \Gamma,\ \mathrm{M}} \quad \overline{\mathrm{PAY}} + \overline{\mathrm{POST}} + \overline{\mathrm{NT}} \tag{11}$$

$$\text{s.t.} \quad \Delta_i(\vec{x}) = \beta \left[\sum_{a=1}^{n!} \sum_{b=0}^{n} \Pi_a(\vec{x}) \Gamma_b(\vec{x}) (1-\beta)^{z(i,a)-1} I_{z(i,a) \le b} \right], \tag{12}$$

$$\delta_i(x_i) \quad \text{nonincreasing in } x_i \ \forall\ i, \tag{13}$$

$$\mathrm{m}_i(x_i) = \mathrm{m}_i(1) - \delta_i(1) + \delta_i(x_i)x_i + \int_{x_i}^{1} \delta_i(z_i)dz_i \quad \forall\ i, \tag{14}$$

$$\mathrm{m}_i(x_i) - \delta_i(x_i)x_i \ \geq\ 0 \quad \forall\ x_i, \tag{15}$$

$$\Pi_a(\vec{x}) \geq 0,\ a = 1, \ldots, n! \quad \text{and} \quad \sum_{a=1}^{n!} \Pi_a(\vec{x}) = 1 \quad \forall\ \vec{x}, \tag{16}$$

$$\Gamma_b(\vec{x}) \geq 0,\ b = 0, \ldots, n \quad \text{and} \quad \sum_{b=0}^{n} \Gamma_b(\vec{x}) = 1 \ \ \forall\ \vec{x}, \tag{17}$$

where $\mathrm{m}_i(z_i) = \int_{\vec{x}_{-i}} \mathrm{M}_i(z_i, \vec{x}_{-i})dF_{-i}(\vec{x}_{-i})$ and $\delta_i(z_i) = \int_{\vec{x}_{-i}} \Delta_i(z_i, \vec{x}_{-i})dF_{-i}(\vec{x}_{-i})$. Constraints (13)-(14) ensure incentive compatibility and (15) imposes individual rationality (see the discussion in the text following (6)). Constraints (16)-(17) ensure that Π and Γ are well defined probability distributions. Using Proposition 1, (11) can be written as

$$\sum_{i=1}^{n} [\mathrm{m}_i(1) - \delta_i(1)] + \int_{\vec{x}} \left\{ \sum_{i=1}^{n} \psi_i(x_i)\Delta_i(\vec{x}) + \sum_{b=1}^{n} E[\mathrm{POST}|b]\Gamma_b(\vec{x}) + C_o \sum_{b=0}^{n} \Gamma_b(\vec{x})(1-\beta)^b \right\} dF(\vec{x}).$$

Setting $\mathrm{m}_i(1) \equiv \delta_i(1)$ for all i ensures (15) and leaves us with only δ_i to worry about. (Note that (14) defines m_i in terms of δ_i once $\mathrm{m}_i(1)$ is fixed.) Further, an optimal Δ is one which minimizes the integrand of the expression above, i.e., we can re-write our objective function as

$$C_o + \sum_{i=1}^{n} [\psi_i(x_i) - C_o]\, \Delta_i(\vec{x}) + \sum_{b=1}^{n} E[\text{POST}|b]\Gamma_b(\vec{x}),$$

where we have used the fact that $\overline{\text{NT}}$ is just C_o, the outside option cost, times the probability of non-transaction, where the latter equals $1 - \sum_{i=1}^{n} \Delta_i(\vec{x})$. Expanding Δ_i using (12), the optimal mechanism minimizes

$$C(\Pi, \Gamma) \triangleq C_o + \beta \sum_{i=1}^{n} \sum_{a=1}^{n!} \sum_{b=0}^{n} \Pi_a(\vec{x})\Gamma_b(\vec{x})(1-\beta)^{z(i,a)-1} I_{z(i,a)\leq b}\, [\psi_i(x_i) - C_o] + \sum_{b=1}^{n} E[\text{POST}|b]\Gamma_b(\vec{x}),$$

subject to (13), (16)-(17). To prove Proposition 2, we first prove by Lemmas 1-2 that permutation $(1, 2, \dots, n)$ (which we index as $a = 1$) is a (weakly) dominant strategy no matter what $\vec{x}$ and Γ is. Then, given that Π^* selects $a = 1$ w.p.1, we show with Lemma 3 that for any $\vec{x}$ there exists b^* such that setting the max number of suppliers to post-qualify as b^* is a (weakly) dominant strategy (i.e., $\Gamma^*_{b^*}(\vec{x}) = 1$).

Lemma 1 *Label suppliers such that* $\psi_1(x_1) \leq \psi_2(x_2) \leq \cdots \leq \psi_n(x_n)$ *and let permutation* $(1, 2, 3, \dots, n)$ *be indexed by* $a = 1$ *. Then for all possible permutations of the* n *suppliers, indexed by* $a = 1, \dots, n!$*, we have*

$$\sum_{i=1}^{n} (1-\beta)^{z(i,1)-1} I_{z(i,1)\leq b}[\psi_i(x_i) - C_o] \leq \sum_{i=1}^{n} (1-\beta)^{z(i,a)-1} I_{z(i,a)\leq b}[\psi_i(x_i) - C_o].$$

Proof. The proof utilizes an interchange argument. For any permutation a, suppose $z(1, a) = r > 1$ and $z(t, a) = 1$ for some $t > 1$. Consider a revised permutation $a^{(1)}$ in which the positions of t and 1 are swapped, so that $z(1, a^{(1)}) = 1$ and $z(t, a^{(1)}) = r$. Then

$$\begin{aligned}
&\sum_{i=1}^{n} (1-\beta)^{z(i,a^{(1)})-1} I_{z(i,a^{(1)})\leq b}[\psi_i(x_i) - C_o] - \sum_{i=1}^{n} (1-\beta)^{z(i,a)-1} I_{z(i,a)\leq b}[\psi_i(x_i) - C_o] \\
&= \sum_{i=1}^{n} \left[(1-\beta)^{z(i,a^{(1)})-1} I_{z(i,a^{(1)})\leq b} - (1-\beta)^{z(i,a)-1} I_{z(i,a)\leq b} \right] [\psi_i(x_i) - C_o], \\
&= [\psi_1(x_1) - C_o] + (1-\beta)^{r-1}[\psi_t(x_t) - C_o] - [\psi_t(x_t) - C_o] - (1-\beta)^{r-1}[\psi_1(x_1) - C_o], \\
&= (1 - (1-\beta)^{r-1})[\psi_1(x_1) - \psi_t(x_t)] \;\leq\; 0.
\end{aligned}$$

For $a^{(1)}$ we can repeat the process, this time swapping supplier 2 into the second position, creating a new permutation $a^{(2)}$ for which an analogous analysis holds. Continuing this process we eventually end up with permutation $a^{(n)}$, which is precisely permutation 1. This proves the lemma. ■

Lemma 2 *Under the assumptions of Lemma 1, $a = 1$ (weakly) dominates any other permutation $a = 1, \ldots, n!$ for any max number to post-qualify rule Γ.*

Proof. Notice that only the second term of $C(\Pi, \Gamma)$ involves a, so it suffices to prove that $a = 1$ minimizes this term.

$$\begin{aligned}
&\sum_{i=1}^{n}\sum_{a=1}^{n!}\sum_{b=0}^{n} \Pi_a(\vec{x})\Gamma_b(\vec{x})(1-\beta)^{z(i,a)-1} I_{z(i,a)\le b}\left[\psi_i(x_i) - C_o\right] \\
&\geq \sum_{a=1}^{n!}\sum_{b=0}^{n} \Pi_a(\vec{x})\Gamma_b(\vec{x}) \sum_{i=1}^{n}(1-\beta)^{z(i,1)-1} I_{z(i,1)\le b}\left[\psi_i(x_i) - C_o\right] \quad \text{by Lemma 1,} \\
&= \sum_{b=0}^{n} \Gamma_b(\vec{x}) \sum_{i=1}^{n}(1-\beta)^{z(i,1)-1} I_{z(i,1)\le b}\left[\psi_i(x_i) - C_o\right],
\end{aligned}$$

which proves Lemma 2. ■

Lemma 2 implies that $\Pi_1^*(\vec{x}) \equiv 1$ for all $\vec{x}$; that is, it is always optimal to rank suppliers on the waiting list according to ascending virtual costs (breaking ties arbitrarily, for example evenly). Hence, it only remains to find Γ^* assuming $\Pi_1^*(\vec{x}) \equiv 1$. Before doing this in the following lemma, we introduce some notation. Write $C(\Pi^*, \Gamma) = \sum_{b=0}^{n} \Gamma_b(\vec{x}) S(b)$, where

$$S(b) \triangleq C_o + \beta \sum_{i=1}^{n}(1-\beta)^{z(i,1)-1} I_{z(i,1)\le b}\left[\psi_i(x_i) - C_o\right] + E[POST|b].$$

Proposition 2 claims that given $\vec{x}$, $\Gamma_{b^*}^*(\vec{x}) \equiv 1$ for the b^* satisfying (8). This is equivalent to proving that, given $\vec{x}$, b^* minimizes $S(b)$, where $b \in \{0, 1, \ldots, n\}$. $S(b)$ is simply the expected value of a b-stage problem where, at stage $i = 1, \ldots, b$ the buyer attempts to buy an asset with value $\psi_i(x_i)$, where the attempt is successful with probability β and fails with probability $1 - \beta$. Failures cost the buyer $\overline{C}_{REJECT}$, and success costs C_{ACCEPT}. The buyer quits either after the first success, or after failing the b stages he pays a terminal cost of C_o. To see this, we write

$$\begin{aligned}
S(b) &= C_o + \beta \sum_{i=1}^{n} (1-\beta)^{z(i,1)-1} I_{z(i,1)\le b} \left[\psi_i(x_i) - C_o\right] + E[POST|b], \\
&= C_o + \beta \sum_{i=1}^{n} (1-\beta)^{i-1} I_{i\le b} \left[\psi_i(x_i) - C_o\right] + E[POST|b] \quad \text{since } z(i,1) \equiv i, \\
&= C_o + \beta \sum_{i=1}^{b} (1-\beta)^{i-1} \left[\psi_i(x_i) - C_o\right] + \sum_{i=1}^{b} \left[(i-1)\overline{C}_{REJECT} + C_{ACCEPT}\right] \beta(1-\beta)^{i-1} \\
&\quad + b(1-\beta)^b \overline{C}_{REJECT} \quad \text{by Proposition 1}, \\
&= \sum_{i=1}^{b} \beta(1-\beta)^{i-1} \left[\psi_i(x_i) + (i-1)\overline{C}_{REJECT} + C_{ACCEPT}\right] + (1-\beta)^b (C_o + b\overline{C}_{REJECT}).
\end{aligned}$$

Lemma 3 *An optimal stopping policy for the above problem is as follows. The buyer should try to buy in period i if and only if $\psi_i(x_i) < C_o - C_{ACCEPT} - \frac{1-\beta}{\beta}\overline{C}_{REJECT}$, and quit otherwise.*

Proof. Let v_i be the optimal value to go at stage i. We prove the result by induction. The statement is true for $i = n$, since

$$v_n = \min\left\{C_o,\ [\psi_i(x_i) + C_{ACCEPT}]\beta + [C_o + \overline{C}_{REJECT}](1-\beta)\right\}, \quad \text{and}$$

$$\begin{aligned}
&\psi_n(x_n) < C_o - C_{ACCEPT} - \frac{(1-\beta)}{\beta}\overline{C}_{REJECT} \\
&\qquad\qquad \iff [\psi_n(x_n) + C_{ACCEPT}]\beta + [C_o + \overline{C}_{REJECT}](1-\beta) \ < \ C_o.
\end{aligned}$$

Suppose the statement is true for $i+1$. Given that

$$v_i = \min\{C_o,\ [\psi_i(x_i) + C_{ACCEPT}]\beta + [v_{i+1} + \overline{C}_{REJECT}](1-\beta)\},$$

there are two cases:

Case 1: $\psi_i(x_i) < C_o - C_{ACCEPT} - \frac{(1-\beta)}{\beta}\overline{C}_{REJECT}$. Since $v_{i+1} \le C_o$, we have

$$\begin{aligned}
&[\psi_i(x_i) + C_{ACCEPT}]\beta + [v_{i+1} + \overline{C}_{REJECT}](1-\beta) \\
&\qquad\qquad \le \ [\psi_i(x_i) + C_{ACCEPT}]\beta + [C_o + \overline{C}_{REJECT}](1-\beta) \ < \ C_o;
\end{aligned}$$

that is, it is optimal to try to buy the asset in this period.

Case 2: $\psi_i(x_i) \geq C_o - C_{ACCEPT} - \frac{(1-\beta)}{\beta}\overline{C}_{REJECT}$. It must be that $\psi_{i+1}(x_{i+1}) \geq \psi_i(x_i) \geq C_o - C_{ACCEPT} - \frac{(1-\beta)}{\beta}\overline{C}_{REJECT}$, which means $v_{i+1} = C_o$ by the induction assumption. Therefore,

$$[\psi_i(x_i) + C_{ACCEPT}]\beta + [v_{i+1} + \overline{C}_{REJECT}](1-\beta)$$
$$= [\psi_i(x_i) + C_{ACCEPT}]\beta + [C_o + \overline{C}_{REJECT}](1-\beta) \geq C_o;$$

that is, it is optimal to quit and choose the outside option. Thus, the statement of the lemma has been proven by induction. ■

Since the optimal stopping policy of Lemma 3 implies b^* satisfies (8), the proof of Proposition 2 is complete once we verify that constraint (13) holds, i.e., $\delta_i(x_i)$ is nonincreasing in x_i. Since the buyer post-qualifies bidders in sequence of ascending virtual costs, δ_i nonincreasing is implied by the virtual costs increasing in x_i, which we assumed on page 14.

B. Proof of Proposition 3

Lemma 3 proves the structure of the optimal policy. It remains to show $\underset{\vec{x}}{E}[v_1(\vec{x})] = \overline{\text{PAY}} + \overline{\text{POST}} + \overline{\text{NT}}$. Using the notation $S(b)$ introduced before Lemma 3 we get $\overline{\text{PAY}} + \overline{\text{POST}} + \overline{\text{NT}} = \underset{\vec{x}}{E}[S(b^*(\vec{x}))]$ where $b^*(\vec{x})$ satisfies equation (8). Since $S(b^*(\vec{x})) = v_1(\vec{x})$, the result follows.

C. Proof of Proposition 4

It is easy to check that $\overline{\text{PRE}}$ as given in equation (2) is strictly increasing in q, and by equation (1) q strictly increases in β.

From Proposition 3 we have $\overline{\text{PAY}} + \overline{\text{POST}} + \overline{\text{NT}} = \underset{\vec{x}}{E} v_1(\vec{x})$, where $v_1(\vec{x})$ is backward inducted from equations (9)-(10). We show that $\underset{\vec{x}}{E} v_1(\vec{x})$ decreases in β because $v_1(\vec{x})$ decreases in β for any realization of $\vec{x}$ (in particular, it strictly decreases in β for any $\vec{x} \in \{\vec{x}|b^*(\vec{x}) > 0\}$, a fact used in proving Proposition 6-7). Given any $\vec{x}$, under the optimal mechanism,

$$v_m(\vec{x}) = v_{m+1}(\vec{x}) = \cdots = v_{n+1}(\vec{x}) = C_o$$
$$v_i(\vec{x}) = [\psi_i(x_i) + C_{ACCEPT}]\beta + [v_{i+1}(\vec{x}) + \overline{C}_{REJECT}](1-\beta)\}, \text{ for } i = 1, ..., m-1$$

where $m = \min\{i \in \{1, ..., n\} | \psi_i(x_i) > C_o - C_{ACCEPT} - \overline{C}_{REJECT}\frac{1-\beta}{\beta}\}$ or $n+1$ if the set is empty. Notice that, m increases with β.

In the remainder of the proof, we first show that $v_i(\vec{x})$ increases in i and then show that $\psi_i(x_i) \leq v_{i+1}(\vec{x}) - C_{ACCEPT} - \overline{C}_{REJECT}\frac{1-\beta}{\beta}$ for $i = 1, ..., m-1$. Using these two facts, we show that $\frac{d}{d\beta}v_i(\vec{x}) \leq 0$ for $i = 1, ..., n$ and hence $v_1(\vec{x})$ decreases in β.

Because $v_m(\vec{x}) = C_o$ and $\psi_{m-1}(x_i) \leq C_o - C_{ACCEPT} - \overline{C}_{REJECT}\frac{1-\beta}{\beta}$, we have $v_{m-1}(\vec{x}) \leq v_m(\vec{x})$. This implies that

$$v_{m-2}(\vec{x}) - v_{m-1}(\vec{x}) \;=\; [\psi_{m-2}(x_{m-2}) - \psi_{m-1}(x_{m-1})]\beta + [v_{m-1}(\vec{x}) - v_m(\vec{x})](1-\beta) \;\leq\; 0,$$

since $\psi_{m-2}(x_{m-2}) \leq \psi_{m-1}(x_{m-1})$ and $v_{m-1}(\vec{x}) \leq v_m(\vec{x})$. By induction, we have $v_i(\vec{x}) \leq v_{i+1}(\vec{x})$ for $i = 1, ..., n$. Next, given that $v_i(\vec{x}) \leq v_{i+1}(\vec{x})$ for $i = 1, ..., m-1$, we have

$$v_{i+1}(\vec{x}) \;\geq\; v_i(\vec{x}) \;=\; [\psi_i(x_i) + C_{ACCEPT}]\beta + [v_{i+1}(\vec{x}) + \overline{C}_{REJECT}](1-\beta)\}, \text{ for } i = 1, ..., m-1,$$

implying that $\psi_i(x_i) \leq v_{i+1}(\vec{x}) - C_{ACCEPT} - \overline{C}_{REJECT}\frac{1-\beta}{\beta}$ for $i = 1, ..., m-1$.

Consider the fact that, for $i = m+1, ..., n$, $v_i(\vec{x}) = C_o$. As β increases, $v_i(\vec{x})$ cannot increase, since — even if m increases — it will still always be the minimum of C_o and another term; thus, $\frac{d}{d\beta}v_i(\vec{x}) \leq 0$ for $i = m+1, \ldots, n$. For $i = m$, $\frac{d}{d\beta}v_m(\vec{x}) \leq 0$ because m increases with β, so we can write for $i = m-1$

$$\begin{aligned}
\frac{d}{d\beta}v_i(\vec{x}) &= [\psi_i(x_i) + C_{ACCEPT}] - [v_{i+1}(\vec{x}) + \overline{C}_{REJECT}] \\
&\quad + \beta\frac{dC_{ACCEPT}}{d\beta} + (1-\beta)\frac{dv_{i+1}(\vec{x})}{d\beta} + (1-\beta)\frac{d\overline{C}_{REJECT}}{d\beta} \\
&\leq v_{i+1}(\vec{x}) - C_{ACCEPT} - \overline{C}_{REJECT}\frac{1-\beta}{\beta} + C_{ACCEPT} - [v_{i+1}(\vec{x}) + \overline{C}_{REJECT}] \\
&\quad + \beta\frac{dC_{ACCEPT}}{d\beta} + (1-\beta)\frac{d\overline{C}_{REJECT}}{d\beta} + (1-\beta)\frac{dv_{i+1}(\vec{x})}{d\beta} \\
&= -\frac{\overline{C}_{REJECT}}{\beta} + \beta\frac{dC_{ACCEPT}}{d\beta} + (1-\beta)\frac{d\overline{C}_{REJECT}}{d\beta} + (1-\beta)\frac{dv_{i+1}(\vec{x})}{d\beta} \\
&< 0,
\end{aligned}$$

where the first inequality is due to $\psi_i(x_i) \leq v_{i+1}(\vec{x}) - C_{ACCEPT} - \overline{C}_{REJECT}\frac{1-\beta}{\beta}$ and the second inequality is due to the fact that $\frac{dv_{i+1}(\vec{x})}{d\beta} \leq 0$, $\frac{dC_{ACCEPT}}{d\beta} < 0$ and $\frac{d\overline{C}_{REJECT}}{d\beta} < 0$. Repeating the above for $i = m-2, m-3, \ldots, 1$ yields $\frac{d}{d\beta} v_1(\vec{x}) \leq 0$ and at least one strict inequality holds if $\vec{x} \in \{\vec{x} | b^*(\vec{x}) > 0\}$.

D. Proof of Proposition 5

We prove Proposition 5 by first establishing the following technical lemma.

Lemma 4 *Let* $\overline{TOTAL} \triangleq \overline{PRE} + \overline{PAY} + \overline{POST} + \overline{NT}$*, the total ex ante expected procurement cost of the buyer. Then* $\frac{\partial^2 \overline{TOTAL}}{\partial C_o \partial \beta} < 0$ *and* $\frac{\partial^2 \overline{TOTAL}}{\partial K \partial \beta} > 0$.

Proof. Letting $\overline{\text{TOTAL}} \triangleq \underset{\vec{x}}{E}[\text{TOTAL}(\vec{x})]$ we prove Lemma 4 by showing $\frac{\partial^2 \text{TOTAL}(\vec{x})}{\partial C_o \partial \beta} < 0$ and $\frac{\partial^2 \text{TOTAL}(\vec{x})}{\partial K \partial \beta} > 0$ for any realization of $\vec{x}$, where $\text{TOTAL}(\vec{x})$ is the expected total cost conditional on the realization of $\vec{x}$. (Note that it is still an expected cost because we take the expectation over bidders' qualification levels.)

From Proposition 1 we see that for fixed $\vec{x}$, when C_o is changed slightly, only $\overline{\text{NT}}$ is affected. Further, $\text{NT}(\vec{x})$ equals C_o times the probability of non-transaction. Let m equal the optimal number of bidders to post-qualify given in equation (8) for a given $\vec{x}$; clearly the probability of non-transaction equals $(1-\beta)^m$. That is, $\frac{\partial \text{TOTAL}(\vec{x})}{\partial C_o} = (1-\beta)^m$. Hence,

$$\frac{\partial^2 \text{TOTAL}(\vec{x})}{\partial C_o \partial \beta} = -m(1-\beta)^{m-1} + (1-\beta)^m \cdot \ln(1-\beta) \cdot \frac{dm}{d\beta} \ < \ 0,$$

where the inequality follows from $0 < \beta < 1$ and the fact that as β increases the m which satisfies equation (8) increases. Simply taking an expectation, we get $\frac{\partial^2 \text{TOTAL}(\vec{x})}{\partial C_o \partial \beta} < 0$ implies that $\frac{\partial^2 \overline{\text{TOTAL}}}{\partial C_o \partial \beta} < 0$.

To show $\frac{\partial^2 \text{TOTAL}(\vec{x})}{\partial K \partial \beta} > 0$, first recall that the parameter K comes into the total cost via the qualification costs, which means that $\frac{\text{TOTAL}(\vec{x})}{\partial K} = \frac{\partial[\text{PRE}(\vec{x}) + \text{POST}(\vec{x})]}{\partial K}$; since qualification costs are assumed linear in K, to show $\frac{\partial^2[\text{PRE}(\vec{x}) + \text{POST}(\vec{x})]}{\partial K \partial \beta} > 0$ it suffices to show $\frac{\partial[\text{PRE}(\vec{x}) + \text{POST}(\vec{x})]}{\partial \beta} > 0$. In other words, it suffices to show that given any realization of $\vec{x}$, the expected total qualification cost strictly increases in β (equivalently, pre-qualification threshold q). A sample path proof is given as follows:

Consider a sample path of the pre-qualification process. When the pre-qualification threshold q equals q^1, denote the the number of suppliers examined (all those who pass plus all those who fail pre-qualification) during pre-qualification as n^1_{pre} and the total (pre plus post) qualification cost as C^1_{qual}. If on the same sample path the buyer instead adopted a higher pre-qualification threshold $q^2 > q^1$, n^2_{pre}, the number of suppliers examined during pre-qualification must be no less than n^1_{pre}. Noticing that the total (pre plus post) qualification cost spent on the first n^1_{pre} suppliers is greater than or equal to C^1_{qual}, we conclude that C^2_{qual}, the total qualification cost when $q = q^2$, must be no less than C^1_{qual}. ■

Using Lemma 4 we are now ready to complete the proof of Proposition 5. Let $\beta^*(K, C_o)$ denote the optimal β given K and C_o. Then, by this definition, it must be true that

$$\begin{aligned} \overline{\text{TOTAL}}(K, C_o)|_{\beta=\beta^*(K,C_o)} &\leq \overline{\text{TOTAL}}(K, C_o)|_{\beta=\beta^*(K+dK,C_o)}, \quad \text{and} \\ \overline{\text{TOTAL}}(K + dK, C_o)|_{\beta=\beta^*(K+dK,C_o)} &\leq \overline{\text{TOTAL}}(K + dK, C_o)|_{\beta=\beta^*(K,C_o)}. \end{aligned}$$

$$\begin{aligned} \text{Therefore,} \quad & [\overline{\text{TOTAL}}(K + dK, C_o) - \overline{\text{TOTAL}}(K, C_o)]|_{\beta=\beta^*(K+dK,C_o)} \\ & \leq [\overline{\text{TOTAL}}(K + dK, C_o) - \overline{\text{TOTAL}}(K, C_o)]|_{\beta=\beta^*(K,C_o)}. \end{aligned}$$

Because $\frac{\partial^2 \overline{\text{TOTAL}}}{\partial K \partial \beta} > 0$, we conclude that $\beta^*(K + dK, C_o) \leq \beta^*(K, C_o)$, that is, β^* decreases with K. Similarly, with the fact that $\frac{\partial^2 \overline{\text{TOTAL}}}{\partial C_o \partial \beta} < 0$, we can prove β^* increases with C_o, and the proof of Proposition 5 is complete.

E. Proof of Proposition 6

For shorthand, let $\underline{\beta} \triangleq \int_{s=0}^{1} [1 - H(q_0|s)]dG(s)$ be the lower bound on β and let $\overline{\text{TOTAL}}(n, \beta)$, $\overline{\text{PRE}}(n, \beta)$, $\overline{\text{PAY}}(n, \beta)$, $\overline{\text{POST}}(n, \beta)$, and $\overline{\text{NT}}(n, \beta)$ be the expected total cost, the expected pre-qualification cost, the expected payment, the expected post-qualification cost and the expected non-transaction cost respectively, given the probability of bidder qualification β and n bidders.

Above all, we show the expected total cost has a uniform lower bound. For any finite n and any β,

$$\begin{aligned}
\overline{\text{TOTAL}}(n<\infty,\beta) &= [\overline{\text{PRE}}+\overline{\text{PAY}}+\overline{\text{POST}}+\overline{\text{NT}}](n<\infty,\beta),\\
&\geq \overline{\text{PAY}}(n<\infty,\beta)+\overline{\text{POST}}(n<\infty,\underline{\beta})+\overline{\text{NT}}(n<\infty,\beta),\\
&= \overline{\text{PAY}}(n<\infty,\beta)\\
&\quad+\overline{\text{POST}}(n=\infty,\underline{\beta})\cdot[1-\prod_{i=1}^{n}(1-\beta F_i(r_i))]+C_o\prod_{i=1}^{n}(1-\beta F_i(r_i)), \qquad (18)\\
&\geq \overline{\text{PAY}}(n<\infty,\beta)+\overline{\text{POST}}(n=\infty,\underline{\beta}) \qquad (19)\\
&\geq \overline{\text{POST}}(n=\infty,\underline{\beta}).
\end{aligned}$$

The first inequality is due to the fact that $\overline{\text{PRE}}(n<\infty,\beta)+\overline{\text{POST}}(n<\infty,\beta)\geq\overline{\text{POST}}(n<\infty,\underline{\beta})$, which is because $\frac{\partial[\text{PRE}(\vec{x})+\text{POST}(\vec{x})]}{\partial\beta}>0$ (shown during the proof of Proposition 5). The second equality uses the fact that the buyer transacts (finds a qualified supplier i bidding below its reserve price r_i) with probability $\prod_{i=1}^{n}(1-\beta F_i(r_i))$ and the second inequality then follows because in the optimal post-qualification stopping problem (see Proposition 3) the buyer can always defect to the outside option, in particular, $\overline{\text{POST}}(n=\infty,\underline{\beta})\leq C_o$. The last inequality is due to $\overline{\text{PAY}}(n<\infty,\beta)\geq 0$ for any finite n.

In the following, we will first show the existence of K^{nt} and $K^{nt}<\infty$ by showing K^{nt} is just the K such that $C_o-C_{ACCEPT}-\frac{(1-\underline{\beta})}{\underline{\beta}}\overline{C}_{REJECT}=0$, and next show the existence of K^{pre} and $K^{pre}>0$, and then show the existence of K^{post} and $K^{post}<K^{nt}$. Since $K^{pre}\leq K^{post}$ is straightforward by Proposition 5, we then conclude $0<K^{pre}\leq K^{post}<K^{nt}<\infty$. Finally, K^{pre},K^{post} being increasing in C_o is by Proposition 5.

Since $C_o-C_{ACCEPT}-\frac{(1-\underline{\beta})}{\underline{\beta}}\overline{C}_{REJECT}$ is linearly decreasing in K, there exists a unique K such that $C_o-C_{ACCEPT}-\frac{(1-\underline{\beta})}{\underline{\beta}}\overline{C}_{REJECT}=0$. Denote this K by K^{nt}, and we will show that it is optimal to run the auction whenever $K<K^{nt}$ and it is optimal to forego the auction in favor of the outside option whenever $K\geq K^{nt}$.

Consider any $K<K^{nt}$, we have

$$\overline{\text{TOTAL}}|_{n<\infty,\underline{\beta}}(K,C_o) \quad < \quad \overline{\text{TOTAL}}|_{n<\infty,\underline{\beta}}(K^{nt},C_o)=C_o,$$

where the inequality is due to the fact that $\overline{\text{PAY}}+\overline{\text{PAY}}+\overline{\text{NT}}$ strictly increases in K when $K < K^{nt}$ (by equation (9)-(10)), and the equality follows from $\{\vec{x}|b^*(\vec{x}) > 0\} = \emptyset$. This implies that running the auction with post-qualification only is better than taking the outside option when $K < K^{nt}$.

Then, consider any β and $K \geq K^{nt} > K'$, we have

$$\begin{aligned}\overline{\text{TOTAL}}|_{n<\infty,\beta}(K, C_o) &\geq \overline{\text{TOTAL}}|_{n<\infty,\beta}(K', C_o) \\ &\geq \overline{\text{POST}}|_{n=\infty,\underline{\beta}}(K', C_o) \cdot [1 - \prod_{i=1}^{n}(1 - \beta F_i(r_i))] + C_o \prod_{i=1}^{n}(1 - \beta F_i(r_i)),\end{aligned}$$

where the first inequality is due to the fact that the total cost increases in K (since $\overline{\text{PRE}}$ increases in K by equation (2) and $\overline{\text{PAY}} + \overline{\text{POST}} + \overline{\text{NT}}$ increases in K by equation (9)-(10)), the second inequality is from equation (18). Since the limit from the left $\lim_{K'\uparrow K^{nt}} \overline{\text{POST}}|_{n=\infty,\underline{\beta}}(K', C_o) = C_o$ (since the buyer becomes indifferent between post qualifying bidders and taking the outside option as K' approaches K^{nt} from the left), we have $\overline{\text{TOTAL}}|_{n<\infty,\beta}(K, C_o) \geq C_o$.

Therefore, K^{nt} as defined is just the threshold switching between "running the auction" and "taking the outside option". Further, because C_o is finite and $C_o - C_{ACCEPT} - \frac{(1-\underline{\beta})}{\underline{\beta}}\overline{C}_{REJECT}$ is linearly decreasing in K, K^{nt} is finite and increasing in C_o.

The existence of K^{pre} is due to the existence of K^{nt} and the fact that it is always optimal to do pre-only when $K = 0$. This fact is true because $\overline{\text{PRE}} = 0$ when $K = 0$ and hence $\overline{\text{TOTAL}} = \overline{\text{PAY}} + \overline{\text{POST}} + \overline{\text{NT}}$, which strictly decreases in β because $\frac{\partial \overline{\text{TOTAL}}}{\partial \beta}|_{K=0} = \frac{\partial \overline{\text{PAY}}+\overline{\text{POST}}+\overline{\text{NT}}}{\partial \beta}|_{K=0} < 0$ for all β on $[\underline{\beta}, 1]$ (by the proof of Proposition 4 and the fact that $\{\vec{x}|b^*(\vec{x}) > 0\} \neq \emptyset$ when $K = 0$). Further, $K^{pre} > 0$ because $\overline{\text{TOTAL}}$ strictly decreases in β on the full support of $[\underline{\beta}, 1]$ when K is positive and close enough to zero, given that $\frac{\partial \overline{\text{TOTAL}}}{\partial \beta}$ is uniformly continuous in K and β on $[0, K^{nt}] \times [\underline{\beta}, 1]$.

The existence of K^{post} and $K^{post} < K^{nt}$ are due to the fact that it is optimal to run the auction with post-only when K is less than but close enough to K^{nt}. That is, there exists $\Delta K > 0$ such that $\overline{\text{TOTAL}}(K^{nt} - \Delta K, \beta) > \overline{\text{TOTAL}}(K^{nt} - \Delta K, \underline{\beta})$, $\forall \beta > \underline{\beta}$. ($\overline{\text{TOTAL}}(K, \beta)$ is the expected total cost as a function of K and β, for fixed n and $C_o > 0$.) This is true because i) $\overline{\text{TOTAL}}(K^{nt}, \beta) > \overline{\text{TOTAL}}(K^{nt}, \underline{\beta}), \forall \beta > \underline{\beta}$; ii) $\frac{\partial \overline{\text{TOTAL}}}{\partial \beta}(K^{nt}, \beta)|_{\beta=\underline{\beta}} > 0$.

In the following, we first show why i) and ii) are true and then complete the proof by showing why i) and ii) together imply the existence of ΔK.

Fact i) is true because for all $\beta > \underline{\beta}$ and $K' < K^{nt}$, $\overline{\text{TOTAL}}(K', \beta) \geq \overline{\text{PAY}}(K', \beta) + \overline{\text{POST}}|_{n=\infty}(K', \underline{\beta})$ (by inequality (19)) and $\lim_{K' \uparrow K^{nt}} \overline{\text{PAY}}(K', \beta) > 0$ (since $\{\vec{x}|b^*(\vec{x}) > 0\} \neq \emptyset$ when $K = K^{nt}$ and $\beta > \underline{\beta}$); however, $\lim_{K' \uparrow K^{nt}} \overline{\text{POST}}|_{n=\infty}(K', \underline{\beta}) = C_o$ and $\overline{\text{TOTAL}}(K^{nt}, \underline{\beta}) = C_o$.

ii) is true. Note that $\frac{\partial \overline{\text{TOTAL}}}{\partial \beta}(K^{nt}, \beta)|_{\beta=\underline{\beta}} = \frac{\partial \overline{\text{PRE}}}{\partial \beta}(K^{nt}, \beta)|_{\beta=\underline{\beta}} + \frac{\partial(\overline{\text{POST}}+\overline{\text{PAY}}+\overline{\text{NT}})}{\partial \beta}(K^{nt}, \beta)|_{\beta=\underline{\beta}}$. By Lemma 4 $\frac{\partial \overline{\text{PRE}}}{\partial \beta}(K^{nt}, \beta)|_{\beta=\underline{\beta}} > 0$. To show i), we show $\frac{\partial(\overline{\text{POST}}+\overline{\text{PAY}}+\overline{\text{NT}})}{\partial \beta}(K^{nt}, \beta)|_{\beta=\underline{\beta}} = 0$ as follows.

$$\begin{aligned}
&\frac{\partial(\overline{\text{PAY}} + \overline{\text{POST}} + \overline{\text{NT}})}{\partial \beta}|_{\beta=\underline{\beta}} \\
&\quad = \frac{\partial \underset{\vec{x}}{E} v_1(\vec{x})}{\partial \beta}|_{\beta=\underline{\beta}} = \int_{\vec{x}} \left[\frac{\partial v_1(\vec{x})}{\partial \beta}|_{\beta=\underline{\beta}}\right] dF(\vec{x}) = \int_{\{\vec{x}|b^*(\vec{x})>0\}} \left[\frac{\partial v_1(\vec{x})}{\partial \beta}|_{\beta=\underline{\beta}}\right] dF(\vec{x}),
\end{aligned}$$

where the last equality is due to the fact that $\frac{\partial v_1(\vec{x})}{\partial \beta}|_{\beta=\underline{\beta}} = 0$ for $\vec{x} \in \{\vec{x}|b^*(\vec{x}) = 0\}$ (because $v_1(\vec{x}) = C_o$ for any $\vec{x} \in \{\vec{x}|b^*(\vec{x}) = 0\}$).

We can show $\frac{\partial v_1(\vec{x})}{\partial \beta}|_{\beta=\underline{\beta}}$ is uniformly bounded for all n and $\vec{x}$. Clearly $\frac{\partial v_1(\vec{x})}{\partial \beta}|_{\beta=\underline{\beta}}$ is bounded from above by zero (Proposition 4); we show that for any $\vec{x}$ it is bounded from below as well.

$$\begin{aligned}
\frac{\partial v_1(\vec{x})}{\partial \beta}|_{\beta=\underline{\beta}} &= \frac{\partial\{\sum_{i=1}^{b^*(\vec{x})} \psi_i(x_i)\beta(1-\beta)^{i-1} + E[\text{POST}|b^*(\vec{x})] + C_o(1-\beta)^{b^*(\vec{x})}\}}{\partial \beta}|_{\beta=\underline{\beta}}, \\
&= \sum_{i=1}^{b^*(\vec{x})} \psi_i(x_i) \frac{\partial[\beta(1-\beta)^{i-1}]}{\partial \beta}|_{\beta=\underline{\beta}} + \frac{\partial\{E[\text{POST}|b^*(\vec{x})] + C_o(1-\beta)^{b^*(\vec{x})}\}}{\partial \beta}|_{\beta=\underline{\beta}} \\
&\geq \sum_{i=1}^{b^*(\vec{x})} \psi_i(x_i) \frac{\partial[\beta(1-\beta)^{i-1}]}{\partial \beta}|_{\beta=\underline{\beta}} + \min_{b=0,\ldots,n} \left\{\frac{\partial\{E[\text{POST}|b] + C_o(1-\beta)^b\}}{\partial \beta}|_{\beta=\underline{\beta}}\right\} \\
&\geq \sum_{i=1}^{b^*(\vec{x})} \psi_i(x_i) \min\left\{0, \frac{\partial[\beta(1-\beta)^{i-1}]}{\partial \beta}|_{\beta=\underline{\beta}}\right\} + \min_{b=0,\ldots,n} \left\{\frac{\partial\{E[\text{POST}|b] + C_o(1-\beta)^b\}}{\partial \beta}|_{\beta=\underline{\beta}}\right\}
\end{aligned}$$

$$\begin{aligned}
&\geq \quad [C_o - C_{ACCEPT} - \frac{\overline{C}_{REJECT}(1-\beta)}{\beta}] \sum_{i=1}^{b^*(\vec{x})} \min\left\{0, \frac{\partial\left[\beta(1-\beta)^{i-1}\right]}{\partial\beta}|_{\beta=\underline{\beta}}\right\} \\
&\qquad + \min_{b=0,\ldots,n}\left\{\frac{\partial\{E[\text{POST}|b] + C_o(1-\beta)^b\}}{\partial\beta}|_{\beta=\underline{\beta}}\right\} \\
&\geq \quad [C_o - C_{ACCEPT} - \frac{\overline{C}_{REJECT}(1-\beta)}{\beta}] \sum_{i=1}^{\infty} \min\left\{0, \frac{\partial[\beta(1-\beta)^{i-1}]}{\partial\beta}|_{\beta=\underline{\beta}}\right\} \\
&\qquad + \min_{b=0,\ldots,\infty}\left\{\frac{\partial\{E[\text{POST}|b] + C_o(1-\beta)^b\}}{\partial\beta}|_{\beta=\underline{\beta}}\right\}. \qquad (20)
\end{aligned}$$

Note: the first inequality is due to $\frac{\partial\{E[\text{POST}|b^*(\vec{x})]+C_o(1-\beta)^{b^*(\vec{x})}\}}{\partial\beta}|_{\beta=\underline{\beta}} \geq \min_{b=0,\ldots,n}\{\frac{\partial\{E[\text{POST}|b]+C_o(1-\beta)^b\}}{\partial\beta}|_{\beta=\underline{\beta}}\}$; the second inequality is due to $\frac{\partial[\beta(1-\beta)^{i-1}]}{\partial\beta}|_{\beta=\underline{\beta}} \geq \min\{0, \frac{\partial[\beta(1-\beta)^{i-1}]}{\partial\beta}|_{\beta=\underline{\beta}}\}$ for all i; the third inequality is due to $\min\{0, \frac{\partial[\beta(1-\beta)^{i-1}]}{\partial\beta}|_{\beta=\underline{\beta}}\} \leq 0$ and $0 \leq \psi_i(x_i) \leq C_o - C_{ACCEPT} - \frac{\overline{C}_{REJECT}(1-\beta)}{\beta}$ for all $i \leq b^*(\vec{x})$; and the last inequality is due to $C_o - C_{ACCEPT} - \frac{\overline{C}_{REJECT}(1-\beta)}{\beta} \geq 0$ (by $b^*(\vec{x}) > 0$) and $\sum_{i=1}^{b^*(\vec{x})} \min\{0, \frac{\partial[\beta(1-\beta)^{i-1}]}{\partial\beta}|_{\beta=\underline{\beta}}\} \geq \sum_{i=1}^{\infty} \min\{0, \frac{\partial[\beta(1-\beta)^{i-1}]}{\partial\beta}|_{\beta=\underline{\beta}}\}$ and $\min_{b=0,\ldots,n}\left\{\frac{\partial\{E[\text{POST}|b]+C_o(1-\beta)^b\}}{\partial\beta}|_{\beta=\underline{\beta}}\right\} \geq \min_{b=0,\ldots,\infty}\left\{\frac{\partial\{E[\text{POST}|b]+C_o(1-\beta)^b\}}{\partial\beta}|_{\beta=\underline{\beta}}\right\}$. Since the RHS of the last inequality can be shown (independent of $\vec{x}$ and n) to be bounded, $\frac{\partial[\overline{\text{PAY}}(n,\beta)+\overline{\text{POST}}(n,\beta)+\overline{\text{NT}}(n,\beta)]}{\partial\beta}|_{\beta=\underline{\beta}}$ is bounded from below uniformly for all n.

Given that as $K \uparrow K^{nt}$, $\{\vec{x}|b^*(\vec{x}) > 0\} \to \emptyset$ (by the definition of K^{nt} and Proposition 2) and $\frac{\partial v_1(\vec{x})}{\partial\beta}|_{\beta=\underline{\beta}}$ is bounded, the dominated convergence theorem implies $\frac{\partial(\overline{\text{PAY}}+\overline{\text{POST}}+\overline{\text{NT}})}{\partial\beta}|_{\beta=\underline{\beta}} \downarrow 0$ as $K \uparrow K^{nt}$.

Given i) and ii), to complete this proof, we can construct a $\Delta K > 0$ such that $\overline{\text{TOTAL}}(K^{nt} - \Delta K, \beta) > \overline{\text{TOTAL}}(K^{nt} - \Delta K, \underline{\beta})$, $\forall\beta > \underline{\beta}$.

Considering that $\frac{\partial\overline{\text{TOTAL}}}{\partial\beta}(K^{nt}, \beta)$ is continuous in β and positive (by ii)) at $\underline{\beta}$, there exists $\delta > 0$ such that $\varepsilon_1 \triangleq \min\{\frac{\partial\overline{\text{TOTAL}}}{\partial\beta}(K^{nt}, \beta) : \beta \in [\underline{\beta}, \underline{\beta} + \delta]\} > 0$. Given that $\frac{\partial\overline{\text{TOTAL}}}{\partial\beta}(K, \beta)$ is continuous in (K, β) on $[0, K^{nt}] \times [\underline{\beta}, \underline{\beta} + \delta]$ and hence uniformly continuous, there exists a $\Delta K_1 > 0$ such that $|\frac{\partial\overline{\text{TOTAL}}}{\partial\beta}(K, \beta) - \frac{\partial\overline{\text{TOTAL}}}{\partial\beta}(K^{nt}, \beta)| < \frac{\varepsilon_1}{2}$ for all $K \in [K^{nt} - \Delta K_1, K^{nt}]$ and $\beta \in [\underline{\beta}, \underline{\beta} + \delta]$. Further, it is easy to check that $\frac{\partial\overline{\text{TOTAL}}}{\partial\beta}(K, \beta) > 0$ for all $K \in [K^{nt} - \Delta K_1, K^{nt}]$ and $\beta \in [\underline{\beta}, \underline{\beta} + \delta]$.

Considering that $\overline{\text{TOTAL}}(K^{nt}, \beta) - \overline{\text{TOTAL}}(K^{nt}, \underline{\beta})$ is positive (by i)) and continuous in β on $[\underline{\beta} + \delta, 1]$, there exists $\varepsilon_2 \triangleq \min\{\overline{\text{TOTAL}}(K^{nt}, \beta) - \overline{\text{TOTAL}}(K^{nt}, \underline{\beta}) : \beta \in [\underline{\beta} + \delta, 1]\} > 0$. Given that

$\overline{\text{TOTAL}}(K,\beta) - \overline{\text{TOTAL}}(K,\underline{\beta})$ is continuous in (K,β) on $[0,K^{nt}] \times [\underline{\beta}+\delta,1]$ and hence uniformly continuous, there exists a $\Delta K_2 > 0$ such that $|[\overline{\text{TOTAL}}(K,\beta) - \overline{\text{TOTAL}}(K,\underline{\beta})] - [\overline{\text{TOTAL}}(K^{nt},\beta) - \overline{\text{TOTAL}}(K^{nt},\underline{\beta})]| < \frac{\varepsilon_2}{2}$ for all $K \in [K^{nt} - \Delta K_2, K^{nt}]$ and $\beta \in [\underline{\beta}+\delta,1]$. It is easy to check that $\overline{\text{TOTAL}}(K,\beta) - \overline{\text{TOTAL}}(K,\underline{\beta}) > 0$ for all $K \in [K^{nt} - \Delta K_2, K^{nt}]$ and $\beta \in [\underline{\beta}+\delta,1]$.

Therefore, $\overline{\text{TOTAL}}(K^{nt} - \Delta K, \beta) > \overline{\text{TOTAL}}(K^{nt} - \Delta K, \underline{\beta})$ for all $\beta > \underline{\beta}$ and $\Delta K < min\{\Delta K_1, \Delta K_2\}$. This establishes the existence of K^{post} and hence completes the proof.

F. Proof of Proposition 7

When post-only is used the buyer's cost strictly decreases in the number of invited bidders; thus, once post-only is optimal ($N > \bar{N}_2$) we must also have that inviting up to capacity is optimal ($N > \bar{N}_1$), so the existence of $\bar{N}_1$ and $\bar{N}_1 \leq \bar{N}_2$ are established as long as the existence of $\bar{N}_2$ is established.

The existence of $\bar{N}_2$ is shown by showing two facts: first, $\lim_{n' \to \infty} \overline{\text{TOTAL}}(n', \underline{\beta}) < \overline{\text{TOTAL}}(n, \beta)$ for any β and finite n; and second, $\overline{\text{TOTAL}}(n, \underline{\beta}) < \overline{\text{TOTAL}}(n, \beta)$ for all $\beta > \underline{\beta}$ when n large enough.

First fact. When the buyer is able to invite an infinite number of bidders and perform post-qualification only, the expected total cost approaches expected post-qualification cost, $\overline{\text{POST}}(n = \infty, \underline{\beta})$, because all other costs are zero. To see this, note that the post-only strategy avoids all pre-qualification costs, the infinite bidder auction with fixed probability of bidder qualification ensures the buyer can find a qualified bidder, and infinite numbers of bidders drives bids to the left endpoint of the cost distribution (zero). Hence, $\lim_{n' \to \infty} \overline{\text{TOTAL}}(n', \underline{\beta}) = \overline{\text{POST}}(n = \infty, \underline{\beta}) < \overline{\text{TOTAL}}(n, \beta)$ for any β and finite n, where the strict inequality is from inequality (19) given that $\overline{\text{PAY}}(n < \infty, \beta) > 0$ when $K < K^{nt}$.

Second fact. Since $\overline{\text{PRE}}(n, \beta) = n\overline{\text{PRE}}(1, \beta)$ (by equation (2)), $\beta > \underline{\beta}$ cannot be optimal when $n > \frac{C_o}{\overline{\text{PRE}}(1,\beta)}$. However, since $\overline{\text{PRE}}(1, \beta)$ approaches zero as β approaches $\underline{\beta}$, this bound is not uniform for all β. Fortunately, we can prove $\frac{\partial[\overline{\text{TOTAL}}(n,\beta)]}{\partial\beta}|_{\beta=\underline{\beta}} > 0$ when n is large, which helps establish a uniform bound. Given that $\frac{\partial[\overline{\text{PRE}}(1,\beta)]}{\partial\beta}|_{\beta=\underline{\beta}} > 0$ (by Proposition 4), we have $\frac{\partial[\overline{\text{PRE}}(n,\beta)]}{\partial\beta}|_{\beta=\underline{\beta}} = n\frac{\partial[\overline{\text{PRE}}(1,\beta)]}{\partial\beta}|_{\beta=\underline{\beta}} \to \infty$ as $n \to \infty$. This implies $\frac{\partial[\overline{\text{TOTAL}}(n,\beta)]}{\partial\beta}|_{\beta=\underline{\beta}} > 0$ for n large enough, provided that $\frac{\partial[\overline{\text{PAY}}(n,\beta)+\overline{\text{POST}}(n,\beta)+\overline{\text{NT}}(n,\beta)]}{\partial\beta}|_{\beta=\underline{\beta}}$ is uniformly bounded for all n (by inequality (20)). Therefore, there exist n_0 and $\delta > 0$ such that $\overline{\text{TOTAL}}$ is strictly increasing on $[\underline{\beta}, \underline{\beta} + \delta]$ for all $n > n_0$. Thus $\overline{\text{TOTAL}}(n, \underline{\beta}) < \overline{\text{TOTAL}}(n, \beta)$ for all $\beta > \underline{\beta}$ when $n > \max\{n_0, \frac{C_o}{\overline{\text{PRE}}(1,\underline{\beta}+\delta)}\}$, which is finite given that $\overline{\text{PRE}}(1, \underline{\beta} + \delta) > 0$ when $K > 0$.

Showing $\bar{N}_2$ exists. Suppose not; then there must exist a sequence $N^{(1)} < N^{(2)} < N^{(3)} \ldots$, such that $N^{(m)} \to \infty$ as m goes to infinity but $\beta^*(N^{(m)}) > \underline{\beta}$ for all m. Then by the second fact it must be that the sequence $\{n^*(N^{(m)})\}$ is bounded. By the first fact and $\overline{\text{TOTAL}}(n, \beta)$ continuous over the compact set $[\underline{\beta}, 1]$, $\min_{\beta\in[\underline{\beta},1],n\in\{n^*(N^{(1)}),n^*(N^{(2)}),\ldots\}} \overline{\text{TOTAL}}(n, \beta)$ exists and strictly exceeds $\overline{\text{POST}}(n = \infty, \underline{\beta}) = \lim_{n'\to\infty} \overline{\text{TOTAL}}(n', \underline{\beta})$. Hence there exists an m such that $\overline{\text{TOTAL}}(N^{(m)}, \underline{\beta}) < \overline{\text{TOTAL}}(n^*(N^{(m)}), \beta^*(N^{(m)}))$, a contradiction to $(n^*(N^{(m)}), \beta^*(N^{(m)}))$ optimal.