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RHO DOMINANCE AND A NEW FAMILY OF SUM RULES FOR PION BARYON SCATTERING

H. Banerjee *)
CERN - Geneva

S. Mallik **)
Saha Institute of Nuclear Physics, Calcutta

and

B. Dutta-Roy *)
Stevens Institute of Technology, Hoboken, N.J.

A B S T R A C T

A family of sum rules for the scattering of pions from spin-half baryon targets is presented. The derivation of the sum rules is based on the assumptions of i) unsubtracted dispersion relations for both the forward and the backward amplitudes (non-flip, flip and the A amplitudes) corresponding to unit isospin in the t channel, and ii) rho dominance (near direct channel threshold) of the t channel contributions in the dispersion relations for the backward amplitudes. In the πN sector, where a direct verification of our sum rules is possible, they are in satisfactory agreement with the available experimental data. Resonance saturation of our sum rules in the $\pi\Sigma$ sector together with the assumption of universal rho-hadron coupling yields the isovector magnetic moment of Σ in good agreement with the SU(3) prediction. As for pion-baryon coupling our sum rules in both the $\pi\Sigma$ and $\pi\Xi$ sectors are consistent with SU(3) symmetry with F/D ratio ~2/3. Our analysis of the $\pi\Xi$ sum rules further suggests the spin-parity assignment $J^P = 5/2^-$ for $\Xi^*(1930)$.

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*) On leave of absence from Saha Institute of Nuclear Physics, Calcutta, India.

**) Senior Research Fellow of the Department of Atomic Energy, India.

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1. INTRODUCTION

As early as 1960 Sakurai [1] observed that the threshold contribution from the Feynman diagram corresponding to the exchange of rho meson in t channel (hereinafter referred to as RECT) reproduces quite closely the s wave pion nucleon scattering length, $a^{(-)}$, antisymmetric in the isospin indices of the pion. The observation, interesting as it is, has defied all attempts towards a dynamical interpretation for quite some time. With the advent of the current algebra results for pion nucleon scattering interest in Sakurai's observation was revived [1] since the former seemed to provide a theoretical basis for the latter via the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin [2] (KSRF) relation. However, it was soon pointed out [3] that the derivation of the KSRF relation was not quite satisfactory and the problem of understanding the validity of this relation and hence of Sakurai's observation is still an open question.

Even if the KSRF relation is accepted as empirical, the RECT cannot, in general, be expected to yield the combination of s wave pion-hadron scattering lengths which correspond to the t channel quantum numbers $I^G = 1^+$ where I and G denote the isospin and G parity respectively. The current algebra approach provides us with an estimate of the pion-hadron s wave scattering length only in the "soft meson limit". It has been emphasized by Fubini and Furlan [4], as well as by Von Hippel and Kim [4] that if the low energy s wave phase shifts are large and/or there are low lying s wave resonances the current algebra estimate for the scattering length may not even be qualitatively correct.

From these observations emerges the puzzle what is the role of the RECT in the context of low energy meson-baryon scattering and how is this role to be reconciled with the empirical success of Sakurai's observation. A solution to this puzzle within the framework of on-shell dynamics is contained in a sum rule recently derived by the present authors [5]. The sum rule relates the rho pole (which is assumed to dominate the t channel singularities with isospin and G parity $I^G = 1^+$) contribution [6] to the backward non-flip amplitude at threshold to those from the direct channel singularities. The important point to be observed is that of the low-lying singularities in the direct channel only those in the odd partial waves make significant contributions to the sum rule, the even waves being kinematically suppressed. In the important case of πN scattering we thus have the result that the contributions from nucleon pole, $\Delta(1236)$, etc., to the dispersion theoretic expression for $a^{(-)}$ is, to a very good approximation, half of the rho pole contribution. It is thus clear that the connection between the rho pole contribution and $a^{(-)}$, required for an understanding of the empirical success of Sakurai's prescription for the πN scattering length, is to be attributed to the smallness of the contribution to dispersion theoretic expression for $a^{(-)}$ from even partial waves in the low energy region.

In the present work we investigate in detail the sum rules for πN, $\pi \Sigma$, and $\pi \Xi$ scattering. In each of these cases we examine the sum rules for the t channel helicity non-flip amplitude, the helicity flip amplitude (which in the usual notation is denoted by B), and the amplitude A. Apart from lending further support to our observations regarding the significance of the RECT, these sum rules open up new grounds for testing some of the familiar hypotheses of strong interaction physics. Thus, within the framework of our sum rules one may verify the validity of the assumption of universality of rho coupling to hadrons [1]. On the other hand, if the universality hypothesis is taken for granted one can obtain information about the meson baryon coupling constants and the isovector part of the anomalous magnetic moments of baryons from these sum rules. In some favourable cases it is even possible to specify the as yet unidentified quantum numbers of some resonances on the basis of our sum rules.

In Section 2 we present the sum rules, the details of their derivations being relegated to the Appendix. Sections 3, 4 and 5 are devoted to detailed investigation and discussion of the sum rules for pion scattering on N, Σ , and Ξ, respectively. Section 6 concludes by summarizing the main results obtained.

2. SUM RULES

We start from the Feynman matrix elements for the scattering of pion from a spin-half baryon target H (of mass M) which in the usual notation may be written as

$$T = \bar{u}(p')\left[-A(s,t) + i\frac{(\not{k}+\not{k}')}{2}B(s,t)\right]u(p) , \tag{1}$$

where k(k') and p(p') are the initial (final) meson and baryon four-momenta respectively and s, t, u denote, as usual, the Mandelstam variables. As mentioned in the introductory remarks, we consider amplitudes characterized by isospin and G parity $I^G = 1^+$ in the t channel. This is ensured for the combination $(\pi^- H - \pi^+ H)$ denoted in the following by a superscript (-); thus

$$A^- \equiv \frac{1}{2}\left(A_{\pi^- H} - A_{\pi^+ H}\right) .$$

By convention we choose H to be the member with the highest weight in the isospin multiplet to which it belongs.

The general procedure for the derivation of our sum rules for the helicity non-flip amplitude has been outlined in our earlier works [5)]. As for the amplitudes A^- and B^- the method of derivation of the appropriate sum rules is quite analogous. For the sake of completeness, however, we have explained the main steps in the derivation of these sum rules in the Appendix. Here we present the sum rules in a form suitable for our discussions in the following Sections. Using natural units $m_\pi = \hbar = c = 1$, we write

$$F_\rho^-(t=0) = \text{pole terms} + \frac{2}{\pi}\int_{W_0}^{\infty}\frac{dW}{q^2}\left[\left(1+\frac{E}{M}\right)\mathrm{Im}\, f_o^- + \left(1-\frac{E}{M}\right)\mathrm{Im}\, f_e^-\right], \tag{2}$$

$$\frac{1}{4\pi} A_\rho^-(t=0) = \frac{M}{2\pi^2}\int_{W_0}^{\infty}\frac{dW}{Wq^2}\left[A^-\right], \tag{3}$$

$$\frac{1}{4\pi} B_\rho^-(t=0) = \text{pole terms} + \frac{1}{2\pi^2}\int_{W_0}^{\infty}\frac{dW}{Wq^2}\, E\omega\left[B^-\right]. \tag{4}$$

Here the subscript ρ indicates the contribution to the sum rule from the rho meson pole in t channel so that

$$F_\rho^-(t) = \frac{g_{\rho\pi\pi}\, g_{\rho NN}\left(1+K^V m_\rho^2/4M^2\right)}{2\pi\left(m_\rho^2 - t\right)}, \tag{5}$$

$$\frac{1}{4\pi} A_\rho^-(t) = -\frac{g_{\rho\pi\pi}\, g_{\rho NN}}{2\pi}\cdot\frac{K^V}{(m_\rho^2-t)}, \tag{6}$$

and

$$\frac{1}{4\pi} B_\rho^-(t) = \frac{g_{\rho\pi\pi}\, g_{\rho NN}}{2\pi}\cdot\frac{(K^V+1)}{(m_\rho^2-t)}. \tag{7}$$

In the above q is the momentum and $W\,[E(\omega)]$ is the total [baryon (pion)] energy in the centre-of-mass system. The lower limit W_0 of the integrals corresponds to the threshold of the direct channel cut. The quantities appearing in the integrands are given by

$$f_{o,e}^- = \sum_{l\,(\text{odd, even})}\left[(l+1)\, f_{l+}^- + l\, f_{l-}^-\right], \tag{8}$$

in terms of the partial wave amplitudes

$$f_{\ell\pm} = \frac{e^{i\delta_{\ell\pm}}\sin\delta_{\ell\pm}}{q} ,$$

and

$$[A^-] \equiv \mathrm{Im}\left[A^-(s,t=0) - A^-(s,t=-4q^2)\right], \tag{9}$$

$$[B^-] \equiv \mathrm{Im}\left[\left(1+\frac{q^2}{E\omega}\right)B^-(s,t=0) - B^-(s,t=-4q^2)\right]. \tag{10}$$

The pole terms correspond to the contributions from single baryon intermediate states.

On the left-hand side of each sum rule appears K^V, the isovector part of the anomalous magnetic moment of H and the coupling constants $g_{\rho\pi\pi}$ and $g_{\rho HH}$ of the rho meson to the pion and the baryon respectively. It must be mentioned that the above identification of K^V is possible only through the assumption of rho dominance of the electromagnetic form factors of hadrons. According to the hypothesis of universal rho meson coupling to hadrons

$$g_{\rho HH} = I_H\, g_{\rho\pi\pi} , \tag{11}$$

where I_H is the isospin of the baryon H. Numerically $g_{\rho\pi\pi}$ may be determined from the width and mass of ρ. There are considerable uncertainties regarding the experimental numbers. For our discussions in the following we take [7] $\Gamma_\rho = 125 \pm 20$ MeV and $m_\rho = 765 \pm 10$ MeV, yielding

$$\frac{g^2_{\rho\pi\pi}}{4\pi} = 2.43 \pm 0.4 . \tag{12}$$

It may be noted that although there are only two independent invariant amplitudes for pion-baryon scattering we have three different sum rules. This comes about from the fact that the amplitudes free from kinematic singularities appropriate for backward dispersion relations have different kinematic coefficients [see Eqs. (A1) and (A2) in the Appendix] from those used for forward dispersion relations. The distinctive feature of our sum rule for the non-flip amplitude consists in the suppression of even partial waves in the low energy region. The sum rule for the A^- amplitude does not get any contribution from s and p waves. Thus, it is possible to test this sum rule even if one does not have adequate information of pion-baryon coupling.

3. SUM RULES FOR πN SCATTERING

In order to verify our sum rules for $\pi\Sigma$ and $\pi\Xi$ scattering, where we hardly have more information than the knowledge of the positions and the widths of resonances, the procedure of resonance saturation is unavoidable. It is of some interest to check on the accuracy of such a procedure in the case of πN scattering where we can estimate the right-hand sides of our sum rules independently by using phase shifts in the low energy region and Regge parametrization in the high energy regime.

In Table 1, the contributions to our sum rules from the region p_L (laboratory momentum of pion) ≤ 1.8 GeV/c are evaluated using the CERN phase shifts [8]. As for the Regge parameters, we have used those given by Rarita et al. [9] for the forward amplitudes, and by Barger and Cline [10] for the backward amplitudes from fits to the experimental data in the high energy regime ($p_L \leq 20$ GeV/c). It should be pointed out that, apart from some amount of uncertainties arising from the divergences in the different fits, both in the phase shift [11] and in the high energy [12] regions, the use of Regge parametrization right from $p_L \approx 1.8$ GeV/c, though unavoidable, is difficult to justify. In order to make a simple estimate of errors arising from all these sources we observe that the threshold forward amplitudes appearing on the right-hand sides of our sum rules may be expressed in terms of the s and/or p wave πN scattering lengths and make the conservative assumption that errors in the evaluation of the contributions from the backward amplitudes are comparable to those in the corresponding forward amplitudes. Our error estimates in Table 1 are based on the quoted errors in πN scattering lengths and coupling constant [13]. It is evident from Table 1 that the discrepancy between the two sides of our sum rules are well within our estimated errors. Comparison of the results from resonance saturation displayed in Table 2, with those in Table 1, suggests that the procedure of resonance saturation is fairly reliable and may be followed in $\pi\Sigma$ and $\pi\Xi$ scattering.

Our results justify the validity of the rho dominance assumption viz., that the rho pole contribution evaluated in the manner outlined in the preceding Section should provide a very good approximation for the entire t channel contribution to the backward dispersion relations in the neighbourhood of the πN threshold. Further evidences in support of this assumption may be derived from fixed u finite energy sum rules (FESR). Indeed, if non-rho terms in the t channel are simulated by an effective 3^- G meson (1660 MeV) pole, the pole parameters obtained [14] from FESR suggest that the contribution from the non-rho terms in our sum rules should be less than ten per cent of our estimate for ρ contribution.

Perhaps the most appropriate place to look for evidences in support of our rho dominance assumption is the energy dependence of

$$F_t^-(t=4(1-\omega^2)) \equiv F^-(\omega^2) + \frac{g_r^2}{8\pi M^2}\cdot\frac{1-\frac{1}{2}M^2}{(\omega^2-\frac{1}{4}M^2)} - \frac{1}{\pi}\int_1^\infty \frac{\mathrm{Im}\,F^-(\omega'^2)}{\omega'^2-\omega^2}d\omega'^2, \tag{13}$$

in the neighbourhood of the πN threshold. In the above, $F^-(\omega^2)$ is the backward amplitude as given in Eq. (A.2). In the Figure we have plotted the points representing $[F_t^-(t)]^{-1}$ as determined by Engels et al. [11] using CERN phase shifts. These points are very nicely fitted [at least up to $t = -4(\omega^2 - 1) \gg -40$] with a straight line corresponding to the inverse of $F_\rho^-(t)$ given in Eq. (5) with $g^2_{\rho\pi\pi}/4\pi \approx 2.65$ (i.e., $\Gamma_\rho = 138$ MeV). We are thus led to conclude that t dependence of $F_t^-(t)$ amply justifies our rho dominance assumption. The same observation holds for their evaluation of the t channel contributions to the backward B^- amplitude if one allows for some uncertainties consistent with our error estimates in Table 1. It is, therefore, remarkable that on the basis of their estimate for the rho contribution at threshold from the formula

$$F_\rho^-(t=0) = \frac{3}{\pi M} \int_4^\infty \frac{\mathrm{Im}\, f_+^1(t')}{t'} \, dt' ,$$

where $f_+^1(t)$ is the non-flip $J = 1$ $\pi\pi$-$N\bar{N}$ partial wave amplitude, these authors conclude that the sum rule (2) is badly satisfied and, therefore, the assumption of rho dominance is not valid. The source of the trouble, as they observe, is that the partial wave projection of the nucleon exchange contribution

$$B_N^- = g_r^2 \left(\frac{1}{M^2 - s} + \frac{1}{M^2 - u} \right) , \quad A_N^- = 0 , \tag{14}$$

is quite large and appreciably distorts, via t channel unitarity, $\mathrm{Im}\, f_+^1(t)$ in the neighbourhood of the rho mass. However, if this is really the case it is difficult to understand why a linear fit to their determination of $[F_t^-(t)]^{-1}$ should work. It may be pointed out that the nucleon pole contributions appropriate for our unsubtracted backward dispersion relations are strongly suppressed in the region $t \gg 4$ in contrast [15] with what one obtains from a direct use of the expressions in Eq. (14) for nucleon exchange contributions in the backward amplitudes. Moreover, it has been emphasized by Hamilton et al. [16] that the neglect of multipion ($\omega\pi$ etc.) intermediate states in the t channel unitarity may be quite serious in the determination of $\mathrm{Im}\, f_+^1(t)$ in the rho meson spectral region. Further, the behaviour of $J_+(t)$ [$J_+(t) \equiv f_+^1(t)/F_\pi(t)$, where $F_\pi(t)$ is the pion electromagnetic form factor] for $t > 4$

would be very much sensitive to the subtraction constants they use in the dispersion relation for $J_+(t)$. We would, therefore, like to conclude that the above uncertainties may easily account for the 25% discrepancy between their estimate of rho contribution and ours.

4. SUM RULES FOR $\pi\Sigma$ SCATTERING

Among the applications of our sum rules in the pion-baryon sector, the example of $\pi\Sigma$ scattering is distinguished by the fact that the inelastic channel $\pi\Sigma \to \pi\Lambda$ is open even below the physical threshold and by the presence of strong s wave scattering manifested through the s wave $Y_0^*(1405)$ resonance. It has been observed [17] that the current algebra sum rule for $\pi\Sigma$ scattering fails, and this, of course, is not surprising in view of the strong low energy s wave scattering and possibly because of the presence of inelastic channel open at threshold. Our sum rules for the helicity non-flip amplitude, the A^- amplitude and the helicity flip amplitude B^- respectively, read as follows:

$$\frac{g^2_{\rho\pi\pi}}{2\pi m^2_\rho}\left(1+\frac{m^2_\rho K^V_\Sigma}{4m^2_\Sigma}\right) = \frac{g^2_r}{2\pi}\left[\frac{(1-\alpha)^2}{3m_\Lambda m_\Sigma} + \frac{\alpha^2}{m^2_\Sigma}\right] + \cdot 049 \pm \cdot 004 \quad \text{(resonance and unphysical cut contribution)} \qquad (15)$$

$$-\frac{g^2_{\rho\pi\pi} K^V_\Sigma}{2\pi m^2_\rho} = -\cdot 123 \pm \cdot 008 \quad \text{(resonance and unphysical cut contribution)} \qquad (16)$$

$$-\frac{g^2_{\rho\pi\pi}(1+K^V_\Sigma)}{2\pi m^2_\rho} = \frac{g^2_r}{2\pi}\left[\frac{(1-\alpha)^2}{3m^2_\Lambda} + \frac{\alpha^2}{m^2_\Sigma}\right] + \cdot 179 \pm \cdot 008 \quad \text{(resonance and unphysical cut contribution)} \qquad (17)$$

Here K^V_Σ is the isovector anomalous magnetic moment of the Σ measured in Σ magnetons and the baryon pole term contributions to the sum rule have been written down on the assumption of SU(3) symmetry for the meson-baryon (ps) coupling in terms of the pion-nucleon coupling constant g_r and the F/D ratio specified by the parameter α defined through $\alpha \equiv F/(F+D)$. The composition of the resonance and the unphysical cut contributions are presented in Table 3.

Equation (16) may be solved for K_Σ^V. Using the value of $g^2_{\rho K K}/4\pi$ given by Eq. (12) we obtain $K_\Sigma^V = 0.76 \pm 0.12$ which conforms favourably with the SU(3) value $K_\Sigma^V = K_p + \frac{1}{2}K_n = 0.84$, so that henceforth we shall use SU(3) symmetry for the anomalous magnetic moments of baryons. As for the parameter α we shall use the value given by SU(6) symmetry, namely, $\alpha = 2/5$, which agrees with the determination $\alpha = 0.4 \pm 0.02$ from Kp forward dispersion relations [18], placing us in a position where we can test our sum rules (15), (16) and (17), which read

$$
\begin{aligned}
\cdot 177 \pm \cdot 03 &= \cdot 051 \pm \cdot 003\,[\Lambda\text{-pole}] + \cdot 064 \pm \cdot 004\,[\Sigma\text{-pole}] + \cdot 049 \pm \cdot 009 \\
&= \cdot 164 \pm \cdot 006,
\end{aligned}
\tag{18}
$$

$$
-\cdot 137 \pm \cdot 02 = -\cdot 123 \pm \cdot 008, \tag{19}
$$

and

$$
\begin{aligned}
\cdot 300 \pm \cdot 05 &= \cdot 055 \pm \cdot 003\,[\Lambda\text{-pole}] + \cdot 064 \pm \cdot 004\,[\Sigma\text{-pole}] + \cdot 179 \pm \cdot 008 \\
&= \cdot 298 \pm \cdot 01
\end{aligned}
\tag{20}
$$

respectively. The agreement within errors as evident in Eqs. (18), (19) and (20) justifies the use of SU(3) symmetry for the magnetic moments of baryons and pion-baryon coupling constants.

5. SUM RULES FOR $\pi\Xi$ SCATTERING

The $\pi\Xi$ sector (baryon number one and strangeness -2) is characterised by the fact that, apart from the relevant members of $\frac{1}{2}^+$ baryon octet and $\frac{3}{2}^+$ decuplet of baryon resonances, there are two other identified resonances, namely the $\Xi^*(1815)$ and $\Xi^*(1930)$, the spin and parity of which are as yet unknown. The recently observed [7] resonance $\Xi^*(2030)$ is not relevant for the present discussion since it does not seem to decay in the $\pi\Xi$ channel.

In this case the sum rules for the full amplitude, A^- and B^- amplitudes respectively are

$$\frac{g^2_{\xi\pi\pi}}{4\pi m_\rho^2}\left(1+\frac{m_\xi^2 K_\Xi^V}{4m_\Xi^2}\right) = \frac{g_Y^2}{4\pi m_\Xi^2}(1-2\alpha)^2 + \cdot 051 \pm \cdot 013 \left[\Xi^*(1530)\right]$$

+ contributions of $\Xi^*(1815)$ and $\Xi^*(1930)$,

(21)

$$-\frac{g^2_{\xi\pi\pi}}{4\pi m_\xi^2} K_\Xi^V = \cdot 071 \pm \cdot 018 \left[\Xi^*(1530)\right]$$

+ contributions of $\Xi^*(1815)$ and $\Xi^*(1930)$,

(22)

$$\frac{g^2_{\xi\pi\pi}}{4\pi m_\xi^2}\left(1+K_\Xi^V\right) = \frac{g_Y^2}{4\pi m_\Xi^2}(1-2\alpha)^2 - \cdot 017 \pm \cdot 004 \left[\Xi^*(1530)\right]$$

+ contributions of $\Xi^*(1815)$ and $\Xi^*(1930)$,

(23)

in an obvious notation. From SU(3) symmetry, which was found to be quite reliable in our consideration of sum rules for $\pi\Sigma$ scattering we have $K_\Xi^V = 2K_n + K_p \simeq -2.033$. If we also use the value $\alpha = 2/5$ the sum rules (21), (22) and (23) read

$$\cdot 068 \pm \cdot 011 = \cdot 007\,(\Xi\text{-pole}) + \cdot 051 \pm \cdot 013 \left[\Xi^*(1530)\right]$$

+ other resonance contributions ,

(24)

$$\cdot 166 \pm \cdot 027 = \cdot 071 \pm \cdot 018 \left[\Xi^*(1530)\right]$$

+ other resonance contributions ,

(25)

$$-\cdot 084 \pm \cdot 014 = \cdot 007\,(\Xi\text{-pole}) - \cdot 017 \pm \cdot 004 \left[\Xi^*(1530)\right]$$

+ other resonance contributions .

(26)

It is immediately evident that the sum rules for A^- and B^- amplitudes are very badly satisfied if we include contributions from $\Xi(1318)$ and $\Xi^*(1530)$ only; the discrepancy between the two sides seems to be too large to be accounted for by the inaccuracy involved in resonance saturation or by plausible high energy contribution beyond the resonance region. This implies that there must be other resonances which significantly contribute to these sum rules. Indeed, there are two more resonances: one is $\Xi^*(1815)$ with width $\Gamma \approx 16$ MeV and $\pi\Xi$ branching ratio $\sim 10\%$ and the other is $\Xi^*(1930)$ with width $\Gamma = 120$ MeV and the $\pi\Xi$ mode observed but the branching ratio as yet undetermined [7]. It is clear that $\Xi^*(1815)$, in view of its small width and small branching ratio for the decay mode relevant to our considerations, cannot play an important role in our sum rules; contrariwise, $\Xi^*(1930)$ being a broad resonance may be expected to provide significant contributions. The observation that sum rules (25) and (26) are badly satisfied, whereas the sum rule (24) is well satisfied leads us to conclude that $\Xi^*(1930)$, if it has to bring agreement to these sum rules, should be a negative parity (even partial wave) resonance. Furthermore, in view of the fact that s wave resonances do not contribute at all to the $A^{(-)}$ sum rule and their contributions to the sum rule (26) are suppressed compared to contributions from d or higher partial waves, it is most likely that $\Xi^*(1930)$ be a d wave resonance. It is interesting to note that our arguments may be pushed even further. We observe that a $d_{3/2}$ resonance in contrast to a $d_{5/2}$ resonance would make a negative contribution to the A^- sum rule and a positive contribution to B^- sum rule, which is exactly the opposite of what we wish to achieve. If, however, the spin parity of $\Xi^*(1930)$ were $5/2^-$, our sum rules would read

$$.068 \pm .01 = .058 \pm .013 \, [\Xi\text{-pole and } \Xi^*(1530)], \tag{27}$$

$$.166 \pm .027 = -.071 \pm .018 \, [\Xi^*(1530)] + .187x \, [\Xi^*(1930)] , \tag{28}$$

$$-.084 \pm .014 = -.010\,[\Xi\text{-pole and } \Xi^*(1530)] - .197x\,[\Xi^*(1930)] \tag{29}$$

where x is the $\pi\Xi$ branching ratio in $\Xi^*(1930)$ decay. We may thus conclude that if $\Xi^*(1930)$ is given the assignment $5/2^-$ and $\pi\Xi$ branching ratio (as yet undetermined from experiment) is $\sim 45\%$, the discrepancies in our sum rules disappear.

The spin parity assignment for $\Xi^*(1930)$ arrived at on the basis of our sum rules lends support to a recent speculation [19] that this resonance together with N(1680), Λ(1830) and Σ(1770) may constitute a $J^P = 5/2^-$ baryon octet. Indeed these authors point out that the Gell-Mann-Okubo mass formula is well satisfied, and furthermore the known widths and branching ratios for N(1680), Λ(1830) and Σ(1770) are well reproduced. These authors predict the partial width for $\Xi^*(1930) \to \pi\Xi$ to be 41 MeV which is in general agreement with our requirement.

It may be pointed out that $\Xi^*(1930)$ has also been speculated [20] as the Regge recurrence of Ξ(1318) and, therefore, as having the spin parity assignment $\frac{5}{2}^+$. Indeed, the slope of this trajectory $\alpha'_{\Xi} = 1.01/\mathrm{BeV}^2$ required for such an assignment agrees quite closely with those of the Regge trajectories for other members of the baryon octet, e.g., $\alpha'_N = 1.01/\mathrm{GeV}^2$, $\alpha'_\Lambda = 0.98/\mathrm{GeV}^2$. Moreover, $\Xi^*(1930)$ along with N(1690), Λ(1815), and Σ(1910) would complete an octet of $\frac{5}{2}^+$ resonances and the Gell-Mann-Okubo mass formula would again be well satisfied. It is, therefore, difficult to rule out unambiguously the $\frac{5}{2}^+$ assignment for $\Xi^*(1930)$ on the basis of branching ratios only, (which are in any case not well known), without taking symmetry breaking effects into account. However, our sum rules definitely disfavours such an assignment, the arguments being essentially the same as those used against the $\frac{3}{2}^-$ assignment.

6. CONCLUSION

We have presented above a set of dispersion theoretic sum rules for pion-baryon scattering. The underlying assumptions are the following. First, we assume unsubtracted dispersion relations for the forward as well as the backward pion-baryon elastic amplitudes. It should be emphasized that Regge asymptotics are sufficient but not necessary to justify this assumption. Our second assumption is that rho contribution dominates those from the entire t channel singularities in the dispersion relations for the appropriate backward amplitudes in the neighbourhood of the pion-baryon threshold.

The best place to test the validity of our rho dominance assumption is naturally the case of πN scattering, which is one of the best studied hadronic processes. Here, the close agreement of the two sides of our sum rules taking into account the uncertainties in the evaluation of the relevant dispersion integrals, the results[14] of analyses of fixed u finite energy sum rules, and lastly the t dependence of $[F_t^-(t)]^{-1}$ defined in Eq. (13) as evaluated by Engels et al.[11] provide the required justification for our assumptions.

In the $\pi\Sigma$ sector, from the sum rule (16) for the A^- amplitude, we have obtained the isovector part of the anomalous magnetic moment of Σ in close agreement with the prediction of the SU(3) symmetry. As for the $BB\pi$ couplings the sum rules are consistent with the SU(3) symmetry and the value of the parameter α $(=F/(F+D))$ as determined by Kim[18] from an analysis of Kp forward dispersion relations. Finally, our sum rules for $\pi\Xi$ scattering favour the J^p assignment as $\frac{5}{2}^-$ for $\Xi^*(1930)$.

In conclusion we would like to point out that the successes of the present approach are not confined to the sum rules for pion-baryon scattering only. The sum rules appropriate for kaon-nucleon scattering [21] are in satisfactory agreement with the available data. It has also been demonstrated elsewhere [22] that because of the suppression of even partial waves [see Eq. (2); in the case of the sum rules for meson-meson scattering [5] the even partial waves do not contribute at all] our sum rules for the non-flip amplitudes provide a convenient framework for a dynamical derivation of the higher symmetry relations among the trilinear hadronic coupling constants.

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APPENDIX

We shall outline here the derivation of the sum rules. In the forward direction the non-flip amplitude is

$$D^-(\omega_L) \equiv \frac{1}{4\pi}\left[A^-(s,t=0) + \omega_L B^-(s,t=0)\right], \tag{A.1}$$

in terms of the invariant amplitudes $A(s,t,u)$ and $B(s,t,u)$, which are analytic functions of the Mandelstam variables $s = M^2+1+2M\omega_L$ and $t = -2q^2(1-\cos\Theta)$. From the symmetry property of the amplitudes A^- and B^- in s, u variables, it follows that the amplitude $D^-(\omega_L)/\omega_L$ is free from kinematic singularities in the ω_L^2 plane and satisfies, in view of Pomeranchuk's theorem, an unsubtracted dispersion relation. In the backward direction, the non-flip amplitude is defined as

$$F^-(\omega^2) \equiv \frac{1}{4\pi}\left[\frac{E}{M\omega} A^-(s,t=-4q^2) + B^-(s,t=-4q^2)\right]. \tag{A.2}$$

$F^-(\omega^2)$ is again free from kinematic singularities and considerations based on Regge asymptotic allow writing an unsubtracted dispersion relation. Rho pole along with other t channel singularities contribute to the dispersion relation for $F^-(\omega^2)$ and because of the assumed lack of subtraction our rho dominance assumption is just equivalent to the statement that nearby singularities dominate.

Similarly, we can write dispersion relations for the amplitudes $A^-(s,t)/(s-u)$ and $B^-(s,t)$ which are free from kinematic singularities both in the forward direction in ω_L^2 plane and in the backward direction in ω^2 plane. Again our assumption of unsubtracted dispersion relations is justified from considerations based on leading Regge poles in the crossed channels.

TABLE 1

Contributions to πN sum rules from the "phase shift" and the high energy regions

Amplitudes	Nucleon pole $g_r^2/4\pi = 14.5$	Phase shift region $p_L \leqslant 1.8$ GeV/c	High energy region $1.8 \leqslant p_L \leqslant 20$ GeV/c	Total right-hand side with estimated errors*	Left-hand side
Non-flip	0.320	-0.162	0.003	0.161±0.020	0.132±0.022
A^-	0	-0.301	-0.061	-0.362±0.090	-0.304±0.050
B^-	0.320	0.115	0.058	0.493±0.090	0.386±0.064

* For a discussion on the estimation of the errors, see the text.

TABLE 2

Resonance [7] contributions to πN sum rules

J^P	Resonance (Mass in MeV)	Contributions to the sum rules for		
		non-flip amplitude	A^- amplitude	B^- amplitude
1/2$^+$	N'(1460)	0.025*	0	0.016*
3/2$^-$	N (1515	-0.001	-0.086	0.094
1/2$^-$	N (1525)	-0	0	0
5/2$^-$	N (1675)	-0.001	0.038	-0.043
5/2$^+$	N (1690)	0.017	-0.011	0.015
1/2$^-$	N'(1715)	-0.001	0	0.001
1/2$^+$	N"(1785)	-0.001	0	0.004
7/2$^-$	N (2190)	-0.002	-0.030	0.039
3/2$^+$	Δ (1236)	-0.189*	-0.239*	0.037*
1/2$^-$	Δ (1630)	0	0	-0
3/2$^-$	Δ (1670)	0	0.020	-0.023
5/2$^+$	Δ (1880)	-0.007	0.004	-0.005
1/2$^+$	Δ (1905)	-0.003	0	-0.002
7/2$^+$	Δ (1940)	-0.013	-0.018	0.001
11/2$^+$	Δ (2420)	-0.003	-0.005	-0
Total resonance contribution		-0.179	-0.327	0.134
Nucleon pole contribution		0.320±0.020	0	0.320±0.020
Total (right-hand side)		0.141±0.020	-0.327	0.454±0.020

* Calculated from phase shift analysis data.

TABLE 3

Resonance [7] contributions to $\bar{K}\Sigma$ sum rules

J^P	Resonance (Mass in MeV)	Contributions to the sum rules for non-flip amplitude	A^- amplitude	B^- amplitude
$1/2^-$	$\Lambda(1405)$	-0	0	0
$3/2^-$	$\Lambda(1520)$	-0	-0.082±0.007	0.082±0.007
$1/2^-$	$\Lambda(1670)$	0	0	0
$3/2^-$	$\Lambda(1690)$	-0	-0.021	0.022
$5/2^-$	$\Lambda(1815)$	0.001	-0.001	0.002
$5/2^-$	$\Lambda(1830)$	-0	0.016	-0.017
$7/2^-$	$\Lambda(2100)$	0	-0.003	0.003
$3/2^+$	$\Sigma(1385)$	0.042*	0.058*	-0.013*
$3/2^-$	$\Sigma(1660)$	-0	-0.093	0.098
$5/2^+$	$\Sigma(1700)$	0.004±0.004	-0.003±0.003	0.004±0.004
$5/2^-$	$\Sigma(1765)$	-0	0.001	-0.001
$5/2^+$	$\Sigma(1905)$	0.002	-0	0
$7/2^+$	$\Sigma(2030)$	0	0.003	-0.001
Total resonance contribution		0.049±0.004	-0.123±0.008	0.179±0.008

* Calculated à la Kim [18] and includes contribution from unphysical region; the contribution from other s and p waves in this region being negligible.

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FIGURE CAPTION

The points ● represent the inverse of $F_t^-(t)$, defined in Eq. (13) as determined by Engels et al., rising CERN phase shifts. The straight line represents $1/F_\rho^-(t)$ calculated according to Eq. (5) with $\Gamma_\rho = 138$ MeV.