Korean J. Math. **29** (2021), No. 2, pp. 345–353
http://dx.doi.org/10.11568/kjm.2021.29.2.345

RIEMANN SOLITONS ON CERTAIN TYPE OF KENMOTSU MANIFOLD

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Abstract. The object of the present paper is to investigate the nature of Riemann solitons on generelized D-conformally deformed Kenmotsu manifold with hyper generalized pseudo symmetric curvature conditions.

1. Introduction

Let the symbols ∇ and ∇^d stand for the Riemann connection and the generalized D-conformally deformed connection respectively. Also, let R, S, Q, r and R^d, S^d, Q^d, r^d respectively stand for curvature tensor, Ricci tensor, Ricci operator, scalar curvature with respect to ∇ and ∇^d respectively. In this study, we consider an almost contact metric manifold $(M^{2n+1}, \phi, \xi, \eta, g)$ that consists of a $(1,1)$-tensor field ϕ, a vector field ξ and a 1-form η called respectively the structure endomorphism, the characteristic vector field and the contact form. In a recent paper, the authors ([2]) has introduced a new type of space called hyper generalized weaky symmetric manifold. Then the authors studied ([8]) hyper generalized pseudo Q-symmetric semi-Riemanian manifold. In Section 3 of this paper we extend this concept to generalized D-conformally deformed structure of a $(2n+1)$-dimensional Kenmotsu manifold.

A $(2n+1)$-dimensional Kenmotsu manifold is said to be hyper generalized pseudo symmetric (which will be abbreviated hereafter as $[H(GPS)_n, \nabla]$) if it admits the equation

$$
\begin{aligned}
&(\nabla_X \bar{R})(Y,U,V,\ W)\\
= \;& 2\alpha(X)\bar{R}(Y,U,V,\ W) + \alpha(Y)\bar{R}(X,U,V,\ W)\\
&+\alpha(U)\bar{R}(Y,X,V,\ W) + \alpha(V)\bar{R}(Y,U,X,\ W)\\
&+\alpha(W)\bar{R}(Y,U,V,\ X) + 2\beta(X)(g\wedge S)(Y,U,V,W)\\
&+\beta(Y)(g\wedge S)(X,U,V,W) + \beta(U)\ (g\wedge S)(Y,X,V,W)\\
&+\beta(V)\ (g\wedge S)(Y,U,X,W) + \beta(W)\ (g\wedge S)(Y,U,V,X), \qquad (1)
\end{aligned}
$$

Received January 20, 2021. Revised March 30, 2021. Accepted April 6, 2021.
2010 Mathematics Subject Classification: 53C15, 53C25.
Key words and phrases: Almost contact metric structure, generelized D-conformally deformation, Riemann solitons, η-Einstein manifolds.
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where

$$(g \wedge S)(Y,U,V,W) = g(Y,W)S(U,V) + g(U,V)S(Y,W) - g(Y,V)S(U,W) - g(U,W)S(Y,V), \tag{2}$$

and α, β being non-zero 1-forms defined as $\alpha(X) = g(X,\theta_1)$ and $\beta(X) = g(X,\theta_2)$.

Ricci flow was first introduced by R. S. Hamilton ([17]) in 1982 which generalizes the notion of Riemann flow ([20], [19]). Keeping the tune with Ricci soliton, Hirica and Udriste ([18]) introduced and studied Riemann soliton. The Riemann flow is an evolution equation for metrics on a Riemannian manifold defined as follows

$$\frac{\partial}{\partial t} G(t) = -2R(g(t)), \qquad t \in [0, I],$$

where $G = \frac{1}{2} g \circledast g$, $\circledast$ is the Kulkarni-Nomizu product and R is the Riemann curvature tensor associated to the metric g. For $(0,2)$-tensors A and B, the Kulkarni-Nomizu product $(A \circledast B)$ is given by

$$(A \circledast B)(Y,U,V,Z) = A(Y,Z)B(U,V) + A(U,V)B(Y,Z) - A(Y,V)B(U,Z) - A(U,Z)B(Y,V). \tag{3}$$

Recently, the present authors studied the Riemann solitons in the frame of $(LCS)_n$-manifolds ([4]) and α-cosymplectic manifolds ([5]). The Riemann soliton is a smooth manifold M together with Riemannian metric g that satisfies

$$2R + (g \circledast \pounds_W g) = 2\kappa(g \circledast g), \tag{4}$$

where W is a potential vector field, $\pounds_W$ denotes the Lie-derivative along the vector field W and κ is a constant. A Riemann soliton is called expanding, steady and shrinking when $\kappa < 0$, $\kappa = 0$ and $\kappa > 0$ respectively.

We organize our present paper as follows: After Introduction, in Section 2, we briefly recall some known results for Kenmotsu manifolds and generalized D-conformally deformed of a Kenmotsu manifold and established some properties of the deformed Kenmotsu manifold. In Section 3, we discuss the properties of a generalized D-conformally deformed Kenmotsu manifold under hyper generalized pseudo symmetric curvature condition equipped with Riemann solitions. Finally, e determine a necessary condition for shrinking, steady and expanding of the soliton.

2. Preliminaries

According to the definition of Blair ([11]), an *almost contact structure* (ϕ, ξ, η) on a $(2n+1)$-dimensional Riemannian manifold satisfies the following conditions

$$\phi^2 = -I + \eta \otimes \xi, \tag{5}$$

$$\eta(\xi) = 1, \tag{6}$$

$$\phi\xi = 0, \ \eta \circ \phi = 0, \ \text{rank } \phi = n - 1. \tag{7}$$

Moreover, if g is a Riemannian metric on M^{2n+1} satisfying

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \tag{8}$$

$$g(X,\xi) = \eta(X), \tag{9}$$

$$g(\phi X, Y) = -g(X, \phi Y), \tag{10}$$

for any vector fields X, Y on M^{2n+1}, then the manifold M^{2n+1} ([11]) is said to admit an *almost contact metric structure* (ϕ, ξ, η, g).

DEFINITION 2.1. [15] If in an almost contact metric structure (ϕ, ξ, η, g) on M^{2n+1}, the Riemann connection ∇ of g satisfies $(\nabla_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X$, for any vector fields X, Y on M^{2n+1}, then the structure is called Kenmotsu.

PROPOSITION 2.2. [3,9,15] *If $(M^{2n+1}, \phi, \xi, \eta, g)$ is a Kenmotsu manifold, then for any vector fields X, Y, Z on M^{2n+1}, the following relations hold*

$$\nabla_X \xi = X - \eta(X)\xi, \tag{11}$$

$$(\nabla_X \eta)Y = g(X, Y) - \eta(X)\eta(Y), \tag{12}$$

$$S(X, \xi) = -2n\eta(X), \tag{13}$$

$$\eta(R(X, Y)Z) = g(X, Z)\eta(Y) - g(Y, Z)\eta(X), \tag{14}$$

$$R(\xi, X)Y = \eta(Y)X - g(X, Y)\xi, \tag{15}$$

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X. \tag{16}$$

DEFINITION 2.3. [1] If a contact metric manifold M^{2n+1} with the almost contact metric structure (ϕ, ξ, η, g) is transformed into $(\phi^d, \xi^d, \eta^d, g^d)$, where

$$\phi^d = \phi, \ \ \xi^d = \frac{1}{p}\xi, \ \ \eta^d = p\eta, \ \ g^d = qg + (p^2 - q)\eta \otimes \eta, \tag{17}$$

where p and q are constants such that $p \neq 0$ and $q > 0$, then the transformation is called a generalized D-conformal deformation.

Note that the generalized D-conformal deformation give rise to conformal deformation (for $p^2 = q$) and D-homothetic deformation (for $p = q =$ constant) ([16], [6], [10]). The generalized D-conformal deformation are studied by various authors in ([22], [23], [21], [24]).

The relation between the Levi-Civita connections ∇ of g and ∇^d of g^d is given by ([1])

$$\nabla^d_X Y = \nabla_X Y + \frac{(p^2 - q)}{p^2} g(\phi X, \phi Y)\xi, \tag{18}$$

for any vector fields X, Y on M^{2n+1}.

In view of (17), (18) and definition of Riemannian curvature tensor, Ricci tensor, scalar curvature, we get the following:

PROPOSITION 2.4. [1] *If a Kenmotsu structure (ϕ, ξ, η, g) on M^{2n+1} is transformed into $(\phi^d, \xi^d, \eta^d, g^d)$ under a generalized D-conformal deformation, then R, R^d, S, S^d, r and r^d are related by*

$$R^d(X, Y)Z = R(X, Y)Z + \frac{(p^2 - q)}{p^2}[g(\phi Y, \phi Z)X - g(\phi X, \phi Z)Y], \tag{19}$$

$$S^d(X, Y) = S(X, Y) + \frac{2n(p^2 - q)}{p^2} g(\phi X, \phi Y), \tag{20}$$

$$r^d = \frac{r}{q} + \frac{2n(2n+1)(p^2-q)}{p^2}, \tag{21}$$

for any vector fields X, Y, Z on M^{2n+1}.

Now we shall bring out some properties of a generalized D-conformally deformed structure (ϕ^d, ξ^d, η^d, g^d) of a Kenmotsu manifold M^{2n+1} as follows:

Proposition 2.5. *Under a generalized D-conformal deformation of a Kenmotsu structure (ϕ, ξ, η, g) on M^{2n+1} is transformed into (ϕ^d, ξ^d, η^d, g^d), then for any vector fields X, Y, Z on M^{2n+1}, we have*

$$\phi^d = -I + \eta^d \otimes \xi^d, \tag{22}$$

$$\eta^d(\xi^d) = 1, \tag{23}$$

$$\phi^d \xi^d = 0, \;\; \eta^d \circ \phi^d = 0, \tag{24}$$

$$g^d(\phi^d X, \phi^d Y) = g^d(X,Y) - \eta^d(X)\eta^d(Y), \tag{25}$$

$$g^d(X, \xi^d) = \eta^d(X), \tag{26}$$

$$\nabla^d_X \xi^d = \frac{1}{p}[X - \eta^d(X)\xi^d], \tag{27}$$

$$(\nabla^d_X \eta^d)Y = \frac{1}{p}[g^d(X,Y) - \eta^d(X)\eta^d(Y)], \tag{28}$$

$$S^d(X, \xi^d) = -\frac{2n}{p^2}\eta^d(X), \tag{29}$$

$$\eta^d(R^d(X,Y)Z) = \frac{1}{p^2}[g^d(X,Z)\eta^d(Y) - g^d(Y,Z)\eta^d(X)], \tag{30}$$

$$R^d(\xi^d, X)Y = \frac{1}{p^2}[\eta^d(Y)X - g^d(X,Y)\xi^d], \tag{31}$$

$$R^d(X,Y)\xi^d = \frac{1}{p^2}[\eta^d(X)Y - \eta^d(Y)X]. \tag{32}$$

Now using (28) and (29), we obtain

$$(\nabla^d_X S^d)(Y, \xi^d) = -\frac{1}{p}[\frac{2n}{p^2} g^d(X,Y) + S^d(X,Y)], \tag{33}$$

for any vector fields X ,Y and Z on M^{2n+1}.

3. Main results

In the beginning, we shall define a hyper generalized pseudo symmetric space on a generalized D-conformally deformed structure (ϕ^d, ξ^d, η^d, g^d) of a Kenmotsu manifold M^{2n+1}.

3.1. Hyper generalized pseudo symmetric deformed Kenmotsu manifold.

Definition 3.1. A generalized D-conformally deformed structure $(\phi^d, \xi^d, \eta^d, g^d)$ of a Kenmotsu manifold M^{2n+1} is said to be hyper generalized pseudo symmetric if it satisfies the condition

$$\begin{aligned}
&(\nabla^d_X \bar{R}^d)(Y,U,V,W)\\
=\;& 2\alpha^d(X)\bar{R}^d(Y,U,V,\ W) + \alpha^d(Y)\bar{R}^d(X,U,V,\ W)\\
&+\alpha^d(U)\bar{R}^d(Y,X,V,\ W) + \alpha^d(V)\bar{R}^d(Y,U,X,\ W)\\
&+\alpha^d(W)\bar{R}^d(Y,U,V,\ X) + 2\beta^d(X)(g^d \wedge S^d)(Y,U,V,W)\\
&+\beta^d(Y)(g^d \wedge S^d)(X,U,V,W) + \beta^d(U)\ (g^d \wedge S^d)(Y,X,V,W)\\
&+\beta^d(V)\ (g^d \wedge S^d)(Y,U,X,W) + \beta^d(W)\ (g^d \wedge S^d)(Y,U,V,X). \quad (34)
\end{aligned}$$

where

$$\begin{aligned}
(g^d \wedge S^d)(Y,U,V,W) =\;& g^d(Y,W)S^d(U,V) + g^d(U,V)S^d(Y,W)\\
&-g^d(Y,V)S^d(U,W) - g^d(U,W)S^d(Y,V), \quad (35)
\end{aligned}$$

and A^d_i are non-zero 1-forms defined by $A^d_i(X) = g^d(X, \sigma_i)$, for $i = 1, 2$.

In this section, we consider a Kenmotsu manifold (M^{2n+1}, g) $n \geq 1$ which is hyper generalized pseudo symmetric. Now, making use of (35) in (34), we find

$$\begin{aligned}
&(\nabla_X \bar{R}^d)(Y,U,V,\ W)\\
=\;& 2\alpha^d(X)\bar{R}^d(Y,U,V,\ W) + \alpha^d(Y)\bar{R}^d(X,U,V,\ W)\\
&+\alpha^d(U)\bar{R}^d(Y,X,V,\ W) + \alpha^d(V)\bar{R}^d(Y,U,X,\ W)\\
&+\alpha^d(W)\bar{R}^d(Y,U,V,\ X) + 2\beta^d(X)[g^d(Y,W)S^d(U,V)\\
&+g^d(U,V)S^d(Y,W) - g^d(Y,V)S^d(U,W)\\
&-g^d(U,W)S^d(Y,V)] + \beta^d(Y)[g^d(X,W)S^d(U,V)\\
&+g^d(U,V)S^d(X,W) - g^d(X,V)S^d(U,W)\\
&-g^d(U,W)S^d(X,V)] + \beta^d(U)\ [g^d(Y,W)S^d(X,V)\\
&+g^d(X,V)S^d(Y,W) - g^d(Y,V)S^d(X,W)\\
&-g^d(X,W)S^d(Y,V)] + \beta^d(V)\ [g^d(Y,W)S^d(U,X)\\
&+g^d(U,X)S^d(Y,W) - g^d(Y,X)S^d(U,W) - g^d(U,W)S^d(Y,X)]\\
&+\beta^d(W)[g^d(Y,X)S^d(U,V) + g^d(U,V)S^d(Y,X)\\
&-g^d(Y,V)S^d(U,X) - g^d(U,X)S^d(Y,V)]. \quad (36)
\end{aligned}$$

Now, contracting over Y and W in (36), we get

$$\begin{aligned}
&(\nabla^d_X S^d)(U,V)\\
=\;& 2\alpha^d(X)S^d(U,V)+\alpha^d(U)S^d(X,V)\\
&+\alpha^d(R^d(X,U)V)+2\beta^d(Q^dX)g^d(U,V)\\
&+\alpha^d(R^d(X,V)U)+\alpha^d(V)S^d(X,U)\\
&+2\beta^d(X)[2nS^d(U,V)+r^dg^d(U,V)]\\
&+\beta^d(U)\ [(2n-2)S^d(X,V)+r^dg^d(X,V)]\\
&+\beta^d(V)\ [(2n-2)S^d(X,U)+r^dg^d(X,U)]\\
&-\beta^d(Q^dU)g^d(X,V)-\beta^d(Q^dV)g^d(X,U).
\end{aligned}\tag{37}$$

Now setting $V=\xi^d$ and using (29), (31), (32) in the foregoing equation, we obtain

$$\begin{aligned}
&(\nabla^d_X S^d)(U,\xi^d)\\
=\;& -\frac{2n}{p^2}[2\alpha^d(X)\eta^d(U)+\alpha^d(U)\eta^d(X)]+\alpha^d(\xi^d)S^d(X,U)\\
&+\frac{1}{p^2}[g^d(X,U)\alpha^d(\xi^d)-2\eta^d(U)\alpha^d(X)+\eta^d(X)\alpha^d(U)]\\
&+2\beta^d(X)(r^d-\frac{4n^2}{p^2})\eta^d(U)+\beta^d(U)(r^d-\frac{4n(n-1)}{p^2})\eta^d(X)\\
&+\beta^d(\xi^d)\ [2(n-1)S^d(U,X)+r^dg^d(U,X)]\\
&+2\beta^d(Q^dX)\eta^d(U)-\beta^d(Q^dU)\eta^d(X)+\frac{2n}{p^2}\beta^d(\xi^d)g^d(U,X)
\end{aligned}\tag{38}$$

which yields by using (33)

$$\begin{aligned}
&-\frac{1}{p}[\frac{2n}{p^2}g^d\,(X,U)+S^d\,(X,U)]\\
=\;& [-\frac{2(2n+1)}{p^2}\alpha^d(X)+2\beta^d(X)(r^d-\frac{4n^2}{p^2})+2\beta^d(Q^dX)]\eta^d(U)\\
&+[-\frac{(2n-1)}{p^2}\alpha^d(U)+(r^d-\frac{4n(n-1)}{p^2})\beta^d(U)-\beta^d(Q^dU)]\eta^d(X)\\
&+\frac{1}{p^2}\alpha^d(\xi^d)g^d(X,U)+\alpha^d(\xi^d)S^d(X,U)+\frac{2n}{p^2}\beta^d(\xi^d)g^d(U,X)\\
&+\beta^d(\xi^d)\ [2(n-1)S^d(U,X)+r^dg^d(U,X)].
\end{aligned}\tag{39}$$

Putting $X=U=\xi^d$ in succession, we obtain from (39) that

$$[r^d-\frac{2n(2n-1)}{p^2}]\beta^d(\xi^d)=\frac{2n}{p^2}\alpha^d(\xi^d).\tag{40}$$

$$\begin{aligned}
&-\frac{2(2n+1)}{p^2}\alpha^d(X)+2\beta^d(X)(r^d-\frac{4n^2}{p^2})+2\beta^d(Q^dX)\\
=\;& [\frac{2(2n-1)}{p^2}\alpha^d(\xi^d)-2\beta^d(\xi^d)(r^d-\frac{4n(n-1)}{p^2})+2\beta^d(Q^d\xi^d)]\eta^d(X).
\end{aligned}\tag{41}$$

and

$$-\frac{(2n-1)}{p^2}\alpha^d(U)+(r^d-\frac{4n(n-1)}{p^2})\beta^d(U)-\beta^d(Q^dU)$$
$$(42)\qquad = [\frac{(6n+1)}{p^2}\alpha^d(\xi^d)-(3r^d-\frac{12n^2-2n}{p^2})\beta^d(\xi^d)]\eta^d(U).$$

By virtue of (40), (41) and (42), the equation (39) yields

$$S^d(X,U)$$
$$= -\left(\frac{\frac{1}{p^2}\alpha^d(\xi^d)+(r^d+\frac{2n}{p^2})\beta^d(\xi^d)+\frac{2n}{p^3}}{\frac{1}{p}+\alpha^d(\xi^d)+2(n-1)\beta^d(\xi^d)}\right)g^d(U,X)$$
$$(43)\qquad -\left(\frac{\frac{(10n-1)}{p^2}\alpha^d(\xi^d)-(5r^d-\frac{20n^2-14n}{p^2})\beta^d(\xi^d)}{\frac{1}{p}+\alpha^d(\xi^d)+2(n-1)\beta^d(\xi^d)}\right)\eta^d(U)\eta^d(X).$$

and (40) gives

$$(44)\qquad r^d=\frac{2n}{p^2}[\frac{\alpha^d(\xi^d)}{\beta^d(\xi^d)}+(2n-1)].$$

Next using (44) in (43) we have

$$S^d(X,U)$$
$$= -\left(\frac{\frac{(2n+1)}{p^2}\alpha^d(\xi^d)+\frac{4n^2}{p^2}\beta^d(\xi^d)+\frac{2n}{p^3}}{\frac{1}{p}+\alpha^d(\xi^d)+2(n-1)\beta^d(\xi^d)}\right)g^d(U,X)$$
$$(45)\qquad +\left(\frac{\frac{1}{p^2}\alpha^d(\xi^d)+\frac{4n}{p^2}\beta^d(\xi^d)}{\frac{1}{p}+\alpha^d(\xi^d)+2(n-1)\beta^d(\xi^d)}\right)\eta^d(U)\eta^d(X).$$

This motivate us to state:

Theorem 3.2. *Let* $(\phi^d, \xi^d, \eta^d, g^d)$ *be a generalized* D*-conformally deformed hyper generalized pseudo symmetric Kenmotsu manifold* M^{2n+1}*. Then such a space is conformally flat provided* $\frac{1}{p}+\alpha^d(\xi^d)+2(n-1)\beta^d(\xi^d)\neq 0$.

Theorem 3.3. *The scalar curvature of a hyper generalized pseudo symmetric generalized* D*-conformally deformed Kenmotsu manifold is constant.*

Corollary 3.4. *Let* $(\phi^d, \xi^d, \eta^d, g^d)$ *be a generalized* D*-conformally deformed pseudo symmetric Kenmotsu manifold* M^{2n+1}*. Then such a space is conformally flat provided* $p\alpha^d(\xi^d)\neq -1$.

4. Generalized D-conformally deformed hyper generalized pseudo symmetric Kenmotsu manifold and the case of Riemann soliton

In this section, we consider a generalized D-conformally deformed Kenmotsu manifold $(\phi^d, \xi^d, \eta^d, g^d)$ admitting a Riemann soliton. Then with the aid of (3) and (4), we obtain

$$\begin{aligned} & 2R^d(Y,U,V,Z) + g^d(Y,Z)(\pounds_{\xi^d} g^d)(U,V) \\ & + g^d(U,V)(\pounds_{\xi^d} g^d)(Y,Z) \\ & - g^d(Y,V)(\pounds_{\xi^d} g^d)(U,Z) - g^d(U,Z)(\pounds_{\xi^d} g^d)(Y,V) \\ = \; & 2\kappa\left[g^d(Y,Z)\, g^d(U,V) - g^d(Y,V)\, g^d(U,Z)\right]. \end{aligned} \tag{46}$$

Now by contraction over Y and Z we get

$$\frac{1}{2}(\pounds_{\xi^d} g^d)(U,V) + \frac{1}{2n-1} S^d(U,V) = \frac{2n\kappa - div(\xi^d)}{2n-1} g^d(U,V). \tag{47}$$

and then using (47) in (46), we get

$$r^d = 2n[(2n+1)\kappa - \frac{4n}{p}]. \tag{48}$$

Comparing (44) with (48) we have

$$(2n+1)p^2\kappa = 4np + (2n-1) + \frac{\alpha^d(\xi^d)}{\beta^d(\xi^d)}. \tag{49}$$

This leads to the following:

THEOREM 4.1. *Assume that a Kenmotsu structure* (ϕ, ξ, η, g) *on* M^{2n+1} *is transformed into* $(\phi^d, \xi^d, \eta^d, g^d)$ *under a generalized D-conformally deformation which is a hyper generalized pseudo symmetric space. Then the Riemann soliton is expanding, steady and shrinking as* $\frac{\alpha^d(\xi^d)}{\beta^d(\xi^d)} + 2n(2p+1) <=> 1$.

Acknowledgement. The first named author gratefully acknowledges to UGC, F.No. 16-6(DEC.2018)/2019(NET/CSIR) and UGC-Ref.No. 1147/(CSIR-UGC NET DEC. 2018) for financial assistance.

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