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James A. Lock
Cleveland State University, j.lock@csuohio.edu

Gérard Gouesbet

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Rigorous justification of the localized approximation to the beam-shape coefficients in generalized Lorenz–Mie theory.   I.   On-axis beams

James A. Lock

Department of Physics, Cleveland State University, Cleveland, Ohio 44115

Gérard Gouesbet

Laboratoire d'Energétique des Systèmes and Procédés, Unité de Recherche Associée au Centre National de la Recherche Scientifique, No. 230, Institut National des Sciences Appliquées de Rouen, B.P. 08, 76131 Mont-Saint-Aignan Cedex, France

Generalized Lorenz–Mie theory describes electromagnetic scattering of an arbitrary light beam by a spherical particle. The computationally most expensive feature of the theory is the evaluation of the beam-shape coefficients, which give the decomposition of the incident light beam into partial waves. The so-called localized approximation to these coefficients for a focused Gaussian beam is an analytical function whose use greatly simplifies Gaussian-beam scattering calculations. A mathematical justification and physical interpretation of the localized approximation is presented for on-axis beams.

1.   INTRODUCTION

In the analysis of many applications of light scattering, such as optical particle sizing, optical particle levitation, and the production of nonlinear optical effects in microdroplets, it is of great importance to have both an accurate and a computationally efficient method of calculating the scattering of an arbitrary light beam by a spherical particle. Of particular interest is the scattering of a focused Gaussian laser beam. There are a number of mathematically equivalent theories for arbitrary beam scattering.[1-7] An element common to each of these theories is the decomposition of the incident beam into an infinite series of elementary constituents, such as partial waves[1-5] or plane waves,[6,7] each of which is scattered by the spherical particle in a well-known and easily calculable way. The amplitude and the phase of each elementary constituent in the decomposition of the beam are given by a set of beam-shape coefficients. The most time-consuming task in the numerical implementation of arbitrary beam-scattering theory is the evaluation of these coefficients.

Generalized Lorenz–Mie theory (GLMT) is a direct extension of the methodology of plane-wave Mie theory to the case of arbitrary incident-beam scattering.[1-5] In this formalism the scalar radiation potential[8] of the incident, scattered, and interior electromagnetic fields is expanded in partial waves. The continuity of the tangential components of the electric and magnetic fields at the surface of the spherical particle allows one to solve for the scattered and interior partial-wave amplitudes in terms of the beam-shape coefficients. The beam-shape coefficients themselves may be calculated as three-dimensional integrals over all space[4,9,10] or as two-dimensional integrals over a spherical surface[5,11,12] of the radial component of the incident electric and magnetic fields, or they may be calculated by a finite-series technique.[10,13] Since

the radial component of the incident fields is a rapidly varying function, the computation of the beam-shape coefficients by numerical integration requires substantial computer run time.

In the past few years a highly accurate analytical approximation to the beam-shape coefficients for a focused Gaussian beam was discovered, first for a beam propagating along the positive z axis (i.e., an on-axis beam)[3] and then for a beam propagating parallel to the z axis (i.e., an off-axis beam).[14] This analytical approximation has been called the localized approximation, in analogy to van de Hulst's localization principle in plane-wave Mie theory.[15] Although various semiquantitative motivations of the localized approximation for an on-axis Gaussian beam have been given previously,[3,12,16] a rigorous theoretical justification of the approximation for both on-axis and off-axis beams is still lacking. The purpose of this paper is to provide such a justification for the on-axis case. The justification of the off-axis localized approximation is considered in a separate paper in this issue.[17]

The body of this paper proceeds as follows. In Section 2 we briefly review the GLMT formalism. In Section 3 we summarize the Davis approximations[18] to a focused TEM$_{00}$ Gaussian laser beam. We also give the localized approximation to the beam-shape coefficients for an on-axis Davis first-order beam. In Section 4 we calculate the Taylor series expansion of the beam-shape coefficients for on-axis Davis first-, third-, and fifth-order beams and compare it with the localized approximation. The results underscore the approximate nature of the Davis beams and suggest a way that such approximate descriptions of the beam may be circumvented. Finally, in Section 5 we examine the profile of the beam generated by the localized approximation and comment on its physical interpretation. We find that the beam defined by the localized approximation coefficients is

both an exact solution of Maxwell's equations and very close in shape to a focused TEM$_{00}$ Gaussian laser beam. Thus our analytical expression for the beam-shape coefficients should more properly be called the localized beam model for Gaussian beam scattering rather than the localized approximation.

2. GENERALIZED LORENZ–MIE THEORY FOR AN ARBITRARY INCIDENT BEAM

Consider a monochromatic, linearly polarized electromagnetic wave traveling in the positive z direction in a medium of refractive index n, whose electric and magnetic fields are given by $\mathbf{E}_{\text{inc}}(r, \theta, \phi)\exp(i\omega t)$ and $\mathbf{B}_{\text{inc}}(r, \theta, \phi)\exp(i\omega t)$, respectively. The wave number of the monochromatic beam is

$$k = \frac{2\pi n}{\lambda} = \frac{\omega n}{c}, \tag{1}$$

and its scalar radiation potential $\psi_{\text{inc}}(r, \theta, \phi)\exp(i\omega t)$ (which is proportional to what some authors[2,4] call the Bromwich potential) satisfies the wave equation[8]

$$\nabla^2 \psi_{\text{inc}} + k^2 \psi_{\text{inc}} = 0. \tag{2}$$

As a result, the beam may be decomposed into partial waves (n) having all possible azimuthal components (m) by means of

$$\psi_{\text{inc}}{}^{\text{TE}}(R, \theta, \phi) = -E_0 \sum_{n=1}^{\infty} \sum_{m=-n}^{n} (-i)^n \frac{2n+1}{n(n+1)} (g_n{}^m)_{\text{TE}}$$
$$\times j_n(R) P_n{}^{|m|}(\cos\theta)\exp(im\phi),$$

$$\psi_{\text{inc}}{}^{\text{TM}}(R, \theta, \phi) = -E_0 \sum_{n=1}^{\infty} \sum_{m=-n}^{n} (-i)^n \frac{2n+1}{n(n+1)} (g_n{}^m)_{\text{TM}}$$
$$\times j_n(R) P_n{}^{|m|}(\cos\theta)\exp(im\phi), \tag{3}$$

where the associated Lengendre polynomials $P_n{}^{|m|}(\cos\theta)$ are defined as in Eq. (5) of Ref. 4,

$$R \equiv kr, \tag{4}$$

$j_n(R)$ are spherical Bessel functions, and the electric-field strength is E_0. The quantities $(g_n{}^m)_{\text{TM}}$ are the transverse magnetic beam-shape coefficients, and $(g_n{}^m)_{\text{TE}}$ are the transverse electric beam-shape coefficients. These coefficients are two-dimensional integrals of the radial component of the incident electric and magnetic fields[5]:

$$(g_n{}^m)_{\text{TE}} = \frac{-1}{4\pi}(i^{n-1})\frac{R}{j_n(R)}\frac{(n-|m|)!}{(n+|m|)!}$$
$$\times \int_0^{\pi} \sin\theta \, \mathrm{d}\theta \int_0^{2\pi} \mathrm{d}\phi \, P_n{}^{|m|}(\cos\theta)$$
$$\times \exp(-im\phi)\frac{cB_{\text{rad}}{}^{\text{inc}}(R, \theta, \phi)}{nE_0},$$

$$(g_n{}^m)_{\text{TM}} = \frac{-1}{4\pi}(i^{n-1})\frac{R}{j_n(R)}\frac{(n-|m|)!}{(n+|m|)!}$$
$$\times \int_0^{\pi} \sin\theta \, \mathrm{d}\theta \int_0^{2\pi} \mathrm{d}\phi \, P_n{}^{|m|}(\cos\theta)$$
$$\times \exp(-im\phi)\frac{E_{\text{rad}}{}^{\text{inc}}(R, \theta, \phi)}{E_0}. \tag{5}$$

The value of R is arbitrary if $E_{\text{rad}}{}^{\text{inc}}$ and $B_{\text{rad}}{}^{\text{inc}}$ correspond to an exact solution of Maxwell's equations, because in

such a case the integrals over θ and ϕ are proportional to $j_n(R)/R$. One may write the coefficients of Eq. (5) alternatively as three-dimensional integrals of the radial component of the incident fields,[4]

$$(g_n{}^m)_{\text{TE}} = \frac{-1}{2\pi^2}(i^{n-1})(2n+1)\frac{(n-|m|)!}{(n+|m|)!}$$
$$\times \int_0^{\infty} R\mathrm{d}R \int_0^{\pi} \sin\theta\mathrm{d}\theta \int_0^{2\pi} \mathrm{d}\phi \, j_n(R)$$
$$\times P_n{}^{|m|}(\cos\theta)\exp(-im\phi)\frac{cB_{\text{rad}}{}^{\text{inc}}(R, \theta, \phi)}{nE_0},$$

$$(g_n{}^m)_{\text{TM}} = \frac{-1}{2\pi^2}(i^{n-1})(2n+1)\frac{(n-|m|)!}{(n+|m|)!}$$
$$\times \int_0^{\infty} R\mathrm{d}R \int_0^{\pi} \sin\theta\mathrm{d}\theta \int_0^{2\pi} \mathrm{d}\phi \, j_n(R)$$
$$\times P_n{}^{|m|}(\cos\theta)\exp(-im\phi)\frac{E_{\text{rad}}{}^{\text{inc}}(R, \theta, \phi)}{E_0}, \tag{6}$$

by moving $j_n(R)$ to the left-hand side of Eq. (5), multiplying by another factor of $j_n(R)$, integrating, and using Ref. 19 to evaluate the integral on the left-hand side.

The beam is incident upon a spherical particle of radius a whose center is at the origin of coordinates and that has complex refractive index N relative to the external medium. The far-field scattered intensity is

$$\lim_{r\to\infty} I(r, \theta, \phi) = \frac{nE_0^2}{2\mu_0 c}\frac{1}{R^2}[|S_1(\theta, \phi)|^2 + |S_2(\theta, \phi)|^2], \tag{7}$$

where μ_0 is the permeability constant of free space. The scattering amplitudes S_1 and S_2 are given by

$$S_1(\theta, \phi) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \frac{2n+1}{n(n+1)}[(g_n{}^m)_{\text{TM}}a_n m\pi_n{}^{|m|}(\theta)$$
$$+ i(g_n{}^m)_{\text{TE}}b_n\tau_n{}^{|m|}(\theta)]\exp(im\phi),$$

$$S_2(\theta, \phi) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \frac{2n+1}{n(n+1)}[i(g_n{}^m)_{\text{TE}}b_n m\pi_n{}^{|m|}(\theta)$$
$$+ (g_n{}^m)_{\text{TM}}a_n\tau_n{}^{|m|}(\theta)]\exp(im\phi), \tag{8}$$

where the angular functions are

$$\pi_n{}^{|m|}(\theta) = \frac{1}{\sin\theta}P_n{}^{|m|}(\cos\theta),$$

$$\tau_n{}^{|m|}(\theta) = \frac{\mathrm{d}}{\mathrm{d}\theta}P_n{}^{|m|}(\cos\theta), \tag{9}$$

and the coefficients a_n and b_n are the partial-wave scattering amplitudes of the plane-wave Mie theory.

Of special interest is an incident beam of field strength E_0 propagating along the z axis that strikes the spherical particle head on. This is known as an on-axis beam, and the radial component of the incident fields for this geometry reduces to

$$E_{\text{rad}}{}^{\text{inc}}(R, \theta, \phi) = E_0 \exp(-iR\cos\theta)f_e(R, \theta)\sin\theta\cos\phi,$$

$$B_{\text{rad}}{}^{\text{inc}}(R, \theta, \phi) = B_0 \exp(-iR\cos\theta)f_b(R, \theta)\sin\theta\sin\phi,$$

$$B_0 = nE_0/c. \tag{10}$$

As a result, the ϕ integration in Eqs. (5) and (6) may be performed analytically. Only the $m = \pm 1$ beam-shape coefficients are nonzero; i.e.,

$$(g_n{}^m)_{\text{TE}} = \mp i/2(g_n)_b \delta_{m,\pm 1}, \qquad (g_n{}^m)_{\text{TM}} = 1/2(g_n)_e \delta_{m,\pm 1},$$
$$(11)$$

where

$$(g_n)_e = -1/2(i^{n-1})\frac{R}{j_n(R)}\frac{1}{n(n+1)}\int_0^\pi \sin^2\theta d\theta f_e(R,\theta)$$
$$\times \exp(-iR\cos\theta)P_n{}^1(\cos\theta),$$

$$(g_n)_b = -1/2(i^{n-1})\frac{R}{j_n(R)}\frac{1}{n(n+1)}\int_0^\pi \sin^2\theta d\theta f_b(R,\theta)$$
$$\times \exp(-iR\cos\theta)P_n{}^1(\cos\theta) \qquad (12)$$

from Eq. (5), or, equivalently,

$$(g_n)_e = -\frac{1}{\pi}(i^{n-1})\frac{2n+1}{n(n+1)}\int_0^\infty R dR \int_0^\pi \sin^2\theta d\theta f_e(R,\theta)$$
$$\times j_n(R)\exp(-iR\cos\theta)P_n{}^1(\cos\theta),$$

$$(g_n)_b = -\frac{1}{\pi}(i^{n-1})\frac{2n+1}{n(n+1)}\int_0^\infty R dR \int_0^\pi \sin^2\theta d\theta f_b(R,\theta)$$
$$\times j_n(R)\exp(-iR\cos\theta)P_n{}^1(\cos\theta) \qquad (13)$$

from Eq. (6). For this situation the scattering amplitudes become

$$S_1(\theta,\phi) = \sin\phi \sum_{n=1}^\infty \frac{2n+1}{n(n+1)}$$
$$\times [(g_n)_e a_n \pi_n{}^1(\theta) + (g_n)_b b_n \tau_n{}^1(\theta)],$$

$$S_2(\theta,\phi) = \cos\phi \sum_{n=1}^\infty \frac{2n+1}{n(n+1)}$$
$$\times [(g_n)_e a_n \tau_n{}^1(\theta) + (g_n)_b b_n \pi_n{}^1(\theta)]. \qquad (14)$$

At this point it might seem appropriate to give the localized approximation to the beam-shape coefficients, thus completing our summary of the GLMT formalism. But because the specific form of the localized approximation for a focused Gaussian beam is closely related to the expressions for the beam's electric- and magnetic-field strength, we defer stating the localized approximation until the end of Section 3, after the expressions for the focused Gaussian fields have been presented.

3. DAVIS ON-AXIS-BEAM APPROXIMATIONS

In this section we examine an on-axis Gaussian beam polarized in the x direction and propagating along the z axis, which is focused by a lens to a half-width w_0 in the $z = 0$ plane. The so-called Davis procedure[18-20] allows one to construct the expression for the focused beam as a series expansion in powers of

$$s = \frac{1}{kw_0} \qquad (15)$$

that satisfies Maxwell's equations exactly. Truncating the series at any given power of s produces an

approximation to the actual beam. The higher the power of s that is retained, the better the approximation.[21,22]

In the Davis procedure one assumes that the vector potential of the beam is linearly polarized in the x direction; i.e.,

$$\mathbf{A}(\mathbf{r}, t) = A(\mathbf{r})\hat{u}_x \exp(i\omega t). \qquad (16)$$

The vector potential then satisfies the scalar-wave equation

$$\nabla^2 A(\mathbf{r}) + k^2 A(\mathbf{r}) = 0. \qquad (17)$$

For a focused Gaussian beam, $A(\mathbf{r})$ is taken to be of the form

$$A(\mathbf{r}) = \frac{iE_0}{ck}\exp(-ikz)\alpha(\mathbf{r}). \qquad (18)$$

The scaling factor iE_0/ck produces an electric field of peak strength E_0. In terms of rectangular coordinates normalized to the width of the beam focal waist and its spreading length ($l = w_0/s$),

$$\xi \equiv \frac{x}{w_0}, \qquad (19)$$

$$\eta \equiv \frac{y}{w_0}, \qquad (20)$$

$$\zeta \equiv \frac{sz}{w_0}, \qquad (21)$$

the differential equation satisfied by $\alpha(\mathbf{r})$ is

$$\frac{\partial^2\alpha}{\partial\xi^2} + \frac{\partial^2\alpha}{\partial\eta^2} + s^2\frac{\partial^2\alpha}{\partial\zeta^2} - 2i\frac{\partial\alpha}{2\zeta} = 0. \qquad (22)$$

The solution of this equation may be written[18,22] as a series in powers of s^2 as

$$\alpha(\xi,\eta,\zeta) = D_0\exp(-\nu D_0)[1 + s^2(2D_0 - \nu^2 D_0{}^3)$$
$$+ s^4(6D_0{}^2 - 3\nu^2 D_0{}^4 - 2\nu^3 D_0{}^5 + 1/2\nu^4 D_0{}^6)$$
$$+ O(s^6)], \qquad (23)$$

where

$$\nu = \xi^2 + \eta^2, \qquad (24)$$
$$D_0 = (1 - 2i\zeta)^{-1}. \qquad (25)$$

Once $\alpha(\mathbf{r})$ is truncated at a certain power s^2, the electric and magnetic fields of the beam are determined to the same order of approximation by substitution of Eq. (18) into

$$\mathbf{E}(\mathbf{r}, t) = c\left[\left(A + s^2\frac{\partial^2 A}{\partial\xi^2}\right)\hat{u}_x + s^2\frac{\partial^2 A}{\partial\xi\partial}\hat{u}_y + s^3\frac{\partial^2 A}{\partial\xi\partial\zeta}\hat{u}_z\right]$$
$$\times \exp(i\omega t), \qquad (26)$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{1}{w_0}\left(s\frac{\partial A}{\partial\zeta}\hat{u}_y - \frac{\partial A}{\partial\eta}\hat{u}_z\right)\exp(i\omega t). \qquad (27)$$

If we use the form for $\alpha(\mathbf{r})$ in Eq. (23), the field expressions become

$$\mathbf{E}(\mathbf{r}, t) = E_0 \exp(-ikz + i\omega t)D_0 \exp(-\nu D_0)$$
$$\times (e_x \hat{u}_x + e_y \hat{u}_y + 2iD_0 \xi e_z \hat{u}_z), \quad (28)$$

$$\mathbf{B}(\mathbf{r}, t) = B_0 \exp(-ikz + i\omega t)D_0 \exp(-\nu D_0)$$
$$\times (b_x \hat{u}_x + b_y \hat{u}_y + 2iD_0 \eta b_z \hat{u}_z), \quad (29)$$

where e_x, e_y, e_z, b_x, b_y, and b_z are series expansions in powers of s.

As was mentioned in Section 2, the beam-shape coefficients are integrals of the radial component of the electric and magnetic fields. These radial components are given in terms of the rectangular components of Eqs. (28) and (29) by

$$E_{\text{rad}}(r, \theta, \phi) = E_x \sin\theta \cos\phi + E_y \sin\theta \sin\phi$$
$$+ E_z \cos\theta, \quad (30)$$

$$B_{\text{rad}}(r, \theta, \phi) = B_x \sin\theta \cos\phi + B_y \sin\theta \sin\phi$$
$$+ B_z \cos\theta. \quad (31)$$

Substituting this into Eq. (10), we obtain

$$f_e(R, \theta) = D_0 \exp(-s^2 R^2 D_0 \sin^2\theta)h_e(R, \theta), \quad (32)$$

$$f_b(R, \theta) = D_0 \exp(-s^2 R^2 D_0 \sin^2\theta)h_b(R, \theta), \quad (33)$$

where h_e and h_b are series expansions in powers of s^2. In Section 4, f_e and f_b will be integrated into Eq. (12) to yield the on-axis beam-shape coefficients.

The Davis first-order beam approximation corresponds to truncation of Eq. (23) at the power s^0, giving

$$\alpha^{D1}(\mathbf{r}) = D_0 \exp(-\nu D_0). \quad (34)$$

If we substitute this into Eqs. (18), (26), and (27), the resulting expressions for $\mathbf{E}$ and $\mathbf{B}$ contain terms of powers s^0 through s^3. However, the s^2 term in $\alpha(\mathbf{r})$ that has not yet been considered produces additional contributions to the fields of powers s^2 through s^5. Thus one may obtain the Davis first-order fields by truncating the expressions for $\mathbf{E}$ and $\mathbf{B}$ at the power s^1. We call this truncation the mathematically conservative (MC) version of the fields, since higher-order contributions to $\alpha(\mathbf{r})$ will not change these terms in the field expressions. The result is

$$e_x^{MC1} = 1, \qquad e_y^{MC1} = 0, \qquad e_z^{MC1} = s, \quad (35)$$

$$b_x^{MC1} = 0, \qquad b_y^{MC1} = 1, \qquad b_z^{MC1} = s, \quad (36)$$

$$h^{MC1} \equiv h_e^{MC1} = h_b^{MC1} = 1. \quad (37)$$

The Davis first-order beam approximation describes a laser beam in the TEM$_{00}$ mode quite accurately when the beam is weakly focused.[21] For example, when $\lambda = 0.6328$ μm and $w_0 \approx 30$ μm, $s = 0.0034$ and the s^2 and s^3 corrections to Eqs. (35)–(37) are only one part in 10^5. On the other hand, if the beam is tightly focused with $w_0 \approx 1$ μm, then $s = 0.1$ and the higher-order beam corrections become important, since the expressions for the fields[22] are a slowly convergent series in s.

The Davis third-order beam approximation is the first of these higher-order corrections, since it truncates $\alpha(\mathbf{r})$ at the power s^2, giving

$$\alpha^{D3}(\mathbf{r}) = D_0 \exp(-\nu D_0)[1 + s^2(2D_0 - \nu^2 D_0^3)]. \quad (38)$$

If we substitute Eq. (38) into Eqs. (18), (26), and (27), the resulting expressions for $\mathbf{E}$ and $\mathbf{B}$ contain new terms of powers s^2 through s^5. Again, the s^4 term in $\alpha(\mathbf{r})$ that has not yet been considered produces additional contributions to the fields of powers s^4 through s^7. Thus the Davis third-order MC fields are given by truncation of the expressions for $\mathbf{E}$ and $\mathbf{B}$ at the power s^3:

$$e_x^{MC3} = 1 + s^2(4\xi^2 D_0^2 - \nu^2 D_0^3),$$
$$e_y^{MC3} = s^2(4\xi\eta D_0^2),$$
$$e_z^{MC3} = s + s^3(-2D_0 + 4\nu D_0^2 - \nu^2 D_0^3), \quad (39)$$

$$b_x^{MC3} = 0,$$
$$b_y^{MC3} = 1 + s^2(2\nu D_0^2 - \nu^2 D_0^3),$$
$$b_z^{MC3} = s + s^3(2D_0 + 2\nu D_0^2 - \nu^2 D_0^3), \quad (40)$$

$$h^{MC3} \equiv h_e^{MC3} = h_b^{MC3} = 1 + 2is^2 RD_0 \cos\theta. \quad (41)$$

The Davis fifth-order beam approximation truncates $\alpha(\mathbf{r})$ at s^4, giving

$$\alpha^{D5}(\mathbf{r}) = D_0 \exp(-\nu D_0)[1 + s^2(2D_0 - \nu^2 D_0^3)$$
$$+ s^4(6D_0^2 - 3\nu^2 D_0^4 - 2\nu^3 D_0^5 + \tfrac{1}{2}\nu^4 D_0^6)]. \quad (42)$$

To the same order of approximation, the MC versions of the electric and the magnetic fields are

$$e_x^{MC5} = 1 + s^2(4\xi^2 D_0^2 - \nu^2 D_0^3) + s^4(2D_0^2 - 4\nu D_0^3$$
$$- \nu^2 D_0^4 + 16\xi^2 \nu D_0^4 - 2\nu^3 D_0^5 - 4\xi^2 \nu^2 D_0^5$$
$$+ \tfrac{1}{2}\nu^4 D_0^6),$$

$$e_y^{MC5} = s^2(4\xi\eta D_0^2) + s^4(4\xi\eta D_0^2)(4\nu D_0^2 - \nu^2 D_0^3),$$

$$e_z^{MC5} = s + s^3(-2D_0 + 4\nu D_0^2 - \nu^2 D_0^3) + s^5(-6D_0^2$$
$$- 6\nu D_0^3 + 17\nu^2 D_0^4 - 6\nu^3 D_0^5 + \tfrac{1}{2}\nu^4 D_0^6), \quad (43)$$

$$b_x^{MC5} = 0,$$

$$b_y^{MC5} = 1 + s^2(2\nu D_0^2 - \nu^2 D_0^3) + s^4(-2D_0^2 + 4\nu D_0^3$$
$$+ 5\nu^2 D_0^4 - 4\nu^3 D_0^5 + \tfrac{1}{2}\nu^4 D_0^6),$$

$$b_z^{MC5} = s + s^3(2D_0 + 2\nu D_0^2 - \nu^2 D_0^3) + s^5(6D_0^2$$
$$+ 6\nu D_0^3 + 3\nu^2 D_0^4 - 4\nu^3 D_0^5 + \tfrac{1}{2}\nu^4 D_0^6), \quad (44)$$

respectively, and

$$h_e^{MC5} = 1 + s^2 D_0(2iR \cos\theta)$$
$$+ s^4 D_0^2(2 - 4iR \cos\theta + 4R^2 \sin^2\theta),$$

$$h_b^{MC5} = 1 + s^2 D_0(2iR \cos\theta)$$
$$+ s^4 D_0^2(-2 + 4iR \cos\theta + 2R^2 \sin^2\theta), \quad (45)$$

respectively.

Since the Davis beam approximations are not exact solutions of Maxwell's equations, the beam-shape coefficients derived from them through Eqs. (5) and (12) are

Table 1. Terms in the Series Expansion of g_n in Powers of s That Are Independent of R for $z_f = 0$

Beam Shape Coefficients	$k = 1$ First Order	$k = 3$ Third Order	$k = 5$ Fifth Order
$g_n{}^{MCk}$	s^0	$s^0 s^2$	$s^0 s^2 s^4$
$g_n{}^{Lk}$	$s^0 s^2$	$s^0 s^2 s^4$	$s^0 s^2 s^4 s^6$
$g_n{}^{Bk}$	$s^0 s^2$	$s^0 s^2 s^4 s^6$	$s^0 s^2 s^4 s^6 s^8 s^{10}$

not independent of R. Since the radial component of $\mathbf{E}$ and $\mathbf{B}$ is written as a power series in s, the beam-shape coefficients will also be a power series in s. The lowest power of s that multiplies an R-dependent term in the beam-shape coefficients provides a measure of how close $h_e(R, \theta)$ and $h_b(R, \theta)$ come to representing an exact solution of Maxwell's equations. The precise powers of s that are independent of R will be examined below and are collected in Table 1.

The R dependence represents a potential shortcoming in the implementation of GLMT, since the theory relies on the fact that the beam-shape coefficients are constants. There is, however, a way to minimize this potential difficulty. It consists of employing either of two other versions of the radial component of the Davis fields that lead to g_n coefficients with weaker R dependence than those produced by Eqs. (37), (41), and (45) and thus are more appropriate for the parameterization of tightly focused beams.

In Refs. 1–4 the first of the two additional versions is called the L-type radial fields. One obtains this version by substituting the Davis rectangular coordinate components of the fields given by Eqs. (35) and (36), (39) and (40), and (43) and (44) into Eqs. (28) and (29) and retaining terms of all powers of s, even though the coefficients of some powers of s will change when the next-higher-order fields are calculated. This procedure gives

$$h^{L1} \equiv h_e{}^{L1} = h_b{}^{L1} = 1 + s^2 D_0(2iR \cos \theta) = D_0 \quad (46)$$

for the L-type first-order fields,

$$
\begin{aligned}
h_e{}^{L3} &= s^4 D_0{}^2(-4iR \cos \theta) + D_0(1 + 4s^4 D_0{}^2 R^2 \sin^2 \theta \\
&\quad - s^6 D_0{}^3 R^4 \sin^4 \theta), \\
h_b{}^{L3} &= s^4 D_0{}^2(4iR \cos \theta) + D_0(1 + 2s^4 D_0{}^2 R^2 \sin^2 \theta \\
&\quad - s^6 D_0{}^3 R^4 \sin^4 \theta)
\end{aligned}
\quad (47)
$$

for the L-type third-order fields, and

$$
\begin{aligned}
h_e{}^{L5} &= s^4 D_0{}^2(2 - 4iR \cos \theta) + s^6 D_0{}^3(-12iR \cos \theta \\
&\quad - 4R^2 \sin^2 \theta) + s^8 D_0{}^4(-12iR^3 \sin^2 \theta \cos \theta \\
&\quad - 2R^4 \sin^4 \theta) + D_0(1 + 4s^4 D_0{}^2 R^2 \sin^2 \theta \\
&\quad - s^6 D_0{}^3 R^4 \sin^4 \theta + 17s^8 D_0{}^4 R^4 \sin^4 \theta \\
&\quad - 6s^{10} D_0{}^5 R^6 \sin^6 \theta + \tfrac{1}{2} s^{12} D_0{}^6 R^8 \sin^8 \theta), \\
h_b{}^{L5} &= s^4 D_0{}^2(-2 + 4iR \cos \theta) + s^6 D_0{}^3(12iR \cos \theta \\
&\quad + 4R^2 \sin^2 \theta) + s^8 D_0{}^4(12iR^3 \sin^2 \theta \cos \theta \\
&\quad + 2R^4 \sin^4 \theta) + D_0(1 + 2s^4 D_0{}^2 R^2 \sin^2 \theta \\
&\quad - s^6 D_0{}^3 R^4 \sin^4 \theta + 3s^8 D_0{}^4 R^4 \sin^4 \theta \\
&\quad - 4s^{10} D_0{}^5 R^6 \sin^6 \theta + \tfrac{1}{2} s^{12} D_0{}^6 R^8 \sin^8 \theta)
\end{aligned}
\quad (48)
$$

for the L-type fifth-order fields.

The second of the two additional versions is the Barton symmetrized version of the Davis fields.[22] Although the Davis first-order electric and magnetic fields of Eqs. (35) and (36) are symmetrical, i.e.,

$$e_x = b_y, \qquad e_y = b_x, \qquad e_z = b_z, \quad (49)$$

the Davis third- and fifth-order fields are not. Specifically, since the vector potential is polarized in the x direction and $\mathbf{B} = \nabla \times \mathbf{A}$, the x component of the magnetic field vanishes. If a vector potential that is identical but polarized in the y direction is added to Eq. (16), the resulting electric and magnetic fields are symmetric for all powers of s, giving

$$
\begin{aligned}
e_x{}^{B1} &= b_y{}^{B1} = 1, \\
e_y{}^{B1} &= b_x{}^{B1} = 0, \\
e_z{}^{B1} &= b_z{}^{B1} = s
\end{aligned}
\quad (50)
$$

for the Barton symmetrized first-order fields,

$$
\begin{aligned}
e_x{}^{B3} &= b_y{}^{B3} = 1 + s^2(2\xi^2 D_0{}^2 + \nu D_0{}^2 - \nu^2 D_0{}^3), \\
e_y{}^{B3} &= b_x{}^{B3} = s^2(2\xi\eta D_0{}^2), \\
e_z{}^{B3} &= b_z{}^{B3} = s + s^3(3\nu D_0{}^2 - \nu^2 D_0{}^3)
\end{aligned}
\quad (51)
$$

for the Barton symmetrized third-order fields, and

$$
\begin{aligned}
e_x{}^{B5} &= b_y{}^{B5} = 1 + s^2(2\xi^2 D_0{}^2 + \nu D_0{}^2 - \nu^2 D_0{}^3) \\
&\quad + s^4(2\nu^2 D_0{}^4 + 8\xi^2 \nu D_0{}^4 - 3\nu^3 D_0{}^5 \\
&\quad - 2\xi^2 \nu^2 D_0{}^5 + \tfrac{1}{2}\nu^4 D_0{}^6), \\
e_y{}^{B5} &= b_x{}^{B5} = s^2(2\xi\eta D_0{}^2) + s^4(2\xi\eta D_0{}^2)(4\nu D_0{}^2 - \nu^2 D_0{}^3), \\
e_z{}^{B5} &= b_z{}^{B5} = s + s^3(3\nu D_0{}^2 - \nu^2 D_0{}^3) \\
&\quad + s^5(10\nu^2 D_0{}^4 - 5\nu^3 D_0{}^5 + \tfrac{1}{2}\nu^4 D_0{}^6)
\end{aligned}
\quad (52)
$$

for the Barton symmetrized fifth-order fields. If these expressions are then substituted into Eqs. (28) and (29) for the electric and the magnetic fields, respectively, and terms of all powers in s are retained, we obtain

$$h^{B1} \equiv h_e{}^{B1} = h_b{}^{B1} = D_0, \quad (53)$$

$$
\begin{aligned}
h^{B3} \equiv h_e{}^{B3} = h_b{}^{B3} &= D_0(1 + 3s^4 D_0{}^2 R^2 \sin^2 \theta \\
&\quad - s^6 D_0{}^3 R^4 \sin^4 \theta),
\end{aligned}
\quad (54)
$$

$$
\begin{aligned}
h^{B5} \equiv h_e{}^{B5} = h_b{}^{B5} &= D_0(1 + 3s^4 D_0{}^2 R^2 \sin^2 \theta \\
&\quad - s^6 D_0{}^3 R^4 \sin^4 \theta + 10s^8 D_0{}^4 R^4 \sin^4 \theta \\
&\quad - 5s^{10} D_0{}^5 R^6 \sin^6 \theta + \tfrac{1}{2} s^{12} D_0{}^6 R^8 \sin^8 \theta).
\end{aligned}
\quad (55)
$$

It should be noticed that h^{B1} is identical to h^{L1}. This equivalence between the B and the L versions of the fields does not persist for the higher-order fields. It turns out, however, that h^{B3} is the average of $h_e{}^{L3}$ and $h_b{}^{L3}$ and that h^{B5} is the average of $h_e{}^{L5}$ and $h_b{}^{L5}$. This relationship also holds for the beam-shape coefficients. Thus the Barton symmetrized fields possess the virtue that one does not require two separate sets of beam-shape coefficients in Eqs. (14). The TE coefficients $(g_n)_b$ and the TM coefficients $(g_n)_e$ are equal.

Up to this point we have assumed that the center of the focal waist of the beam is at the origin of coordinates. If the center of the focal waist is placed instead at the coordinate $(0, 0, z_f)$, all the above equations remain valid, with the replacements

$$z \rightarrow z - z_f \tag{56}$$

made in Eqs. (10), (28), and (29) and

$$D_0 \rightarrow D = \left(1 + 2is\frac{z_f}{w_0} - 2i\zeta\right)^{-1} \tag{57}$$

in Eqs. (32)–(55).

In analogy with van de Hulst's localization principle,[15] the localized approximation to the beam-shape coefficients for an on-axis Gaussian beam associates an incident light ray at a transverse distance r from the z axis with the partial wave n according to prescription

$$kr \rightarrow n + 1/2,$$
$$\theta \rightarrow \pi/2. \tag{58}$$

If we use the notation of Eqs. (10), (32), and (33), the localized approximation to the beam-shape coefficients for an on-axis Davis first-order beam is then[3]

$$(g_n)^{\text{loc}} = f^{\text{loc}} \exp(ikz_f) = [1 + 2is(z_f/w_0)]^{-1} \exp(ikz_f)$$
$$\times \exp\left[\frac{-s^2(n + 1/2)^2}{1 + 2is(z_f/w_0)}\right],$$
$$f^{\text{loc}} = f^{\text{MC1}}(R = n + 1/2, \theta = \pi/2). \tag{59}$$

These coefficients have been compared[12] with the values of the beam-shape coefficients obtained by numerical integration of the on-axis Davis first-order fields in Eqs. (12). The two sets of coefficients were found to agree to a few parts in 10^5 for $s \leq 0.007$. The appropriateness of the localized approximation for larger values of s is examined in Sections 4 and 5.

4. JUSTIFICATION OF THE LOCALIZED APPROXIMATION FOR AN ON-AXIS GAUSSIAN BEAM

There are two ways to assess the accuracy of the localized approximation for a focused Gaussian beam. First, one may calculate the beam-shape coefficients for a Davis beam by use of Eqs. (12) or (13) and compare the values of the localized approximation coefficients with them. This is the approach taken in this section. But inserting Eqs. (11) and (59) into Eq. (3) provides an exact solution of Maxwell's equations that defines a focused beam in its own right. A second and perhaps more physically meaningful way to assess the validity of the localized approximation is to compare the profile of this localized beam with that of a TEM_{00} focused laser beam. This approach is examined in Section 5 below.

We now evaluate Eqs. (12) for an on-axis Davis first-, third-, and fifth-order beam by expanding $f_e(R, \theta)$ and $f_b(R, \theta)$ in powers of s and integrating term by term. Consider the expression for g_n of Eqs. (12) for an on-axis Davis first-order beam in the MC version and focused at

the origin, where $f^{\text{MC1}}(R, \theta)$ and $h^{\text{MC1}}(R, \theta)$ are given by Eqs. (32) and (33) and Eq. (37). A series expansion of $f^{\text{MC1}}(R, \theta)$ in powers of s^2 gives

$$f^{\text{MC1}}(R, \theta) = 1 + s^2(2iR\cos\theta - R^2\sin^2\theta)$$
$$+ s^4(1/2R^4\sin^4\theta - 4R^2\cos^2\theta$$
$$- 4iR^3\sin^2\theta\cos\theta) + s^6(-8iR^3\cos^3\theta$$
$$+ 12R^4\sin^2\theta\cos^2\theta + 3iR^5\sin^4\theta\cos\theta$$
$$- 1/6R^6\sin^6\theta) + \mathcal{O}(s^8). \tag{60}$$

When this series is substituted into Eqs. (12), a number of integrals of the form

$$\int_0^\pi \sin^2\theta d\theta T(\theta)\exp(-iR\cos\theta)P_l^1(\cos\theta), \tag{61}$$

where

$$T(\theta) = 1, \cos\theta, \cos^2\theta, \sin^2\theta, \cos^3\theta, \sin^2\theta\cos\theta,$$
$$\sin^2\theta\cos^2\theta, \sin^4\theta, \sin^4\theta\cos\theta, \sin^6\theta, \tag{62}$$

must be evaluated. The simplest of these is[12]

$$\int_0^\pi \sin^2\theta d\theta \exp(-iR\cos\theta)P_n^1(\cos\theta)$$
$$= -2(-i)^{n-1}n(n + 1)G_n(R), \tag{63}$$

where

$$G_n(R) \equiv \frac{j_n(R)}{R}. \tag{64}$$

One evaluates all the other integrals encountered by taking successive derivatives of Eq. (63) with respect to R. The resulting expression for g_n^{MC1} may then be simplified by use of the differential equation for $G_n(R)$,

$$R^2G_n'' + 4RG_n' + R^2G_n = (n - 1)(n + 2)G_n, \tag{65}$$

and various derivatives of it to eliminate second and higher derivatives of G_n. When this procedure is carried out we obtain

$$(g_n^{\text{MC1}})_e = (g_n^{\text{MC1}})_b = g_n^{\text{MC1}} = 1 - s^2(n - 1)(n + 2)$$
$$+ \frac{2s^2RG_n'(R)}{G_n(R)} + \mathcal{O}(s^4) \tag{66}$$

for the on-axis Davis first-order beam-shape coefficients in the MC version. Since the Davis first-order beam approximation is not an exact solution of Maxwell's equations, the θ integral in Eqs. (12) is not exactly proportional to $j_n(R)/R$. As a result, g_n^{MC1} is not independent of R. The R-dependent terms occur at powers s^2 and higher.

In order to gain further insight into the Davis beam approximations, we calculate g_n^{MC3}, using Eqs. (12) with the Davis third-order beam approximation. Taylor series expanding Eqs. (32) and (33) and Eq. (41) in powers of

s^2 and integrating the resulting expression term by term gives

$$g_n{}^{MC3} = 1 - s^2(n-1)(n+2)$$
$$+ \frac{s^4}{2}(n-3)(n-1)(n+2)(n+4)$$
$$+ \frac{12s^4 R G_n{}'(R)}{G_n(R)} + \mathcal{O}(s^6). \qquad (67)$$

This set of beam-shape coefficients contains R-dependent terms of powers s^4 and higher. Use of the Davis fifth-order beam approximation of Eqs. (32), (33), and (45) gives

$$(g_n{}^{MC5})_e = 1 - s^2(n-1)(n+2) + \frac{s^4}{2}[(n-1)^2(n+2)^2$$
$$- 2(n-1)(n+2) + 4] + \mathrm{NCT}\left[\frac{s^6 R G_n{}'(R)}{G_n(R)}, s^6 R^2 \right],$$

$$(g_n{}^{MC5})_b = 1 - s^2(n-1)(n+2) + \frac{s^4}{2}[(n-1)^2(n+2)^2$$
$$- 6(n-1)(n+2) - 4] + \mathrm{NCT}\left[\frac{s^6 R G_n{}'(R)}{G_n(R)}, s^6 R^2 \right],$$
$$(68)$$

which contain R-dependent or nonconstant terms, NCT, of powers s^6 and higher.

The on-axis beam-shape coefficients of Eqs. (66)–(68) are adequate for a weakly focused Gaussian beam with $s \approx 0.001$. But for a tightly focused Gaussian beam with $s \approx 0.1$, the nonconstant terms in g_n must be shifted to higher powers of s in order that the R dependence of the coefficients be minimized. One accomplishes this by employing either of the two alternative versions of the radial fields described in Section 3. For the L-type first-order radial fields of Eq. (46), Taylor series expanding f^{L1} and integrating term by term gives

$$g_n{}^{L1} = 1 - s^2(n-1)(n+2) + \mathrm{NCT}\left[\frac{s^4 G_n{}'(R)}{G_n(R)} \right], \quad (69)$$

which is nonconstant beginning at s^4, whereas $g_n{}^{MC1}$ was nonconstant beginning at s^2. For the L-type third-order radial fields of Eqs. (47) we obtain

$$(g_n{}^{L3})_e = 1 - s^2(n-1)(n+2) + \frac{s^4}{2}(n-1)(n+2)$$
$$\times (n^2 + n - 4) + \mathrm{NCT}\left[\frac{s^6 R G_n{}'(R)}{G_n(R)} \right],$$

$$(g_n{}^{L3})_b = 1 - s^2(n-1)(n+2) + \frac{s^4}{2}(n-1)(n+2)$$
$$\times (n^2 + n - 8) + \mathrm{NCT}\left[\frac{s^6 R G_n{}'(R)}{G_n(R)} \right], \quad (70)$$

which are nonconstant beginning at s^6, whereas $g_n{}^{MC3}$ was nonconstant beginning at s^4. For the L-type fifth-order radial fields of Eqs. (48) we obtain

$$(g_n{}^{L5})_e = 1 - s^2(n-1)(n+2)$$
$$+ \frac{s^4}{2}[(n-1)^2(n+2)^2 - 2(n-1)(n+2) + 4]$$
$$- \frac{s^6}{6}(n-2)(n-1)(n+2)(n+3)(n^2 + n - 6)$$
$$+ \mathrm{NCT}\left[s^8 \frac{R G_n{}'(R)}{G_n(R)} \right],$$

$$(g_n{}^{L5})_b = 1 - s^2(n-1)(n+2)$$
$$+ \frac{s^4}{2}[(n-1)^2(n+2)^2 - 6(n-1)(n+2) - 4]$$
$$- \frac{s^6}{6}(n-2)(n-1)(n+2)(n+3)(n^2 + n - 18)$$
$$+ \mathrm{NCT}\left[s^8 \frac{R G_n{}'(R)}{G_n(R)} \right], \qquad (71)$$

which are nonconstant beginning at s^8, whereas $g_n{}^{MC5}$ was nonconstant beginning at s^6.

The R dependence of the beam-shape coefficients obtained from the Barton symmetrized version of the Davis radial fields is even weaker. For the Barton symmetrized first-order radial field of Eq. (53), Taylor series expanding f^{B1} in powers of s and integrating term by term gives

$$g_n{}^{B1} = 1 - s^2(n-1)(n+2) + \mathrm{NCT}\left[s^4 \frac{R G_n{}'(R)}{G_n(R)} \right]. \quad (72)$$

For the Barton symmetrized third-order radial field of Eq. (54) we obtain

$$g_n{}^{B3} = 1 - s^2(n-1)(n+2) + \frac{s^4}{2}(n-2)(n-1)(n+2)$$
$$\times (n+3) - \frac{s^6}{6}(n-3)(n-2)(n-1)(n+2)(n+3)$$
$$\times (n+4) + \mathrm{NCT}\left[s^8 \frac{R G_n{}'(R)}{G_n(R)}, s^8 R^2 \right], \qquad (73)$$

and for the Barton symmetrized fifth-order radial field of Eq. (55) we obtain

$$g_n{}^{B5} = 1 - s^2(n-1)(n+2) + \frac{s^4}{2}(n-2)(n-1)$$
$$\times (n+2)(n+3) - \frac{s^6}{6}(n-3)(n-2)(n-1)$$
$$\times (n+2)(n+3)(n+4) + \frac{s^8}{24}(n-4)(n-3)$$
$$\times (n-2)(n-1)(n+2)(n+3)(n+4)(n+5)$$
$$- \frac{s^{10}}{120}(n-5)(n-4)(n-3)(n-2)$$
$$\times (n-1)(n+2)(n+3)(n+4)(n+5)(n+6)$$
$$+ \mathrm{NCT}\left[s^{12} \frac{R G_n{}'(R)}{G_n(R)}, s^{12} R^2, s^{12} \frac{R^3 G_n{}'(R)}{G_n(R)} \right].$$
$$(74)$$

In each case the TE and TM coefficients are equal. These results are summarized by

$$g_n^{Bk} = \sum_{l=0}^{k} \frac{(-1)^l s^{2l}}{l!} \frac{(n-1)!}{(n-1-l)!} \frac{(n+1+l)!}{(n+1)!}$$
$$+ \text{NCT}(s^{2k+2}), \tag{75}$$

where $k = 1, 3, 5$ and $\text{NCT}(s^{2k+2})$ indicates that there are a number of different types of nonconstant term beginning at the power s^{2k+2}. Even for a tightly focused beam with $w_0 \approx \lambda$, the on-axis Barton symmetrized beam-shape coefficients as calculated by Eq. (74) are constants, as required by GLMT, to 1 part in 10^8. The R dependences of the MC-type, L-type, and Barton symmetrized beam-shape coefficients are compared in Table 1.

The results of Eqs. (69)–(74) indicate that the method by which both the L-type and the Barton k-order radial fields are generated from the rectangular components of the Davis fields anticipates the form that the MC-type radial fields will have at the $k + 1$ order. At the present time, we do not completely understand why this is so. We do, however, recognize that this procedure enables us to weaken substantially the R dependence of the beam-shape coefficients for the Davis approximations. This weakening is especially important for off-axis beams for which the field expressions are sufficiently complicated that analytical calculations beyond the Davis first-order beam are quite lengthy.

The beam-shape coefficients of Eqs. (66)–(75) were derived under the assumption that the Gaussian beam was focused at the origin. If instead the center of the focal waist is at $(0, 0, z_f)$, the Barton symmetrized beam-shape coefficients become, after much algebra,

$$g_n^{Bk}(z_f) = \sum_{j=0}^{j+2l=2k+1} \sum_{l=0} \left(-2is \frac{z_f}{w_0} \right)^j (-1)^l s^{2l} \frac{(l+j)!}{l!\,j!} \frac{1}{l!}$$
$$\times \frac{(n-1)!}{(n-1-l)!} \frac{(n+1+l)!}{(n+1)!} \exp(ikz_f)$$
$$+ \text{NCT}(s^{2k+2}) \tag{76}$$

for $k = 1, 3, 5$. This is our most general result for the analytical evaluation of the beam-shape coefficients for an on-axis focused Gaussian beam.

It now remains to compare the value of the beam-shape coefficients of Eq. (76) with the beam-shape coefficient of the localized approximation of Eqs. (59). For $z_f = 0$ the localized approximation becomes

$$(g_n)^{\text{loc}} = \exp[-s^2(n+1/2)^2], \tag{77}$$

and Eq. (76) with the nonconstant terms removed reduces to

$$g_n^{B5}(0) = \sum_{l=0}^{5} \frac{(-1)^l s^{2l}}{l!} \frac{(n-1)!}{(n-1-l)!} \frac{(n+1+l)!}{(n+1)!}. \tag{78}$$

The ratio of the factorials containing n and l in Eq. (78) may be written as a product of l pairs of factors, each pair having the form

$$(n-1-q)(n+2+q) = n^2 + n - (2 + 3q + q^2), \tag{79}$$

where $0 \le q \le l - 1$. If $n \gg l^2$, these factors may be approximated by

$$(n + 1/2)^2 = n^2 + n + 1/4, \tag{80}$$

$$g_n^{B5}(0) \approx \sum_{l=0}^{5} \frac{1}{l!} [-s^2(n+1/2)^2]^l \approx \exp[-s^2(n+1/2)^2], \tag{81}$$

which verifies the localized approximation on-axis for $z_f = 0$. For $z_f \ne 0$ Eq. (76) becomes approximately

$$g_n^{B5}(z_f) = \sum_{l=0}^{j+2l=11} \sum_{j=0} \left(-2is \frac{z_f}{w_0} \right)^j \frac{(l+j)!}{l!\,j!} \frac{[-s^2(n+1/2)^2]^l}{l!}$$
$$\times \exp(ikz_f). \tag{82}$$

But since

$$(1 + \epsilon)^{-p} = 1 - p\epsilon + \frac{p(p+1)}{2!} \epsilon^2$$
$$- \frac{p(p+1)(p+2)}{3!} \epsilon^3 + \cdots$$
$$= \sum_{j=0}^{\infty} \frac{(p+j-1)!}{(p-1)!\,j!} (-\epsilon)^j, \tag{83}$$

Eqs. (59) become

$$(g_n)^{\text{loc}} = \frac{1}{1 + 2is \dfrac{z_f}{w_0}} \sum_{l=0}^{\infty} \left[\frac{-s^2(n+1/2)^2}{1 + 2is \dfrac{z_f}{w_0}} \right]^l \frac{1}{l!} \exp(ikz_f)$$
$$= \sum_{l=0}^{\infty} \frac{1}{\left(1 + 2is \dfrac{z_f}{w_0} \right)^{l+1}} \frac{[-s^2(n+1/2)^2]^l}{l!} \exp(ikz_f)$$
$$= \sum_{j=0}^{\infty} \sum_{l=0}^{\infty} \frac{(l+j)!}{l!\,j!} \left(-2is \frac{z_f}{w_0} \right)^j \frac{[-s^2(n+1/2)^2]^l}{l!}$$
$$\times \exp(ikz_f), \tag{84}$$

which verifies the localized approximation on-axis for $z_f \ne 0$. It is surprising that f^{loc} of Eqs. (59) for a Davis first-order beam is so closely related to the g_n^{B5} coefficients derived from the Barton symmetrized fifth-order beam approximation. This close relation is not shared by $f^{\text{MC3}}(n + 1/2, \pi/2)$ or $f^{\text{MC5}}(n + 1/2, \pi/2)$.

On the basis of the form of Eqs. (72)–(74), we propose a modified localized approximation to the beam-shape coefficients for a focused Gaussian beam:

$$(g_n)^{m.\text{loc}} = f^{m.\text{loc}} \exp(ikz_f)$$
$$= \left(1 + 2is \frac{z_f}{w_0} \right)^{-1} \exp(ikz_f)$$
$$\times \exp \left[\frac{-s^2(n-1)(n+2)}{1 + 2is \dfrac{z_f}{w_0}} \right]$$
$$f^{m.\text{loc}} = f^{\text{MC1}}\{R = [(n-1)(n+2)]^{1/2}, \theta = \pi/2\}. \tag{85}$$

The new radial evaluation point $R = [(n-1)(n+2)]^{1/2}$ arises from the form of the differential equation for $G_n(R)$ in Eq. (65). The justification of the modified localized approximation proceeds identically as in Eqs. (78)–(84).

All the results of this section were obtained from Eqs. (12), which express the beam-shape coefficients as a two-dimensional integral over a spherical surface. Might a different perspective on the coefficients be obtained if Eqs. (13), which express them as a three-dimensional integral over all space were used? If the radial component of the incident fields is an exact solution of Maxwell's equations, Eqs. (12) and (13) are identical. But if the radial component is only an approximate solution to Maxwell's equations, as is the case for the Davis fields, then Eqs. (12) and (13) give different results. The R-dependent terms arising from Eqs. (12) are integrated over in Eqs. (13), producing beam-shape coefficients that should presumably be constants for all powers of s. But some of the nonconstant terms diverge when integrated over R. Thus for the Davis beams the use of Eqs. (13) does not circumvent the problems that arose with use of Eqs. (12) to evaluate the beam-shape coefficients.

5. SHAPE OF THE LOCALIZED BEAM

Thus far we have considered the beam-shape coefficients given by the localized approximation of Eqs. (59) and the modified localized approximation of Eqs. (85) to be exactly that: approximations to the coefficients derived from a Davis beam. But the Davis beams are themselves approximate solutions to Maxwell's equations. Thus, by comparing the numerical values of the g_n coefficients, we have been comparing one approximation with another. In this section we take a different point of view, namely, that the choice of any arbitrary set of beam-shape coefficients can be used to construe on the basis of Eqs. (12) an *exact* solution of Maxwell's equations. This solution then assumes the form

$$E_x = F_1 - F_2 \sin^2 \phi, \qquad cB_x/n = F_2 \sin \phi \cos \phi,$$

$$E_y = F_2 \sin \phi \cos \phi, \qquad cB_y/n = F_1 - F_2 \cos^2 \phi,$$

$$E_z = F_3 \cos \phi, \qquad cB_z/n = F_3 \sin \phi, \qquad (86)$$

where

$$F_1 = G_1 \sin \theta + G_2 \cos \theta,$$

$$F_2 = G_1 \sin \theta + G_2 \cos \theta - G_3,$$

$$F_3 = G_1 \cos \theta - G_2 \sin \theta, \qquad (87)$$

$$G_1 = \frac{-i}{R} \sin \theta \sum_{n=1}^{\infty} (-i)^n (2n+1) g_n \frac{J_n(R)}{R} \pi_n{}^1(\theta),$$

$$G_2 = \frac{-1}{R} \sum_{n=1}^{\infty} (-i)^n \frac{2n+1}{n(n+1)} g_n [J_n(R) \tau_n{}^1(\theta)$$
$$+ i J_n{}'(R) \pi_n{}^1(\theta)],$$

$$G_3 = \frac{-1}{R} \sum_{n=1}^{\infty} (-i)^n \frac{2n+1}{n(n+1)} g_n [J_n(R) \tau_n{}^1(\theta)$$
$$+ i J_n{}'(R) \pi_n{}^1(\theta)], \qquad (88)$$

with the Ricatti–Bessel function J defined by

$$J_n(R) \equiv R j_n(R). \qquad (89)$$

We call this point of view the localized beam model for the incident fields in GLMT.

It should be noted that the fields of Eqs. (86)–(89) are obtained from the scalar radiation potential or the Bromwich potential of Eqs. (2) and (3), whereas the radial components of the Davis fields that led to the beam-shape coefficients of Eqs. (59) and (85) are obtained from the vector potential of Eqs. (16) and (17). This suggests that there is a fundamental connection between the Bromwich potential and vector potential descriptions of the incident beam. Although we were able to work out all the details of this connection for a plane wave, we were unable to do so for a focused Gaussian beam. This connection between the two beam descriptions for a focused Gaussian beam warrants further study.

In this section we examine the focal waist of the localized beams defined by the beam-shape coefficients of Eqs. (59) and (85) and compare its properties with the focal waist of a Barton symmetrized fifth-order beam. At this point we also define what we call the S beam by the infinite series generalization of Eq. (76):

$$g_n{}^S(z_f) = \sum_{j=0}^{\infty} \sum_{l=0}^{\infty} \left(-2is \frac{z_f}{w_0}\right)^j (-1)^l s^{2l} \frac{(l+j)!}{l! \, j!} \frac{1}{l!}$$
$$\times \frac{(n-1)!}{(n-1-l)!} \frac{(n+1+l)!}{(n+1)!} \exp(ikz_f). \qquad (90)$$

We conjecture that this beam would be the limit of Eq. (76) if the Davis procedure were to be carried out to all orders, thus producing an exact solution of Maxwell's equations. Since the beam-shape coefficients for the Barton symmetrized fifth-order beam differ from Eq. (90) by terms of the powers s^{12} and higher, the comparison of Eqs. (59) and (85) with $g_n{}^{B5}$ for $w_0 \gtrsim \lambda$ or $s \lesssim 0.16$ is nearly identical to that with Eq. (90).

In the $z = 0$ focal plane the beam-profile functions of Eqs. (87) and (88) become

$$F_1\left(R, \frac{\pi}{2}\right) = \sum_{n=1}^{\infty} (2n+1) g_n \frac{j_n(R)}{R} \left| \pi_n{}^1\left(\frac{\pi}{2}\right) \right|, \qquad (91)$$

$$F_2\left(R, \frac{\pi}{2}\right)$$
$$= \sum_{n=1}^{\infty} \left(\frac{(n+1)g_{n+2} - (2n+3)g_{n+1} + (n+2)g_n}{(n+1)} \right) j_{n+1}(R)$$
$$\times \left| \pi_n{}^1\left(\frac{\pi}{2}\right) \right|, \qquad (92)$$

$$F_3\left(R, \frac{\pi}{2}\right) = \frac{-3}{2} (g_2 - g_1) j_1(R) + \sum_{n=1}^{\infty} \left[\frac{-(n+2)(n+4)g_{n+3} + (2n+5)g_{n+2} + (n+1)(n+3)g_{n+1}}{(n+1)(n+3)} \right] j_{n+2}(R) \left| \pi_n{}^1\left(\frac{\pi}{2}\right) \right|. \qquad (93)$$
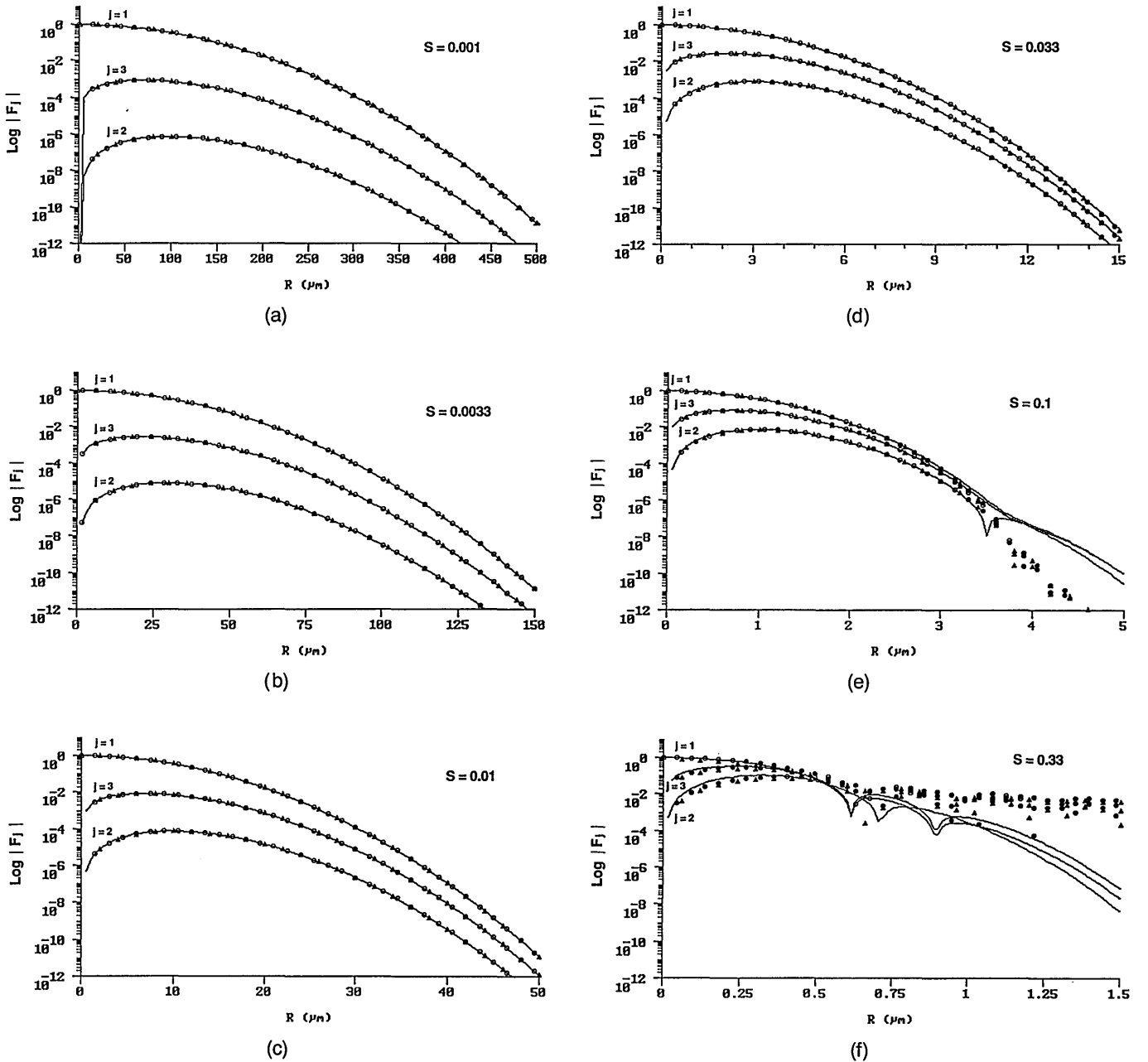
Fig. 1. Magnitude of the beam-profile functions of Eqs. (91)–(96) as a function of r for $\lambda = 0.6328 \ \mu$m and for six values of s. For (a)–(d) the localized beam profiles (circles), the modified localized beam profiles (triangles), and the Barton fifth-order beam profiles (solid curves) are indistinguishable. For (a)–(d), $F_1(r)$ and a Gaussian function are indistinguishable.

For a plane wave with $g_n = 1$, these equations reduce to $F_1(R, \pi/2) = 1$ and $F_2(R, \pi/2) = F_3(R, \pi/2) = 0$, as expected. The focal-plane beam profile of the on-axis Barton symmetrized fifth-order beam approximation is given in analytical form by[11]

$$F_1{}^{B5}(R, \pi/2) = (1 + 3s^4R^2 - s^6R^4 + 10s^8R^4$$
$$- 5s^{10}R^6 + \tfrac{1}{2}s^{12}R^8)\exp(-s^2R^2), \qquad (94)$$

$$F_2{}^{B5}(R, \pi/2) = (2s^4R^2 + 8s^8R^4 - 2s^{10}R^6)\exp(-s^2R^2),$$
$$(95)$$

$$F_3{}^{B5}(R, \pi/2) = (2s^2R + 6s^6R^3 - 2s^8R^5 + 20s^{10}R^5$$
$$- 5s^{12}R^7 + s^{14}R^9)\exp(-s^2R^2). \qquad (96)$$

The beam-profile functions $F_1(R)$, $F_2(R)$, and $F_3(R)$ for

the localized beam of Eqs. (59), the modified localized beam of Eqs. (85), and the Barton symmetrized fifth-order beam approximation of Eqs. (94)–(96) are shown in Figs. 1(a), 1(b), 1(c), 1(d), 1(e), and 1(f) for $s = 0.001$, $s = 0.0033$, $s = 0.01$, $s = 0.033$, $s = 0.1$, and $s = 0.33$, respectively. The function $F_1(R)$ describes the dominant shape of the beam at its focal waist. The profile functions $F_2(R)$ and $F_3(R)$ describe the fields induced by the Gaussian falloff of E_x and B_y in the x and y directions, respectively. For $s \leq 0.033$, $F_1(R)$ is almost exactly a Gaussian. For larger values of s, $F_1(R)$ deviates from a Gaussian because of the falloff of E_y, E_z, B_x, and B_z in the x and y directions and induces additional E_x and B_y fields proportional to various powers of s. The more times the fields induce each other back and forth, the higher the

Table 2. Average of the Magnitude of the Deviation of the Ratio F_i^{loc}/F_i^s from Unity in Parts per 10^6 for $i = 1, 2, 3^a$

s	$|F_1^{loc}/F_1^s - 1|_{ave}$	$|F_2^{loc}/F_2^s - 1|_{ave}$	$|F_3^{loc}/F_3^s - 1|_{ave}$
0.001	1	3	2
0.0033	13	34	20
0.01	117	306	178
0.033	1322	3409	1994
0.084	9366	21,954	13,270
0.1	14,167	31,399	19,382
	= 1.4%	= 3.1%	= 1.9%
0.15	4.8%	7.6%	5.5%

^{a}The average extends over $0 \le R \le 2.625/s$ or $1.0 \ge \exp(-s^2R^2) \ge 0.001$. F_i^s for the S beam is obtained from Eqs. (90)–(93) with use of a 52-term sum for g_n^s. loc, localized beam.

power of s in the contribution to Eqs. (94)–(96). As the beam width decreases and s correspondingly increases, the induced fields become stronger and the profile functions $F_2(R)$ and $F_3(R)$ increase with respect to $F_1(R)$ but are still dominated by it.

For $s \le 0.033$ in Fig. 1, the localized beam, the modified localized beam, and the Barton symmetrized fifth-order beam approximation are virtually identical. A quantitative comparison of the localized beam and the S beam, which we consider a generalization of the Barton fifth-order beam, is given in Tables 2 and 3. For tightly focused beams with $s > 0.1$ in Fig. 1, the localized beam and the modified localized beam continue to be virtually identical but behave differently from the Barton symmetrized fifth-order beam when the magnitude of $F_1(R)$ falls below a certain value. In each case $F_1(R)$ initially decreases more slowly than a Gaussian because of the strong in-

duced components in E_x and B_y and then becomes oscillatory. In Fig. 1 the localized and the modified localized beams are superposed on an oscillatory background of amplitude 10^{-18} as a result of roundoff errors in the computation of Eqs. (91)–(93). This low-level background is 6 orders of magnitude below the minimum values plotted in Fig. 1. When $w_0 < \lambda$, the localized beam has difficulty being confined in the focal plane, as is demonstrated in Table 4, where the value of the actual rms beam width w_0^{rms}, defined by

$$kw_0^{rms} = \left[\frac{2}{3} \frac{\int_0^\infty R^4(E_x^2 + E_y^2 + E_z^2)^{1/2}dR}{\int_0^\infty R^2(E_x^2 + E_y^2 + E_z^2)^{1/2}dR} \right]^{1/2}, \quad (97)$$

is given as a function of the intended beam width w_0. For a purely Gaussian beam profile, w_0^{rms} and w_0 should be equal. Table 4 shows that the beam-shape coefficients of Eqs. (59) and (85) that define the localized beams are incapable of producing a beam localized to any less than $w_0^{rms} \approx 0.8\lambda$. The Barton symmetrized fifth-order beam has difficulty localizing to any less than $w_0^{rms} \approx 0.3\lambda$. The results shown in Table 4 may be taken as a qualitative demonstration that a light beam cannot be focused to a width any narrower than approximately its wavelength.

In previous studies of Gaussian-beam scattering, the beam-shape coefficients were obtained by direct integration of Eqs. (5) and (12) for $s = 0.084$ with use of the Davis first-order beam approximation[5,23] and for $s = 0.082$ with use of the Barton symmetrized fifth-order beam approximation[24] or by decomposition of a Davis first-order Gaussian beam with $s = 0.084$ into an angular spectrum

Table 3. Average of the Magnitude of the Deviation of the Ratio F_i^{mloc}/F_i^s From Unity in Parts per 10^6 for $i = 1, 2, 3^a$

s	$|F_1^{mloc}/F_1^s - 1|_{ave}$	$|F_2^{mloc}/F_2^s - 1|_{ave}$	$|F_3^{mloc}/F_3^s - 1|_{ave}$
0.001	2	3	2
0.0033	17	31	21
0.01	157	277	186
0.033	1754	3085	2068
0.084	11,737	20,095	13,208
0.1	17,301	28,858	18,943
	= 1.7%	= 2.9%	= 1.9%
0.15	5.4%	6.9%	5.2%

^{a}The average extends over $0 \le R \le 2.625/s$ or $1.0 \ge \exp(-s^2R^2) \ge 0.001$. F_i^s for the S beam is obtained from Eqs. (90)–(93) with use of a 52-term sum for g_n^s. mloc, modified localized beam.

Table 4. Actual rms Half-Width of the Focal Waist of the Localized Beam, the Modified Localized Beam, and the Barton Symmetrized Fifth-Order Beam Approximation

w_0 (μm)	s	$(w_0^{rms})_{loc}$ (μm)	$(w_0^{rms})_{mloc}$ (μm)	$(w_0^{rms})_{B5}$ (μm)
1000	0.001	999.999	999.999	999.999
300	0.003333	299.997	300.000	300.000
100	0.01	99.992	99.992	99.990
30	0.03333	29.972	29.972	29.967
10	0.1	9.917	9.917	9.899
5	0.2	4.922	4.922	4.824
4	0.25	4.432	4.432	3.876
3	0.3333	5.497	5.497	3.121
2	0.5	5.283	5.283	2.622
1	1.0	—	—	1.655

of many thousands of plane waves.[6,7,25] In each of these cases the computation of the beam-shape coefficients required substantial computer run time. The results of Tables 2 and 3 indicate that for $s = 0.084$ the focused beams defined by the localized and the modified localized approximations, i.e., what we are now calling the localized beams, differ by only ~1% from a Barton symmetrized fifth-order beam in the region in which the electric field satisfies $E_0 \geq E \geq 10^{-3}E_0$. Similarly, Fig. 1(e) indicates that the comparison remains reasonably good for $E \gtrsim 10^{-6}E_0$. As a result, we claim that for $s = 0.084$ the parameterization of the incident beam by the localized or the modified localized approximations of Eqs. (59) and (85) is equal in validity to the parameterizations of the computationally more expensive methods of Refs. 5–7 and 23–25. A consequence of this claim is that, for either weakly focused or tightly focused beams, use of the localized beams of Eq. (59) or Eq. (85) represents a great simplification in Gaussian-beam scattering and reduces manyfold the computer run time required for its implementation.

We make our claim concerning the validity of the localized beams on a number of grounds. Experimental laser-beam profiles are rarely measured beyond their $1/e$ or $1/e^2$ points because of limitations in detector dynamic range, background illumination, and detector noise. Unless the beam has been spatially filtered, optical noise and diffraction rings from dust on the laser mirrors corrupt the Gaussian-beam profile.[26] Even if the beam has been spatially filtered, forward scattering from dust along the beam path and inhomogeneities in the focusing lens cause small deviations from the ideal Gaussian profile. With these limitations on the quality of an experimental beam, any member of a family of beam models that deviates from other members by less than a fraction of a percent for $E \gtrsim 10^{-6}E_0$ or, equivalently $I \gtrsim 10^{-12}I_0$, but may deviate more substantially for $E \lesssim 10^{-6}E_0$ is an equally valid candidate to be an acceptable model of the experimental beam. From the point of view of the scattered light, beam models that deviate from each other at the level $10^{-6}E_0$ produce scattered fields that deviate from each other at a level of 10^{-6} of the maximum field strength, as a result of geometrical ray scattering. These small differences in the scattered intensity are well below background noise levels and detector dynamic range. The only conditions for which differences among various models might be detectable is in focused scattering,[27] i.e., forward diffraction, rainbows, and glory scattering.

A related issue is the determination of the largest value of s for which the localized beams defined by Eqs. (59) or (85) may be considered good approximations to a focused Gaussian laser beam. Figure 1 and Tables 2–4 give an indication of the answer. For $s = 0.1$, Fig. 1(e) shows that the oscillatory behavior in $F_1(R)$, $F_2(R)$, and $F_3(R)$ occurs at the level $F_1(R) \lesssim 10^{-7}$. For $s = 0.2$ it occurs at $F_1(R) \lesssim 10^{-3}$. Also, the actual width of the localized beam w_0^{rms} remains close to the idealized width w_0 for $s < 0.2$, as shown in Table 4. These two results provide evidence that the localized beam model is achieving the required degree of localization for $s < 0.2$. On this basis we believe that the on-axis localized beams defined by Eqs. (59) or (85) are accurate approximations to an experimental focused TEM_{00} laser beam for $s \leq 0.15$, and possibly for localizations as tight as $s = 0.2$. By comparison, $w_0 = \lambda$ corresponds to $s = 1/2\pi \approx 0.16$. As a result, the localized beam model provides a useful practical tool for simplifying and speeding up GLMT computations of the scattering of either a weakly or tightly focused Gaussian laser beam by a spherical particle.

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REFERENCES

1. G. Gouesbet and G. Gréhan, "Sur la généralisation de la théorie de Lorenz–Mie," J. Opt. (Paris) **13**, 97–103 (1982).
2. G. Gouesbet, G. Gréhan, and B. Maheu, "Scattering of a Gaussian beam by a Mie scatter center using a Bromwich formalism," J. Opt. (Paris) **16**, 83–93 (1985).
3. G. Gréhan, B. Maheu, and G. Gouesbet, "Scattering of laser beams by Mie scatter centers: numerical results using a localized approximation," Appl. Opt. **25**, 3539–3548 (1986).
4. G. Gouesbet, B. Maheu, and G. Gréhan, "Light scattering from a sphere arbitrarily located in a Gaussian beam, using a Bromwich formulation," J. Opt. Soc. Am. A **5**, 1427–1443 (1988).
5. J. P. Barton, D. R. Alexander, and S. A. Schaub, "Internal and near-surface electromagnetic fields for a spherical particle irradiated by a focused laser beam," J. Appl. Phys. **64**, 1632–1639 (1988).
6. E. E. M. Khaled, S. C. Hill, P. W. Barber, and D. Q. Chowdhury, "Near-resonance excitation of dielectric spheres with plane waves and off-axis Gaussian beams," Appl. Opt. **31**, 1166–1169 (1992).
7. E. E. Khaled, S. C. Hill, and P. W. Barber, "Scattered and internal intensity of a sphere illuminated with a Gaussian beam," IEEE Trans. Antennas Propag. **41**, 295–303 (1993).
8. J. R. Reitz, F. J. Milford, and R. W. Christy, *Foundations of Electromagnetic Theory*, 3rd ed. (Addison-Wesley, Reading, Mass., 1979), Sec. 17.5.
9. B. Maheu, G. Gréhan, and G. Gouesbet, "Generalized Lorenz–Mie theory: first exact values and comparisons with the localized approximation," Appl. Opt. **26**, 23–25 (1987).
10. G. Gouesbet, G. Gréhan, and B. Maheu, "Computations of the g_n coefficients in the generalized Lorenz–Mie theory using three different methods," Appl. Opt. **27**, 4874–4883 (1988).
11. S. A. Schaub, J. P. Barton, and D. R. Alexander, "Simplified scattering coefficient expressions for a spherical particle located on the propagation axis of a fifth-order Gaussian beam," Appl. Phys. Lett. **55**, 2709–2711 (1989).
12. J. A. Lock, "The contribution of high-order rainbows to the scattering of a Gaussian laser beam by a spherical particle," J. Opt. Soc. Am. A **10**, 693–706 (1993).
13. G. Gouesbet, G. Gréhan, and B. Maheu, "Expressions to compute the coefficients g_n^m in the generalized Lorenz–Mie theory using finite series," J. Opt. (Paris) **19**, 35–48 (1988).
14. G. Gouesbet, G. Gréhan, and B. Maheu, "Localized interpretation to compute all the coefficients g_n^m in the generalized Lorenz–Mie theory," J. Opt. Soc. Am. A **7**, 998–1007 (1990).
15. H. C. van de Hulst, *Light Scattering by Small Particles* (Dover, New York, 1981), Secs. 12.31 and 12.33.
16. B. Maheu, G. Gréhan, and G. Gouesbet, "Ray localization in Gaussian beams," Opt. Commun. **70**, 259–262 (1989).
17. G. Gouesbet and J. A. Lock, "Rigorous justification of the localized approximation to the beam-shape coefficients in generalized Lorenz–Mie theory. II. Off-axis beams," J. Opt. Soc. Am. A. **11**, 2516–2525 (1994).
18. L. W. Davis, "Theory of electromagnetic beams," Phys. Rev. A **19**, 1177–1179 (1979).
19. G. Arfken, *Mathematical Methods for Physicists*, 3rd ed. (Academic, New York, 1985), Eq. (11.175).

20. A. Wunsche, "Transition from the paraxial approximation to exact solutions of the wave equation and application to Gaussian beams," J. Opt. Soc. Am. A **9,** 765–774 (1992).

21. G. Gouesbet, B. Maheu, and G. Gréhan, "The order of approximation in a theory of the scattering of a Gaussian beam by a Mie scatter center," J. Opt. (Paris) **16,** 239–247 (1985).

22. J. P. Barton and D. R. Alexander, "Fifth order corrected electromagnetic field components for a fundamental Gaussian beam," J. Appl. Phys. **66,** 2800–2802 (1989).

23. J. P. Barton, D. R. Alexander, and S. A. Schaub, "Internal fields of a spherical particle illuminated by a tightly focused laser beam: focal point positioning effects at resonance," J. Appl. Phys. **65,** 2900–2906 (1989).

24. J. P. Barton, D. R. Alexander, and S. A. Schaub, "Theoretical determination of net radiation force and torque for a spherical particle illuminated by a focused laser beam," J. Appl. Phys. **66,** 4594–4602 (1989).

25. S. C. Hill, Army Research Laboratory/BED, White Sands Missile Range, New Mexico 88002 (personal communication, December 1992).

26. M. V. Klein, *Optics* (Wiley, New York, 1970), Fig. 9.19.

27. M. V. Berry and K. E. Mount, "Semiclassical approximations in wave mechanics," Rep. Prog. Phys. **35,** 315–397 (1972), Sec. 6.