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*"To the solid ground
Of Nature trusts the mind which builds for aye."*—WORDSWORTH

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*RIPPLES AND WAVES**

YOU have always considered cohesion of water (capillary attraction) as a force which would seriously disturb such experiments as you were making, if on too small a scale. Part of its effect would be to modify the waves generated by towing your models through the water. I have often had in my mind the question of waves as affected by gravity and cohesion jointly, but have only been led to bring it to an issue by a curious phenomenon which we noticed at the surface of the water round a fishing-line one day slipping out of Oban (becalmed) at about half a mile an hour through the water. The speed was so small that the lead kept the line almost vertically downwards; so that the experimental arrangement was merely a thin straight rod held nearly vertical, and moved through smooth water at speeds from about a quarter to three-quarters of a mile per hour. I tried boat-hooks, oars, and other forms of moving solids, but they seemed to give, none of them, so good a result as the fishing-line. The small diameter of the fishing-line seemed to favour the result, and I do not think its roughness interfered much with it. I shall, however, take another opportunity of trying a smooth round rod like a pencil, kept vertical by a lead weight hanging down under water from one end, while it is held up by the other end. The fishing-line, however, without any other appliance proved amply sufficient to give very good results.

What we first noticed was an extremely fine and numerous set of short waves preceding the solid much longer waves following it right in the rear, and oblique waves streaming off in the usual manner at a definite angle on each side, into which the waves in front and the waves in the rear merged so as to form a beautiful and symmetrical pattern, the tactics of which I have not been able thoroughly to follow hitherto. The diameter of the "solid" (that is to say the fishing-line) being only two or three millimetres and the longest of the observed waves five or six centimetres, it is clear that the waves at distances in any directions from the solid exceeding fifteen or twenty centimetres, were sensibly unforced (that is to say moving each as if it were part of an endless series of uniform parallel waves undisturbed by any solid). Hence the waves seen right in front and right in rear showed (what became immediately an obvious result of theory) two different wave-lengths with the same velocity of propagation. The speed of the vessel falling off, the waves in rear of the fishing-line became shorter and those in advance longer, showing another obvious result of theory. The speed further diminishing, one set of waves shorten and the other lengthen, until they become, as nearly as I can distinguish, of the same lengths, and the oblique lines of waves in the intervening pattern open out to an obtuse angle of nearly two right angles. For a very short time a set of parallel waves some before and some behind the fishing-line, and all advancing direct with the same velocity, were seen. The speed further diminishing the pattern of waves disappeared altogether. Then slight tremors of the fishing-line (produced for example by striking it above water) caused circular rings of waves to diverge in all directions, those in front advancing at a greater speed relatively to the water than that of the fishing-line. All these phenomena illustrated very remarkably a geometry of ripples communicated a good many years ago to the *Philosophical Magazine* by Hirst, in which, however, so far as I can recollect, the dynamics of the subject were not discussed. The speed of the solid which gives the uniform system of parallel waves before and behind it, was clearly an absolute minimum wave-velocity, being the limiting velocity to which the common velocity of the larger waves in rear and shorter waves in front was reduced by shortening the former and lengthening the latter to an equality of wave-length.

Taking ·074 of a gramme weight per centimetre of breadth for the cohesive tension of a water surface (calculated from experiments by Gay Lussac, contained in Poisson's theory of capillary attraction, for pure water at a temperature, so far as I recollect, of about 9° Cent.), and one gramme as the mass of a cubic centimetre, I find, for the minimum velocity of propagation of surface waves, 23 centimetres per second.* The mini-

* Extract from a letter to Mr. W. Froude, by Sir W. Thomson.

* One nautical mile per hour, the only other measurement of velocity, except the French metrical reckoning, which ought to be used in any practical measurement, is 51·6 centimetres per second.

mum wave velocity for sea-water may be expected to be not very different from this. (It would of course be the same if the cohesive tension of sea water were greater than that of pure water in precisely the same ratio as the density.)

About three weeks later, being becalmed in the Sound of Mull, I had an excellent opportunity, with the assistance of Prof. Helmholtz, and my brother from Belfast, of determining by observation the minimum wave velocity with some approach to accuracy. The fishing-line was hung at a distance of two or three feet from the vessel's side, so as to cut the water at a point not sensibly disturbed by the motion of the vessel. The speed was determined by throwing into the sea pieces of paper previously wetted, and observing their times of transit across parallel planes, at a distance of 912 centimetres asunder, fixed relatively to the vessel by marks on the deck and gunwale. By watching carefully the pattern of ripples and waves, which connected the ripples in front with the waves in rear, I had seen that it included a set of parallel waves slanting off obliquely on each side, and presenting appearances which proved them to be waves of the critical length and corresponding minimum speed of propagation. Hence the component velocity of the fishing-line perpendicular to the fronts of these waves was the true minimum velocity. To measure it, therefore, all that was necessary was to measure the angle between the two sets of parallel lines of ridges and hollows, sloping away on the two sides of the wake, and at the same time to measure the velocity with which the fishing-line was dragged through the water. The angle was measured by holding a jointed two-foot rule, with its two branches, as nearly as could be judged, by the eye, parallel to the sets of lines of wave-ridges. The angle to which the ruler had to be opened in this adjustment was the angle sought. By laying it down on paper, drawing two straight lines by its two edges, and completing a simple geometrical construction with a length properly introduced to represent the measured velocity of the moving solid, the required minimum wave-velocity was readily obtained. Six observations of this kind were made, of which two were rejected as not satisfactory. The following are the results of the other four:—

Velocity of Moving Solid.	Deduced Minimum Wave-Velocity.
51 centimetres per second.	23·0 centimetres per second.
38 ,, ,,	23·8 ,, ,,
26 ,, ,,	23·2 ,, ,,
24 ,, ,,	22·9 ,, ,,
	Mean 23·22

The extreme closeness of this result to the theoretical estimate (23 centimetres per second) was, of course, merely a coincidence, but it proved that the cohesive force of sea-water at the temperature (not noted) of the observation cannot be very different from that which I had estimated from Gay Lussac's observations for pure water.

I need not trouble you with the theoretical formulæ just now, as they are given in a paper which I have communicated to the Royal Society of Edinburgh, and which will probably appear soon in the *Philosophical Magazine*. If 23 centimetres per second be taken as the minimum speed they give 1·7 centimetres for the corresponding wave-length.

I propose, if you approve, to call ripples, waves of lengths less than this critical value, and generally to restrict the name waves to waves of lengths exceeding it. If this distinction is adopted, ripples will be undulations such that the shorter the length from crest to crest the greater the velocity of propagation; while for waves the greater the length the greater the velocity of propagation. The motive force of ripples is chiefly cohesion; that of waves chiefly gravity. In ripples of lengths less than half a centimetre the influence of gravity is scarcely sensible; cohesion is nearly paramount. Thus the motive of ripples is the same as that of the trembling of a dew drop and of the spherical tendency of a drop of rain or spherule of mist. In all waves of lengths exceeding five or six centimetres, the effect of cohesion is practically insensible, and the moving force may be regarded as wholly gravity. This seems amply to confirm the choice you have made of dimensions in your models, so far as concerns escaping disturbances due to cohesion.

The introduction of cohesion into the theory of waves explains a difficulty which has often been felt in considering the patterns of standing ripples seen on the surface of water in a finger-glass made to sound by rubbing a moist finger on its lip. If no other levelling force than gravity were concerned, the length from crest to crest corresponding to 256 vibrations per second would be a fortieth of a millimetre. The ripples would be quite undistinguishable without the aid of a microscope, and the disturbance of the surface could only be perceived as a dimming of the reflections seen from it. But taking cohesion into account, I find the length from crest to crest corresponding to the period of $\frac{1}{256}$ of a second to be 1·9 millimetres, a length which quite corresponds to ordinary experience on the subject.

When gravity is neglected the formula for the period (P) in terms of the wave-length (l), the cohesive tension of the surface (T), and the density of the fluid (ρ), is

$$P = \sqrt{\frac{l^3 \rho}{2 \pi T}},$$

where T must be measured in kinetic units. For water we have $\rho = 1$, and (according to the estimate I have taken from Poisson and Gay Lussac) $T = 982^* \times \cdot 074 = 73$. Hence for water

$$P = \frac{l^{\frac{3}{2}}}{\sqrt{2 \pi \times 73}} = \frac{l^{\frac{3}{2}}}{21 \cdot 4}$$

When l is anything less than half a centimetre the error from thus neglecting gravity is less than 5 per cent. of P. When l exceeds $5\frac{1}{2}$ centimetres the error from neglecting cohesion is less than five per cent. of the period. It is to be remarked that, while for waves of sufficient length to be insensible to cohesion, the period is proportional to the square-root of the length, for ripples short enough to be insensible to gravity, the period varies in the sesquiplicate ratio of the length.

WILLIAM THOMSON

Mr. Froude having called my attention to Mr. Scott Russell's Report on Waves (British Association, York, 1844) as containing observations on some of the phenomena which formed the subject of the preceding letter to him, I find in it, under the heading "Waves of the Third Order," or, "Capillary Waves," a most interesting account of the

* 982 being the weight of one gramme in kinetic units of force-centimetres per second.