Rise Time of Pulsed Parametric Oscillators

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Abstract—General relations are derived that describe the rise time of the output power of parametric oscillators driven by a time-dependent pump. Both singly resonant and doubly resonant oscillators are discussed. Results of computer calculations using a Gaussian time envelope for the pump pulse are presented and, in the case of the doubly resonant oscillator, are compared to predictions of a steady-state analysis. Agreement between calculated rise times and published values is good.

I. INTRODUCTION

A PARAMETER of great practical importance in the design and operation of a parametric oscillator is the rise time of the output power of such a device. In CW operation the rise time is a measure of the time the oscillator requires to reach steady state in response to perturbations. In pulsed operation, the rise time may limit the amount of power transferred from the pump to the signal and idler fields, possibly even preventing the oscillator from turning on.

Previous calculations of the rise time of parametric oscillators [1], [2] have assumed a constant pump power and have neglected spatial variations of the oscillator signal and idler fields. Such results are not strictly applicable for pulsed oscillators and are inadequate to describe the rise time of a singly resonant oscillator (SRO) where the nonresonant field has a strong spatial variation. In addition, because of the large pump power requirements of most oscillators, the pulsed parametric oscillator driven by a Q-switched laser has emerged as the most practical form of this device [3]–[14].

The use of a pulsed pump, however, changes the whole nature of the oscillator operation, affecting both the effective pump threshold and the power conversion efficiency from pump to signal and idler. With a pulsed pump it is possible for the peak power of the pump pulse to exceed the CW threshold by a large factor and yet result in a negligible power transfer to the oscillator fields. This will happen if the buildup time of the resonant signal (or idler) power is comparable to the time the pump power remains above threshold.

We undertake in this paper the task of calculating the rise time of pulsed singly and doubly resonant oscillators (DRO). The results are then used to derive

relationships that must be obeyed by the various oscillator parameters to ensure efficient power conversion. Numerical results using a Gaussian time envelope for the pump pulse are presented and significant differences are found between these results and rise-time values computed using a square pump pulse and a steady-state analysis [1], [2]. Finally, experimental rise-time values found in the literature are compared to computed values and good agreement is found.

II. NONLINEAR INTERACTION EQUATIONS

In the most general case, the fields involved in a parametric oscillator are functions of both space and time. For the rise-time calculations to be considered here, however, the pump field of angular frequency ω_3 is assumed to be unaffected by the parametric interaction (nondepleted pump) so that only the signal (ω_1) and idler (ω_2) fields will be considered. The pump field is also assumed to be monochromatic and to have a "slowly varying" time envelope characteristic of a Q-switched laser pulse.

Using these assumptions in Maxwell's equations with nonlinear polarization terms, requiring $\omega_3 = \omega_1 + \omega_2$ and making the usual slowly varying envelope approximations gives

$$\frac{\partial E_1}{\partial z} + \frac{n_1}{c}\frac{\partial E_1}{\partial t} + a_1 E_1 = -iK_1 \exp[i\Delta\phi]\exp[-i\Delta kz]E_2 E_3$$
(1)

$$\frac{\partial E_2}{\partial z} + \frac{n_2}{c}\frac{\partial E_2}{\partial t} + a_2 E_2 = -iK_2 \exp[i\Delta\phi]\exp[-i\Delta kz]E_1 E_3$$
(2)

where $E_j = E_j(z, t)$ is the real traveling-wave field amplitude and

$$\Delta\phi = \phi_3 - \phi_2 - \phi_1, \qquad \Delta k = k_3 - k_2 - k_1$$

$$K_j = \frac{\omega_j d_e M}{2c\epsilon_0 n_j}, \qquad j = 1, 2.$$

The effective nonlinear coefficient [15], [16] is d_e, the index of refraction at ω_j is n_j, and the constant M results from integrating the equations over the transverse coordinates. The value of M for Gaussian beams and a near-field approximation is

$$M = 2\sqrt{2/\pi}\ W_1 W_2 W_3 / [(W_1 W_2)^2 + (W_1 W_3)^2 + (W_2 W_3)^2].$$

Double refraction effects can also be included in M since they influence the threshold of a parametric oscillator [15]. The definition of E_j used in (1) and (2) gives

the time averaged power as $P_j = \epsilon_o n_j c E^2_j(z_1 t)/2$. For resonant fields, $a_j = \alpha_j/L'_j$ where α_j is the fractional round-trip field loss or one-way power loss and L'_j is the optical length of the cavity at ω_j. For nonabsorbing high-optical-quality crystals, a_j can be set equal to zero for nonresonant fields; the absence of feedback is then accounted for by the boundary conditions.

The pulsed nature of the pump field is accounted for by writing it as

$$E_3(z, t) = E_{3p} f(t) \tag{3}$$

where E_{3p} is the peak value of the field at ω_3 and $f(t)$ has a peak value of unity. Note that the pump envelope is assumed independent of z. The envelope function $f(t)$ will later be taken to the Gaussian but for the moment is left arbitrary. Defining $t = 0$ as the time when the oscillator first reaches threshold, a pumping ratio N can be defined by

$$N = \frac{P_{3p}}{P_{3T}} = \frac{E_{3p}{}^2}{E_{3T}{}^2} = \frac{1}{f^2(0)}. \tag{4}$$

The threshold[1] for oscillation is P_{3T} and N is the factor by which the pump peak power exceeds threshold. The quantity P_{3T} as defined here is what is usually considered to be the CW threshold [15] (gain = loss) and may differ considerably from the experimentally observed threshold (minimum peak pump power for oscillation to occur). The separate cases of singly resonant and doubly resonant oscillators will now be considered.

III. Singly Resonant Oscillator

In this case the signal field (ω_1) is taken to be resonant and its amplitude assumed independent of z so $\partial E_1/\partial z \approx 0$. There is no feedback at the idler frequency so that for a lossless crystal $a_2 \approx 0$ and $E_2(0, t) = 0$. Exact phase matching is assumed ($\Delta k = 0$) and the relative phase $\Delta\phi$ is assumed to be that which yields maximum conversion of pump energy into the signal and idler fields. For $\Delta k = 0$, the condition is $\Delta\phi = \pi/2$. Equation (1) is then integrated over the cavity length L, noting that $n_1 = 1$ and $K_1 = 0$ except over the length l of the nonlinear crystal and the result combined with (2) to give a single integral equation for $E_1(t)$. Using the boundary conditions $E_2 (0, t) = 0$ and $E_1(t = nl/c) = E_{10}$, the result is

$$R_1{}^{1/2}(t) = \exp\left[-\left(\frac{c\alpha_1 t}{L'}\right)\right]\left[\exp\left(\frac{n\alpha_1 l}{L'}\right) + I(t)\right],$$

$$t \geq \frac{nl}{c} \tag{5}$$

where

$$R_1(t) = E_1{}^2(t)/E_{10}{}^2 = P_1(t)/P_{10}$$

and

$$I(t) \equiv \frac{2c^3\alpha_1 N}{n^2 l^2 L'} \int_{nl/c}^{t} \int_{z-(nl/c)}^{z} \exp\left[\alpha_1 \frac{c}{L'} x\right] f(x) f(y)$$
$$\cdot \left(\frac{nl}{c} + y - x\right) R_1{}^{1/2}(y) \, dy \, dx. \tag{6}$$

The optical length of the cavity is L', assumed equal at ω_1 and ω_2, $n = n_1 \simeq n_2$, and α_1 is the one-way fractional power loss at ω_1.

Note that the initial condition on E_1 has been specified at $t = nl/c$ rather than at $t = 0$. We have thus assumed $E_1 \simeq E_{10}$ for the first transit time through the crystal. This procedure ensures that E_2 is nonzero throughout the crystal for all times t under consideration and allows the integral over l to be performed. Because of the slow growth rate of the resonant signal near threshold, this definition of the initial condition on E_1 introduces negligible error and eliminates the cumbersome mathematical formalism that results from assuming $E_2(0, t) = 0$ and thus $E_2(l, t) = 0$ for times less than nl/c.[2]

The time dependence of the pump field is now assumed to be Gaussian, a form that closely approximates output pulse of a Q-switched laser. The envelope function $f(t)$ is thus

$$f(t) = \exp\left[-(t - \tau_T)^2/a^2\right] \tag{7}$$

where

$$a^2 = \frac{\tau_p{}^2}{2 \ln (2)}$$

and τ_p is defined as the full width at half maximum of the pump pulse intensity. The quantity τ_T is the time between the oscillator threshold level ($t = 0$) and the peak of the pump pulse and can be written as

$$\tau_T{}^2 = \frac{a^2 \ln (N)}{2} = \frac{\tau_p{}^2 \ln (N)}{4 \ln (2)} \tag{8}$$

with N being defined by (4). The relationships between the various quantities are illustrated in Fig. 1(a).

The rise time of a parametric oscillator can be defined in a number of ways. A convenient operational approach is to define the rise time τ_R as the time required for the signal output power to reach a predetermined value starting from the moment the pump reaches threshold. Defining P_{1e} as the signal output power and assuming the only loss is due to output coupling, this definition of τ_R can be written as

$$P_{1e}(\tau_R) \equiv \alpha_1 P_1(\tau_R) = \text{constant}. \tag{9}$$

Since the initial internal signal power $P_1(0) = P_{10}$ is due to spontaneous parametric fluorescence driven by the threshold pump power, we have $P_{10} \propto P_{3T}/\alpha_1$. However, for the SRO,[1] $P_{3T} \propto \alpha_1$ so that P_{10} is independent of the

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[1] For the SRO it is straightforward to show that $E^2_{3T} = 2\alpha_1/K_1 l^2$. For the DRO the threshold condition is $E^2_{3T} \simeq \alpha^2/K_1 K_2 l^2$ for $\alpha_1 = \alpha_2$.

[2] This condition is not strictly true because some parametric fluorescence at ω_2 is always present at the output of the nonlinear crystal.
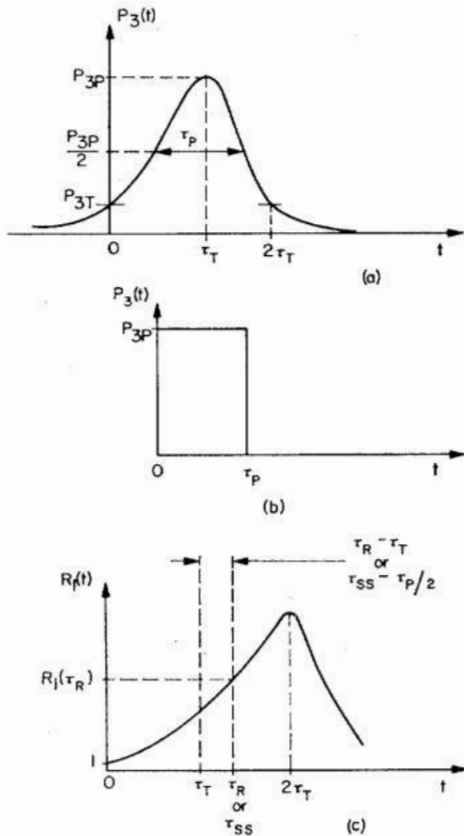
Fig. 1. (a) Gaussian pump pulse defining the quantities τ_T, P_{3T}, and P_{3p}. Note $t = 0$ is defined by the oscillator threshold level. (b) Square-pulse approximation to the Gaussian used with steady-state equations for the DRO. (c) Qualitative plot of $R_1(t) = P_1(t)/P_{10}$ showing the relationship between τ_R and τ_T. Note that in the undepleted pump approximation used here the signal power continues to grow until the pump falls below threshold ($t = 2\tau_T$).
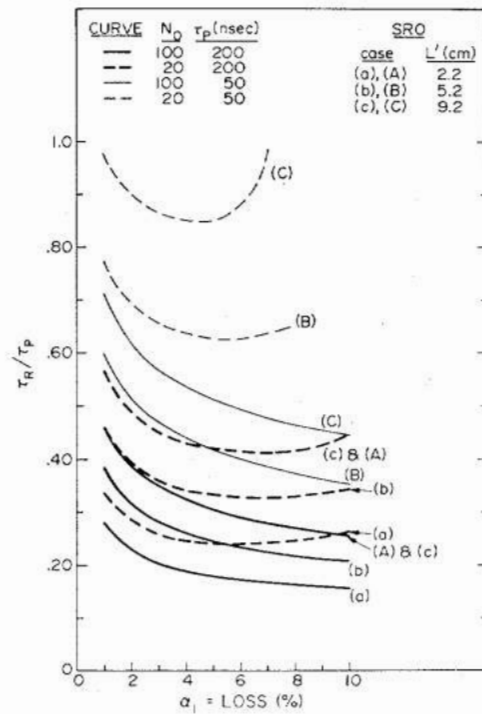

Fig. 2. SRO absolute rise time divided by pump pulsewidth versus resonant loss for various values of cavity optical length and normalized pump ratio. $\tau_p = 200$ ns—heavy lines or 50 ns—light lines and $r = 10^8$.

coupling loss α_1.[3] Consequently, the constant appearing in (9) can be replaced by rP_{10} where r is some constant. The rise time τ_R is now defined by $\alpha_1 P_1(\tau_R) = rP_{10}$ or using $R_1(t) = P_1(t)/P_{10}$

$$R_1(\tau_R) = r/\alpha_1. \qquad (10)$$

Once a value for r is chosen, (10) is used with (5) and (6) to find τ_R.

Although in the sense defined here the oscillator is not ON until (10) is satisfied, the oscillator is always ON in the sense of the pump being above the CW threshold for all times in the range $t = 0$ to $t = 2\tau_T$. The signal and idler fields experience a net round-trip gain and thus grow in amplitude during this time period. The value of r chosen for calculating rise time is arbitrary, but fortunately, because of the "explosive" nature of the signal buildup, τ_R is not a strong function of the ratio r/α_1. A change of two orders of magnitude in r results in a change of τ_R by only about 10 percent.

For experimental comparisons a useful parameter

is the quantity $P_{3P}/(P_{3T}/\alpha_1) = N\alpha_1$ since the peak pump power is usually fixed and knowledge of the crystal parameters allows computation of the quality P_{3T}/α_1. Fixing the quantity $N\alpha_1$ is equivalent to fixing the peak pump power and allows the effect of different cavity losses on rise time to be predicted. For all the numerical results of this paper, therefore, a normalized pump ratio N_0 has been used instead of N as defined by (4). For the SRO, N_0 is defined as the pumping ratio multiplied by the resonant loss in percent:

$$N_0 = N(100\,\alpha_1). \qquad (11)$$

Equation (5) cannot be solved in closed form for $R_1(t)$ but can be solved numerically. Using digital computer techniques, the rise time for a singly resonant oscillator has been computed using (5). The results for two values of τ_p are shown in Figs. 2–4. Figs. 2 and 3 plot the ratio of the rise time to the pump pulsewidth. The ordinate axis in Fig. 4 is the time between the peak of the pump pulse and the moment the oscillator turns ON [see Fig. 1(c)] divided by τ_p. This is the most convenient to measure experimentally and so is the most useful comparison to experiment. A value of $r = 10^8$ has been used in all calculations. For an initial internal power $P_{10} = 10^{-10}$ W, this choice of r corresponds to an external power of 10^{-2} W and for $\alpha_1 = 1$ percent an internal power of 1 W.

IV. DOUBLY RESONANT OSCILLATOR

The simplest case to consider for the DRO is the one for equal losses at ω_1 and ω_2. This is the usual case ex-

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[3] This conclusion is true only if the feedback at the signal (and idler in the case of the DRO) is small enough so that the effective input to the spontaneous fluorescence remains approximately one photon per mode [17].
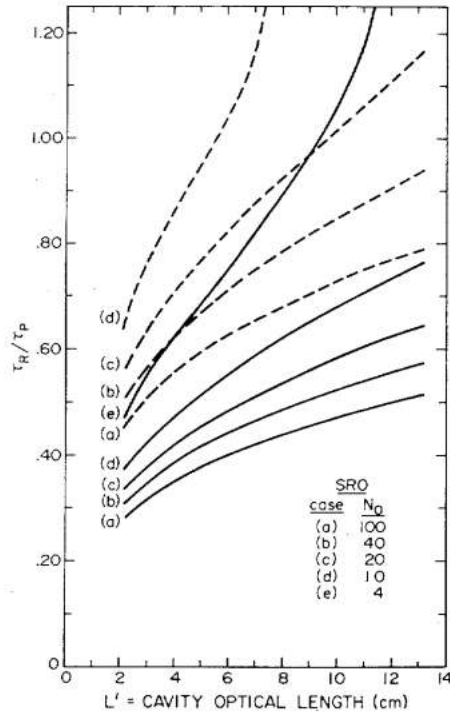
Fig. 3. SRO absolute rise time divided by pump pulsewidth as a function of cavity optical length for various normalized pump ratios. $\tau_p = 200$ ns—solid lines or 50 ns—broken lines; $\alpha_1 = 1$ percent, $r = 10^8$.

Fig. 4. SRO rise time relative to the peak of the Gaussian pump pulse divided by τ_p plotted as a function of cavity optical length. $\tau_p = 200$ ns—solid lines or 50 ns—broken lines; $\alpha_1 = 1$ percent, $r = 10^8$.

perimentally and will be the only one treated here. Setting $\partial/\partial z = 0$ in (1) and (2), integrating each over the cavity length, and solving the resulting time-dependent equations with initial conditions $E_1(0) = E_{10}$ and $E_2(0) = E_{20}$ gives $E_1(t)$ as

$$E_1(t) = \tfrac{1}{2}[(E_{10} + \sqrt{(K'_1/K'_2)}E_{20})\exp[ts_+(t)]$$
$$+ (E_{10} - \sqrt{(K'_1/K'_2)}E_{20})\exp[ts_-(t)]] \quad (12)$$

where $\beta = c\alpha/L'$, $K'_j = K_j\sin(\Delta kl/2)/(\Delta kl/2)$ and

$$ts_\pm(t) = \beta[N^{1/2}g(t) - t]. \quad (13)$$

The relative phase has been chosen as $\Delta\phi = \Delta kl/2 + (\pi/2)$ to maximize the interaction and the function $g(t)$ is defined by

$$g(t) = \int_0^t f(t)\,dt = \frac{a\sqrt{\pi}}{2}\left[\operatorname{erf}\left(\frac{t - \tau_T}{a}\right) + \operatorname{erf}\left(\frac{\tau_T}{a}\right)\right].$$
$$(14)$$

The integral in (14) has been evaluated for $f(t)$ as defined by (7) and (8) and

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\,du.$$

For the DRO as well as the SRO, the initial values of the resonant fields are found from the parametric fluorescence power inside the resonator at threshold and within the bandwidth of the oscillator [2], [17]. Since these values are many orders of magnitude below the
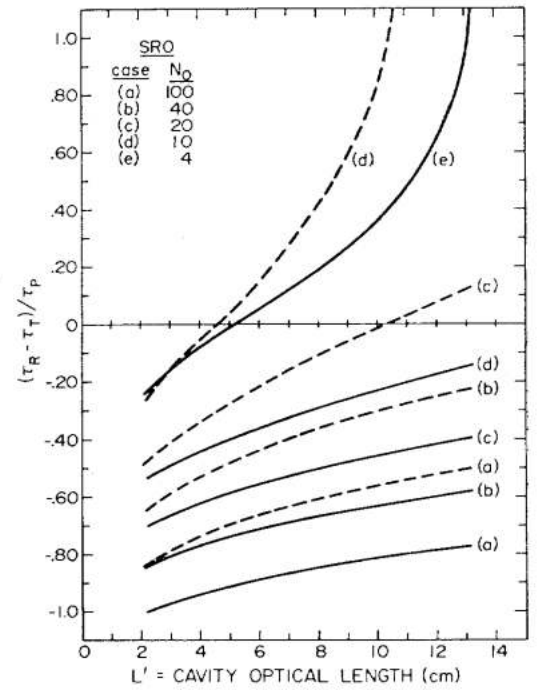
oscillator operation level, only the rapidly growing part of (12) will contribute to the field at ω_1. Neglecting the second term of (12), the power output at ω_1 can be written as

$$P_1(t) = \tfrac{1}{4}(1 + \gamma)^2 P_1(0)\exp[2ts(t)] = P_{10}\exp[2ts(t)] \quad (15)$$

where from (12) and [17],

$$\gamma = \sqrt{\frac{K_1}{K_2}}\frac{E_{20}}{E_{10}} = \left[\frac{\omega_1 P_2(0)}{\omega_2 P_1(0)}\right]^{1/2} = \frac{\omega_2 n_2}{\omega_1 n_1}$$

and

$$P_{10} = \frac{(1 + \gamma)^2}{4}P_1(0).$$

P_{10} is the initial input to the signal oscillator mode.

Contrary to the case of the SRO, P_{10} is not independent of the loss of the DRO because now $P_{3T} \propto \alpha^2$. As a result, P_{10}/α is a constant. Defining the rise time in the same way as for the SRO, the constant in (9) can be replaced by rP_{10}/α for the DRO. The rise time τ_R is thus defined by the following equation:

$$R_1(\tau_R) \equiv \frac{P_1(\tau_R)}{P_{10}} = \frac{r}{\alpha^2}. \quad (16)$$

Using (16), (13) and (15) give

$$\tau_R s(\tau_R) = \frac{c\alpha}{L'}(N^{1/2}g(\tau_R) - \tau_R) = \frac{\ln(r/\alpha^2)}{2}. \quad (17)$$

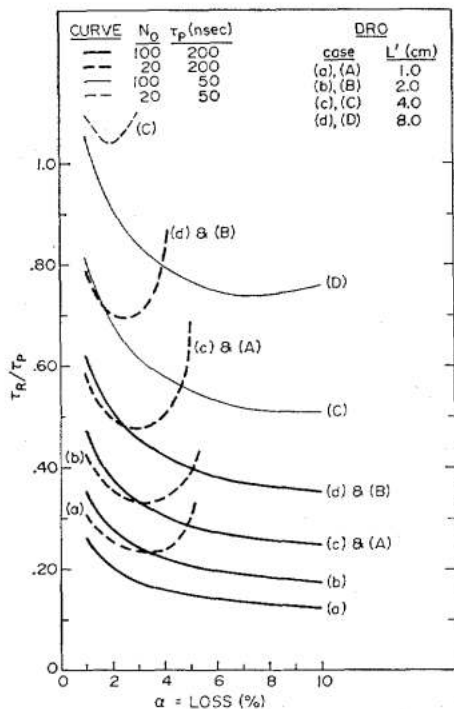Note that the ratio r/α^2 enters as the argument of a

Fig. 5. DRO absolute rise time divided by pump pulsewidth versus resonant loss for various values of cavity optical length and normalized pumping ratio. $\tau_p = 200$ ns—heavy lines or 50 ns—light lines; $r = 10^8$.
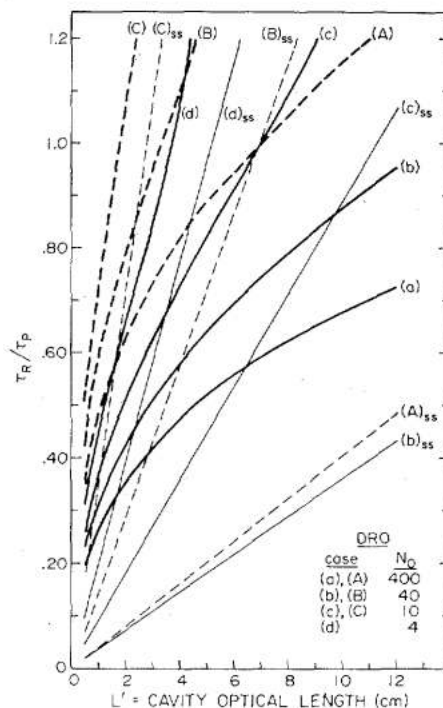

Fig. 6. DRO absolute rise time divided by pump pulsewidth as a function of cavity optical length for various normalized pump ratios. $\tau_p = 200$ ns—solid lines or 50 ns—broken lines; $\alpha = 1$ percent, $r = 10^8$. Light lines are results using the steady-state formula (18).

logarithm, so that as in the case of the SRO, the rise time is not strongly dependent on the value chosen for r.

As noted in the Introduction, earlier calculations [1], [2] of the doubly resonant parametric oscillator rise time have assumed a constant pump power. This result can be found by setting $f(t) = 1$ so $g(t) = t$; (17) then gives

$$\tau_{ss} = \frac{L' \ln (r_{ss}/\alpha^2)}{2c\alpha(N^{1/2} - 1)}. \tag{18}$$

r_{ss}/α^2 is the ratio of the external cavity steady-state power to the initial internal power and τ_{ss} is the time required to reach steady state. This result has previously been derived by Kreuzer [1] and by Byer [2] using a slightly different definition of rise time. One possible way of using (18) to predict the rise time of a pulsed oscillator is to assume the pump pulse is a square pulse of width τ_p as shown in Fig. 1(b). The magnitude of the square pulse is taken to be the same as the peak of the Gaussian pulse defining τ_p. For the sake of comparison, the predictions obtained using this quasi-steady-state approach are also shown as light curves in Figs. 6 and 7.

Results for the DRO for two values of τ_p are shown in Figs. 5–7. Fig. 5 shows the effect of losses on DRO rise time and Fig. 6 gives the ratio τ_R/τ_p (or τ_{ss}/τ_p) as a function of cavity length for different pump ratios. Fig. 7 plots the quantity $(\tau_R - \tau_T)/\tau_p$ or $(\tau_{ss} - \tau_p/2)/\tau_p$. The value $r = 10^8$ has been used throughout. As in the case of the SRO, a normalized pumping parameter N_0, which

is independent of α, is used in the figures. For the case of the DRO, N_0 is given by

$$N_0 = N(100\alpha)^2. \tag{19}$$

V. Discussion

A number of interesting results are apparent from the figures. Figs. 2 and 5 show that increasing cavity loss may actually decrease the rise time even though larger loss means larger oscillator threshold. This curious behavior results from the use of a Gaussian pump pulse. The Gaussian function e^{-x^2} has the property that for $x^2 > \frac{1}{2}$ its slope increases in magnitude as x^2 decreases. Since the rate of growth of the resonant fields is proportional to the amount by which the pump exceeds threshold, the oscillator fields will grow most rapidly when the pump ratio $P_3(t)/P_{3T}$ grows most rapidly. Lower losses imply a lower oscillator threshold and thus the threshold pump level occurs further out on the "tail" of the Gaussian, where the pump ratio changes more slowly per unit time. Consequently the rise time can be longer for smaller losses. However, the time $(\tau_R - \tau_T)$ increases monotonically with increasing loss reflecting the fact that for larger losses, the pump is above threshold for a shorter total time.

As mentioned previously, the curves shown above were computed using the value $r = 10^8$. Rise-time values have also been computed using values of $r = 10^6$, 10^{10}, and 10^{12}. A change of two orders of magnitude in r changes
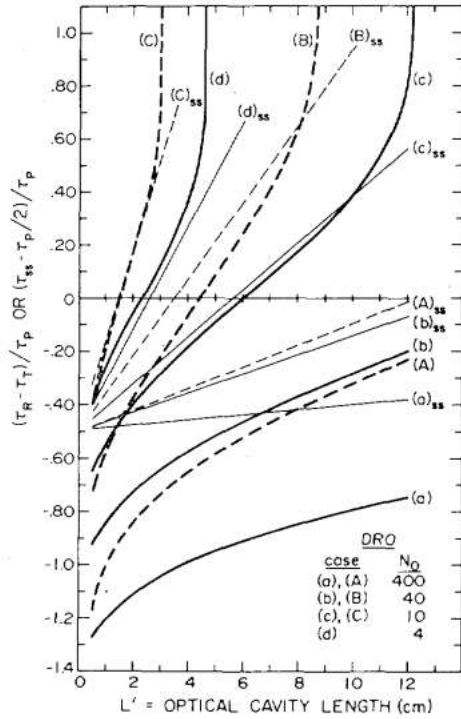
Fig. 7. DRO rise time relative to peak of Gaussian pump pulse divided by τ_p plotted as a function of cavity optical length. $\tau_p = 200$ ns—solid lines or 50 ns—broken lines; $\alpha = 1$ percent, $r = 10^8$. Light lines are results using steady-state formula (18).

Fig. 8. Minimum pump ratio for a DRO to reach the condition $r = 10^8$. Various cases of α and τ_p are shown versus cavity optical length. Dashed lines are results of the steady-state formula (22). The value $N_{\min} = 1$ at $L' = 0$ corresponds to CW threshold.

τ_R by roughly 10 percent. The only time the value of r makes a significant difference in the calculated values of τ_R is when the parameters τ_p, L', α, and N_0 are such that the ON condition can barely be achieved.

It is apparent from the figures that the rise time can be longer than the pump pulsewidth τ_p. The important quantity, however, is τ_R compared to τ_T [see Fig. 1(c)]. If $\tau_R > 2\tau_T$, the oscillator will not turn ON in the practical sense that the output power will never reach the preselected level. Inspection of (17) shows that the maximum value of $\tau_R s(\tau_R)$ occurs for $\tau_R = 2\tau_T$ at which point the pump power again crosses the threshold level [see Fig. 1(a)]. Use of this value for τ_R in (17) will define a minimum allowable pump ratio $N_{\min}$ given α and L' or, conversely, a maximum cavity optical length $L'_{\max}$ given N and α. These extrema are given by the solutions to the following equations:

$$e^{x^2} \operatorname{erf}(x) - \frac{2}{\sqrt{\pi}} x = \frac{L' \ln(r/\alpha^2)}{2c\alpha a \sqrt{\pi}} \quad (20)$$

where

$$x^2 = \ln(N_{\min})/2$$

and

$$L'_{\max} = \frac{2c\alpha a \sqrt{\pi}}{\ln(r/\alpha^2)} \left[N^{1/2} \operatorname{erf}(y) - \frac{2}{\sqrt{\pi}} y \right] \quad (21)$$

with $y = \ln(N)/2$.

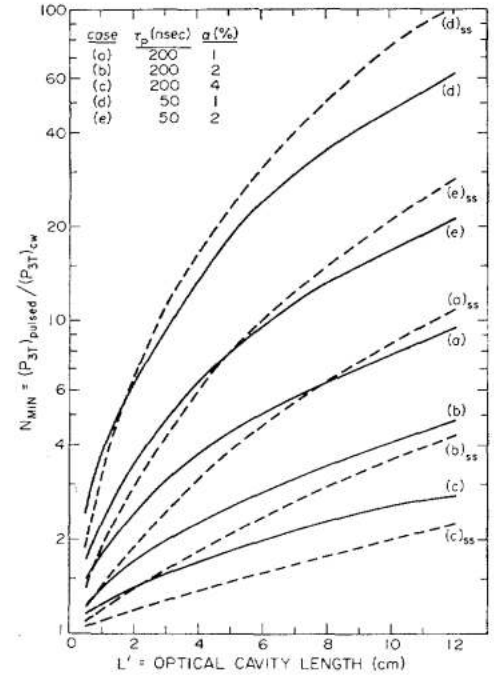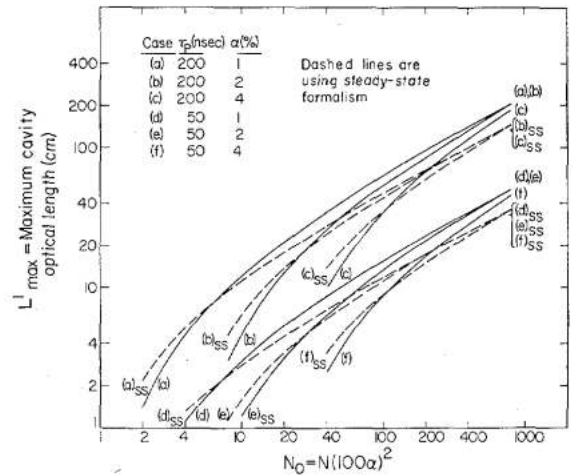The corresponding quantities using the steady-state

Fig. 9. Maximum cavity optical length for a DRO to reach the condition $r = 10^8$. Various cases of α and τ_p are shown versus cavity optical length. Dashed lines are results of a steady-state formula (23).

approximation of a square pulse and (18) are

$$(N_{\min})_{ss} = \left(\frac{L' \ln(r/\alpha^2)}{2c\alpha\tau_p} + 1 \right)^2 \quad (22)$$

$$(L'_{\max})_{ss} = \frac{2c\alpha\tau_p(N^{1/2} - 1)}{\ln(r/\alpha^2)}. \quad (23)$$

Predictions of (20)–(23) are shown in Figs. 8 and 9 for two values of τ_p. Note that the normalized pump ratio

TABLE I
RELATIVE RISE-TIME VALUES FOUND IN THE LITERATURE COMPARED TO CALCULATED VALUES

N^a (approximate)	α^b (%)	τ_p (ns)	n	l (cm)	L (cm)	$\tau_R - \tau_T$ Measured (ns)	$\tau_R - \tau_T$ Calculated (ns)	References
3	15(SRO)	16	2.2	0.94	1.7	0	−1	[6]
20	15(DRO)	22	2.2	0.94	1.7	−12	−15	[6]
16	17(DRO)	15	2.2	0.94	1.7	−6	−9	[18]
4	14(SRO)	12	2.2	0.94	1.7	0	−2	[18]
20–40	6(SRO)	22	2.2	1.4	12	−4 to −8	−6 to −11	[7]
10–20	1.25(DRO)	180	2.2	0.35	5	0 to −25	−21 to −64	[10]
1.5	16(SRO)	25	1.8	0.8	1.5	−5 to 0	−15	[9]

[a] Care must be taken in defining measured threshold for pulsed oscillators (see [18]). The results discussed in [18] have been used where possible in selecting the value of N used here.

[b] In cases of the DRO where $\alpha_1 \neq \alpha_2$, a value of $\alpha = \sqrt{\alpha_1\alpha_2}$ is assumed.

N_0 defined by (19) is used in Fig. 9. From the curves in Figs. 6–9 it can be seen that the results of the steady-state approximation are generally unreliable but might be used for a rough estimate.

The quantity $N_{\min}$ as defined by (20) and plotted in Fig. 8 is the factor by which the pulsed threshold is increased due to the finite oscillator rise time. Once the CW threshold has been calculated (using the results of [15], for example) and a cavity length chosen, the actual pulsed threshold is found by multiplying the CW result by $N_{\min}$ to obtain the threshold peak pump power. The pulsed oscillator threshold is thus defined by the minimum peak pump power for which the oscillator will turn on rather than the pump power that gives a net gain at the signal and idler frequencies. Note that if rise time were not an important consideration (i.e., CW case), $N_{\min} = 1$. If the peak pump power is fixed (N_0 is given), the maximum allowable cavity length can be found from Fig. 9. Note that for $L' = L'_{\max}$ or $N = N_{\min}$ the oscillator is driven at the "pulsed threshold" level.

From Figs. 2, 5, and 8 it is apparent that the effect of a finite oscillator rise time is often reduced for larger losses. It might appear advantageous to increase the losses in a given experimental situation. One should be careful in applying such logic, however, because the larger CW threshold resulting from increased loss could also prevent the oscillator from working. There is a definite tradeoff between increased threshold and reduced rise time and the optimum combination will have to be determined for each experimental situation.

Although the results shown in Figs. 8 and 9 are strictly applicable only to doubly resonant oscillators, they can also be used to give an estimate of the effect of rise time on SRO operation. For the SRO the abscissa of Fig. 9 should be $N_0 = N(100\ \alpha_1)$. It should be noted that the minimum pump values shown in Fig. 8 would apply for a perfectly adjusted oscillator pumped by a single-frequency pump. In a practical situation, a safety margin of about an order of magnitude in the pumping ratio would ensure ease of cavity alignment and stable reproducible operation, as well as ensuring significant energy conversion from the pump to the signal and idler fields.

VI. COMPARISON TO EXPERIMENT AND CONCLUSIONS

Several experimental parametric oscillator results have been published and have included oscilloscope traces of the pump and signal output pulses. From these data, values of $(\tau_R - \tau_T)$ can be found and compared to theoretical predictions. From the experimental photographs, the time $(\tau_R - \tau_T)$ is defined by the onset of pump depletion or by the leading edge of the oscillator pulse if no depletion is evident. Table I sumarizes the result of a number of authors. The value of $r = 10^8$ has been used for the predicted values.

The values of N shown in Table I are quite approximate. Where enough information was supplied, the measured peak pump power and the theoretical CW threshold were used to find N. Otherwise N was computed using the measured threshold power. Considering the approximate nature of the experimental data, the agreement with theory is considered reasonable.

The results presented here can be used to determine whether in a given experimental situation the signal power will approach the steady-state level corresponding to the peak pump power. The importance of this consideration is that as a result of the "explosive" nature of the signal buildup under pulsed excitation, failure to reach the quasi-steady-state level also implies a negligible conversion of pump power to signal power even though the peak pump power may exceed the CW threshold (gain = loss) by a large factor. The parameters $N_{\min}$ and $L'_{\max}$ can be used to set practical design limits on pulsed parametric oscillator systems.

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Influence of the Lifetime and Degeneracy of the $^4I_{11/2}$ Level on Nd–Glass Amplifiers

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Abstract—Gain saturation of an Nd–glass amplifier by 1.06-μm light pulses is determined by the lifetimes and degeneracies of the two laser states as well as by thermalization rates among the components of the two multiplets to which the laser levels belong. Reported are measurements showing that although small signal gain was the same for amplified microsecond and picosecond pulses, comparable energy gain saturation occurred when the energy density of the mode-locked pulses was 1.9 times smaller than the amount for the microsecond pulses indicating terminal level filling with the shorter pulses. Our measurements indicate the terminal laser level, which is one of two unresolved groups in the $^4I_{11/2}$ multiplet, has a lifetime greater than 50 ns and a degeneracy likely to be one or two. Pulse train distortion due to saturating amplification was measured to find whether the thermalization rates among the levels of the laser multiplets were fast or slow compared to the 10^{-11}-s pulses, but experimental uncertainties prohibit a determination.

I. Introduction

THE ONSET of gain saturation in an Nd–glass amplifier occurs when an optical pulse traveling in the inverted medium becomes strong enough to change appreciably the population of the laser levels [1]. The rate of increase and extent of saturation depend not only on the optical pulse power density and duration,

but also on the lifetimes and degeneracies of the levels in the 1.06-μm transition.

The upper laser level shown in Fig. 1 is the lower lying component of the $^4F_{3/2}$ pair with a several-hundred microsecond spontaneous emission lifetime. Not counting the sublevels that would be split by a magnetic field, the degeneracy of either member of the $^4F_{3/2}$ multiplet is one. The terminal laser level is the lower lying of two groups in the six-fold degenerate $^4I_{11/2}$ multiplet. Spectroscopically the components of either group in this manifold have been unresolved leaving the degeneracy of the terminal laser level for us to guess. The $^4I_{11/2}$ group empties spontaneously by a radiationless phonon transition [2] to the $^4I_{9/2}$ ground state about 2000 cm⁻¹ below. Measurements with Nd–glass at room temperature show the lifetime of the $^4I_{11/2}$–$^4I_{9/2}$ transition likely to be of the order of or greater than 100 ns [3]. This lifetime and also assumed degeneracies for the terminal laser level have been used as parameters in evaluating measurements of gain saturation for Q-switched pulses in Nd–glass amplifiers [4].

Our own measurements have been made with pulses of different durations. When the pulses for amplification were generated by a laser oscillator operating in the random spiking mode with individual spikes longer than a microsecond, our results were in accord with those of