Risk Allocation and Information : Some Recent Theoretical Developments

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I

I propose to set forth here some new developments in the theory of risk allocation in a market economy. In contrast to earlier work, these revolve about the effects of information on the viability and efficiency of risk-bearing markets. Specific questions raised are the allocative effects of additional publicly available information and of possibly differing private information.

Markets are means for the mutually beneficial exchange of goods and for inducing the transformation of goods from one form to another. We will take it as axiomatic that individuals are risk-averse, so that the bearing of risks is a cost and the shifting of risks to others a good. The existence of insurance, common stocks, and many other devices testifies to the validity of the assumption of risk aversion, though it must be admitted that gambling and perhaps some speculative activity might be regarded as evidence for risk preference in some contexts.

As part, then, of the general use of market for exchanging goods, we expect to find markets in which risks are traded. The risks are shifted to those more able to bear them until at the margin the cost to the risk-bearer is equal to the benefit to the risk-shifter. More specifically, there are, in addition to the usual commodities, a set of *contingent commodities* ; a unit contingent commodity is an agreement to deliver one unit of a specific good or (more generally in practice) to pay one unit of money if and only if a specified event has occured. An insurance policy is a good example.

If an event is certain to occur, then a commodity contract contingent on its occurrence is identical with the corresponding unconditional contract, and creation of the contingent commodity market has no economic significance. If an event is certain not to occur, then a commodity contingent on that event must have price zero, and again existence of the contingent market has no economic significance. However, if there is uncertainty about the event, then in general a market for contracts contingent upon that event will be viable ; there will be a price at which supply and demand will balance with some buyers and some sellers.

In fact, if markets are created for every commodity for every contingency, then the general competitive equilibrium leads to an efficient allocation of risk-bearing (Arrow [1953] ; Debreu [1953], [1959, Chapter VII]).

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The existence of competitive equilibrium with universal contingent contracts follows by suitable reinterpretation of the usual existence results. Even if the markets only exist for some contingencies, existence of equilibrium can still be demonstrated (Radner [1968]). However, as is usual when markets are absent, the market allocation of risk-bearing is no longer efficient; there exists conceivable reallocations which would make everyone better off.

It is a matter of some controversy how to represent the concept of uncertainty. The most usual doctrine represents uncertainty by probabilities, and I shall follow that convention here. It certainly is the only theory that has shown itself to be useful in deriving any results. Within this framework, however, is a point which has created controversy for generations; is probability objective or subjective? I.e., given an event, is there one probability to which all reasonable people must subscribe or can individuals differ? The currently most accepted doctrine is the subjective probability theory; probabilities express individual beliefs just as utilities express individual tastes. Individual behavior is determined by maximizing expected utility, where the expectation is computed according to the individual's own probabilities.

Under the subjective probability theory, there are two motives for trading in contingent commodities. One is the desire to avoid risks fundamental to insurance; this depends on the existence of uncertainty and would hold even if everyone agreed on the probabilities. The other motive derives from differences of opinion; if I judge an event more probable than you do, then, other things being equal, there will be a price at which I am willing to sell a contract contingent on that event and you are willing to buy it. Betting on horse races is a pure example. For a more serious example, it is clear that many of the participants on the commodity futures or stock markets are basing their actions on anticipations of the future; since some sell and some buy, these anticipations must differ.

From this viewpoint, the efficiency of the competitive market in contingent claims remains valid, in the sense that there is no alternative feasible allocation which will yield every individual a higher expected utility based on his or her own probabilities.

Both the subjective and the objective probability theories, however, recognize there may be differences of opinion based on different observations. As part of the complete structure of probability beliefs about the world, every individual has conditional probabilities. The probability that an event A occurs will in general be changed by the knowledge that another event B has occured. Thus, the probability that it will snow on 1 January 1980 in Cambridge, Massachusetts will be some number which will be approximately the proportion of the times it has snowed on 1 January over the years for which observations have been made. But on 31 December 1979, a more relevant probability will be that given by the weather map on that date; roughly, the proportion of times that a similar weather map has been followed by snow the next day.

Therefore, two individuals with the same probability beliefs may nevertheless have different probabilities for the same event when entering the market, because they have observed different other events. The existence of information derived from observations can have profound effects on the working of the risk-sharing markets. Indeed, the problem of differential information has long been known in the insurance literature under such headings as "moral hazard" and "adverse selection."

In this paper, I want to survey some aspects of the effects of information on the markets for contingent goods, by means of a toy example studied under different informational assumptions. The theory is still under development and is due to many scholars I give specific references in appropriate places.

II

First, some definitions. By " information ", I mean any observation which effectively changes probabilities according to the principles of conditional probability. The prior probabilities are defined for all events, an event being described by statements about both the variables that are relevant to individual welfare and those that define the range of possible observations. Given an observation, there is a conditional or posterior distribution of possible values of the welfare-relevant variables.

Consider first the case of a single individual. Suppose information is offered to him or her at no cost. Should the individual accept it? Clearly, the answer is yes. He or she can't be worse off because the information can always be disregarded. In more technical language, the individual, in the absence of information, will have to make a decision. That is, he or she will have to choose among a set of alternative actions. Since the consequences of each action are uncertain, the choice will be made so as to maximize expected utility. Now suppose the individual is told that information will be made available. That is, an observation which specifies which of a number of possible alternative events occurred will be made and transmitted to him. The decision can be made after receiving the observation and therefore maximizes expected utility computed according to the probabilities conditional on the observation. The decision made will now depend on the actual event observed. Put slightly differently, the individual can, before receiving the information, choose a *decision function*, which specifies the decision made for each possible observation. The decision made in the absence of information is a special kind of decision function, one which specifies the same decision for any possible observation. Therefore, the optimal decision function must be at least as good, in the expected utility sense, as a constant decision, simply, from the definition of the word, " optimal." In general, the best decision function will be better, in the expected utility sense, than the best decision which disregards the observation.

This analysis neglects the cost of acquiring and using the information. The most important and most stubborn of these costs are the limits on individual information-processing capacity. These costs are extremely important in actual economic life; but I wish to neglect them for the present discussion. The point to be stressed here is rather the difference between the individual and social values of information, even apart from costs. As will be seen, when risks are allocated by the market, information may be harmful rather than beneficial.

III

Let us review briefly the theory of allocation under contingent contracts by means of the simplest possible example. Suppose the world can be in only two possible states, though it is not known to any party which is the truth. Further, suppose there are only two individuals. More precisely, to preserve the competitive flavor, we should assume two types of individuals, with indefinitely many in each type and with identity of tastes and endowments among all individuals within a type. The effect is the same as if there were two individuals, each of whom, however, behaved like a perfect competitor in taking prices as parameters. Call the members of our tiny economy Walras and Böhm-Bawerk. Suppose further that their subjective probabilities for the two events are the same. (I shall maintain throughout the hypothesis that the subjective probabilities of the events held *prior* to any observations are the same to all individuals. Any difference in probability judgments when entering a market are attributed to different observations.) In fact, to make the examples even simpler, it will be assumed that the probability of each event is 1/2 for each individual. Suppose that, of the two states, state 1 is relatively more favorable to Walras than to Böhm-Bawerk. Then Walras sells contracts payable if state 1 occurs and buys contracts payable if state 2 occurs.

Let me state a whimsical, more specific, example, which will be used throughout the lecture as a theme for variations. Suppose there are two commodities, " wheat " and " barley," which are perfect substitutes in consumption but produced under different conditions. If v is the total amount of wheat and barley consumed by an individual, then the individual's von Neumann-Morgenstern utility is assumed to be log v. Walras has initially a stock of 1 unit of wheat, Böhm-Bawerk a stock of 1 unit of barley; there is no production. There are two weather states, with probability 1/2 each: in state W, all the barley is destroyed and none of the wheat; in B, all the wheat is destroyed and none of the barley. Hence, the effective initial holdings of the consumers' good, " wheat plus barley," are given in the following table:

Table 1

Individual	*State of the World*	
	W	B
Walras	1	0
Böhm-Bawerk	0	1

There are two contracts on the market; delivery of the consumers' good if W and delivery of the good if B. Because of the total symmetry of the assumptions, the two contracts have equal unit prices. In equilibrium, Walras sells 1/2 unit of wheat if W and buys 1/2 unit of barley if B. Böhm-Bawerk does the opposite. The expected utilities of Walras and Böhm-Bawerk are the same, both being (1/2) log (1/2) + (1/2) log (1/2) = log (1/2). For ease of understanding, it is preferable to state the *certainty-equivalent income* instead of expected utility, i.e., that income which, if obtained with certainty, would have a utility equal to the given expected utility. If y* is the certainty-equivalent income, then,

$$\log y^* = E(\log y),$$

or,

$$y^* = \text{antilog } E(\log y),$$

where y is the variable consumption.

To cover the general case, let p_W and $p_B = 1 - p_W$ be the probabilities of states W and B, respectively, from the viewpoint of any particular individual. Let y_W and y_B

be the individual's consumption under the respective states. Then, his or her certainty-equivalent income is,

$$y^* = y_W^{p_W} \; y_B^{p_B} .$$

In the present case, $p_W = p_B = 1/2$, so that,

$$y^* = (y_W y_B)^{1/2}. \tag{1}$$

It is seen that with perfect contingent markets but no information,

$$y^*_W = y^*_B = 1/2. \tag{2}$$

IV

A major theme of this lecture is the surprising fact that an increase in information may *lower* the efficiency of the market, as first noted by Hirshleifer [1971]. The simplest illustration already occurs if information is introduced into the above example. Suppose the information is *public*, by which is meant that both parties know it. Because there are only two possible states of the world, information consists of knowing which state will prevail. Clearly, if W is known, Böhm-Bawerk will have no purchasing power; therefore no transactions will take place (this is in fact the competitive equilibrium allocation, with all prices for the contingent commodities as possible equilibria). Then Walras will consume 1 unit, for utility 0, Böhm-Bawerk 0 units, for utility $-\infty$, if W occurs. If B occurs, the allocation is reversed. *Ex ante,* states W and B occur with probability 1/2 each; hence, for each individual, the expected utility is $-\infty$, and therefore,

$$y^*_W = y^*_B = 0. \tag{3}$$

Thus, the existence of public information effectively prevents the sharing of risk-bearing and destroys the corresponding utility gain.

V

In the preceding example, public information effectively reduced the market to autarchy. The information eliminated the possibility of trading risks without doing any offsetting good. This is because the information has no social use in a pure exchange economy. If production is introduced, however, it is reasonable to suppose that information enhances productive capability. If inputs are made before outputs, then, under uncertainty, outputs are a random function of inputs. Information reduces the uncertainty of output for any given input, and therefore should improve the allocation of resources for production.

Let me now introduce a simple example, which generalizes the previous one and at the same time demonstrates the possibility that information increases productivity.

Instead of being endowed with wheat or barley, let each member of the market be endowed with land which can be sown to either wheat or barley according to the decision of the owner. We retain the assumption about the unknown state of the world; in W weather, only wheat grows, in B wheather, only barley. Walras is better off than Böhm-Bawerk in W weather in the sense that if they both plant their entire land (one unit for each) in wheat, then Walras's output is greater than Böhm-Bawerk's; the opposite holds in B weather. The production hypothesis is summarized in the following table.

Table 2

Output as a Function of Production Decision and State of the World

Individual	*State of the World* W	B
Walras	1 if sown to wheat 0 if sown to barley	a if sown to barley 0 if sown to wheat
Böhm-Bawerk	a if sown to wheat 0 if sown to barley	1 if sown to barley 0 if sown to wheat

The parameter a satisfies the condition, $0 \leqslant a < 1$.

The utility functions and prior probabilities of the two states are as before. If $a = 0$, this production model is equivalent to the pure exchange model; Walras's land can only be used for wheat, so that his output is the same as his endowment in the pure exchange model for each state, and the same is true of Böhm-Bawerk.

Each individual can sow part of his land to wheat and part to barley; the amounts produced under each state are proportional to the amounts sown. Thus, if Walras plants 2/3 of his land in wheat and 1/3 in barley, his output is 2/3 if W, a/3 if B.

If $a = 1$, then Walras and Böhm-Bawerk have identical production possibilities. No trade will occur in either the presence or the absence of information. But public information is certainly productive; if it is known that state W obtains, both will plant to wheat, if B both to barley.

VI

Retain the assumption that information is public. We can compare attained welfare levels with or without information and with or without the existence of contingent markets. There are four possible cases.

Case I: *No information, no contingent markets.* Since each has to be autarchic, each plants so as to maximize the expected utility of consumption. Let Walras, for example, plant w in wheat and $b = 1 - w$ in barley. His output is w if W, ab if B. His expected utility is $(1/2) \log w + (1/2) \log (ab) = (1/2) \log w + (1/2) \log b + (1/2) \log a$. The optimal policy is clearly independent of a; it is $w = b = 1/2$. The situation for Böhm-Bawerk is symmetrical. Straightforward calculation shows that,

$$y^*_W = y^*_B = a^{1/2}/2.$$

Case II: *Contingent markets without information.* There are now markets for wheat-claims conditional on W and for barley-claims contingent on B. Because of the symmetry of the assumptions, it is obvious that the prices of the two kinds of claims are the same.

Each individual can be thought of as made up of a firm and a consumer. The consumer's income is the profit of the firm. The firm can be thought of as supplying two joint products, wheat-claims if W and barley-claims if B. As in the usual theory of firms and households under certainty, the firm should maximize its profits independently of the tastes of the owning household. In this case, profits equal the total value of contingent claims sold, We assume honesty in sale, in that the number of claims sold for a given state does not exceed the amount the firm could supply in that state.

Since the prices of the two kinds of claims are equal, it is clear that Walras should sow his entire land to wheat and sell 1 unit of wheat claims; any land transferred to barley would yield a lesser value sold in the ratio a : 1. Similarly, Böhm-Bawerk sows his land to barley and sells one unit of barlay claims. Under expected utility maximization, each individual will spend half his income on claims of each kind; hence, each will receive 1/2 unit of wheat if W and 1/2 unit of barley if B, so that his income is 1/2 with certainty.

Since $a < 1$, $1/2 > a^{1/2}/2$; hence, introducing contingent markets without information increases the welfare of both.

Case III. *Information without contingent markets.* In the absence of contingent markets, there is no trade either before or after realization of the state of the weather. Clearly, both individuals plant wheat if W and barley if B. Walras realizes and consumes 1 if W and a if B, while Böhm-Bawerk has the same up to a permutation of the states. Therefore, each has a certainty-equivalent income of $(1 \cdot a)^{1/2}$. Each is obviously better off than in case I. Introduction of public information increases welfare if there are no contingent markets, because it permits adaptation of production.

Is the outcome of case III better than that of case II? Is it better to introduce public information or contingent markets, if only one is possible? Comparison of the certainty-equivalent incomes shows that case III yields better outcomes if and only if $a^{1/2} > 1/2$, or $a > 1/4$. The coefficient *a* measures what may be termed the *flexibility* of the economy, its ability to increase production in response to information.

Public information is better than the introduction of contingent markets if the economy is sufficiently flexible and not otherwise.

Case IV. *Information and contingent markets.* It has already been pointed out that public information prevents the execution of mutually advantageous contingent contracts. But it should be noted that, technically speaking, the contingent markets are not "destroyed." Rather, the prices are such that each individual finds it most advantageous neither to buy nor to sell. Specifically, if state W obtains, barley claims have zero price. Then both individuals plant only wheat. Neither can plan to buy wheat claims, since they cannot sell barley claims. On the other hand, since both know that W obtains, neither will want to sell wheat claims, since all they could do with the proceeds would be to buy barley claims, which have no-use. Hence supply and demand balance on both markets. However, from the welfare point of view, the situation is identical with that with public information and no markets.

The results of this section can be set forth in the following table.

Table 3

Certainty-Equivalent Incomes for Different Combinations of Public Information and Contingent Markets

Public Information	*Contingent Markets*	*Certainty-Equivalent Income*
No	No	$a^{1/2}/2$
No	Yes	1/2
Yes	No	$a^{1/2}$
Yes	Yes	$a^{1/2}$

Certainty-equivalent income for (No, No) is less than any of the other three.

Certainty-equivalent income for (Yes, No) is greater than that for (No, Yes) if and only if $a > 1/4$.

VII

Now suppose that information is not public; specifically assume that one member of our toy economy has information that the other does not have. For definiteness, we suppose that Walras knows the state of the weather while Böhm-Bawerk does not. Call this the hypothesis of *differential* information.

The concept of differential information is becoming increasingly recognized as central to many features of economic organization. The problems of adverse selection and moral hazard in insurance are special cases, as will be discussed below; their implications for the general theory of risk-bearing were suggested in Arrow [1965, Lecture 3] and have been greatly developed in recent years. Akerlof [1970] showed how markets might disappear altogether when the parties have different information and know it.

When there is differential information, the prices obtaining on markets may be a means of transmitting information. The existence and efficiency properties of equilibria under these circumstances have been studied by Green [1973], Kihlstrom and Mirman [1975], Grossman [1976], Grossman and Stiglitz [1976], and Shubik [1977], among many others. These concepts will be illustrated in our very simple model.

Case V. *Differential information without contingent markets.* Walras plants entirely to wheat if W, to barley if B. His output is 1 if W, a if B; hence, his certainty-equivalent income is $a^{1/2}$, just as in the case of public information and no contingent markets. Böhm-Bawerk is in the same situation as if there were no public information and no contingent markets; hence, his certainty-equivalent income is $a^{1/2}/2$.

If there are contingent markets, then all depends on what Böhm-Bawerk can infer about what Walras knows. First, we must note that in any case, the equilibrium on the contingent markets cannot be the same as if there were no information. In the last case, prices for the two kinds of contingent contracts (wheat if W, barley if B) must be equal. But suppose the state is in fact W; Walras will buy no barley claims and will therefore sell no wheat claims. At the original prices, Böhm-Bawerk's demands and supplies are unchanged. Hence, there is now excess supply of barley claims and excess demand for wheat claims, so the original prices can no longer be in equilibrium.

How does Böhm-Bawerk respond to the discovery that the previous equilibrium no longer holds? There are (at least) two possibilities. One is in the spirit of competitive theory — he simply takes market prices as parameters and responds to them by profit- and utility-maximization. We may call this the *parametric price* assumption. Alternatively, he may be aware that Walras knows the true state; since the market prices reflect Walras's behavior, Böhm-Bawerk can infer the true state and act accordingly. The equilibrium price for each state must be such that if it is read as a signal for the truth of that state, the resulting behavior will sustain it as an equilibrium. This is the assumption of *rational expectations.*

Before taking up the two cases, some terminology and general remarks are in order. In either case, there will be in general a different set of prices if W occurs than if B occurs; this was also true in the (trivial) case of public information and contingent markets (Case IV). Take barley-claims to be numeraire in each state of nature. Then the price system in each state reduces to the price of wheat-claims in terms of barley-claims (possibly infinite if barley-claims are free goods). Let p^W and p^B be the prices of wheat-claims in terms of barley-claims in the states W and B, respectively; the pair (p^W, p^B) will be referred to as the *equilibrium price system.*

It is important to note that the supply conditions are independent of information once the prices are given. This is a fundamental property of contingent markets. Each of our agents can be thought of as combining a firm and a consumer. The firm sells claims contingent on the state of the weather. Its aim is to maximize its profits in the transactions; given the contingent prices, it cannot do better by using information. To validate any planned sales of contingent claims, the firms must actually plant the corresponding amounts.

Note that demands indeed respond to information; hence, the system as a whole is influenced by information, and supplies may indeed respond to information, but only through the prices of the contingent contracts.

In our simple linear technologies, there will be for each firm one wheat-claim price, p, at which it is indifferent between producing barley and wheat (and therefore between selling barley-claims and wheat-claims). For all smaller wheat-claim prices it produces only barley, for all larger ones only wheat. Walras would realize p if he produces only wheat, a if he produces only barley. Therefore, he produces barley if $p < a$, wheat if $p > a$, and is indifferent between them when $p = a$. In the last case, he is also indifferent among all alternatives which allocate a fraction r of his land to wheat and $1-r$ to barley and therefore produces r of wheat and $a(1-r)$ of barley for any r, $0 \leqslant r \leqslant 1$. It also follows that Walras's income as a function of the wheat-claim price is,

$$Y_W(p) = \max(p, a). \tag{4}$$

Similarly, Böhm-Bawerk produces only barley if $p < a^{-1}$, only wheat if $p > a^{-1}$. If $p = a^{-1}$, he is equally willing to produce ar of wheat and $1-r$ of barley for any r, $0 \leqslant r \leqslant 1$. His income is,

$$Y_B(p) = \max(ap, 1). \tag{5}$$

With these remarks, we can study the equilibrium price systems and associated quantity allocations under the alternative assumptions of parametric prices and rational expectations.

Case VI. *Differential information with contingent markets and parametric prices.* The determination of the equilibrium price system is elementary but slightly tedious; it will be found in the Appendix, the results are,

$$p^W = a^{-1}, \qquad p^B = a \text{ if } a \geqslant 1/2, \quad = 1/2 \text{ if } a < 1/2. \tag{6}$$

The production and consumption allocations are interesting. Since $p^W = a^{-1} > a$, Walras will produce only wheat-claims if W prevails. His supply will be 1. Also, from (4), his income is p^W; since he only buys wheat-claims (knowing that W prevails), he buys 1 unit of wheat-claims, and therefore exactly consumes his own supply. Böhm-Bawerk, not knowing which state prevails, buys both kinds of claims. To meet his demand for wheat-claims, he must therefore produce some of his own, as well as some barley-claims. At $p^W = a^{-1}$, he will in fact be willing to produce both. Precisely, he plants one-half his land in wheat, one-half in barley, and therefore consumes $a/2$ if W prevails.

Suppose B is true, so that the price is p^B. The case $a < 1/2$, where the technology is relatively inflexible, is most interesting. Since $p^B = 1/2 > a$, Walras will find it profitable to plant only wheat and therefore sell only wheat-claims, even though he knows that B is true! Since $p^B = 1/2 < a^{-1}$, Böhm-Bawerk plants only barley and sells barley-claims. Therefore, one unit of barley is produced if B holds. However, Walras's income is $1/2$, which he spends entirely on barley-claims. Walras and Böhm-Bawerk therefore consume $1/2$ each. The production, though not the consumption, reflects a striking misallocation.

If $a \geqslant 1/2$, then still $p^B < a^{-1}$, so that Böhm-Bawerk plants only barley. However, Walras will plant some of his land in each, $(2a)^{-1}$ in wheat and $1 - (2a)^{-1}$ in barley. His product, if B prevails, is $a - (1/2)$ of barley, while Böhm-Bawerk's output of barley if 1, for a total output of $a + (1/2)$. Walras's income is a, spent entirely on barley-claims, so that he receives a units of barley, while Böhm-Bawerk's consumption is the remainder, $1/2$.

If we work through the certainty-equivalent incomes, we find,

$$\begin{aligned} y_W^* &= 2^{-1/2} \text{ if } a < 1/2, \\ &= a^{1/2} \text{ if } a \geqslant 1/2; \\ y_B^* &= a^{1/2}/2 \text{ in any case.} \end{aligned} \tag{7}$$

Comparison with the results in Table 3 shows that Walras is at least as well off under differential information as in any previous case. Böhm-Bawerk, on the other hand, has the same welfare level as in the absence of both contingent markets and information and is worse off than in the presence of either.

Case VII. *Differential information with contingent market and rational expectations.* It is obvious that if both wind up knowing the true state, the situation is the same as with public information and contingent markets (Case IV). Barley claims are free goods in state W, wheat claims in state B; the contingent markets are ineffective, and the real outcome is the same as with public information and no markets (Case III).

It may be a useful exercise to restate this conclusion more formally. It is asserted that the equilibrium price system is $p^W = \infty$, $p^B = 0$. Clearly, with these prices, both individuals plant only wheat if W, only barley if B. Further, if W holds, both know it, Walras by assumption and Böhm-Bawerk by inference from observing p^W, and therefore both demand only wheat claims; the same holds, *mutatis mutandis*, if state B holds.

In the spirit of general equilibrium theory, one may ask if the equilibrium price system is unique. In this case, it is not, and the formal argument is instructive. Suppose there is another equilibrium price system, p^W, p^B. First suppose that $p^W \neq p^B$. Then Böhm-Bawerk can infer from the market price which state holds. If W is true, both demand only wheat-claims. Hence, both must supply only wheat-claims. But this policy will be profit-maximizing for both whenever $p^W \geqslant a^{-1}$. Similarly, equilibrium will hold if B is true whenever $p^B \leqslant a$. Hence, any pair satisfying these conditions is an equilibrium price system.

Is it possible that $p^W = p^B$ in some equilibrium price system? If it did, then Böhm-Bawerk would *not* be able to infer which state prevailed. His demands for the two kinds of claims would therefore be the same as if he took prices parametrically. Since the rational expectations and parametric price models differ only with respect to Böhm-Bawerk's demand, it follows that, if $p^W = p^B$, the pair of prices would be a rational expectations price equilibrium system if and only if it is a parametric price equilibrium system. But, from (6), there is not in this example any parametric price equilibrium system with $p^W = p^B$.

In this example, there is no rational expectations equilibrium with $p^W = p^B$. But it is certainly possible to have such equilibria in other contexts. Therefore, rational expectations equilibria do not necessarily convey information held by one individual to the uninformed.

Table 4

Assumption on Markets	*Productivity Parameter* a	*Individual* Walras	Böhm-Bawerk
None	$0 \leqslant a < 1$	$a^{1/2}$	$a^{1/2}/2$
Parametric prices	$1/2 \leqslant a < 1$	$a^{1/2}$	$a^{1/2}/2$
Parametric prices	$0 \leqslant a < 1/2$	$2^{-1/2}$	$a^{1/2}/2$
Rational expectations	$0 \leqslant a < 1$	$a^{1/2}$	$a^{1/2}$

* Walras knows the state of the world, Böhm-Bawerk does not.

The informed party, Walras, is as well off with differential information as with public information and better off if the uninformed party takes prices parametrically and if the technology is not very flexible. The uninformed party, on the contrary, is as badly off as if there were neither information nor contingent markets unless he uses the observed prices to form rational expectations; in that case, the situation is essentially the same as with public information.

VIII

These examples all show compromise between spreading the risk-bearing and efficiency of production. Can both kinds of efficiency be achieved?

First, what is the optimal allocation in an ideal system? Clearly, if W obtains, both parties should plant to wheat; the total production would be $1+a$. Similarly, if B obtains, a total output of $1+a$ is also obtainable. If, in each state, the total output is divided equally between the two, then each consumes $(1+a)/2$ with certainty. Since $(1+a)/2 > a^{1/2}$ and also $(1+a)/2 > 1/2$ whenever $0 < a < 1$, this is better than any allocation achieved through the market structures analyzed so far. The allocation is Pareto optimal; it is the only one which is also symmetric between the participants.

Could this allocation be achieved through a market mechanism? It can, if the contingent markets operate *before* information is available while production takes place after. In this case, Walras knows that he could produce 1 unit if W, a units if B, and will sell claims accordingly. The market prices for the two kinds of claims are equal, say to 1. Then Walras's and Böhm-Bawerk's incomes are each $1+a$; they each purchase $(1+a)/2$ of each kind of claim, thereby realizing the optimal allocation.

Whether this principle has significant application can only be determined in individual cases. It depends on the possibility of insuring that contingent markets are in existence before information can be known.

IX

To conclude, I give some real-world cases where problems of market organization are illustrated by the new theoretical developments.

Examples of differential information abound. In the field of insurance, both adverse selection and moral hazard arise from differential information. In the case of adverse selection, the insured has a greater knowledge of the risks than the insurer. His or her demand behavior will change accordingly. In equilibrium, the insurer is correct on the average but cannot distinguish among insured with varying risks. Those with higher risks find their insurance underpriced relative to the true risks and will therefore buy more insurance than is efficient. Therefore, the average risk per dollar of insurance is higher; the premium must rise and might conceivably rise to the point of eliminating low-risk individuals from the market. If the insurance company can observe the total amount of insurance purchased by an individual, it might infer his risk status and use that information in setting rates. The differential information is that which enables the individual insured to know his or her particular risks.

In the case of moral hazard, the individual can make decisions which cannot be monitored successfully by the insurance company. Thus, in the case of health insurance, the ill person demands medical services based on his or her perception of his illness. But if the cost of medical services is partly or wholly covered by the insurance, he or she will demand more than if the full cost were covered. The insurer is not, however, able to distinguish among medical needs.

It is for reasons such as these that other kinds of cost controls are widely proposed and beginning to be enforced, at least in the United States. The market is no longer regarded as thoroughly efficient, and non-market controls are invoked.

Differential information is manifested in another form in connection with quality of product. The quality of medical services is a particularly acute form of this problem. Here, the seller in general knows much more than the buyer; the latter is not in a strong position to insist on quality standards. It is unlikely that a market mechanism of any kind will be very useful. Society has been adapting to this example of market by regulation and by ethical codes, such as those governing the practices of medicine and law.

For a final example among a vast number of possibilities, I mention the allocation of resources to the production of information itself. Research and development of new products is the main form. There are conflicting tendencies to over- and under-investment. In the case where public information renders contingent markets useless, there can be over-investment by social standards. Since differential information is advantageous (at least in the parametric price case), each individual may be willing to expand resources to find by the information; but if both succeed, they may have made both worse off. On the other hand, information is hard to make into a private good; if discovered, it is likely to leak in some form, and therefore the investor will not get the full reward.

My main stress in this lecture is not on the particular applications, most of which remain to be worked out, but to exemplify as simply as possible some new tendencies in economic thinking about risk-bearing.

APPENDIX

As promised in the text, I work out here in detail the equilibrium price system and quantity allocations for Case VI, where there are contingent markets, differential information, and parametric prices.

First, note that Böhm-Bawerk, who receives no information directly and makes no inferences from observed prices, is uncertain about the state of the wheather. He therefore demands some claims of each kind. Hence, in equilibrium, there must be a willingness to supply claims of both kinds. From the discussion of supply in the text, we see that at any equilibrium, we must have,

$$a \leqslant p \leqslant a^{-1}. \tag{8}$$

With this range of prices, the incomes of the two individuals are,

$$Y_W(p) = p, \qquad Y_B(p) = 1. \tag{9}$$

Let d stand for demand for wheat-claims, with subscripts representing the agents. Böhm-Bawerk's demand is independent of state. For any market price, p, his demands come from maximizing,

$$(1/2) \log w + (1/2) \log b,$$

subject to $pw + b = 1$, where w and b are the demands for wheat- and barley-claims respectively. Then,

$$d_B(p) = w = (2p)^{-1} \tag{10}$$

Walras's demand for wheat-claims depends on the state, to be indicated by a superscript. If W holds, Walras spends his entire income on wheat-claims; if B holds, he spends none of his income on wheat-claims.

$$d_W{}^W(p) = 1, \ d_W{}^B(p) = 0 \tag{11}$$

If d^W and d^B represent total demands in states W and B, respectively,

$$d^W(p) = 1 + (2p)^{-1}, \ d^B(p) = (2p)^{-1}. \tag{12}$$

Walras will supply 1 unit of wheat-claims if $p > a$; he will supply any amount from 0 to 1 indifferently if $p = a$. Similarly, Böhm-Bawerk will supply no wheat-claims if $p < a^{-1}$ and be indifferent at any amount from 0 to a if $p = a^{-1}$. Hence, total supply is indifferent over the interval $< 0, 1 >$ if $p = a$, equals 1 if $a < p < a^{-1}$, and is indifferently anything in the interval $< 1, 1+a >$ if $p = a^{-1}$.

If d(p) is the demand function, then p is an equilibrium in any of the three following circumstances: (a) $p = a$, $0 \leqslant d(a) \leqslant 1$; (b) $a < p < a^{-1}$, $d(p) = 1$; (c) $p = a^{-1}$, $1 \leqslant d(a^{-1}) \leqslant 1 + a$. (Note that clearing the wheat-claims market automatically clears the barley-claims market.)

Suppose state W holds. Since $d^W(p) > 1$, the only possible equilibrium is at $p = a^{-1}$. Since $1 < d^W(a^{-1}) = 1 + (a/2) < 1 + a$, it is in fact true that $p^W = a^{-1}$. Total output is $1 + (a/2)$, of which 1 is supplied by Walras. From (11), Walras's demand for wheat-claims is also 1, which is realized.

In state W, then, Böhm-Bawerk commits himself to producing $a/2$ of wheat if W; this can be done by sowing one-half his land in wheat and half in barley. He buys $a/2$ wheat-claims and $1/2$ barley-claims; the former is realized.

In state B, matters are a little more complex. Is it possible that $p^B = a$? From (12) and the equilibrium conditions, $p^B = a$ if and only if $(2a)^{-1} \leqslant 1$, or $a \geqslant 1/2$. In that case, Böhm-Bawerk produces only barley. However, he demands $(2a)^{-1}$ of wheat-claims, which is supplied by Walras. This means that Walras must plant $(2a)^{-1}$ of his land in wheat, even though he knows that no wheat will grow; he plants the remainder in barley, with a realized output of $a[1 - (2a)^{-1}] = a - (1/2)$. Walras's demand for barley-claims, which is realized, is a (his total income). Böhm-Bawerk's demand for barley-claims is $1/2$.

It is impossible that $p^B = a^{-1}$; for $d^B(a^{-1}) = a/2$, from (12), and $a/2 < 1$. However, it is possible that $a < p < a^{-1}$; this occurs only when $d^B(p) = 1$, i.e., $p = 1/2$, and it is an equilibrium when $a < 1/2 < a^{-1}$. Since the second inequality must hold, we have that,

$$p^B = 1/2 \text{ when } a < 1/2.$$

Here, Walras has an income $1/2$ and a demand $1/2$ for barley-claims. Because $p^B > a$, he plants only wheat, although he knows that no wheat will grow, a remarkable inefficiency. Böhm-Bawerk grows only barley; since Walras receives $1/2$, Böhm-Bawerk will also receive $1/2$.

REFERENCES

AKERLOFF, G. A. [1970] "The market for "lemons"; quality uncertainty and the market mechanism." *Quarterly Journal of Economics* 84 : 488-500.

ARROW, K. J. [1953] "Le rôle des valeurs boursières pour la répartition la meilleure des risques." *Econométrie,* Colloques Internationaux du Centre National de la Recherche Scientifique, Vol. XI, Paris, 41-47. English translation [1963-4].

ARROW, K. J. [1963-4] "The role of securities in the optimal allocation of risk-bearing." *Review of Economic Studies* 31:91-96. English translation of [1953]. Reprinted in [1970, Chapter 4].

ARROW, K. J. [1965] *Aspects of the Theory of Risk-Bearing.* Helsinki : Yrjö Jahnssonin säätio. Lecture 3, reprinted in [1970, Chapter 5].

ARROW, K. J. [1970] *Essays in the Theory of Risk-Bearing.* Amsterdam : North-Holland.

DEBREU, G. [1953] "Une économie de l'incertain." Paris : Electricité de France, mimeographed.

DEBREU, G. [1959] *Theory of Value.* New York : Wiley ; New Haven, Conn. : Yale University Press.

GREEN, J. [1973] "Information, efficiency, and equilibrium." Harvard Institute of Economic Research, Discussion Paper No. 284.

GROSSMAN, S. [1976] "On the efficiency of competitive stock markets where traders have diverse information." *Journal of Finance* 31 : 573-585.

GROSSMAN, S., and STIGLITZ, J. [1976] "Information and competitive price systems." *American Economic Review* 66 : 246-252.

HIRSHLEIFER, J. [1971] "The private and social value of information and the reward to inventive activity." *American Economic Review* 61 : 561-574.

KIHLSTROM, R. E., and MIRMAN, L. J. [1975] "Information and market equilibrium." *Bell Journal of Economics and Management Science* 6 : 357-376.

RADNER, R. [1968] "Competitive equilibrium under uncertainty." *Econometrica* 36 : 31-58.

SHUBIK, M. [1977] "Competitive equilibrium, contingent commodities, and information." *Journal of Finance* 32 : 189-193.