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RISK ATTITUDES AND DECISION WEIGHTS[1]

BY AMOS TVERSKY AND PETER WAKKER

To accommodate the observed pattern of risk-aversion and risk-seeking, as well as common violations of expected utility (e.g., the certainty effect), we introduce and characterize a weighting function according to which an event has greater impact when it turns impossibility into possibility, or possibility into certainty, than when it merely makes a possibility more or less likely. We show how to compare such weighting functions (of different individuals) with respect to the degree of departure from expected utility, and we present a method for comparing an individual's weighting functions for risk and for uncertainty.

KEYWORDS: Risk attitude, decision weights, rank dependence, source dependence.

1. INTRODUCTION

THE CLASSICAL THEORY OF DECISION under risk and uncertainty combines the principle of mathematical expectation with the assumption of decreasing marginal utility, which jointly imply risk aversion. Three clusters of phenomena reflecting risk attitudes have challenged the descriptive validity of the classical theory. First, although risk aversion is prevalent, there are situations in which risk seeking is commonly observed. Gambling is a case in point. Second, there is a considerable body of evidence that preferences between risky prospects are not linear in the probabilities. The certainty effect, demonstrated by Allais, is the best-known example of this phenomenon. Third, people's preferences depend not only on the degree of uncertainty but also on the source of uncertainty. For instance, people sometimes prefer to bet on known rather than unknown probabilities, as demonstrated by Ellsberg.

There have been many attempts to explain risk attitudes that are inconsistent with expected utility. Much recent work has been devoted to theories that extend expected utility by introducing nonadditive decision weights (Kahneman and Tversky (1979), Quiggin (1982), Yaari (1987), Gilboa (1987), Schmeidler (1989), Luce and Fishburn (1991)). In these models, preferences are determined jointly by the utility function that measures the subjective value of the outcomes, and by the decision weights that capture what may be called chance attitude.

In this article we present a theoretical analysis of decision weights that is motivated by the observed pattern of risk seeking, nonlinear preferences, and source dependence. This pattern suggests an S-shaped weighting function that overweights small probabilities and underweights moderate and high probabilities (Section 2). The theoretical framework used in this paper is introduced in Section 3. Section 4 establishes the properties of the preference order that are necessary and sufficient for an S-shaped weighting function. This analysis is

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extended to uncertainty in Section 5. In analogy to the Pratt/Arrow analysis of comparative risk aversion, Section 6 introduces the relation more-SA-than between the weighting functions of different individuals, which reflects departure from expected utility. Section 7 introduces a method for comparing the weighting function of the same individual for different sources of uncertainty. This method is used to analyze the observed relation between risk and uncertainty. Proofs are deferred to the Appendix.

2. THE FOURFOLD PATTERN

In order to motivate the present development, we first illustrate some common features of people's attitude toward risk. Consider simple prospects of the form (x, p) that offer $\$x$ with probability p, and nothing otherwise. The study of choice between simple risky prospects has given rise to the fourfold pattern illustrated in Table I. These data are taken from a study by Tversky and Kahneman (1992) in which each subject made a series of choices between a risky prospect and various cash offers. The value of $C(x, p)$ is the median cash offer (in dollars) that was indifferent to the prospect (x, p).

Table I exhibits risk seeking for gains and risk aversion for losses of low probability combined with risk aversion for gains and risk seeking for losses of high probability. This pattern has been observed in numerous studies, with and without contingent payoffs (Fishburn and Kochenberger (1979), Kahneman and Tversky (1979), Hershey and Schoemaker (1980), Payne, Laughhunn, and Crum (1981), Cohen, Jaffray, and Said (1987), Wehrung (1989), Tversky and Kahneman (1992)). Extreme risk seeking for long shots has recently been reported by Kachelmeier and Shehata (1992) in an experiment conducted in China with real payoffs that were considerably higher than the subjects' normal monthly income. Risk seeking for small probabilities of gains is consistent with common observations of gambling and risky ventures, whereas risk seeking for high-probability losses is consistent with the tendency to accept a risk in order to avoid a sure loss.

Friedman and Savage (1948) and Markowitz (1952) have attempted to explain the combination of risk seeking and risk aversion in terms of a utility function with both concave and convex regions. However, because the fourfold pattern arises over a wide range of payoffs, it cannot be explained by the utility function for money. Instead, it suggests a nonlinear transformation of the probability scale.

TABLE I

THE FOURFOLD PATTERN OF RISK ATTITUDES

	Gain	Loss
Low probability	$C(100, .05) = 14$ (Risk Seeking)	$C(-100, .05) = -8$ (Risk Aversion)
High probability	$C(100, .95) = 78$ (Risk Aversion)	$C(-100, .95) = -84$ (Risk Seeking)

Suppose the value of the prospect (x, p) is given by $w(p)v(x)$, where v is the value function for gains and losses, and w is a nonlinear weighting function. Figure 1 presents a typical weighting function obtained by Tversky and Fox (1994). This function exhibits diminishing sensitivity: it is steepest near the endpoints and shallower in the middle, yielding overweighting of small probabilities and underweighting of middle and high probabilities. Thus, people underestimate the impact of an increase in probability from 20% to 25% in comparison to an increase from 0% to 5% or from 95% to 100%. Such a weighting function gives rise to the fourfold pattern described above, under plausible assumptions concerning the value function.

3. BASIC CONCEPTS

We first introduce terminology and notation, and then describe the theoretical framework used in the paper. We distinguish decision under risk, where the probabilities are assumed to be known, and decision under uncertainty, where the probabilities associated with the various outcomes are not given in advance.

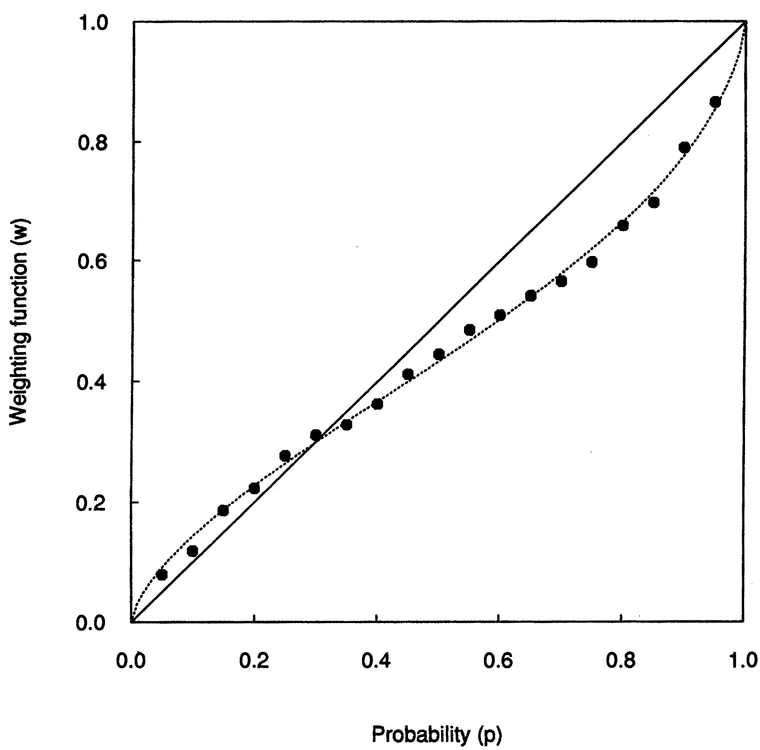

FIGURE 1.—The points represent median estimates, across subjects ($N = 40$), obtained in Tversky and Fox (1994). The smooth curve is obtained by fitting the parametric form $w(p) = \delta p^{\gamma}/(\delta p^{\gamma} + (1-p)^{\gamma})$, suggested by Lattimore, Baker, and Witte (1992). The estimated values of the parameters are $\gamma = .69$, $\delta = .77$.

In both cases, the decision maker has to select between prospects that are described as positive or negative changes with respect to the status quo. To simplify matters, we assume that the outcomes are real numbers designating money, and interpret 0 as the status quo. Gains refer to positive outcomes, and losses refer to negative outcomes. In decision under risk a *prospect* is described by a finite probability distribution. Thus $(x_1, p_1; \ldots; x_n, p_n)$ is the risky prospect yielding outcome x_j with probability p_j, $j = 1, \ldots, n$; the p_j's are nonnegative and sum to one. If there is only one nonzero outcome then the zero outcome is suppressed; for example, $(Z, 1/2)$ is the prospect that yields Z with probability $1/2$ and 0 with probability $1/2$.

Decision under uncertainty is described in terms of a set S, called the *state space*. We assume that exactly one state obtains, but the decision maker is uncertain about this state. Subsets of S are called *events*; $S - A$ is the complement to A. In decision under uncertainty, *prospects* are functions from S to $\mathbb{R}$, taking finitely many values. If state s obtains, then prospect f yields the outcome $f(s)$. An uncertain prospect is described as $(x_1, A_1; \ldots; x_n, A_n)$, where $(A_1, \ldots, A_n)$ is a partition of S and x_j is the outcome associated with the states in A_j. As above, the zero outcome is suppressed if there is only one nonzero outcome; thus (x, A) is the prospect that yields x if A obtains, and 0 if it does not.

Risk can be considered as a special case of uncertainty where probabilities are given for the events in S, and prospects that generate the same probability distribution over the outcomes are treated as identical. In this case, each prospect is described by the probability distribution it induces over the outcomes, with no reference to the state space.

We identify outcomes with degenerate prospects. Thus x can be viewed as a constant function assigning outcome x to all states or as a degenerate probability distribution assigning probability 1 to that outcome. Let $\succcurlyeq$ denote the preference relation over prospects; the relations $\succ$, $\sim$ are defined as usual.

Cumulative Prospect Theory

This article adopts the theoretical framework of cumulative prospect theory, or CPT for short (Tversky and Kahneman (1992)). This theory is more general than the rank-dependent utility model because it permits a different treatment of gains and losses.[2] It assumes a continuous strictly increasing value function v: $\mathbb{R} \to \mathbb{R}$ satisfying $v(0) = 0$. For choice under risk, it invokes two weighting functions, denoted by w^+ and w^-, for gains and losses respectively.[3] A *weighting function* w is a strictly increasing function from $[0,1]$ to $[0,1]$ with $w(0) = 0$ and $w(1) = 1$. For uncertainty, the weighting functions for gains and losses are denoted by W^+ and W^-. Here a *weighting function* (or a capacity) W on S is a

[2] Closely related models were proposed by Starmer and Sugden (1989) and Luce and Fishburn (1991).

[3] In following sections the superscript + is often suppressed.

function on 2^S such that $W(\varnothing)=0$, $W(S)=1$, and $W(A) \geq W(B)$ whenever $A \supset B$. Obviously, if W is additive, i.e., $W(A \cup B) = W(A) + W(B)$ for all disjoint events A, B, then it is a probability measure.

According to CPT, the value of a prospect $(x_1, p_1; \ldots; x_n, p_n)$ in which $x_1 \leq \cdots \leq x_k \leq 0 \leq x_{k+1} \leq \cdots \leq x_n$, is

$$\sum_{j=1}^{k} \pi_j^- v(x_j) + \sum_{j=k+1}^{n} \pi_j^+ v(x_j), \tag{3.1}$$

where the *decision weights* are defined by $\pi_j^- = w^-(p_1 + \cdots + p_j) - w^-(p_1 + \cdots + p_{j-1})$ and $\pi_j^+ = w^+(p_j + \cdots + p_n) - w^+(p_{j+1} + \cdots + p_n)$.[4] Note that these weights do not necessarily sum to one. For uncertainty, the value of a prospect $(x_1, A_1; \ldots; x_n, A_n)$, in which $x_1 \leq \cdots \leq x_k \leq 0 \leq x_{k+1} \leq \cdots \leq x_n$, is

$$\sum_{j=1}^{k} \pi_j^- v(x_j) + \sum_{j=k+1}^{n} \pi_j^+ v(x_j), \tag{3.2}$$

where now *decision weights* are defined by $\pi_j^- = W^-(A_1 \cup \cdots \cup A_j) - W^-(A_1 \cup \cdots \cup A_{j-1})$ and $\pi_j^+ = W^+(A_j \cup \cdots \cup A_n) - W^+(A_{j+1} \cup \cdots \cup A_n)$.[5]

CPT generalizes rank-dependent utility, introduced by Quiggin (1982) and Yaari (1987) in the context of risk, and by Schmeidler (1989) in the context of uncertainty. *Rank-dependent utility* corresponds to the special case where the weighting function for losses is the *dual* of the weighting function for gains, i.e., $w^-(p) = 1 - w^+(1-p)$, and $W^-(A) = 1 - W^+(S-A)$. For prospects with non-negative outcomes, rank-dependent utility coincides with CPT. Tversky and Kahneman (1992) considered another special case of CPT, where $w^+ = w^-$ or $W^+ = W^-$, which provided a reasonably good fit for risk choice. Preference conditions for this property, called *reflection*, are presented in Appendix B.

We assume that the weighting function for risk is continuous, and that the weighting function for uncertainty satisfies *solvability*,[6] i.e., for all events $A \subset C$ and $W(A) \leq p \leq W(C)$ there exists an event B such that $W(B) = p$ and $A \subset B \subset C$. The assumptions made throughout this paper are summarized below; they have been axiomatized in Wakker and Tversky (1993, Section 8.4):

ASSUMPTION 3.1: *Risky prospects are probability distributions over* $\mathbb{R}$. *Uncertain prospects are functions from the state space* S *to the outcome set* $\mathbb{R}$. *Prospects have finitely many outcomes. Preferences between prospects are represented by* (3.1) *or* (3.2). *The value function is continuous and strictly increasing. The weighting function for risk is continuous and strictly increasing; for uncertainty it satisfies solvability.*

[4] Here we follow the usual convention that, for $j=0$, $p_1 + \cdots + p_j = 0$, and for $j = n$, $p_{j+1} + \cdots + p_n = 0$.

[5] For $j=0$, $A_1 \cup \cdots \cup A_j = \varnothing$; for $j=n$, $A_{j+1} \cup \cdots \cup A_n = \varnothing$.

[6] Gilboa (1987) introduced this condition under the name convex-ranged.

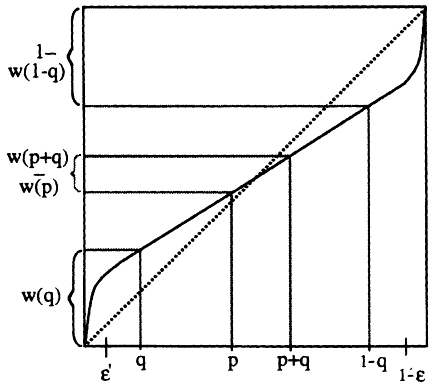

FIGURE 2.—Illustration of SA.

4. SUBADDITIVITY IN CHOICE UNDER RISK

In this article we focus on the weighting function for gains. The analysis for losses is essentially identical and, with few exceptions, will not be discussed separately. Thus we restrict attention to nonnegative outcomes, and suppress the superscript in w^+ and W^+. This section discusses risk; uncertainty is discussed in following sections.

The weighting function presented in Figure 1 is S-shaped: It is steepest near 0 and 1 and shallower in the middle. The experimental evidence is generally consistent with such a weighting function (Camerer (1992), Cohen and Jaffray (1988), Tversky and Kahneman (1992)). Thus, in Figure 1 a "lower" interval $[0, q]$ has more impact than a middle interval $[p, p+q]$ provided the middle interval is bounded away from the upper endpoint 1 (e.g., $p+q \leq .9$). Similarly, an "upper" interval $[1-q, 1]$ has more impact than a middle interval $[p, p+q]$ provided the middle interval is bounded away from the lower endpoint 0 (e.g., $p \geq .1$). The following definition formalizes this notion (see Figure 2): w satisfies *bounded subadditivity*, or *subadditivity* (*SA*) for short,[7] if there exist constants $\varepsilon \geq 0$ and $\varepsilon' \geq 0$ such that

(4.1) $\quad w(q) \geq w(p+q) - w(p) \quad$ whenever $\quad p+q \leq 1-\varepsilon$

and

(4.2) $\quad 1 - w(1-q) \geq w(p+q) - w(p) \quad$ whenever $\quad p \geq \varepsilon'$.

Conditions (4.1) and (4.2) are called *lower SA* and *upper SA*, respectively. Lower SA entails the inequality $w(p+q) \leq w(p) + w(q)$ on the interval $[0, 1-\varepsilon]$. Upper SA implies the same inequality on the interval $[0, 1-\varepsilon']$ for the *dual weighting function* $\hat{w}(p) = 1 - w(1-p)$, as can be seen by substitution. The constants $\varepsilon, \varepsilon'$ are called *boundary constants*, and do not depend on p, q. They serve to ensure that we always compare an interval that includes 0 or 1 with an

[7] For convenience, we use the terms bounded SA or SA instead of the more accurate term $\varepsilon, \varepsilon'$-SA.

interval that does not. Naturally, they may vary from one individual to the other. We are primarily interested in lower SA near 0, say on the interval [0, 0.4], and in upper SA near 1, say on the interval [0.6, 1]. This corresponds to $\varepsilon = \varepsilon' = .6$. Since w is fairly linear in the middle region, lower and upper SA usually hold for larger intervals, and for most functions found in the literature $\varepsilon = .1$ and even $\varepsilon' = 0$ can be chosen. For instance, for the weighting function depicted in Figure 1, SA holds for boundary constants $\varepsilon = .07$ and $\varepsilon' = 0$, and hence for any larger boundary constants.

Preference Conditions

Next we present conditions for preferences that are necessary and sufficient for bounded subadditivity. These conditions are independent of the value function, and thus separate what we have called chance attitude from marginal utility. Previous work assumed a linear value function (Yaari (1987), Chateauneuf (1991)) or a concave value function (Chew, Karni, and Safra (1989), Chew (1989)). Furthermore these papers investigated convex, rather than subadditive, weighting functions.

We begin with the certainty effect, which leads to upper SA (4.2). As demonstrated by Allais, people commonly exhibit the following preferences, where M denotes one million dollar:

$$(1\text{M}, .11) \prec (5\text{M}, .10) \quad \text{and}$$
$$1\text{M} \succ (0, .01; 1\text{M}, .89; 5\text{M}, .10).$$

The certainty effect suggests the preference condition that is needed to characterize upper SA. To illustrate, let us shift probability mass from 5M to 0 in the upper right prospect until the decision maker is indifferent between the upper prospects. Suppose we find

$$(1\text{M}, .11) \sim (5\text{M}., .08).$$

Obviously, from the second preference above it follows by dominance that the same probability shift (.2 from 5M to 0 on the right) yields

$$1M \succ (0, .03; 1\text{M}, .89; 5\text{M}, .08).$$

In general, upper SA requires that

$$(4.3) \qquad (z, 1-q) \sim (Z, p) \quad \text{implies} \quad z \succcurlyeq (0, 1-p-q; z, q; Z, p)$$

for $0 < z < Z$, and $p \geq \varepsilon'$ where $\varepsilon' \geq 0$ is the *boundary constant*.[8] To interpret the condition, recall that in CPT prospects are evaluated in terms of cumulative events, e.g., receiving z or more. According to (4.3), an increase of q in the probability of that event has more impact on the left, where it makes that event certain than on the right where it merely makes the event more probable. It is instructive to note that the CPT difference between the left prospects in (4.3) is

[8] Segal (1987) proposed a similar generalization of the Allais example which implies a convex weighting function.

$(1-w(1-q))v(z)$, and between the right prospects is $(w(p+q)-w(p))v(z)$; hence (4.2) implies (4.3) since $v(z)>0$.

Next we turn to the overweighting of small probabilities. Consider the following preferences.

$$40 \prec (0,.35;40,.05;100,.60)$$
$$(40,.95;100,.05) \succ (100,.65).$$

Let us shift probability mass from \$100 to 0 in the upper right prospect until the decision maker is indifferent between the upper prospects. Suppose we find

$$40 \sim (0,.45;40,.05;100,.50).$$

From the second preference above it follows by dominance that the same probability shift (.10 from 100 to 0 on the right) yields

$$(40,.95;100,.05) \succ (100,.55).$$

In general, lower SA requires that

$$z \sim (0,1-p-q;z,q;Z,p) \text{ implies } (z,1-q;Z,q) \succcurlyeq (Z,p+q), \tag{4.4}$$

for $0<z<Z$ and $p+q \le 1-\varepsilon$, where $\varepsilon \ge 0$ is the boundary constant. Thus a q probability shift from z to Z has more impact on the left, where it makes the receipt of (at least) Z possible, than on the right where it merely increases the probability of receiving (at least) Z. Note that for the right prospects the extreme outcomes, 0 and Z, did not change, whereas for the left prospects the best outcome changed from z to Z. To see that lower SA implies (4.4), note that the CPT difference between the left prospects in (4.4) is $w(q)(v(Z)-v(z))$, and between the right prospects it is $(w(p+q)-w(p))(v(Z)-v(z))$.

PROPOSITION 4.1: *Under Assumption* 3.1, *the weighting function w satisfies SA if and only if the preference relation satisfies* (4.3) *and* (4.4).[9]

Actually, lower SA is equivalent to (4.4), and upper SA to (4.3).

Applications

With the exception of convex weighting functions, explored by several authors, the parametric weighting functions proposed in the literature are generally consistent with bounded subadditivity. Setting $\varepsilon=.1$ and $\varepsilon'=0$ is sufficient to accommodate most of these functions.

There are two approaches for testing SA, axiomatic and parametric. Wakker, Erev, and Weber (1994) found that in its general form the comonotonic independence axiom, which underlies all rank-dependent models, did not fare better than the independence axiom of expected utility, but "CPT with its S-shaped w-function provides the best description of the choice patterns ob-

[9] The same boundary constant ε applies to (4.1) and (4.4), and the same boundary constant ε' to (4.2) and (4.3).

served in this experiment. However, the improvement in prediction does not reach statistical significance" (p. 214). Stronger support for the preference conditions for an S-shaped weighting function was obtained by Wu and Gonzales (1994).

Most studies have estimated decision weights on the basis of various parametric assumptions about the value or the weighting function. The resulting weighting functions generally supported SA (Hogarth and Einhorn (1990), Birnbaum, Coffey, Mellers, and Weiss (1992), Lattimore, Baker, and Witte (1992), Tversky and Kahneman (1992), Gonzales (1993), Camerer and Ho (1994), Tversky and Fox (1994)); for a recent review that includes some earlier literature, see Camerer (1994).

Violations of SA are rare. One of the four subjects in Allais (1988) exhibits a convex ("pessimistic") weighting function, which violates lower SA. In Lattimore, Baker, and Witte (1992), only five out of 114 subjects yielded estimates that were inconsistent with SA. Karmarkar (1978) and Karni and Safra (1990) also considered S-shaped weighting functions.

It is noteworthy that lower SA accounts for the observed tendency (Kahneman and Tversky (1979)) to undervalue probabilistic insurance that reduces the probability of a loss, say from p to $p/2$, relative to regular insurance that reduces it from p to 0.

In order to characterize the degree of departure from expected utility theory, it is useful to devise a measure of the degree of SA. To this end, define for given p, q satisfying $p + q \leq 1$ and the appropriate boundary conditions,

$$D(p,q) = w(p) + w(q) - w(p+q) \quad \text{and}$$

$$D'(p,q) = 1 - w(1-q) + w(p) - w(p+q).$$

Under SA, both D and D' are positive for all p, q that satisfy the boundary conditions, whereas under expected utility D and D' are both zero. Let d and d' denote, respectively, the average values of D over all $p + q \leq 1 - \varepsilon$ and of D' over all $p \geq \varepsilon'$.

Simple graphical interpretations are possible whenever the weighting function is approximately linear except near the endpoints; see Figure 3. In this case, for all p, q in the linear middle range, D and D' are independent of p and q, and D is the lower intercept, D' the upper intercept, of the linear function. Thus the averages d and d' provide estimates for the lower and upper intercepts, and $s = 1 - d - d'$ is an estimate of the slope; s can be interpreted as an index of sensitivity to probability changes. It equals 1 for expected utility, and it is less than 1 under SA. If expected utility is accepted as a standard for rational choice, then s could be interpreted as an index of rationality.

Tversky and Fox (1994) estimated the values of d, d', and s for three studies of risky choice. The median estimates were .07, .16, and .76, in accord with both lower and upper SA. The observation that d' exceeds d suggests that upper SA is generally more pronounced than lower SA. Further discussion of the empirical evidence appears in the final section.

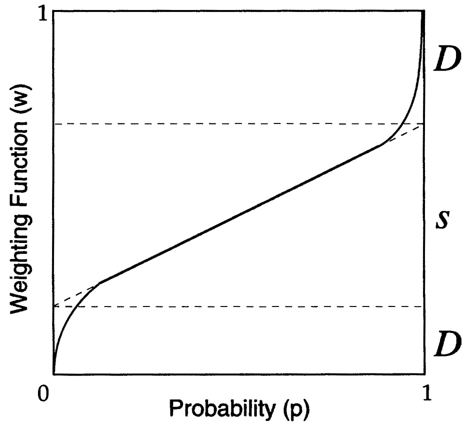

FIGURE 3.—A weighting function that is linear except near the endpoints.

5. SUBADDITIVITY IN CHOICE UNDER UNCERTAINTY

The above definitions of SA naturally extend to uncertainty: W satisfies *bounded subadditivity*, or *subadditivity* (*SA*) for short,[10] if there are events E, E' such that

$$(5.1) \qquad W(B) \geq W(A \cup B) - W(A) \quad \text{whenever} \quad W(A \cup B) \leq W(S - E)$$

and

$$(5.2) \qquad 1 - W(S - B) \geq W(A \cup B) - W(A) \quad \text{whenever} \quad W(A) \geq W(E').$$

Conditions (5.1) and (5.2) are called *lower SA* and *upper SA*, respectively. Lower SA implies that an event B has a greater impact when added to the null event than when it is added to a nonnull event A. Upper SA implies that an event B has a greater impact if it is subtracted from certainty than when it is subtracted from an event $A \cup B$. Upper SA for W is equivalent to lower SA for the dual weighting function $\hat{W}(A) = 1 - W(S - A)$. The events E, E' are called *lower* and *upper boundary events*. They are "small" events that do not depend on A and B. Under SA, an event B has greater impact when it turns impossibility into possibility or possibility into certainty than when it merely makes a possibility more likely. That is, a change from $\varnothing$ to B, or from $S - B$ to S, is more noticeable than a change from A to $A \cup B$.

Preference Conditions

The preference conditions for uncertainty are similar to those for risk, but require one further preparation; because the probabilities are not given, inequalities such as $W(A) \geq W(E')$ must be defined in terms of preferences. This is commonly done by defining $A \succcurlyeq B$ if there exists a gain Z such that

[10] Again, we use the terms bounded SA or SA instead of the more accurate E, E'-SA.

TABLE II

A DEMONSTRATION OF SA. OUTCOMES DEPEND ON THE TEMPERATURE t AT 4 PM ON APRIL 1, 1991 IN NEW YORK CITY. THE PERCENTAGE OF RESPONDENTS (113 STOCK BROKERS) WHO SELECTED EACH PROSPECT IS GIVEN IN BRACKETS, SEPARATELY FOR GAINS ($K = \$1000$) AND FOR LOSSES ($K = -\$1000$).

Problem		A_1 if $t > 80$	A_2 if $80 \geq t \geq 70$	A_3 if $70 > t \geq 60$	A_4 if $60 > t$	Gains	Losses
I	f'	0	0	0	$5K$	[65]	[34]
	g'	$2K$	$2K$	0	0	[35]	[66]
II	f''	$5K$	0	0	0	[58]	[29]
	g''	0	0	$2K$	$2K$	[42]	[71]
III	f	$5K$	0	0	$5K$	[32]	[72]
	g	$2K$	$2K$	$2K$	$2K$	[68]	[28]

$(Z, A) \succcurlyeq (Z, B)$, that is, winning on A is preferred to winning on B.[11] Clearly, $A \succcurlyeq B$ if and only if $W(A) \geq W(B)$. The following two conditions characterize SA:

$$z \sim (0, S-(A \cup B); z, B; Z, A) \quad \text{implies} \quad (z, S-B; Z, B) \succcurlyeq (Z, A \cup B) \tag{5.3}$$

whenever $0 < z < Z$ and $A \cup B \preccurlyeq S - E$, and

$$(z, S-B) \sim (Z, A) \quad \text{implies} \quad z \succcurlyeq (0, S-(A \cup B); z, B; Z, A), \tag{5.4}$$

whenever $0 < z < Z$ and $A \succcurlyeq E'$; here E and E' are the boundary events. The interpretation of (5.3) and (5.4) is similar to that of the corresponding conditions for risk. To see that lower SA implies (5.3), note that the CPT difference between the left prospects in (5.3) is $W(B)(v(Z) - v(z))$, and between the right prospects is $(W(A \cup B) - W(A))(v(Z) - v(z))$. To derive (5.4) from upper SA, note that the CPT difference between the left prospects in (5.4) is $(1 - W(S-B))v(z)$, and between the right prospects it is $(W(A \cup B) - W(A))v(z)$.

PROPOSITION 5.1: *Under Assumption* 3.1, *the weighting function W satisfies SA if and only if* (5.3) *and* (5.4) *are satisfied.*[12]

We conclude the section with several empirical observations. Table II illustrates both upper and lower SA, and provides a novel counterexample to expected utility that does not involve independence or substitution.

Table II shows that for the gain prospects the majority choice favored f' over g', f'' over g'', and g over f, although $f = f' + f''$ and $g = g' + g''$. Furthermore,

[11] For losses a dual definition should be used. That is, A is more likely than B if there exists a loss $-Z$ such that $(-Z, A) \preccurlyeq (-Z, B)$.

[12] The same boundary event E applies to (5.1) and (5.3), and the same boundary event E' applies to (5.2) and (5.4).

(f', f'', g) was the single most popular pattern for gains, exhibited by 31% of the subjects. This pattern violates expected utility, but is consistent with SA. To verify this, note that lower SA implies $W(A_1 \cup A_4) \le W(A_1) + W(A_4)$. Furthermore, if upper SA exceeds lower SA (i.e., $E' = \varnothing$ can be taken), as is commonly the case, then $1 \ge W(A_1 \cup A_2) + W(A_3 \cup A_4)$ follows. Therefore, $V(f) \le V(f') + V(f'')$ but $V(g) \ge V(g') + V(g'')$, where $V(f)$ denotes the value of prospect f according to CPT. Hence the observed pattern is consistent with CPT. Because the data exhibit SA in a strict sense, they are inconsistent with expected utility theory, in which (setting the utility of 0 to 0) all the above inequalities should be equalities.

Note that the modal preferences for the loss prospects are the mirror image of the preferences for the gain prospects, again exhibiting SA in the strict sense contrary to expected utility. Here, (g', g'', f) was the single most popular pattern, exhibited by 35% of the subjects. This pattern is consistent with the reflection assumption ($W^+ = W^-$), characterized in Appendix B.[13]

Although SA is a plausible condition for decision under uncertainty, it is unlikely to hold in some special circumstances in which the union of disjoint events is less "vague" than its constituents. For example, consider an urn with one hundred green and red balls in unknown proportion, which are numbered from 1 to 100. Then the events "even and red" and "even and green" are vague, but their union "even" is no longer vague. If, as suggested by Ellsberg (1961), people prefer to bet on known probabilities, defined by the numbers, rather than on the unknown probabilities involving colors, then SA may not hold in such situations.

6. COMPARATIVE SUBADDITIVITY

The relation more-concave-than or more-risk-averse-than between utility functions of different individuals was introduced and characterized by Pratt (1964) and Arrow (1965). This relation orders individuals by their departure from the (objective) expected value. In this section we develop a similar analysis for weighting functions. Specifically, we introduce and characterize the relation of more-SA-than between the weighting functions of different individuals, which orders them by their departure from expected utility theory. If this theory is taken as the standard of rational behavior, then the more-SA-than relation can be interpreted as an ordering by departure from rationality.

As in Section 4, the present treatment extends previous work (Yaari (1987), Chew, Karni, and Safra (1989), Chew (1989), Chateauneuf (1991), Chateauneuf and Cohen (1994), Wakker (1994)) by considering S-shaped rather than convex weighting functions, and by comparing weighting functions independently of the value functions.

[13] The modal choices in Table II are also at variance with additive regret models (Bell (1982), Loomes and Sugden (1982), and Fishburn (1982)), which are violated by the strict form of (5.3) and (5.4).

A transformation ϕ: $[0,1] \to [0,1]$ is called *SA* if it has the same mathematical properties as an SA weighting function, i.e., $\phi(0)=0$, $\phi(1)=1$, ϕ is continuous and strictly increasing, and ϕ satisfies (4.1) and (4.2). One weighting function is *more SA* than another, if the first is obtained from the second by an SA transformation. This definition applies to both risk and uncertainty, where the weighting functions for uncertainty are defined on the same domain. Figure 4 illustrates this relation.

Decision under Risk

It is readily verified (see Proposition 6.1) that w_2 is more SA than w_1 if and only if the following two conditions hold:

(6.1) $w_1(r) = w_1(p+q) - w_1(p)$ implies
$w_2(r) \geq w_2(p+q) - w_2(p)$

and

(6.2) $1 - w_1(1-r) = w_1(p+q) - w_1(p)$ implies
$1 - w_2(1-r) \geq w_2(p+q) - w_2(p)$;

the boundary condition for (6.1) is $p+q \leq 1-\varepsilon$ for $\varepsilon \geq 0$, and the boundary

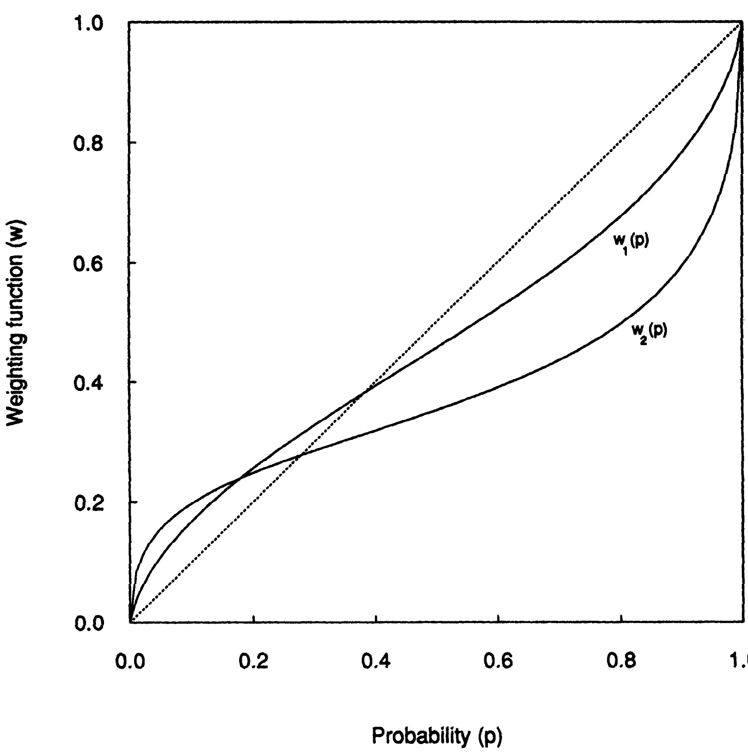

FIGURE 4.—The weighting function w_2 is less additive than w_1.

condition for (6.2) is $p \geq \varepsilon'$, where $\varepsilon' \geq 0$. The proof is based on the observation that, under (6.1), w_2 satisfies lower SA "when its arguments are measured in w_1 units." The same logic applies to (6.2). The relation of ε and ε' to the boundary constants for the transformation carrying w_1 into w_2 is given in Appendix A; see (A1) and (A2). According to the above conditions, $w_2(r)$ and $\hat{w}_2(r) = 1 - w_2(1-r)$ are overweighted more than $w_1(r)$ and $\hat{w}_1(r) = 1 - w_1(1-r)$, respectively.

Next we present the preference conditions for the proposition that $\succcurlyeq_2$ is more SA than $\succcurlyeq_1$, defined for the preference orders of two individuals. We impose no restrictions on the value functions of the two individuals, so their value functions may be different. Two conditions are required, one for upper and one for lower SA. First we consider lower SA:

(6.3) If $x \sim_1 (0, 1-p-q; x, q; X, p)$ and

$$(x, 1-r; X, r) \sim_1 (X, p+q), \text{ then}$$

$$y \sim_2 (0, 1-p-q; y, q; Y, p) \text{ implies}$$

$$(y, 1-r; Y, r) \succcurlyeq_2 (Y, p+q),$$

whenever $0 < x < X$, $0 < y < Y$, and $p + q \leq 1 - \varepsilon$ for the boundary constant $\varepsilon \geq 0$. This condition states that if, for $\succcurlyeq_1$, the improvement on the left (r probability for receiving X instead of x) matches the corresponding improvement on the right, then for $\succcurlyeq_2$ the comparable improvement on the left (r probability for receiving Y instead of y) outweighs the corresponding improvement on the right. To further illustrate the condition, we show how it follows from (6.1). The first two indifferences imply, by comparing the CPT difference of the left prospects to that of the right prospects, that $w_1(r)(v_1(X) - v_1(x)) = (w_1(p+q)\text{-}w_1(p))(v_1(X) - v_1(x))$, i.e., $w_1(r) = w_1(p+q) - w_1(p)$. By (6.1), $w_2(r) \geq w_2(p+q) - w_2(p)$. This implies that $w_2(r)(v_2(Y) - v_2(y))$, i.e., the CPT difference between the left prospects in the lower two lines in (6.3), is at least as large as $(w_2(p+q) - w_2(p))(v_2(Y) - v_2(y))$, which is the difference between the right two prospects. Hence (6.1) implies (6.3).

Second, we consider upper SA:

(6.4) If $(x, 1-r) \sim_1 (X, p)$ and

$$x \sim_1 (0, 1-p-q; x, q; X, p) \text{ then}$$

$$(y, 1-r) \sim_2 (Y, p) \text{ implies}$$

$$y \succcurlyeq_2 (0, 1-p-q; y, q; Y, p),$$

whenever $0 < x < X$, $0 < y < Y$, and $p \geq \varepsilon'$ for the boundary constant $\varepsilon' \geq 0$. This condition states that if, for $\succcurlyeq_1$, the improvement on the left (yielding x with certainty) matches the corresponding improvement on the right, then for $\succcurlyeq_2$ the comparable improvement on the left (yielding y with certainty) outweighs the corresponding improvement on the right. In other words, the certainty effect is more pronounced for the second decision maker than for the first.

PROPOSITION 6.1: *Under Assumption* 3.1 (*for both* $\succcurlyeq_1$ *and* $\succcurlyeq_2$), *the following three statements are equivalent*:[14]

(i) w_2 *is more SA than* w_1;

(ii) *conditions* (6.1) *and* (6.2) *are satisfied*;

(iii) *conditions* (6.3) *and* (6.4) *are satisfied.*

Decision under Uncertainty

We first extend (6.1) and (6.2) to uncertainty. W_2 is more SA than W_1 if and only if (see Proposition 6.2) W_2 is a strictly increasing transform of W_1, and

(6.5) $$W_1(C) = W_1(A \cup B) - W_1(A) \quad \text{implies} \quad W_2(C) \geq W_2(A \cup B) - W_2(A)$$

and

(6.6) $$1 - W_1(S - C) = W_1(A \cup B) - W_1(A) \quad \text{implies} \quad 1 - W_2(S - C) \geq W_2(A \cup B) - W_2(A).$$

The boundary condition for (6.5) is: $W_1(A \cup B) \leq W_1(S - E)$ for some boundary event E; the boundary condition for (6.6) is: $W_1(A) \geq W_1(E')$ for a boundary event E'. The relation of E and E' to the boundary constants for the transformation carrying W_1 into W_2 is given in (A3) and (A4) in the Appendix.

Next we present the corresponding preference conditions for the proposition that $\succcurlyeq_2$ is more SA than $\succcurlyeq_1$. We require three conditions.

First, for all events A, B,

(6.7) $$A \succcurlyeq_1 B \quad \text{if and only if} \quad A \succcurlyeq_2 B.$$

This condition guarantees that W_2 is a strictly increasing transform of W_1. The second condition ensures that W_2 is more lower SA than W_1:

(6.8) $$\text{If } x \sim_1 (0, S - (A \cup B); x, B; X, A) \text{ and } (x, S - C; X, C) \sim_1 (X, A \cup B) \text{ then } y \sim_2 (0, S - (A \cup B); y, B; Y, A) \text{ implies } (y, S - C; Y, C) \succcurlyeq_2 (Y, A \cup B)$$

whenever $0 < x < X$, $0 < y < Y$, and $A \cup B \preccurlyeq_1 S - E$ for the boundary event E. Third, we require that W_2 be more upper SA than W_1:

(6.9) $$\text{If } (x, S - C) \sim_1 (X, A) \text{ and } x \sim_1 (0, S - (A \cup B); x, B; X, A) \text{ then } (y, S - C) \sim_2 (Y, A) \text{ implies } y \succcurlyeq_2 (0, S - (A \cup B); y, B; Y, A)$$

[14] The same boundary constants $\varepsilon, \varepsilon'$ apply to conditions (6.1) and (6.2), and to conditions (6.3) and (6.4). Their relation to the boundary constants for the transformation carrying w_1 into w_2 is described in (A1) and (A2) in the Appendix.

whenever $0 < x < X$, $0 < y < Y$, and $A \succcurlyeq_1 E'$ for the boundary event E'. The interpretation of (6.8) and (6.9) is essentially identical to that of (6.3) and (6.4).

PROPOSITION 6.2: *Under Assumption* 3.1 (*for both* $\succcurlyeq_1$ *and* $\succcurlyeq_2$), *the following three statements are equivalent*:[15]

(i) W_2 *is more SA than* W_1;

(ii) *conditions* (6.5), (6.6), *and* (6.7) *are satisfied*;

(iii) *conditions* (6.7), (6.8), *and* (6.9) *are satisfied.*

7. SOURCE DEPENDENCE

Perhaps the most persistent objection to expected utility theory concerns the distinction between risk and uncertainty. The expectation principle, it has been argued, can be applied to decision under risk where probabilities are known but not to decision under uncertainty or ignorance where the probabilities are unknown. This view, advanced by several authors, notably Keynes (1921) and Knight (1921), has been underscored by Ellsberg (1961), who argued convincingly that people prefer to bet on an urn that contains an equal number of green and red balls than on an urn that contains red and green balls in an unknown proportion. Numerous experiments have confirmed this hypothesis; see Camerer and Weber (1992) for a review. More generally, there is evidence that people's preferences depend not only on their degree of uncertainty but also on the source of uncertainty. This phenomenon has been called source dependence.

In this section we distinguish two aspects of source dependence, which we call source preference and source sensitivity. Let $\mathscr{A}$ and $\mathscr{B}$ be two distinct families of events. For example, one family may be generated by spinning a roulette wheel, the other by the possible outcomes of a horse race. We shall refer to such families as *sources*. We assume that the families are closed under union and complementation, and are rich in the sense that they both satisfy solvability. In decision under risk, we interpret the uncertainty as generated by a standard random device. Although probabilities could be realized by various random devices, we do not distinguish between them and treat risk as a single source.

Source Preference

In the domain of gains, the decision maker exhibits a general *preference for source* $\mathscr{A}$ *over source* $\mathscr{B}$ if, for any event A in $\mathscr{A}$ and B in $\mathscr{B}$, $W^+(A) = W^+(B)$ implies $W^+(S - A) \geq W^+(S - B)$. Expressed in terms of preferences, this means that $(x, A) \sim (x, B)$ implies $(x, S - A) \succcurlyeq (x, S - B)$ for all $x > 0$. Ellsberg's example of preference for the known over the unknown urn illustrates this relation.

To extend source preference to negative outcomes, we start with the preference condition. A preference to bet on source $\mathscr{A}$ rather than $\mathscr{B}$ means that

[15] The same boundary events E, E' apply to conditions (6.5) and (6.6), and to conditions (6.8) and (6.9). Their relation to the boundary constants for the transformation carrying W_1 into W_2 is described in (A3) and (A4) in the Appendix.

$(-x, A) \sim (-x, B)$ implies $(-x, S-A) \succcurlyeq (-x, S-B)$ for any $x > 0$. In terms of the weighting function, therefore, source $\mathscr{A}$ is *preferred to* source $\mathscr{B}$ if $W^{-}(A) = W^{-}(B)$ implies $W^{-}(S-A) \leq W^{-}(S-B)$. Note that the inequality for losses is the opposite of the inequality for gains. Hence source preference reduces the weighting function for losses and enhances the weighting function for gains. Indeed, it can be shown that if reflection holds, then source $\mathscr{A}$ is preferred to source $\mathscr{B}$ for gains if and only if $\mathscr{B}$ is preferred to $\mathscr{A}$ for losses. Consequently, reflection cannot be satisfied if $\mathscr{A}$ is preferred to $\mathscr{B}$ for both gains and losses.

Source Sensitivity

We next examine the concept of source sensitivity (SS). In the domain of gains, the decision maker exhibits *less SS* to source $\mathscr{B}$ than to source $\mathscr{A}$ if the following two conditions hold:

(7.1) if $W^{+}(A_1) = W^{+}(B_1)$ and $W^{+}(A_2) = W^{+}(B_2)$,
then $W^{+}(A_1 \cup A_2) \geq W^{+}(B_1 \cup B_2)$;

(7.2) if $W^{+}(S-A_1) = W^{+}(S-B_1)$ and $W^{+}(S-A_2) = W^{+}(S-B_2)$,
then $W^{+}(S-(A_1 \cup A_2)) \leq W^{+}(S-(B_1 \cup B_2))$

for all disjoint events A_1, A_2 in $\mathscr{A}$ and disjoint events B_1, B_2 in $\mathscr{B}$ satisfying the following boundary conditions. In (7.1), $W^{+}(A_1 \cup A_2) \leq W^{+}(S-E)$ for some boundary event E; in (7.2), $W^{+}(S-(A_1 \cup A_2)) \geq W^{+}(E')$ for some boundary event E'. Conditions (7.1) and (7.2) are dual in the sense that one condition holds if and only if the other holds for the dual weighting function.

To appreciate the above definition, suppose W^{+} is SA on $\mathscr{A}$ and on $\mathscr{B}$. Equation (7.1) means that the union of disjoint $\mathscr{B}$ events "loses" more than the union of the matching disjoint $\mathscr{A}$ events. Hence the decision maker is less sensitive to an increase in likelihood in source $\mathscr{B}$ than in source $\mathscr{A}$. Equation (7.2) imposes the dual condition. The comparative SS relation between sources for the same decision maker is reminiscent of the more-SA-than relation between different decision makers for the same source. Both relations reflect departure from expected utility, but they are formally and conceptually different.

Expressed in terms of preferences, (7.1) is equivalent to:

(7.3) $(x, A_1) \sim (x, B_1)$ and $(x, A_2) \sim (x, B_2)$ implies
$(x, A_1 \cup A_2) \succcurlyeq (x, B_1 \cup B_2)$ for any $x > 0$,

and (7.2) is equivalent to:

(7.4) $(x, S-A_1) \sim (x, S-B_1)$ and $(x, S-A_2) \sim (x, S-B_2)$ implies
$(x, S-(A_1 \cup A_2)) \preccurlyeq (x, S-(B_1 \cup B_2))$ for any $x > 0$,

for disjoint A_1, A_2 and disjoint B_1, B_2, and under the boundary conditions for (7.1) and (7.2), respectively. The relations of source preference and comparative SS are logically independent. If people prefer one source to another, they can exhibit more or less SS for one source than for the other, or neither.

Turning to losses, we say that the decision maker exhibits *less SS* to source $\mathscr{B}$ than to source $\mathscr{A}$ if conditions (7.1) and (7.2) hold for the weighting function for losses. The preference conditions are obtained from (7.3) and (7.4) by interchanging $x > 0$ and $x < 0$, as well as $\succcurlyeq$ and $\preccurlyeq$.

Empirical Evidence

We conclude this section by discussing some experimental demonstrations of SA and source preference. We define, for any disjoint events A, B satisfying the appropriate boundary condition,

$$D(A,B) = W(A) + W(B) - W(A \cup B) \quad \text{and}$$
$$D'(A,B) = 1 - W(S - B) - W(A \cup B) + W(A).$$

Unlike the case of risk, there is no obvious way to define the global measures d or d', because there is no natural prior measure on the event space with respect to which the averages could be taken. However, for any given experiment one can compute the average of the indices d, d' over all disjoint event pairs A, B. Again, $s = 1 - d - d'$ is a measure of correspondence with expected utility.

Tversky and Fox (1994) estimated decision weights for six sources, including risk. The risky events were generated by drawing a ball from an urn with known composition. The other sources consisted of the following uncertain quantities: the point spread in a playoff basketball game, the point spread of the 1992 Superbowl, the difference between the closing values of the Dow Jones on successive weeks, San Francisco temperature, and Beijing temperature. Table III presents the median values of d, d', and s, for all sources of uncertainty in each of the three studies.

All values of d and d' in Table III, which measure lower and upper SA respectively, are significantly greater than zero, confirming SA for all sources including chance. Furthermore, d and d' are greater (and hence s is smaller) for uncertainty than for chance, indicating that people are less sensitive to uncertainty than to chance.

Figure 5 displays, for each subject in one study ($N = 40$), the average s value for the two uncertain sources (Super Bowl and Dow Jones) against the s value for chance. Three features of Figure 5 are noteworthy. First, all values of s for the uncertain sources, and all but two values of s for the risky source are less

TABLE III

MEDIAN VALUES OF d AND d', AND s, ACROSS SUBJECTS, MEASURING THE DEGREE OF LOWER SA, UPPER SA, AND GLOBAL SENSITIVITY RESPECTIVELY.

	Study 1			Study 2			Study 3		
Source	d	d'	s	d	d'	s	d	d'	s
Chance	.06	.10	.81	.05	.19	.75	.11	.14	.72
Basketball	.21	.19	.61						
Super Bowl				.15	.23	.57			
Dow Jones				.12	.22	.67			
S.F. temp.	.20	.26	.51				.27	.23	.50
Beijing temp.							.28	.32	.42

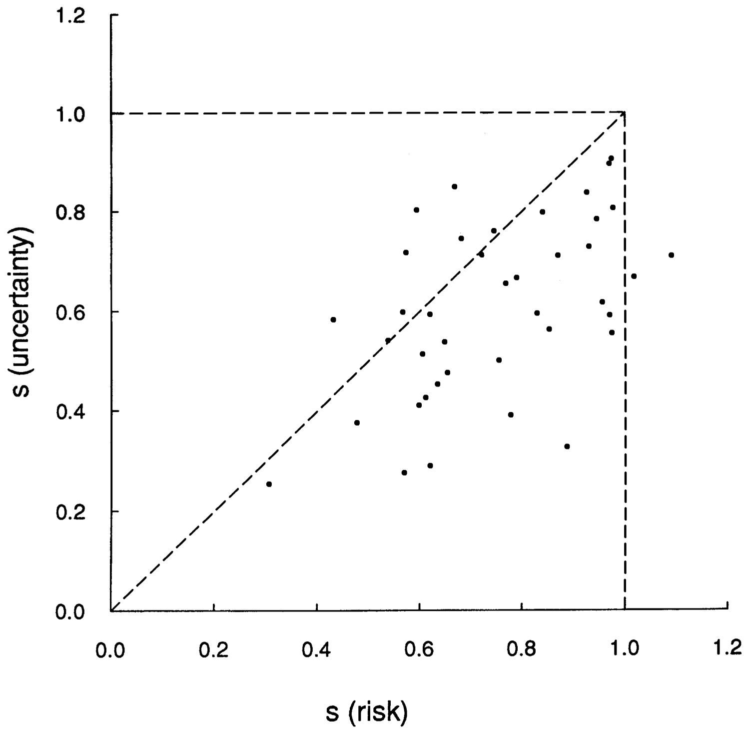

FIGURE 5.—Joint distribution for all subjects in study 2 of the sensitivity measure s for risk and uncertainty.

than 1, as implied by SA. Second, the sensitivity measure s is significantly smaller for uncertainty (mean = .61) than for risk (mean = .75); 32 out of 40 points lie below the identity line. Third, there is a significant correlation ($r = .53$) between the sensitivity measure for risk and for uncertainty, suggesting that the degree of SA (as measured by s) is an important attribute that distinguishes among decision makers.

In addition to the indirect comparisons in terms of s described above, Tversky and Fox also tested the preference conditions for comparative SS. The ordinal analysis confirmed the previous conclusion: subjects exhibited less SS for all five uncertain sources than for chance in the sense that (7.3) and (7.4) were satisfied significantly more often in the predicted than in the opposite direction.

Finally, the evidence indicated that some sources were preferred to risk. For example, Stanford students (who lived near San Francisco) preferred to bet on San Francisco temperature than on risk, but they preferred to bet on risk than on Beijing temperature.

In summary, it appears that the characteristics of the weighting function, which have been observed in studies of risk, tend to hold for uncertainty as well. Hence SA emerges as an important descriptive principle for decision under both risk and uncertainty. Furthermore, the finding that people are less sensitive to uncertainty than to risk indicates that uncertainty enhances the departures from expected utility. Studies of choice under risk therefore provide a lower bound for the departure from expected utility caused by nonadditive weights. Finally, the observation that people often prefer to bet on unknown rather than known probabilities calls for a reassessment of the conclusion commonly drawn from Ellsberg's example. It appears that people prefer risk to uncertainty when they are made to feel ignorant or incompetent. However, in other situations people often prefer betting on an uncertain source (e.g., sports or weather) than on risk (Heath and Tversky (1991)). A comprehensive analysis of the causes and consequences of source dependence awaits further theoretical and experimental research.

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APPENDIX A: PROOFS

PROOF OF PROPOSITION 4.1: That the preference conditions are implied by the corresponding properties for weighting functions, was already explained in the text. Next we assume that (4.4) holds and derive (4.1). Let p, q be such that $p + q \leq 1 - \varepsilon$.

Case 1: $p = 0$. Then (4.1) is trivially satisfied.

Case 2: $p > 0$ and $p + q < 1$. Then, by continuity of v, outcomes $Z > z > 0$ can always be found to give the antecedent in (4.4). For any such outcomes, the CPT difference between the left prospects in (4.4) is $w(q)(v(Z) - v(z))$, the CPT difference between the right prospects is $(w(p+q) - w(p))(v(Z) - v(z))$. By positivity of $v(Z) - v(z)$, the consequent preference in (4.4) holds iff $w(q) \geq w(p+q) - w(p)$. Thus (4.1) follows.

Case 3: $p > 0$, $p + q = 1$. (Note that $p + q = 1$ can only occur if $\varepsilon = 0$.) Define $p_j = p - 1/j$ for j so large that $p_j > 0$. By Case 2, $w(q) \geq w(p_j + q) - w(p_j)$ for all such p_j; obviously, $p_j + q \leq 1 - \varepsilon$. By continuity of w, (4.1) follows for p.

Before turning to the implication (4.3) $\Rightarrow$ (4.2), let us briefly comment on the duality between (4.3) and (4.4). It can be seen that (4.4), when formulated in a perfectly dual manner, would lead to the preference condition $z \sim (0, 1-p-q; z, q; Z, p) \Rightarrow (z, 1-q) \preccurlyeq (Z, p)$ (under appropriate boundary condition). This condition could have been used instead of (4.3) to characterize upper SA of w. We think, however, that the slightly different condition in (4.3) (logically equivalent under Assumption 3.1) is more transparent. Therefore the proof below is not a complete dual to the proof of the implication (4.4) $\Rightarrow$ (4.1).

Let us now assume that (4.3) holds and derive (4.2). Let p, q be such that $p \geq \varepsilon'$.

Case 1: $p = 1 - q$. Then (4.2) is trivially satisfied.

Case 2: $0 < p < 1 - q$. Here outcomes $Z > z > 0$ can always be found to give the antecedent in (4.3). For any such outcomes, the CPT difference between the left prospects in (4.3) is $(1 - w(1-q))v(z)$, the CPT difference between the right prospects is $(w(p+q) - w(p))v(z)$. By positivity of $v(z)$, the consequent preference in (4.3) holds iff $1 - w(1-q) \geq w(p+q) - w(p)$; (4.2) follows.

Case 3: $0 = p < 1 - q$. (Note that $p = 0$ can only occur if $\varepsilon' = 0$.) Define $p_j = 1/j$ for j so large that $p_j < 1 - q$. By Case 2, $1 - w(1-q) \geq w(p_j + q) - w(p_j)$ for all such p_j. By continuity of w, (4.2) follows for $p = 0$. *Q.E.D.*

PROOF OF PROPOSITION 5.1: That the preference conditions are implied by the corresponding properties for weighting functions, was already explained in the text. Next we assume that (5.3) holds and derive (5.1). Suppose that $W(A \cup B) \leq W(S-E)$, i.e., $A \cup B \preccurlyeq S-E$. We may assume that A, B are disjoint (the general case follows by substituting $A-B$ for A).

Case 1: $W(A) > 0$ and $W(A \cup B) < 1$. Then outcomes $Z > z > 0$ can always be found to give the antecedent in (5.3). For any such outcomes, the CPT difference betwen the left prospects in (5.3) is $W(B)(v(Z)-v(z))$, the CPT difference between the right prospects is $(W(A \cup B) - W(A))(v(Z) - v(z))$. By positivity of $v(Z)-v(z)$, the consequent preference in (5.3) holds iff $W(B) \geq W(A \cup B) - W(A)$. (5.1) follows.

Case 2: $W(A) = 0$ and $W(A \cup B) = 1$. (Note that $W(A \cup B) = 1$ can only occur if $W(S-E) = 1$.) In this case the antecedent in (5.3) trivially holds, and the consequent preference in (5.3) implies $W(B) = 1$. This implies (5.1).

Case 3: $W(A) = 0$ and $W(A \cup B) < 1$. This case is derived from Case 1 by a limiting argument and solvability of W. If $W(A \cup B) = 0$ then (5.1) is immediate; therefore we assume $W(A \cup B) > 0$. By solvability there exist, for all n sufficiently large, events A_n such that $A \subset A_n \subset A \cup B$ and $W(A_n) = 1/n$; define $B_n = B - A_n$. By Case 1, $W(B_n) \geq W(A_n \cup B_n) - W(A_n)$; note here that the boundary condition is satisfied for $A_n \cup B_n (= A \cup B)$. Since $W(B) \geq W(B_n)$, it follows that $W(B) \geq W(A \cup B) - W(A_n)$. In the limit (5.1) follows.

Case 4: $W(A) > 0$ and $W(A \cup B) = 1$. If $W(A) = 1$ then (5.1) trivially holds; assume therefore $W(A) < 1$. By solvability, for all n sufficiently large there exist $B_n \subset B$ such that $W(A) < W(A \cup B_n) = 1 - 1/n$. By Case 1, $W(B_n) \geq W(A \cup B_n) - W(A)$; note that the boundary condition is satisfied for $A \cup B_n$. Because $W(B) \geq W(B_n)$, $W(B) \geq W(A \cup B_n) - W(A)$. In the limit, $W(B) \geq W(A \cup B) - W(A)$ follows, i.e., (5.1) holds.

Finally, we assume that (5.4) holds and derive (5.2). Suppose that $W(A) \geq W(E')$, i.e., $A \succcurlyeq E'$. We may assume that A, B are disjoint (the general case follows by substituting $B-A$ for B). Note for the cases below that, by monotonicity of weighting functions, always $W(S-B) \geq W(A)$.

Case 1: $W(S-B) = W(A)$. Then (5.2) trivially holds.

Case 2: $W(S-B) > W(A) > 0$. Then outcomes $Z > z > 0$ can always be found to give the antecedent in (5.4). For any such outcomes, the CPT difference between the left prospects in (5.4) is $(1 - W(S-B))v(z)$; the CPT difference between the right prospects is $(W(A \cup B) - W(A))v(z)$. By positivity of $v(z)$, the consequent preference in (5.4) holds iff $1 - W(S-B)) \geq W(A \cup B) - W(A)$. (5.2) follows.

Case 3: $W(S-B) > W(A) = 0$. (Note that $W(A) = 0$ can only occur if $W(E') = 0$.) By solvability there exist, for all n sufficiently large, events A_n such that $A \subset A_n \subset S-B$ and $W(A_n) = 1/n$. Note that $W(A_n) \geq W(E') = 0$. By Case 2, $1 - W(S-B) \geq W(A_n \cup B) - W(A_n)$. Because $A \subset A_n$, $1 - W(S-B) \geq W(A \cup B) - W(A_n)$. In the limit $1 - W(S-B) \geq W(A \cup B) - W(A)$ follows, i.e., (5.2) holds. *Q.E.D.*

PROOF OF PROPOSITION 6.1: We show how this result can be derived from the similar result for uncertainty, i.e, Proposition 6.2 (the proof of which is given below). Our proof thus illustrates the close relation between risk and uncertainty, and does not take much space. The disadvantage is, of course, that this proof of Proposition 6.1 is not self-contained but relies on Proposition 6.2.

Proposition 6.1 follows from Proposition 6.2 by setting $S = [0,1]$, P is the Lebesgue measure, $W_1 = w_1 \circ P$, and $W_2 = w_2 \circ P$. The only aspect that needs further discussion concerns the boundary constants. It was pointed out in the text that w_2 being more SA than w_1 can be viewed as w_2 satisfying SA "when its arguments are measured in w_1 units." Similarly, the boundary constraints for the SA transformation ϕ such that $w_2 = \phi \circ w_1$ are simply the original boundary constraints, "measured in w_1 units." For an elaboration, denote by $\bar{\varepsilon}$ the boundary constant for condition (4.1), and by $\bar{\varepsilon}'$ the one for (4.2), for transformation ϕ. Then $p \geq \varepsilon'$, the boundary condition for (6.2) and

(6.4), corresponds with $w_1(p) \geq w_1(\varepsilon')(=W_1(E'))$ ("the values of w_1 serve as arguments of ϕ"), and we get:

(A1) $\quad \bar{\varepsilon}' = w_1(\varepsilon')$.

The boundary constraint for (6.1) and (6.3), i.e., $p + q \leq 1 - \varepsilon$, corresponds to $w_1(p+q) \leq w_1(1-\varepsilon)$ $(= W_1(S - E))$; therefore

(A2) $\quad 1 - \bar{\varepsilon} = w_1(1 - \varepsilon)$. *Q.E.D.*

PROOF OF PROPOSITION 6.2: In this proof we use the notation $\pi(B, A)$ for $W(A \cup B) - W(A)$, where it is implicitly assumed that A and B are disjoint; $\pi_1(B, A)$ and $\pi_2(B, A)$ are similar. Condition (6.7) is equivalent to $W_2 = \phi \circ W_1$ for a strictly increasing transformation ϕ, so that will be assumed from now on, and (6.7) is no more discussed. Note also that, because of solvability of both W_1 and W_2, the transformation ϕ is surjective, hence it must be continuous. Conditions (6.5) and (6.6) can be restricted to disjoint A, B, by replacing B by $B - A$; this will be assumed below without further mention.

To derive equivalence of (i) and (ii) in the proposition, we first show that condition (6.5) is equivalent to condition (4.1) for the transformation ϕ—the other conditions for ϕ, $\phi(0) = 0$, $\phi(1) = 1$, and continuity and strict increasingness, have already been established. Condition (4.1) means that $\phi(b) \geq \phi(a + b) - \phi(a)$ ("the value of the difference is at least as large as the difference of values") whenever $a + b \leq 1 - \bar{\varepsilon}$; here $\bar{\varepsilon}$ denotes the boundary constant for ϕ, and is related to the boundary event E in (6.5) by the equation

(A3) $\quad 1 - \bar{\varepsilon} = W_1(S - E)$,

as we shall see.

The proof is by substituting, along with (A3), $a + b = W_1(A \cup B)$ (for $A \cap B = \varnothing$), $a = W_1(A)$, $b = W_1(C)$; by continuity of ϕ and solvability of W_1, these substitutions can always be realized. The equality $W_1(C) = \pi_1(B, A)$ corresponds with $b = a + b - a$, the inequality $W_1(A \cup B) \leq W_1(S - E)$ corresponds with $a + b \leq 1 - \bar{\varepsilon}$, and $W_2(C) \geq \pi_2(B, A)$ corresponds with $\phi(b) \geq \phi(a + b) - \phi(a)$. These substitutions show that (4.1) for ϕ is equivalent to (6.5) for W_1, W_2. It is demonstrated in a dual manner that (6.6) for W_1, W_2 is equivalent to (4.2) for ϕ (now $W_1(S - C) = 1 - b$). The relation between the boundary event E' and the boundary constant $\bar{\varepsilon}'$ for the upper SA transformation ϕ is:[16]

(A4) $\quad \bar{\varepsilon}' = W_1(E')$.

Thus the equivalence (i) $\Leftrightarrow$ (ii) has been established.

Next we turn to the equivalence (ii) $\Leftrightarrow$ (iii). First we derive equivalence of (6.5) and (6.8). That (6.5) implies (6.8) follows from substitution of CPT. So we assume (6.8) and derive (6.5). Suppose $W_1(A \cup B) \leq W_1(S - E)$ and

$$W_1(C) = \pi_1(B, A).$$

We show that $W_2(C) \geq \pi_2(B, A)$.

Case 1: $W_1(A \cup B) < 1$. Because W_1 and W_2 order events the same way, $W_1(F \cup G) = W_1(G)$ if and only if $W_2(F \cup G) = W_2(G)$, so a decision weight $\pi_1(F, G) = 0$ if and only if $\pi_2(F, G) = 0$. This holds in particular for $G = \varnothing$, i.e., $W_1(F) = 0$ if and only if $W_2(F) = 0$. Hence, in view of the assumed $W_1(C) = \pi_1(B, A)$, either all of $W_1(C)$, $\pi_1(B, A)$, $W_2(C)$, $\pi_2(B, A)$ are 0, or none. If they are all 0, then $W_2(C) = \pi_2(B, A)$ and we are done. So assume, from now on:

$$W_1(C), \pi_1(B, A), W_2(C), \pi_2(B, A) \text{ are all positive.}$$

If $W_1(A) = 0$ then also $W_2(A) = 0$, further by (6.5) then $W_1(C) = W_1(A \cup B)$ which implies $W_2(C) = W_2(A \cup B)$; from these equalities the consequent inequality in (6.5) follows as an equality. Therefore assume also that

$$W_1(A) > 0.$$

[16] For this compatibility of boundary constraints it is essential that the boundary constants ε and ε' for upper and lower SA can be different, and that similarly the boundary events E and E' can be different.

Since also $W_1(A \cup B) < 1$, $0 < x < X$ can be found such that

$$x \sim_1 (0, S - (A \cup B); x, B; X, A).$$

Then also

$$(x, S - C; X, C) \sim_1 (X, A \cup B)$$

by the antecedent equality in (6.5), because the CPT difference between the left prospects is the same as between the right, i.e.,

$$W_1(C)(v_1(X) - v_1(x)) = (W_1(A \cup B) - W_1(A))(v_1(X) - v_1(x)).$$

Now $W_2(A) > 0$ and $W_2(A \cup B) < W_2(S) = 1$ follow from the corresponding inequalities for W_1; therefore $0 < y < Y$ can be found such that

$$y \sim_2 (0, S - (A \cup B); y, B; Y, A).$$

Condition (6.8) implies

$$(y, S - C; Y, C) \succcurlyeq_2 (Y, A \cup B).$$

Comparing the CPT difference of the left prospects to that of the right prospects, we get $W_2(C)(v_2(Y) - v_2(y)) \geq (W_2(A \cup B) - W_2(A))(v_2(Y) - v_2(y))$, i.e., $W_2(C) \geq W_2(A \cup B) - W_2(A)$; (6.5) has been demonstrated.

Case 2: $W_1(A \cup B) = 1$. If also $W_1(A) = 1$, then $W_2(A \cup B) = W_2(S) = 1 = W_2(A)$ follows, implying (6.5). Hence assume $W_1(A) < 1$. By solvability we can find, for n sufficiently large, events B_n such that $B_n \subset B$ and $W_1(A \cup B_n) = 1 - 1/n > W_1(A)$, and (recall that $W_1(C) = W_1(A \cup B) - W(A) > 0$) events $C_n \subset C$ such that $W_1(C_n) = W_1(C) - 1/n$. Then $W_1(C_n) = W_1(A \cup B_n) - W_1(A)$. By Case 1, $W_2(C_n) \geq W_2(A \cup B_n) - W_2(A)$. By continuity of ϕ, $W_2(C_n)$ tends to $W_2(C)$, and $W_2(A \cup B_n)$ tends to $W_2(A \cup B)$, hence the inequality $W_2(C) \geq W_2(A \cup B) - W_2(A)$ follows: (6.5) has been proved.

Next we turn to the equivalence (6.6) $\Leftrightarrow$ (6.9). That (6.6) implies (6.9) follows from substitution of CPT. So we assume (6.9) and derive (6.6). Suppose $W_1(A) \geq W_1(E')$ and

$$\pi_1(C, S - C) = \pi_1(B, A).$$

We show that $\pi_2(C, S - C) \geq \pi_2(B, A)$.

Case 1: $W_1(A) > 0$. Because W_1 and W_2 order events the same way, and $\pi_1(C, S - C) = \pi_1(B, A)$, either all of $\pi_1(C, S - C)$, $\pi_1(B, A)$, $\pi_2(C, S - C)$, $\pi_2(B, A)$ are 0, or none. If they are all 0, then $\pi_2(C, S - C) = \pi_2(B, A)$ and we are done. So suppose, from now on:

$$\pi_1(C, S - C), \pi_1(B, A), \pi_2(C, S - C), \pi_2(B, A) \text{ are all positive.}$$

By monotonicity of capacities, $W_1(S - C) \geq W_1(A)$. Now, by the antecedent equality in (6.6), $W_1(S - C) = W_1(A)$ if and only if $W_1(A \cup B) = 1 = W_1(S)$; because W_1 and W_2 order events the same way, similar equalities then hold for W_2, which implies the consequent inequality in (6.6). Therefore we assume that

$$W_1(S - C) > W_1(A).$$

Since also $W_1(A) > 0$, $0 < x < X$ can be found such that

$$(x, S - C) \sim_1 (X, A).$$

Then also

$$x \sim_1 (0, S - (A \cup B); x, B; X, A),$$

by the antecedent equality in (6.6), because the CPT difference between the left prospects is the same as between the right, i.e.,

$$[1 - W_1(S - C)]v_1(x) = [W_1(A \cup B) - W_1(A)]v_1(x).$$

The inequalities $W_2(S - C) > W_2(A) > 0$ follow from the corresponding inequalities for W_1; therefore $0 < y < Y$ can be found such that

$$(y, S - C) \sim_2 (Y, A).$$

Now (6.9) implies

$$y \succcurlyeq_2 (0, S-(A \cup B); y, B; Y, A).$$

This and the $\sim_2$ indifference imply, by taking CPT differences between left and right prospects and dividing by $v_2(y)$, that $1 - W_2(S-C) \geq W_2(A \cup B) - W_2(A)$. (6.6) has been demonstrated.

Case 2: $W_1(A) = 0$. If $W_1(A \cup B) = 0$, then $W_2(A \cup B) = 0$, and (6.6) follows. Suppose therefore that $W_1(A \cup B) > 0$. By solvability we can find, for n sufficiently large, A_n such that $A \subset A_n \subset A \cup B$ and $W_1(A_n) = 1/n$, and next (noting that $1 - W_1(S-C) = W_1(A \cup B) - W_1(A) > 0$, hence $W_1(S-C) < 1$) we can find $C_n \subset C$ such that $W_1(S - C_n) = W_1(S-C) + 1/n$. Setting $B_n = (A \cup B) - A_n$, we get $1 - W_1(S - C_n) = W_1(A_n \cup B_n) - W_1(A_n)$. By Case 1, $1 - W_2(S - C_n) \geq W_2(A_n \cup B_n) - W_2(A_n)$ follows, i.e., $1 - W_2(S - C_n) \geq W_2(A \cup B) - W_2(A_n)$. This inequality implies, by continuity of ϕ, $1 - W_2(S-C) \geq W_2(A \cup B) - W_2(A)$, and (6.6) has been established. *Q.E.D.*

APPENDIX B: REFLECTION

This section presents preference conditions for reflection under risk, i.e., $w^+ = w^-$.

PROPOSITION B1: *Reflection is satisfied if and only if* $\succcurlyeq$ *satisfies the following condition:*

(B1)
$$\text{If } x \sim (0, 1-p-q; x, q; X, p) \text{ and}$$
$$(x, 1-r; X, r) \sim (X, p+q), \text{ then}$$
$$-y \sim (-Y, p; -y, q; 0, 1-p-q) \text{ implies}$$
$$(-Y, r; -y, 1-r) \sim (-Y, p+q),$$

for all $-Y < -y < 0 < x < X$, *and* $p, q, r \in]0,1[$.

PROOF: First we demonstrate necessity of (B1). The first two indifferences in (B1) imply that $w^+(r)(v(X) - v(x)) = (w^+(p+q) - w^+(p))(v(X) - v(x))$, i.e., $w^+(r) = w^+(p+q) - w^+(p)$. By reflection then $w^-(r) = w^-(p+q) - w^-(p)$. This implies $w^-(r)(v(-y) - v(-Y)) = (w^-(p+q) - w^-(p))(v(-y) - v(-Y))$; the left-hand side is the CPT difference between the lower pair of prospects left of $\sim$, the right-hand side is the CPT difference between the lower pair of right prospects. Therefore the third indifference in (B1) implies the fourth. We conclude that reflection implies (B1).

Next we assume that (B1) holds, and derive reflection. Take $p_0, \ldots, p_n$ such that $w^+(p_j) = j/n$ for all j. Then take, for all $2 \leq j \leq n-1$, $X^j > x^j > 0$ such that

$$x^j \sim (0, 1-p_j; x^j, p_j - p_{j-1}; X^j, p_{j-1}).$$

The following indifference is obtained by substitution of CPT, where the CPT value of both prospects is increased by $(v(X^j) - v(x^j))/n$:

$$(x^j, 1-p_1; X^j, p_1) \sim (X^j, p_j).$$

Next take, for all $2 \leq j \leq n-1$, $-Y^j < -y^j < 0$ such that

$$-y^j \sim (-Y^j, p_{j-1}; -y^j, p_j - p_{j-1}; 0, 1-p_j).$$

It follows from the above three indifferences and (B1) that

$$(-Y^j, p_1; -y^j, 1-p_1) \sim (-Y^j, p_j).$$

The two loss-indifferences, and CPT, imply that $w^-(p_1) = w^-(p_j) - w^-(p_{j-1})$. As this holds for all $2 \leq j \leq n-1$, $w^-(p_j) - w^-(p_{j-1}) = w^-(p_1)$ for all such j. So, for a positive constant λ, $w^- = \lambda w^+$ on $\{p_1, \ldots, p_{n-1}\}$, which set is the inverse under w^+ of $\{1/n, \ldots, 1-1/n\}$. It follows that λ is independent of n (for $k \neq m$, compare to $i = k \cdot m$). As n tends to infinity it follows, firstly, that p_{n-1} tends to 1, next, that λ must be 1, then, finally, that the continuous strictly increasing functions w^+ and w^- must be identical. This establishes reflection. *Q.E.D.*

Because the weighting function is S-shaped, it is similar to its dual; hence the qualitative predictions of CPT and rank-dependent utility are not very different. The key difference is that reflection implies $w^{+}(1/2) < 1/2 \Leftrightarrow w^{-}(1/2) < 1/2$, whereas rank-dependent utility implies $w^{+}(1/2) < 1/2 \Leftrightarrow w^{-}(1/2) > 1/2$. In the study of Tversky and Kahneman (1992), the estimated weighting functions for all 25 subjects satisfied the inequalities $w^{+}(1/2) < 1/2$ and $w^{-}(1/2) < 1/2$, in accord with CPT.

Much less is known about the relation between the weighting functions for gains and losses in the context of uncertainty. We suspect that the close correspondence observed in the domain of risk may not always hold for uncertainty.

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