Risk Aversion and Expected Utility Theory: Coherence for Small- and Large-Stakes Gambles

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There is a sizable literature reporting the conclusion that expected utility theory cannot provide a coherent positive theory of risk averse behavior. Recently-published articles have sharpened the criticism with calibrations based on assumed concavity and additivity of initial wealth to income. We demonstrate that the negative conclusions in this literature are valid for only one expected utility model not, as claimed, for expected utility theory. The conclusions are not valid for the expected utility model commonly used in bidding theory nor for a more general expected utility model that we develop by extending the Arrow-Pratt characterization of agents' comparative risk aversion. Clarification of the distinction between expected utility theory and expected utility models also makes it clear that loss aversion is consistent with expected utility theory.

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1. Introduction

A central objective in developing an empirical science is coherence in the application of theory to data from different sources. Thus, Rabin (2000a, 2000b) and Rabin and Thaler (2001) address an important question concerning coherence of the application of concave expected utility theory to explain risk-averse behavior for both small-stakes gambles used in laboratory experiments and large-stakes gambles observed in everyday life. Although Rabin (2000b) singles out the work of experimental economists for special criticism, his "calibration theorem" focuses one's attention on the central issue in a sizable literature that questions the usefulness of expected utility theory for modeling risk-averse behavior.[1]

This criticism has been quite influential, in part because strongly-worded conclusions have been repeated so many times. Thus, Rabin (2000a) tells readers that: "Within the expected-utility framework, the concavity of the utility-of-wealth function is not only sufficient to explain risk aversion – it is also necessary: Diminishing marginal utility of wealth is the *sole* explanation for risk aversion …" (p. 202) Rabin also states that: "The inability of expected utility theory to provide a plausible account of risk aversion over modest stakes has been illustrated in writing in a variety of different contexts …" (pp. 202-203) He continues by stating: "Within the expected-utility framework, for *any* concave utility function, even very little risk aversion over modest stakes implies an absurd degree of risk aversion over large stakes." (p. 203)

Rabin (2000b) proves a calibration theorem and corollary for a concave expected-utility model that he interprets as bringing the following to one's attention: "…this theory implies that people are approximately risk neutral when stakes are small…While not broadly appreciated, the inability of expected-utility theory to provide a plausible account of risk aversion over modest stakes has become oral tradition among some subsets of researchers…" (p. 1281) In addition to proof of the theorem, Rabin discusses at some length its purported implications for empirical methods in economics. He is particularly critical of the use by experimental economists of concave expected-utility theory to explain systematic, one-sided deviations from the predictions of risk-neutral models because of his belief that "…expected-utility theory is manifestly not close to the right explanation of risk attitudes over modest stakes…" (p. 1282) and "…if we think that subjects in experiments *are* risk averse, then we know they are not expected-utility maximizers." (p. 1286) He states that the right explanation is loss aversion, which is said to be inconsistent with expected-utility theory: "…*loss aversion*, is a departure from expected-utility theory that provides a direct explanation for modest scale risk aversion." (p. 1288)

The conclusions about expected-utility theory are stated most forcefully by Rabin and Thaler (2001), as follows: "We are arguing that when people decline gambles with positive expected value for modest stakes, they are violating expected utility theory." (p. 227) They conclude their paper as follows: "…it is time for economists to recognize that expected utility theory is an ex-hypothesis, so that we can concentrate our energies on the important task of developing better descriptive models of choice under uncertainty." (p. 230) This conclusion is immediately preceded by the following: "What should expected utility theory be replaced with? We think it is clear that loss aversion and the tendency to isolate each risky choice must both be key components of a good descriptive theory of risk attitudes." (p. 230)

These criticisms of expected utility theory, and the related discussions of loss aversion, involve some issues for which there is a marked absence of clarity in the literature on decision theory. In order to help clarify the fundamental differences between expected utility theory and alternative decision theories, we draw attention to the distinction between expected utility *theory* and expected utility *models*. We address the coherence question for both of the commonly-used expected utility models, the expected utility of terminal wealth model and the expected utility of income model, and for a more general model that we develop, the expected utility of income and initial wealth model.

We explain that the calibration theorem has no implication for the expected utility of income (EUI) model because of its fixed reference point of zero income. The expected utility of income and initial wealth (EUI&IW) model includes initial wealth as a reference point but the calibration theorem has no implication for this model because initial wealth is not additive to income in the utility function for this model. In contrast, if there is empirical support for the pattern of small-stakes risk aversion hypothesized in Rabin (2000a, 2000b) and Rabin and Thaler (2001), then the calibration theorem has implications for applicability of the expected utility of terminal wealth (EUTW) model.

Since modeling risk aversion with the EUTW model is called into question by Rabin's calibration theorem, and since the calibration theorem has no implication for the EUI&IW model, it is important to ascertain whether central analytical results in the economics of uncertainty that have been derived with the EUTW model have analogues that can be derived with the EUI&IW model. We begin the study of this question by extending the Arrow (1971) and Pratt (1964) characterization of comparative risk aversion to the EUI&IW model. Subsequently, we use the EUI&IW model to derive an analogue of Arrow's (1971) classic two-asset portfolio allocation proposition.

Clarification of the distinction between expected utility theory and specific expected utility models also helps to clarify the implications of loss aversion (Kahneman and Tversky,

1979) for decision theory. We explain that loss aversion is consistent with some expected utility models, hence it is consistent with expected utility theory. Thus, if a researcher observes loss-averse behavior, he should not, *ipso facto*, be motivated to reject expected utility theory in favor of some alternative such as prospect theory (Kahneman and Tversky, 1979), although loss aversion is inconsistent with the expected utility of terminal wealth model.

We mention a small part of the literature that reports data showing risk-averse behavior in laboratory experiments. The common features of these experiments are: (1) consistent one-sided deviations of subjects' choices from the predictions of risk neutrality, in the direction consistent with risk aversion; (2) no relevance of loss aversion; and (3) researchers' use of expected utility models for which the calibration theorem has no implication. Clarification of these issues is important to decision theory and to proper review for journals of papers reporting empirical applications of expected utility theory.[2]

2. Expected Utility Theory vs. Expected Utility Models

We define "expected utility theory" as the theory of decision-making under risk based on a set of axioms for a preference ordering that includes the independence axiom or an alternative that implies that the (expected) utility function that represents the ordering is linear in probabilities. Linearity in probabilities implies that indifference curves in the Machina (1982) triangle diagram and the probability simplex are parallel straight lines. Thus, we include within expected utility theory any model of decision-making under risk that has parallel straight-line indifference curves in the Machina triangle diagram.

Expected utility theory and alternative decision theories are concerned with the properties of preference orderings of probability distributions of "prizes." The identity of the prizes depends on the decision context that is modeled and on assumptions made by the theorist. We shall confine our discussion to decision contexts in which the prizes are amounts of money. Within this context, we identify two expected utility models that are commonly used, the expected utility of terminal wealth model and the expected utility of income model. We also discuss a third model, the expected utility of income and initial wealth model. The distinctions among the models are in the assumed identity of the prizes. All of the models have parallel straight-line indifference curves in the Machina triangle, hence are included within expected utility theory.

2.1 The EUTW Model

The expected utility of terminal wealth (EUTW) model is based on the assumption that the prizes are amounts of terminal wealth. This model was used in the seminal work of Arrow (1971) and Pratt (1964) in which they developed the measures for comparing agents' risk attitudes. This model is commonly used in theoretical and empirical papers on various topics.

It will help explain some essential distinctions between models to briefly review the familiar triangle-diagram representation of indifference curves for simple gambles (Machina, 1987). Consider three amounts of monetary gains (or amounts of income), y_i, i = 1,2,3 such that $y_1 < y_2 < y_3$. Assume that the y_i occur with probabilities, p_i, i = 1,2,3 such that $p_1 + p_2 + p_3 = 1$. If w is the decision-maker's initial wealth, then the expected utility function for the EUTW model is written as

(1) $$U_W = \sum_{i=1}^{3} p_i u(w + y_i) .$$

Since $p_2 = 1 - p_1 - p_3$, indifference curves for expected utility function (1) can be represented in the triangle diagram in Figure 1. Indifference curves are parallel straight lines because expected utility function (1) is linear in probabilities. Higher (respectively lower) Arrow-Pratt absolute risk aversion implies steeper (respectively flatter) slope of the indifference curves. Thus if expected utility function (1) exhibits non-constant absolute risk aversion then the slopes of the indifference curves in Figure 1 depend on the amount of initial wealth, w. In the special cases in which expected utility function (1) exhibits constant absolute risk aversion or the agent is risk neutral, the slope of the indifference curves in Figure 1 is invariant to changes in w.

2.2 The EUI Model

The expected utility of income (EUI) model is based on the assumption that the prizes are amounts of income (or, equivalently, changes in wealth or gains and loses). The EUI model is commonly used in theoretical modeling, most notably in the theory of auctions. Thus, most bidding models are based on the expected utility axioms, the assumption that the prizes are amounts of income, and the assumption of Bayesian-Nash equilibrium.[3]

Considering the same three-outcome lottery as above, the expected utility function for the EUI model is written as

(2) $$U_I = \sum_{i=1}^{3} p_i v(y_i) .$$

Obviously, indifference curves for this model are parallel straight lines whose slope is independent of initial wealth.

2.3 The EUI&IW Model

We here discuss a more general model, the expected utility of income and initial wealth (EUI&IW) model. Assume that the prizes are ordered pairs of amounts of initial wealth and income, (w, y_i), $i = 1,2,3$. Then the expected utility function can be written as

(3) $$U_{IW} = \sum_{i=1}^{3} p_i \upsilon(w, y_i) .$$

Indifference curves for this model are parallel straight lines; therefore it is an expected utility model. Because it appears that the properties of the EUI&IW model have not previously been developed, we extend the Arrow-Pratt theory of comparative risk aversion to this model, as follows.

Let F denote the probability distribution function for the random amount of income, Y. The expected utility function for agent $j = \alpha, \beta$ can be written as

(4) $$U_{IW}^{j} = E_F(\upsilon^j(w,y))$$

for either discrete or continuous distributions of random income. The Arrow-Pratt measure of absolute risk aversion for this model is

(5) $$A^j(w,y) = -\frac{\upsilon_{22}^j(w,y)}{\upsilon_2^j(w,y)}.$$

The risk premium, π^j is defined by

(6) $$\upsilon^j(w, E_F(y) - \pi^j(w,F)) = E_F(\upsilon^j(w,y)).$$

Assume that the function, υ^j is strictly increasing in income, y; then there exists a y-inverse function, ϕ^j, defined by

(7) $$y = \phi^j(w, \upsilon^j(w,y)).$$

Define the function

(8) $$g(w,u) = \upsilon^\alpha(w, \phi^\beta(w,u)).$$

The Arrow-Pratt measures of comparative risk attitudes for agents α and β are as given in the following proposition, which states that: (a) the absolute risk aversion measure for agent α is greater than the absolute risk aversion measure for agent β, if and only if, (b) the risk premium for agent α is greater than the risk premium for agent β, if and only if, (c) the (Bernoulli) utility function for agent α is a strictly increasing and strictly concave transformation of the utility function of agent β of the form given by definition (8) above and statement (c) in the proposition. A version of this proposition that applies to the EUI model is derived simply by specifying that υ^j, and hence all related functions, are constant functions of w. A proof of the proposition is contained in the appendix.

Proposition 1. *If υ^{a} and υ^{β} are strictly increasing in y and twice differentiable then the following statements are equivalent:*

(a) $A^{\alpha}(w,y) > A^{\beta}(w,y)$, *for all* (w,y)*;*

(b) $\pi^{\alpha}(w,F) > \pi^{\beta}(w,F)$, *for all w and* F *;*

(c) $\upsilon^{\alpha}(w,y) = g(w,\upsilon^{\beta}(w,y))$, $g_{2}(w,u) > 0$, $g_{22}(w,u) < 0$, *for all* (w,u).

Proposition 1 makes clear that the Arrow-Pratt characterization of agents' comparative risk aversion can be extended from the EUTW model to the EUI&IW model (and to its special case, the EUI model). Therefore, agents' risk-avoiding behavior can be modeled with the EUI&IW model (or the EUI model) rather than the EUTW model. This is important because the EUTW model is, and the EUI&IW and EUI models are not vulnerable to the Rabin-Thaler critique, as we shall demonstrate in section 3. Before proceeding to the analysis in section 3, we shall compare the properties of the three expected utility models by re-examining one of the original applications by Arrow (1971), the simple two-asset portfolio allocation problem.

2.4 Implications of the Three Models for Portfolio Allocation

Consider the simple portfolio allocation problem involving two assets, one with a risky return and the other with a sure return. Let a denote the amount invested in the risky asset. Let the random variable, *R* be the rate of return per dollar invested in the risky asset. Arrow (1971, Chap. 3) proves the following proposition that relates the optimal amount invested in the risky asset to the absolute risk aversion measure ($-u''/u'$) for the EUTW model.

Proposition 2.a (EUTW model). *The amount,* a *invested in the risky asset increases, does not change, or decreases with initial wealth as* $-d\ell n(u'(w+ar))/dw = -u''(w+ar)/u'(w+ar)$ *decreases, does not change, or increases with the amount of income, ar.*

It might not be immediately obvious that the above is a correct statement of the proposition proved by Arrow. But note that, within the proof of the proposition (Arrow, 1971, Chap.3), *w* is constant and *ar* is variable; hence it is the monotonicity of $-u''(w+ar)/u'(w+ar)$ with respect to *ar* that is used in the proof. (The relevance of this point is that, in the context of the EUTW model there is no distinction between monotonicity in w and monotonicity in *ar*, whereas in the context of the EUI&IW model there is such a distinction.)

The analogous proposition for the EUI model is obvious from inspection of the model. Thus we have:

Proposition 2.b (EUI model). *The amount invested in the risky asset does not vary with initial wealth.*

This implication of the EUI model, that the wealth elasticity of investment in the risky asset is identically zero, seems to be implausible. Prospect theory (Kahneman and Tversky, 1979) has the same implausible implication. This suggests the importance of developing models, such as the EUI&IW model, that have more plausible implications for the wealth-elasticity of risk-taking behavior.

The appendix contains a proof of the following proposition for the EUI&IW model.

Proposition 2.c (EUI&IW model). *The amount invested in the risky asset increases, does not change, or decreases with initial wealth as* $-\partial\ell n(\upsilon_2(w,ar))/\partial w = -\upsilon_{21}(w,ar)/\upsilon_2(w,ar)$ *decreases, does not change, or increases with the amount of income, ar.*

Note that both proposition 2.a and proposition 2.c relate the amount invested in the risky asset to whether the initial-wealth elasticity of the marginal utility of income is decreasing, constant, or increasing in income.[4] The measure of curvature that determines whether investment in the risky asset is increasing or decreasing with initial wealth is the measure,

$$(9) \qquad C(w,y) = -\upsilon_{21}(w,y)/\upsilon_2(w,y).$$

The essential difference between the models comes from whether or not this measure of curvature is distinct from the absolute risk aversion measure.

3. Coherence for Small- and Large-Stakes Gambles

Rabin (2000a, 2000b) and Rabin and Thaler (2001) argue that concave expected utility theory does not provide a coherent theory of both small-stakes and large-stakes risk aversion. Their argument is presented in the form of calibrations based on the theorem and corollary proved in Rabin (2000b). For example, Rabin and Thaler (2001, Table 1) report that if an agent will reject an opportunity to play a 50-50 bet with a loss amount of \$100 and a win amount of \$110, at all initial wealth levels, then that agent will reject the opportunity to play a 50-50 bet with a loss amount of \$1,000 and an arbitrarily large win amount (written as "$\$\infty$").

Part of our argument will be presented in the form of alternative calibrations. We first discuss coherence for the EUI model that is widely applied in bidding theory. Subsequently, we discuss the EUI&IW model and the EUTW model.

3.1 Coherence with the EUI Model

The use of income as the argument of the expected utility function implies a fixed reference point of zero income, which makes it impossible to construct a calibration-theorem argument. Rather than reviewing the construction of the theorem, we will provide an example which demonstrates that the expected utility of income model can provide a coherent explanation of both small-stakes and large-stakes risk aversion. Our example constitutes a counterexample to the belief that Rabin and Thaler's conclusions about incoherence of small-stakes and large-stakes risk aversion hold for the EUI model.

Consider the example used by Rabin (2000b) in which it is assumed that a decision-maker would reject a gamble that yields a gain of $110 and a loss of $100 with equal probability. We will compare implied levels of risk aversion for the EUI model with the levels given in Rabin's Tables I and II for the EUTW model. The second and third columns of our Table I reproduce some results reported in Rabin's Table I and Table II. The "Table I Rejections" we reproduce are implied by the corollary to the calibration theorem if the agent is assumed to reject the 50-50, lose $100/ gain $110 gamble for *all* initial wealth levels. Such an agent would also reject a gamble in which he would, with equal probability, lose $1000 or gain any arbitrarily large amount. The "Table II Rejections" we reproduce are derived from the assumption that the agent will reject the 50-50, lose $100/ gain $110 gamble for all initial wealth levels no greater than $300,000. In that case, the agent would also reject a gamble in which he would, with equal probability, lose $20,000 or gain $160,000,000,000 when his initial wealth is $290,000.

Now consider risk aversion with the EUI model. Assume the von Neumann-Morgenstern expected utility function that "values" amounts of income, y according to:

(10) $$v(y) = \frac{200+y}{20} + 5 - \sqrt{200}, \text{ if } y < -100,$$

$$= \sqrt{200+y} - \sqrt{200}, \text{ if } -100 \leq y \leq 100,$$

$$= \frac{200+y}{2\sqrt{300}} + \frac{\sqrt{300}}{2} - \sqrt{200}, \text{ if } y > 100.$$

This function is globally weakly concave, as assumed for the utility of terminal wealth functions in Rabin's theorem and corollary. Using (10), one finds that an expected utility maximizer would reject the 50-50, lose $100/ gain $110 gamble. But the agent would also *accept* the gambles in the "Equation (10) Acceptance" column of Table 1. Thus, risk aversion over small-stakes gambles does not imply ridiculous levels of risk aversion over large-stakes gambles with the EUI model.

Another example is provided by the function,

(11) $v(y) = (y+a)^{0.9} - d$, if $-a \le y < -10$,

$= \sqrt{20+y} - \sqrt{20}$, if $-10 \le y \le 10$,

$= (y+b)^{0.9} - c$, if $y > 10$,

where: a = 35,704,682.29; d = 6,272,672.839; b = 8,676,235,339; c = 880,031,446.3. This function is *strictly* concave on the domain that is bounded from below by $-a$. Using (11), one finds that an expected utility maximizer would reject the 50-50, lose \$100/ gain \$110 gamble. But the agent would also *accept* the gambles in the "Equation (11) Acceptance" column of Table 1. Thus, once again we observe that risk aversion over small-stakes gambles is consistent with sensible levels of risk aversion over large-stakes gambles with the EUI model.

3.2 Coherence with the EUI&IW Model

Unlike the EUI model, an agent's preferences over risky probability distributions of income can depend on initial wealth in the EUI&IW model. But this does not imply incoherence of small- and large-stakes risk aversion for the EUI&IW model, as shown by the following two examples.

First consider an additive von Neumann-Morgenstern expected utility function that "values" amounts of income, y and initial wealth, w according to

(12) $\upsilon^a(w,y) = \varphi(w) + v(y)$,

where $\varphi(w)$ and $v(y)$ are increasing concave functions of their arguments, and $\varphi''(w)v''(y) \ge (\varphi'(w)v'(y))^2$. It is clear that a decision-maker with such a utility function will reject the 50-50, lose \$100/ gain \$110 gamble for all initial wealth levels, w if 0.5 $v(110)$ +

$0.5\,v(-100) < v(0)$. Assuming that v(y) is given by equation (10), one finds that the decision-maker would *reject* the 50-50, lose \$100/ gain \$110 gamble *for all* initial wealth levels, w. But the agent would also *accept* the gambles in the "Equation (10) Acceptance" column of Table 1, for *all* initial wealth levels, w.

Next consider a multiplicative function that values amounts of income, y and initial wealth, w according to

$$(13) \qquad \upsilon^{m}(w, y) = \varphi^{m}(w)v(y),$$

where $\varphi^{m}(w) > 0$ and $v(y)$ are concave functions of their arguments. Again assuming that v(y) is given by equation (10), one finds that the decision-maker would *reject* the 50-50, lose \$100/ gain \$110 gamble *for all* initial wealth levels, w. Once again, the agent would also *accept* the gambles in the "Equation (10) Acceptance" column of Table 1, *for all* initial wealth levels, w. Thus, risk aversion over small-stakes gambles does not imply ridiculous levels of risk aversion over large-stakes gambles with the EUI&IW Model.

3.3 Incoherence with the EUTW Model

Rabin's (2000a, 2000b) and Rabin and Thaler's (2001) argument is based on the implicit assumption that the "prizes" that are ordered by the preference ordering defined by the expected utility axioms are amounts of terminal wealth. The argument is a logical proposition of the form: *If* an agent rejects a specified small-stakes gamble for all of the assumed values of initial wealth *then* the agent would also reject the following large-stakes gambles. Obviously, the consequent, *then* statement does not follow unless the antecedent, *if* statement is assumed.

Use of the logical proposition to criticize application of models to analysis of data transforms the antecedent of a purely logical proposition into an empirical hypothesis requiring support. Reflection on the type of risk averse behavior that is hypothesized suggests the type of data that are needed to support a conclusion of incoherence. Obviously, it would be impossible to conduct money payoff experiments with subjects whose actual personal wealth varied over the whole real line. Therefore, it is impossible that empirical support for the assumptions in Rabin's (2000a) Table 11.1 or Rabin's (2000b) Table I or Rabin and Thaler's (2001) Table 1 calibrations could ever be provided. Furthermore, it would be virtually impossible to conduct experiments with individual subjects whose actual personal wealth took on all values up to $300,000. In contrast, one could recruit subjects from a subject pool consisting of distinct individuals with wealth levels varying over a range large enough to make use of the logic of the calibration theorem. In that case, the data would confound heterogeneity in subjects' risk attitudes with variability in their individual wealth levels. But such an across-subjects experimental design would be a reasonable approach to providing empirical support for the type of small-stakes risk aversion assumed in Rabin's (2000b) Table II calibration. Furthermore, it is a reasonable conjecture that the (typically-unobserved) variability in wealth across subjects in earlier experiments on risk attitudes may have sufficient variability for this purpose. Therefore, Rabin and Thaler's argument does make questionable the coherence of applications of the EUTW model to small-stakes and large-stakes gambles.

4. Loss Aversion and Expected Utility Theory

We have explained that observations of risk-averse behavior in laboratory experiments can be rationalized by some concave expected utility models without producing incoherence with applications of those models to larger-stakes gambles. But this explanation should not be misinterpreted as a denial of other possible causes of risk-avoiding behavior. Loss aversion is a possible explanation of such behavior in some circumstances. Loss aversion is conventionally represented by a figure, such as Figure 2, showing a payoff valuation function that is strictly concave for gains, strictly convex for losses, and steeper in the loss domain. But the relation between loss aversion and expected utility theory requires some careful re-examination within the context of the issues raised by Rabin and Thaler.

Observing that expected utility theory contains more than the EUTW model helps one to understand the essential differences between that theory and its alternatives. Some researchers, such as Kahneman and Tversky (1979), Rabin (2000b), and Rabin and Thaler (2001), believe that loss aversion distinguishes expected utility theory from alternatives such as prospect theory. But loss aversion is consistent with the EUI and EUI&IW models, hence it is consistent with expected utility theory. Empirical tests that can discriminate on a fundamental level between expected utility theory and its alternatives must be based on differences between the implications of alternative sets of axioms, not different subsidiary assumptions about what the prizes are to which the axioms are applied. Thus, observations of Allais paradox behavior are empirical inconsistencies with expected utility theory because they are inconsistent with the Machina-triangle indifference curves being parallel straight lines (Machina, 1982, 1987). In contrast, observations of loss-averse behavior are consistent with the indifference curves being parallel straight lines, hence they are consistent with expected utility theory.

Explicit integration of loss aversion into expected utility theory has just begun to produce interesting extensions of decision theory. Thus, Nielson (2002) develops an analogue of the characterization of comparative risk aversion ("more risk averse than") stated as one agent's preferences being "more S-shaped than" another agent's preferences over probability distributions of payoffs.

5. Lottery Payoffs in Experiments

Rabin and Thaler (2001, p. 224) criticize the use of lottery payoffs in experiments: "This lottery procedure either isn't necessary, or doesn't work. If subjects are expected utility maximizers then the procedure is unnecessary, since expected utility theory tells us that people will be virtually risk neutral in decisions on the scale of laboratory stakes. If subjects are not expected utility maximizers, then the procedure cannot be relied upon to work, since subjects may not have preferences that are linear in probabilities." Similar assertions are contained in Rabin (2000a) and Rabin (2000b).

Lottery payoffs were introduced into decision theory by Smith (1961). They were used in experiments first by Roth and Malouf (1979) and, subsequently, by Roth and Murnigham (1982), Roth and Schoumaker (1983), and many others. Neither Smith's theoretical argument nor most applications in experiments requires use of the EUTW model rather than the EUI model or the EUI&IW model. Therefore, Rabin and Thaler's argument has no general implication for applicability of lottery payoffs in experiments.

The problem with lottery payoffs does not derive from the calibration theorem but rather from experimental studies that conclude that they do not successfully induce risk attitudes on subjects. Thus, Walker, Smith, and Cox (1990) and Cox and Oaxaca (1995) report that lottery payoffs do not successfully control subjects' risk attitudes in bidding (market) experiments. Selten, Sadrieh, and Abbink (1999) report that lottery payoffs fail to induce risk neutral behavior in non-market experiments. Berg, Dickhaut, and Rietz (2001) report somewhat greater success from using lottery payoffs to induce risk-averse and risk-loving preferences in preference reversal experiments.

It is clear that the empirical failure of lottery payoffs is a failure of expected utility theory. But alternative decision theories have not been subjected to similar tests with types of lottery payoffs that are appropriate in the absence of linearity in probabilities, hence it is unknown whether any of them would fare any better.

6. Exogenous Risky Income

Subjects in experiments may have risky incomes that are exogenous to experimental treatments. Any such risky income would not ordinarily be observed by experimenters. The existence of exogenous risky income raises some questions about rational behavior, and the modeling of rational behavior, that are well beyond the scope of the present paper. These questions revolve around the implications of integration, or non-integration, of risky incomes from different decision contexts. But it is germane to discussion of the issues we are examining to ask whether integration of exogenous large-stakes risks with endogenous small-stakes risks has implications for applicability of concave expected utility theory.

Consider the following example. Assume that an experimental subject rejects the 50-50, lose \$100, gain \$110 gamble. Also assume that this subject has exogenous risky income with a mean value of \$10,000. If the subject does not integrate the endogenous and exogenous risks then the argument in subsection 3.1 tells us that there is no implication of ridiculous large-stakes risk aversion. Thus we will here focus on the case where the subject is assumed to integrate the endogenous and exogenous risky incomes. If the risky incomes are integrated by addition, does concavity necessarily have an untenable implication? In other words, does small-stakes risk aversion that is endogenous to an experiment, together with exogenous large-stakes risky income, have calibration-theorem-like implications for large-stakes risk aversion?

Let the random variables, X and Y denote the exogenous and endogenous risky incomes. The risky income that is assumed to be endogenous to an experiment is the 50-50, lose \$100, gain \$110 gamble. The exogenous income is assumed to have a log-normal distribution with mean of \$10,000 and variance of \$90,000. Assume the von Neumann-Morgenstern utility function that values sums of amounts of income, $x + y$ according to:

(14) $$v(x+y) = \frac{-9{,}800 + x + y}{20} + 5 - \sqrt{200}\text{, if } x + y < 9{,}900$$

$$= \sqrt{-9{,}800 + x + y} - \sqrt{200}\text{, if } 9{,}900 \le x + y \le 10{,}100$$

$$= \frac{-9{,}800 + x + y}{2\sqrt{300}} + \frac{\sqrt{300}}{2} - \sqrt{200}\text{, if } x + y > 10{,}100.$$

It is straightforward to show that equation (14) and the assumed log-normal distribution of *X* imply that the agent will reject the 50-50, lose \$100, gain \$110 gamble. Does the agent have reasonable large-stakes risk aversion? Yes, the minimum gain that would be required to get a decision-maker with expected utility function including (14) and the assumed log-concave distribution of exogenous income to accept a gamble with 0.5 probability of losing \$20,000, and 0.5 probability of receiving the gain, is a gain of \$44,000.

7. Exogenous Risk-Free Income

Rabin's (2000b) calibration theorem is based on the assumption that an agent will reject an opportunity to play a 50-50 bet, such as one with a loss amount of \$100 and a win amount of \$110, *for all initial wealth levels*. This suggests the question of whether an alternative assumption about behavior could support a calibration theorem that would apply to the EUI and EUI&IW models.

Consider the alternative assumption that an agent prefers the certain amount of money, x to playing a 50-50 bet with payoffs $x - \$100$ and $x + \$110$, *for all values of* x. Does this assumption imply a calibration-theorem result that applies to the EUI and EUI&IW models? The answer turns on the interpretation of "initial wealth." Amounts of money that are risk-free, and exogenous to decisions about risky propositions, would seem to be included in any natural interpretation of what is meant by initial wealth In the context of these models. The following example explicates this point.

Consider the EUI&IW model. Suppose that an agent has wealth in the amount, w. Santa Claus then offers the agent a choice between the certain amount of money, x and playing a 50-50 bet with money payoffs $x - \$100$ and $x + \$110$. Since the amount x is not at risk, and is invariant with the agent's choice between alternative decisions about the risky proposition, it is included in the agent's risk-free purchasing power, that is, his initial wealth. The natural representation with the EUI&IW model is that Santa Claus has posed the question of whether $\upsilon(w + x, 0)$ is or is not greater than $\frac{1}{2}\upsilon(w + x, -100) + \frac{1}{2}\upsilon(w + x, 110)$; that is, Santa Claus increased the agent's initial wealth from w to $w+x$ and offered the choice between playing or not playing the 50-50 bet with payoffs of $-\$100$ and $+\$110$. The assumption that the agent declines to play the bet for all amounts, x has no calibration-theorem implications, as shown by examples like those in subsection 3.2 above.

8. Data Supporting Risk Aversion Over Small Stakes

Central elements in the discussion are whether there are data that support decision-makers' risk avoidance with small-stakes gambles and whether, as maintained by Rabin (2000a, pp. 207-8) and Rabin (2000b, pp. 1288-9) and Rabin and Thaler (2001, pp. 226-7), this risk-avoiding behavior can be explained by loss aversion. There is a large literature of experimental economics papers that report behavior which is consistent with concave expected utility models but inconsistent with their risk-neutral special cases. The risk aversion interpretation of much of this data requires maintained hypotheses about Nash equilibrium, rational expectations, etc. Thus, recent experiments by Holt and Laury (forthcoming) are very informative because they do not require complicated maintained hypotheses to interpret the data.

8.1 Risk Aversion in Choices Between Binary Lotteries

In one of their several treatments, Holt and Laury asked subjects to choose one lottery from each of the 10 pairs of binary lotteries in Table 2. First, note that none of the lotteries involves losses, hence loss aversion is irrelevant to interpreting the data. Secondly, note that a risk-neutral expected utility maximizer will choose option A in the first four rows of Table 2 and choose option B in rows five through ten. A risk-preferring expected utility maximizer will "cross over" from choosing option A to choosing option B at some row weakly between row one and row five. And a risk-averse expected utility maximizer will cross over from choosing option A to choosing option B at some row weakly between row five and row ten. Choice of option A in row five and below is consistent only with risk aversion. Results from this experimental treatment involving

175 subjects are reported in Figure 3. Note that 81% of the subjects are still choosing option A in row 5; that is, 81% of the subjects are risk averse. Thus the data clearly provide strong support for risk aversion for small-stakes payoffs.[5]

There is a large literature of experimental economics papers that report risk-averse behavior in contexts that are more complicated than choice between binary lotteries. Because more complicated hypotheses are required to interpret the data in these alternative environments, the conclusion that decision-makers' risk aversion explains one-sided deviations from the predictions of risk-neutral models has been more controversial than for choice over binary lotteries. Private value auctions are one context in which there is a large amount of data that are inconsistent with risk neutrality.

8.2 Risk Aversion in Private Value Auctions

Vickrey (1961) developed a theory of bidding in first-price sealed-bid auctions based on the expected utility axioms, the income assumption about the identity of the prizes, and the assumption of Bayesian-Nash equilibrium. For the case in which private values for the auctioned item are drawn independently from the uniform distribution on $[0, v_h]$, the risk-neutral equilibrium bid function for single-unit auctions with n bidders is

(15) $$b_i = \frac{n-1}{n} v_i .$$

As explained by Vickrey, risk-averse bidders will bid higher amounts than those given by equation (15).

There is a large literature on experiments with first-price private-value auctions that reports data that are inconsistent with bid function (15) because virtually all of the bids are higher than $(n-1)v_i/n$. Bidding more than $(n-1)v_i/n$ cannot be a way to avoid losses; hence such apparently risk-averse bidding behavior cannot be explained by loss aversion. Tests of bidding theory have used both market prices and individual-subject data. For example, Cox, Roberson, and Smith (1982), Cox, Smith, and Walker (1988), and Cox and Oaxaca (1996a) report tests based on auction market prices, individual bids and values, and decision-makers' expected money payoffs from bidding that support the conclusion that almost all subjects in laboratory first-price sealed-bid auctions submit bids that are consistent with risk aversion but inconsistent with risk neutrality. The auction literature does contain some controversy (Kagel, 1995) but the issues are posed within the context of the EUI model, not the EUTW model.

8.3 Risk Aversion in Search Experiments

Sequential job search experiments provide another context in which there are data supporting risk-averse behavior over small stakes. The theory is based on the expected utility axioms, amounts of income as the prizes, and solution of the dynamic search problem by backwards recursion. Subjects cannot lose money in most of these search experiments, hence loss aversion cannot explain the observed behavior.

In search from known distributions without recall opportunities, 77% of 600 search terminations were consistent with the varying predictions of the risk-neutral model over several treatments, whereas 94% were consistent with the weakly risk-averse version of the theory (Cox and Oaxaca, 1989). In search from known distributions with perfect or stochastic recall of past offers, 61% of search terminations were consistent with the risk-neutral model whereas 96% were consistent with the weakly risk-averse model (Cox and Oaxaca, 1996b). In tests with reservation wage data for search from known distributions, the risk-neutral reservation wage path was rejected with data from *all* of the experimental treatments whereas the weakly risk-averse reservation wage path was not rejected with data for any of the three treatments (Cox and Oaxaca, 1992a, 1992b). In search from unknown distributions, 70% of search terminations were consistent with risk-neutral predictions whereas 88% were consistent with the weakly risk-averse model (Cox and Oaxaca, 2000).

Of course, it is impossible to "prove" that many decision-makers stop searching earlier than predicted by risk-neutral search theory, or that their reservation wages are higher than risk-neutral reservation wages, *because of* risk aversion. But alternative explanations embodied in models have not been developed and tested. There are some anomalous data (Sonnemans, 1998, 2000).

9. Concluding Remarks

A central issue in developing an empirical science is coherence in the application of theory to data from different sources. While such coherence is not always possible, it remains a central objective. The principal of parallelism, that the same laws apply inside and outside the laboratory, is a hallmark of an experimental science whether it be natural science (Shapley, 1964) or economics (Smith, 1982). Thus, Rabin (2000a, 2000b), Rabin and Thaler (2001), and the several other authors listed in footnote 1 raise an important question concerning coherence of the application of concave expected utility theory to explain risk-averse behavior for both small-stakes gambles used in laboratory experiments and large-stakes gambles encountered in everyday life. But the central conclusion stated repeatedly by Rabin and Thaler is demonstrably wrong: Calibration-theorem calculations do *not* imply that expected utility theory cannot provide a coherent explanation of both small-stakes and large-stakes risk aversion.

We have addressed the coherence question for both of the commonly-used expected utility models, the expected utility of terminal wealth (EUTW) model and the expected utility of income (EUI) model, and for a more general model, the expected utility of income and initial wealth (EUI&IW) model. We have explained that calibration-theorem arguments based on varying initial wealth levels have no implication for the EUI model because of its fixed reference point of zero income. The EUI&IW model includes initial wealth as a reference point but calibration-theorem arguments based on varying initial wealth levels have no implication for this model because initial wealth is not additive to income in the utility function. In contrast, with empirical support for the hypothesized type of small-stakes risk aversion, the calibration-theorem arguments do have implications for applicability of the EUTW model.

Let us suppose that the across-subjects comparisons that can be made with existing data provide empirical support for Rabin's (2000a, 2000b) and Rabin and Thaler's (2001) assumed pattern of risk-avoiding behavior. What experimental studies are then called into question? We have already explained that typical experimental papers on auctions and search are immune to criticisms based on the calibration theorem because they test hypotheses derived with the EUI model. But what about other experiments involving decisions in risky environments? If the authors used the EUTW model and strict concavity to derive the tested hypotheses then their papers are subject to the criticisms stated by Rabin and Thaler. This brings up the question of whether similar hypotheses could be derived from the EUI&IW model, and tested with the same data. Key insights into answering this question are provided by our extension of the Arrow (1971) and Pratt (1964) characterization of agents' comparative risk aversion to the EUI&IW model and the use of this model to derive an analogue of Arrow's (1971) classic two-asset portfolio allocation proposition. Proposition 1, in section 2.3, informs us that hypotheses based on agents' comparative risk aversion that were derived from the EUTW model have analogues that can be derived from the EUI&IW model. Proposition 2.c, in section 2.4, provides the EUI&IW-model analogue of Arrow's proposition relating the amount invested in the risky asset to changes in initial wealth.

The calibration theorem does not provide a general critique of lottery payoffs designed to control subjects' risk attitudes. The reason is that the theoretical basis for lottery payoffs is linearity in probabilities; it does not require terminal wealth to be the assumed argument of an expected utility function. The main critique of lottery payoffs is provided by experimental data that support conclusions that they do not succeed in inducing predicted behavior. These data are, of course, inconsistent with expected utility theory. But the data appear to be inconsistent with the compound lottery axiom (Walker, Smith, and Cox, 1990; Cox and Oaxaca, 1995), not the independence axiom that gives expected utility functions their defining characteristic of linearity in probabilities. Alternative decision theories must include analogues of the compound lottery axiom if they are to be applicable to compound lotteries. Thus there is no valid presumption that the data from experiments designed to test the effectiveness of lottery payoffs for controlling subjects' risk-avoiding behavior are consistent with alternatives to expected utility theory.

Rabin and Thaler attribute risk-avoiding behavior in experiments to loss aversion. While loss aversion may be empirically significant in some contexts, it cannot explain choices between binary lotteries when the possible payoffs for the lotteries exclude losses. Experiments with such lotteries provide unambiguous support for risk aversion. Bids in first-price private-value auctions that exceed risk-neutral theoretical bids cannot be explained by loss aversion because such bids do not avoid or minimize any possible losses. Search terminations earlier than predicted by risk-neutral search theory and reservation wages that exceed risk-neutral reservation wages cannot be explained by loss aversion because no losses are possible in most of the sequential search experiments.

Both loss aversion and risk aversion are consistent with expected utility theory. Loss aversion is inconsistent with the EUTW model but it is consistent with the EUI and EUI&IW models. The common view that loss aversion is inconsistent with expected utility theory confuses an inconsistency with a secondary assumption about the identity of "prizes" with an inconsistency with a set of axioms. The Allais paradox is inconsistent with the axioms, loss aversion is not.

Observations of risk-averse behavior in laboratory experiments can be rationalized by concave expected utility theory without producing incoherence with applications of that theory to large-stakes gambles. Thus, expected utility theory can provide a coherent explanation of risk aversion in both laboratory and field environments. Then the decision about whether to apply expected utility theory in both environments or neither environment can be based upon whether or not known inconsistencies with the axioms, such as the Allais paradox, are judged to be of sufficient importance to justify sacrificing the simplicity of the theory. There is a substantial literature that reports data from experiments that are inconsistent with parallel straight-line indifference curves in the Machina triangle. But that, in itself, does not imply that expected utility theory should be rejected. The relevant question is whether there is an alternative theory that performs so much better in predicting data that it justifies paying the cost of abandoning the parsimony of expected utility theory. Papers reporting direct tests of the implications of expected utility theory's linear indifference curves, and the implications of the alternative indifference maps provided by other theories, typically conclude that the data are significantly inconsistent with all of the decision theories (see, for example, Harless and Camerer, 1994; Hey and Orme, 1994; Hey, 2001).

Endnotes

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1. See, for examples, Hansson (1988), Segal and Spivak (1990), Epstein and Zin (1990), Kandel and Stambaugh (1991), Epstein (1992), and Loomes and Segal (1994).

2. In just one three-week period the following was observed. A colleague of one of the authors received a referee report stating that the concave expected utility model he was using to explain systematic one-sided deviations from the predictions of expected value maximization could not be correct, "… as shown by Rabin (2000)." Friends of the authors had a paper reporting experiments with matching pennies games involving more and less risky alternatives rejected because the referees claimed that Rabin (2000b) had shown that risk aversion could not explain the one-sided deviations from risk neutral predictions. The authors received in the mail a working paper reporting experiments with private value auctions in which is was stated that the higher-than-risk-neutral bids that were observed could not be explained by subjects' risk aversion because "…if we take the implied estimates of risk aversion seriously, they imply pathologically risk-averse behavior over larger sums of money (Rabin, 2000)." Friends of the authors had a paper reporting auction experiments rejected by a journal in large part because a referee stated that their use of risk aversion had been shown to be incorrect by Rabin (2000b).

3. See, for examples: Vickrey (1961); Riley and Samuelson (1981); Milgrom and Weber (1982); McAfee and McMillan (1987), and Milgrom (1989).

4. The statement in terms of elasticity is correct because one can multiply $-u''(w+ar)/u'(w+ar)$ by w in Arrow's proof on proposition 2.a, and still derive his conclusion, and one can multiply $-\upsilon_{21}(w,ar)/\upsilon_2(w,ar)$ by w in our proof of proposition 2.c and still derive our conclusion.
5. Holt and Laury (forthcoming) also report results from several other treatments, including ones involving lower and higher money payoffs and others involving small and large hypothetical payoffs.

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Appendix: Proofs of the Propositions

Proof of Proposition 1.

We first show that (a) $\leftrightarrow$ (c). Differentiation of (c) with respect to y yields

(a.1) $\upsilon_2^\alpha = g_2 \upsilon_2^\beta$

and

(a.2) $\upsilon_{22}^\alpha = g_{22}(\upsilon_2^\beta)^2 + g_2 \upsilon_{22}^\beta$.

The definition of $A^j(w,y)$ and statements (a.1) and (a.2) imply

(a.3) $$g_{22} = \frac{\upsilon_{22}^\alpha - g_2 \upsilon_{22}^\beta}{\left(\upsilon_2^\beta\right)^2} = \left[\frac{\upsilon_{22}^\alpha}{\upsilon_2^\alpha} - \frac{\upsilon_{22}^\beta}{\upsilon_2^\beta}\right] \frac{\upsilon_2^\alpha}{\left(\upsilon_2^\beta\right)^2} = \left[\mathrm{A}^\beta - A^\alpha\right] \frac{\upsilon_2^\alpha}{\left(\upsilon_2^\beta\right)^2}.$$

Statement (a.1) implies that $g_2(w,u) > 0, \forall (w,u),$ because $\upsilon_2^\alpha(w,y) > 0$ and $\upsilon_2^\beta(w,y) > 0$, $\forall (w,y)$. Statement (a.3) implies that $g_{22}(w,u) < 0, \forall (w,u),$ if and only if $A^\alpha(w,y) > A^\beta(w,y), \forall (w,y)$.

We next show that (c) $\rightarrow$ (b). One has

(a.4) $$\begin{aligned} &\upsilon^\alpha(w, \phi^\beta(w, E_F(\upsilon^\beta(w,y)))) = g(w, E_F(\upsilon^\beta(w,y))) \\ &> E(g(w, \upsilon^\beta(w,y))) = E(\upsilon^\alpha(w, \phi^\beta(w, \upsilon^\beta(w,y)))) = E(\upsilon^\alpha(w,y)). \end{aligned}$$

Therefore

(a.5) $$\begin{aligned} &E_F(y) - \pi^\beta(w,F) = \phi^\beta(w, E_F(\upsilon^\beta(w,y))) = \phi^\alpha(w, \upsilon^\alpha(w, \phi^\beta(w, E_F(\upsilon^\beta(w,y))))) \\ &> \phi^\alpha(w, E_F(\upsilon^\alpha(w,y))) = E_F(y) - \pi^\alpha(w,F). \end{aligned}$$

Therefore (c) $\rightarrow$ (b).

We next show that (b) $\rightarrow$ (c). One has

(a.6) $$\begin{aligned} &\upsilon^\alpha(w, \phi^\beta(w, E_F(\upsilon^\beta(w,y)))) = \upsilon^\alpha(w, E_F(y) - \pi^\beta(w,F)) \\ &> \upsilon^\alpha(w, E_F(y) - \pi^\alpha(w,F)) = \upsilon^\alpha(w, \phi^\alpha(w, E_F(\upsilon^\alpha(w,y)))) = E_F(\upsilon^\alpha(w,y)). \end{aligned}$$

Therefore

(a.7)
$$g(w,E_F(\upsilon^\beta(w,y))) = \upsilon^\alpha(w,\phi^\beta(w,E_F(\upsilon^\beta(w,y))))$$
$$> E_F(\upsilon^\alpha(w,y)) = E_F(\upsilon^\alpha(w,\phi^\beta(w,\upsilon^\beta(w,y)))) = E_F(g(w,\upsilon^\beta(w,y))).$$

Therefore, g is strictly concave in u: $g_{22}(w,u) < 0, \forall(w,u)$.

Proof of Proposition 2.c.

Let *V(a)* denote the expected utility of investing an amount, *a* in the risky asset with cumulative distribution function, F for the random rate of return per dollar invested, R ; that is.

(a.8) $V(a) = E_F(\upsilon(w,ar))$.

Assume that $E_F(r) > 0$ and hence $a > 0$. Let *a(w)* be the optimal choice of *a* as a function of the initial wealth, *w*; this will satisfy the first-order condition as an identity:

(a.9) $V'(a) = E_F(\upsilon_2(w,ar)r) \equiv 0$.

Differentiating with respect to *w* gives

(a.10) $$\frac{da}{dw} = -\frac{E_F(\upsilon_{21}(w,ar)r)}{E_F(\upsilon_{22}(w,ar)r^2)}.$$

Since $\upsilon_{22}(w,ar) < 0$, one has

(a.11) $$sign\left(\frac{da}{dw}\right) = sign\big(E_F(\upsilon_{21}(w,ar)r)\big).$$

We show that if $-\upsilon_{21}(w,y)/\upsilon_2(w,y)$ decreases in y then $sign\big(E_F(\upsilon_{21}(w,ar)r)\big) > 0$, and hence from (a.11) the amount, *a* invested in the risky asset increases. The other cases can be derived by similar arguments.

Consider first the case where $r > 0$. Since $-\upsilon_{21}(w,y)/\upsilon_2(w,y)$ decreases in y, one has

(a.12) $$-\frac{\upsilon_{21}(w,ar)}{\upsilon_2(w,ar)} < -\frac{\upsilon_{21}(w,0)}{\upsilon_2(w,0)},$$

which can be rewritten as

(a.13) $$\upsilon_{21}(w,ar) > \frac{\upsilon_{21}(w,0)}{\upsilon_2(w,0)}\upsilon_2(w,ar).$$

Multiplying both sides by $r > 0$, one has

(a.14) $$\upsilon_{21}(w,ar)r > \frac{\upsilon_{21}(w,0)}{\upsilon_2(w,0)}\upsilon_2(w,ar)r.$$

A similar argument shows that the inequality in (a.14) is satisfied for $r < 0$ as well. Therefore, taking expectations over all values of the random rate of return, and using (a.9), gives one

(a.15) $$E_F(\upsilon_{21}(w,ar)r) > \frac{\upsilon_{21}(w,0)}{\upsilon_2(w,0)}E_F(\upsilon_2(w,ar)r) = 0.$$

Table 1. If Averse to 50-50 Lose $100 / Gain $110, Will Reject or Accept 50-50 $Lose /$Gain Bets

$ Lose	**$ Gain**			
	Table I Rejections	**Table II Rejections**	**Eq. (10) Acceptances**	**Eq. (11) Acceptances**
400	550	550	655	690
600	990	990	1,000	1,040
800	2,090	2,090	1,350	1,385
1,000	∞	718,190	1,695	1,730
2,000	∞	12,210,880	3,425	3,460
4,000	∞	60,528,930	6,890	6,925
6,000	∞	180,000,000	10,355	10,390
8,000	∞	510,000,000	13,820	13,855
10,000	∞	1,300,000,000	17,285	17,320
20,000	∞	160,000,000,000	34,605	34,640

Table 2. Ten Paired Lottery-Choice Decisions

Option A	Option B	Expected Payoff Difference
1/10 of $40.00, 9/10 of $32.00	1/10 of $77.00, 9/10 of $2.00	$23.30
2/10 of $40.00, 8/10 of $32.00	2/10 of $77.00, 8/10 of $2.00	$16.60
3/10 of $40.00, 7/10 of $32.00	3/10 of $77.00, 7/10 of $2.00	$9.90
4/10 of $40.00, 6/10 of $32.00	4/10 of $77.00, 6/10 of $2.00	$3.20
5/10 of $40.00, 5/10 of $32.00	5/10 of $77.00, 5/10 of $2.00	-$3.50
6/10 of $40.00, 4/10 of $32.00	6/10 of $77.00, 4/10 of $2.00	-$10.20
7/10 of $40.00, 3/10 of $32.00	7/10 of $77.00, 3/10 of $2.00	-$16.90
8/10 of $40.00, 2/10 of $32.00	8/10 of $77.00, 2/10 of $2.00	-$23.60
9/10 of $40.00, 1/10 of $32.00	9/10 of $77.00, 1/10 of $2.00	-$30.30
10/10 of $40.00, 0/10 of $32.00	10/10 of $77.00, 0/10 of $2.00	-$37.00

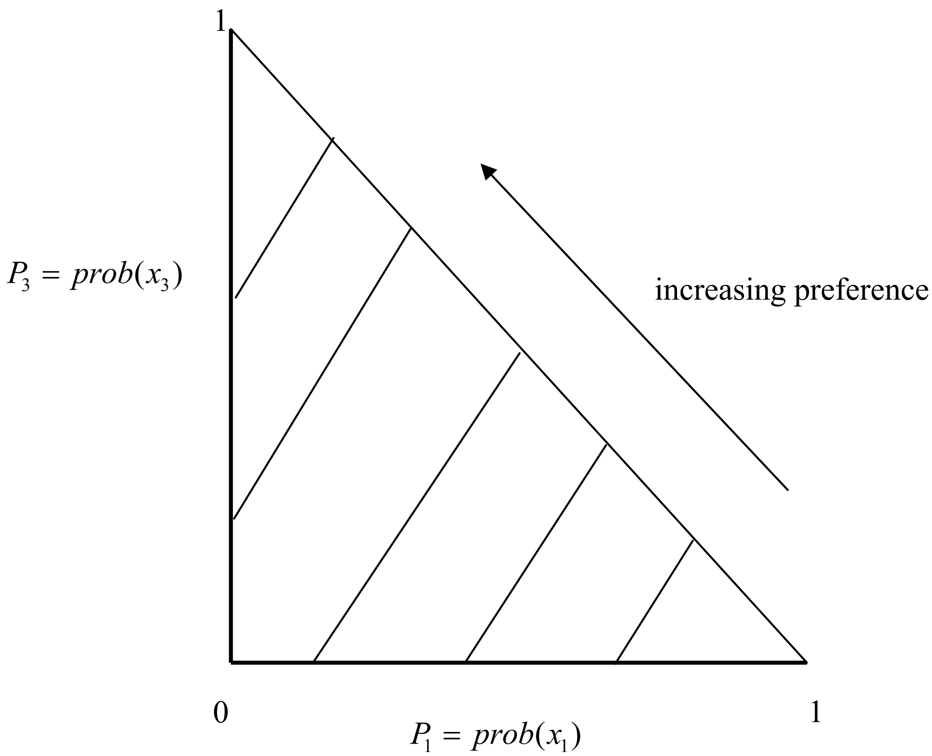

Figure 1. Expected Utility Indifference Curves in the Triangle Diagram

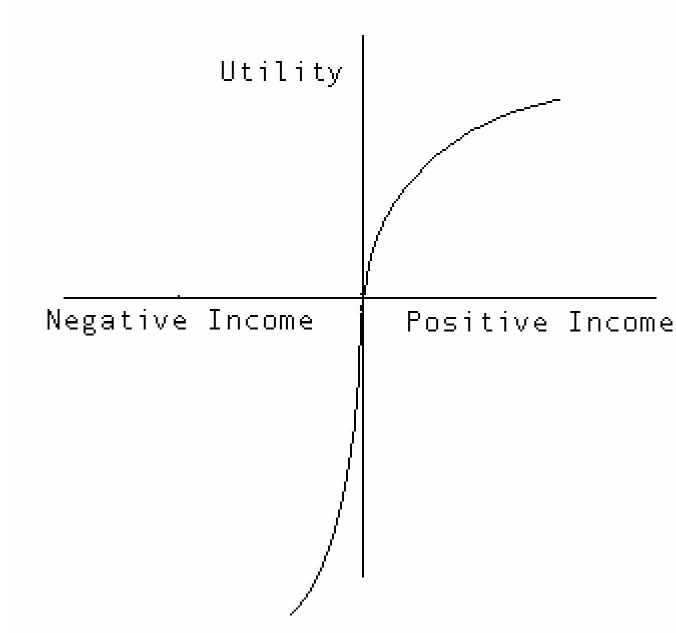

Figure 2. Loss Aversion

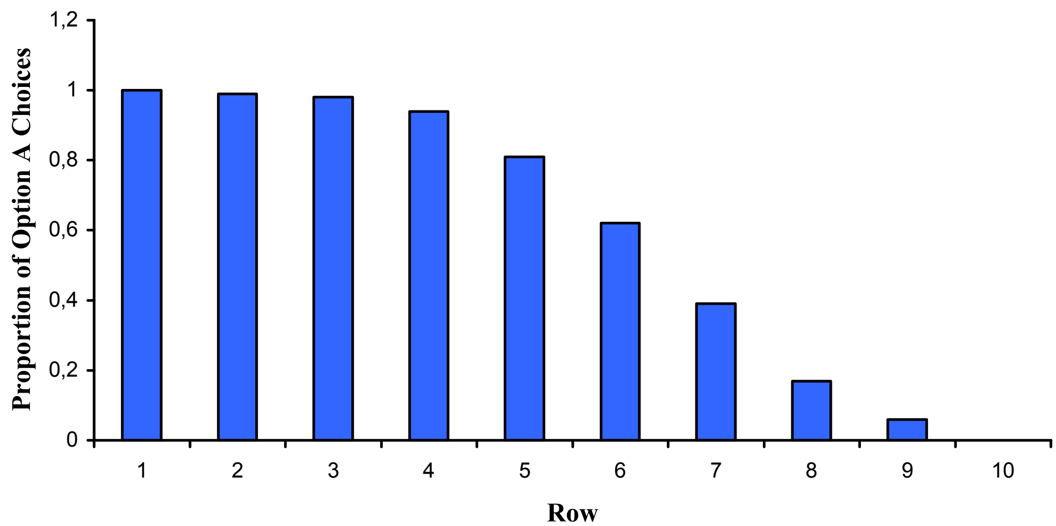

Figure 3. Risk-Averse Choices from Pairs of Binary Lotteries