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Risk Aversion in International Relations Theory

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Abstract

When international relations theorists use the concept of risk aversion, they usually cite the economics conception involving concave utility functions. However, concavity is meaningful only when the goal is measurable on an interval scale. International decisions are usually not of this type, so that many statements appearing in the literature are formally meaningless. Applications of prospect theory face this difficulty especially, as risk aversion and acceptance are at their center. This paper gives two definitions of risk attitude that do not require an interval scale. The second and more distinctive one uses the property of submodularity in place of concavity. R. D. Luce has devised a theory of choice with features of prospect theory but not requiring on an interval scale, and the second definition in combination with this theory yields the traditional claim that decision makers are risk-averse for gains and risk-seeking for losses.

Keywords: risk aversion, prospect theory, international relations, joint receipts, measurement theory.

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1 Introduction

Theories about risk aversion and acceptance have appeared often in international relations research, probably because they touch the most crucial issues -- an action that is too daring can start a war. The idea of an attitude to risk is attractive because it reconciles two opposing notions: it allows leaders to have different personal decision-making styles and still act as calculating goal-seekers. Khrushchev was risk-acceptant but Brezhnev was risk-averse, and one can say this without implying that one was less than rational than the other. Risk attitudes have received considerable attention in formal utility theory and prospect theory approaches.[1]

This paper argues that most of these applications have a serious conceptual problem. When we label someone risk-averse or acceptant, our superficial grammar suggests that it is a trait inherent in the person. Some reflection makes it clear, however, that we are invoking another element: a baseline of neutral decision making. We are suggesting that the person is avoiding or accepting risk *relative to some standard.* In everyday speech, saying someone is "greedy" or "tall" is meant relative to the average, but international relations writers are not claiming that a risk-averse leader avoids uncertainty more than a typical person. They intend their standard to be a theoretical one, for the sake of understanding leaders' decision making in an explanatory way, not a comparison with a population. International applications either leave the risk-neutral standard unclear, or use the economics standard in which risk neutrality means indifference between playing a gamble and simply receiving its expected face value. This rule is defined when the goal involves some measurable commodity like money. However in most international relations decisions, "expected face value" cannot be defined, since the goal is not measurable with respect to an external amount or metric, separate from the individual's utility. (More precisely, the goal is not an interval scale. A loose term will sometimes be used here, that it is not a "commodity.") Assertions of risk aversion are then meaningless, deductions drawn from them do not really follow, and the international behaviours recruited to support one's theory are not relevant to its validity.

The first section describes why risk attitude is relative to an external standard and why the standard economics definition requires an interval-scale measure of the goal. It goes on to indicate why most current ways of avoiding the problem are not successful. The second section, as a step to a definition, distinguishes absolute conceptions of risk attitude from

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[1] Examples of the many non-formal uses of risk-aversion are Russett (1980) on Roosevelt, Huth and Russett (1984) on Hitler and Chamberlain, and Kupchan (1988) on Soviet behaviour. Some utility theory applications of risk aversion are Alsharabati and Kugler (1997), Bueno de Mesquita (1980, 1981, 1985), Fearon (1995), Huth, Gelpi and Bennett (1993), Huth, Bennett and Gelpi (1992), Kilgour and Zagare (1991), Kugler (1987), Kugler and Zagare (1987), Morrow (1985, 1986, 1987, 1988, 1994, 1996), Niou and Ordeshook (1990), Powell (1995), Sandler, Tschirhart and Cauley (1983), Snidal (1993) and Wagner (1985, 1991). Some prospect theory applications or discussions are Bauer and Rotte (1997), Berejikian (1997), Boettcher (1995, 1997), Davis (1997), Davis and Arquilla (1991), Farnham (1992, 1994, 1997), Gause and Feldman (1998), Jervis (1988, 1989, 1992), Kanwisher (1989), Kowert and Hermann (1997), Lebow and Stein (1989), Levy (1987, 1989, 1992a,b, 1996, 1997a,b, 1998), McDermott (1994, 1998), McInerney (1992), Mintz and Geva (1998), Richardson (1992), Stein (1992), Stein and Welch (1997), Taliaferro (1997a,b), Weber (1991), and Weyland (1996).

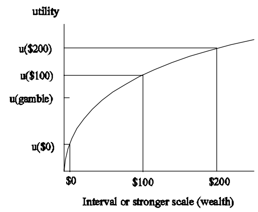
relative ones. An absolute conception asserts simply that someone is risk-averse or risk-acceptant as if there were a zeropoint of neutrality, while a relative conception labels one person as more risk-averse than another without implying a zeropoint. The simplest economics conception is an absolute one, and the section gives a corresponding relative definition. Using a relative definition avoids the need for a risk-neutral standard and fits with much previous international relations theories, but it requires that their claims be reformulated.

The next section defines risk aversion in an absolute sense. Different from the economics use of concavity, it uses the idea of submodularity. The decision maker's goal is assumed to involve several kinds of benefits received jointly, and I describe a theory of choice for such situations, which shares many features of prospect theory but not the need for a money-like commondity. It as developed by Luce and his co-workers with the goal of accounting for experimental data on joint benefits, not to avoid interval scales or represent risk aversion, but the relation is straightforward under the revised definition. The next section describes a usual metaphor that depicts risk as an external substance that can be "shared," "transferred," or "avoided." This can have misleading implications, and the section shows that the revised definitions avoid them and clarify the nature of risk.

The most common goal for introducing risk attitude into international relations theory has been to understand decision making in general, rather than to answer a particular policy problem. Applications of prospect theory have helped us understand international decision making by introducing psychological research. It is important however to recognize the distinctive aspects of international decisions, and modify the approach accordingly.

2 The Need for an Interval Scale

A theory that calls leaders risk-averse or acceptant should state its standard of neutrality, and both prospect theory and utility theory applications use the microeconomic one, of making decisions entirely by expected value. Utility theory's version, suggested by di Finetti (1952) and developed by Arrow (1964) and Pratt (1964), is simpler than prospect theory's and will be reviewed briefly. A **risk-neutral** person is one who is indifferent between playing a gamble and receiving its expected value in cash. For example, the person will be indifferent between receiving $100 for sure or playing a 50/50 gamble for $0 or $200. (Since it is assumed that more money is better, the risk-neutral person would choose the gamble over any sure payment less than $100, and the reverse for a payment that is higher.) Someone who chooses the gamble over its expected value is **risk-acceptant** or **risk-seeking**, and someone choosing the expected value is **risk-averse**. The general trait is the person's **risk attitude**. The definition uses a criterion involving the gamble's face value, and this is a crucial. If it had said that a risk-neutral person is indifferent between a gamble and a sure thing without qualification, it would be empty since anyone can be induced to choose one or the other by making it sufficiently attractive. This version of the definition is stated in terms of money, but any goal will do -- bushels of wheat produced, literacy rate, etc- -- as long as one can calculate a expected value.



The next step in the utility approach is to connect risk aversion with the shape of the decision maker's utility function for the commodity: a decision maker who is risk-averse must have a utility for it that is concave. (A **concave** function bulges upwards between any two points; or more exactly, for any two points on it, the straight line segment connecting them lies below the function.[2]) In the context of a desirable good like money, concavity means that utility is increasing, but increasing at a decreasing rate. Figure 1 shows why a concave function means choosing $100 for certain over the gamble: u($100) lies on the vertical axis above the gamble's utility, which is the 50/50 weighted average of u($200) and u($0). The sure thing is preferred because of the upward bulge between the loss and win utilities. If the utility function were convex and sagged there, it would represent a risk-seeker who would choose the gamble.[3, 4]

These definitions of risk aversion and acceptance put constraints on the measurement scales that can appear on the x-axis, and it is worthwhile to go through some fundamentals to see why. A **measurement scale** is an assignment of numbers to a set of empirical events such that

certain arithmetical relations among the numbers mirror some significant empirical relations among the corresponding events. In a typical scale, only some of the numerical relations mean

[2] A consequence of the definition that will be useful is that convexity and concavity can still apply if a function is defined at only a finite number of points.

[3] It is possible to be risk-averse at one level of assets and risk-seeking at another. Also, with a "wavy" utility function, it is possible to be risk-averse for small gambles and the risk-seeking for large ones, or the reverse. To avoid this one could specify that the property is a "local" one, applying to small gambles.

[4] The next step in the economic theory is to define degree of risk aversion, the most accepted definition being Arrow and Pratt's: $r(x) = -u''(x)/u'(x)$, and a further element is to define the riskiness of a gamble (Rothschild and Stiglitz, 1970). The set of these concepts should be defined **coherently** in the sense that the riskiness of a gamble should be what risk-averse people are averse to. The usual definitions are coherent, but alternative systems are possible (e.g., Landsberger and Meilijson, 1990).

anything in the world; the rest have no interpretation. In the context of calendar years, for example, it is empirically significant that the number 1936 is greater than 1919 -- the numerical order of these numbers indicates the temporal order of the corresponding years. However, the fact that 1936 is a perfect square and 1919 is not, or that the ratio 1936/1919 has a certain value, is empirically meaningless with no external significance.[5]

Measurement scales are classified by their "strength." The stronger the scale, the more of its numerical relationships possess empirical interpretations. In an **interval scale,** an empirical meaning is attached to comparisons of one interval (the numerical difference between a pair of scale values) with another, in terms of being smaller, equal or greater. The scale involving amounts of money qualifies as an interval scale (in fact, it is stronger, a ratio scale[6]), since the numerical equality of the difference between $2 and $3 and that between $5 and $6 has an empirical interpretation. It is so simple that it is easy to overlook: adding a dollar bill to $2 will bring the amount to $3, and the same operation will bring $5 up to $6. The additional dollar functions like a ruler in space that marks off equal distances in "money space." IQ scores are an example of a scale that is not interval, as it is unclear how to interpret interval comparisons. In what sense is a jump from 130 to 140 points equivalent to one from 60 to 70? IQ might be claimed to be an ordinal scale, if those who do better on the test really have greater analytical abilities in life, but it does not qualify as interval.

Returning to the definitions of risk attitudes, concavity means that utility increments are shrinking *for equal increases* in the desired commodity. This phrase involves comparison of intervals, so to support assertions about risk aversion, the x-axis must be at least an interval scale. The typical international decision -- over intervening in a conflict, forming an alliance, or signing a treaty -- has a goal that is not measurable on a interval scale. The problem is not that the objective is non-monetary, since risk attitudes can be and have been used for goals like the percent of work force unemployed, the number of lives lost in an epidemic (Quatronne and Tversky, 1988), or deaths in election violence (Kowert and Hermann, 1997). These scales are interval -- an increase from 4% to 7% in unemployment is the same as from 10% to 13%, in the sense that the same number of additional people are out of work. The problem is that an international decision is usually over a bundle of objectives, some with interval scales and some

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[5] “Meaningless” is used here in the well-defined sense of formal measurement theory (Coombs, Dawes and Tversky, 1978; Roberts, 1984; Narens, 1985.)

[6] In a ratio scale, the ratio of a pair of values has an empirical interpretation.

without, but it does not have a single goal where one can mark off equal increments or otherwise compare intervals.

Multiple Interval Scales

The problem arises when there is no interval scale, but it is broader than that. A decision can often be described in different ways involving alternative interval scales, with the decision maker showing risk aversion in one formulation and risk acceptance in another. Suppose a person has $10,000 to invest in one of two ways. An investment will gain value for ten years at a certain annual rate i, then it is withdrawn and spent on a vacation. For one opportunity, the investor knows i, but for the other i is uncertain in the investor's mind. For the latter, it will be assumed that i is learned right after the investment is made and remains fixed over the ten-year period. It is also assumed that the individual's utility for the interest rate is $u(i) = i^{3/2}$. This function is convex, so the individual is risk-acceptant with respect to the interest rate. However, the problem can also be viewed as uncertainty over the money earned. There is clearly a one-to-one relationship between interest rate and final amount: rate i produces wealth w of $\$10,000 (1 + i)^{10}$. Since utilities are determined by outcomes, assuming a utility function for the interest rate commits one to a utility function for wealth, and straightforward calculation gives it as $u^*(w) = (w^{1/10} - 1)^{3/2}$. This function is concave -- now the person is risk-averse. These are not two different decisions, but two different descriptions of the same decision. Since the framing is arbitrary, so is the risk attitude.

A counterargument might be raised that only the second formulation in final wealth is appropriate, that money is what the individual is "really" after.[7] But this is insupportable -- money has no innate value; it is usually acquired to spend. This individual wants to use it for a vacation, and we might assume that the number of vacation days is a non-linear function of the money, such that the individual has a risk-acceptant utility function for days. Money would then be clearly an intermediate step, again the new description would imply the opposite risk attitude. Money is the metric favoured by economics because it can be transferred back and forth and is conserved in such interactions. This property is important for economics because that discipline studies social interaction for goods, but it is not about the individual's personality as reflected in decision making. The example shows that even given the context and the decision, risk attitude in the sense of concave utility functions is not a personality trait,

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[7] In the finance literature risk attitude is often applied to interest rates, and some experiments on prospect theory involve the inflation rate, which is equivalent (Quattrone and Tversky, 1988).

but a relationship between the decision maker's choice pattern and the measurement scale of the goal. The scale is crucial.

Prospect Theory Applications

In utility theory, the decision maker attaches a value to each possible outcome, but prospect theory looks at the difference between the outcome and some mental reference point. The reference point depends on how the decision is set up in the individual's mind, that is, on how it is "framed." A "value," which is the prospect theory analogue for a utility, is attached to each degree of change from the reference point. The approach was developed by Kahnemann and Tversky (1979; an updating is Kahnemann and Tversky, 1992). Further postulates are that the loss side of the reference point is steeper than the gain side, termed loss aversion, and that sensitivity to losses or gains is marginally decreasing with the amount lost or gained.[8] The implication is that the value function is S-shaped -- the decision maker is risk-averse for gains and risk-seeking for losses. It is also assumed that if the problem admits objective probabilities, the decision maker does not use these directly as weights for the values, but transforms them according to a certain function.[9] Various auxiliary hypotheses are added for particular contexts, such as assumptions about how people edit the input to set up their decision or how they change their reference point over experience and time.

Prospect theory is often presented as the psychological alternative to rational choice, so it is ironical that it comes with relatively more economics baggage about the shapes of functions over interval scales. Accordingly, the lack of an interval-scale goal has greater impact on prospect theory. Also, prospect theory applications have been more empirically-based than utility theory's, and the closer that researchers have gotten to historical cases, the more they have met difficulties that stem from the scale problem. Some case studies that went to great detail on leaders' decisions have been strongly criticized on conceptual grounds. Several authors have expressed second thoughts about prospect theory (e.g., Boettcher, 1995; Jervis, 1992; Levy, 1992b, 1997a; Shafir, 1992; Stein, 1992.) In my view, their doubts are connected with the scale problem, although they have not identified them as such. They often

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[8] Many of the criticisms raised here also apply to loss aversion. It would be worthwhile to work out alternative definitions there too, but this paper will be confined to risk attitudes.

[9] Another sense of risk aversion occasionally used in prospect theory applications is based on the probability weighting function. Someone who understates the probability of a loss will take excessive chances and thus be risk-seeking. This sense is less often used because "aversion" seems to imply a deliberate avoidance, not a cognitive mistake of misunderstood probabilities.

attribute prospect theory's difficulty to the impossibility of getting accurate measurements of leaders' constructions of the situation and their utilities. Levy, for example, has been a major proponent of the approach in international relations, but has expressed reservations on several grounds. His main concern is that laboratory experiments supporting prospect theory gave their subjects simpler, more structured and measurable decisions than national leaders face. Subjects are offered two-way choices with values and likelihoods of success. Levy notes the absence of interval scales in the international setting, but sees it as a lack of laboratory control (1997a, 99). However, the arguments here indicate that the issue is not the practical one of measuring risk attitudes, but the conceptual one of defining them.

Boettcher also sees prospect theory's problem as its data requirements, the difficulty of specifying the utilities and probabilities of foreign policy decision makers. His proposed solution for the probability aspect is to collect data on the relation of frame to the decision maker's nonnumerical probability expressions, such as "possible" and "likely." However, he were able to measure probabilities and utilities precisely, assertions about risk attitude would still be meaningless in these contexts, without an interval scale on the objective.

3 Responses to the Lack of an Interval Scale

The scale difficulty has generally not been named in the literature but it has been present, and international relations writers have used a variety of methods to attribute risk attitudes to states and leaders nonetheless. This section puts their approaches into six categories and describes the difficulties. The purpose is to show that only a few work, and that these work only in restricted contexts, and so to motivate the interval-scale-free definition.

The first common pattern is to describe an historical case where a leader had two possible courses, one of which had a predictable outcome, and the other an uncertain one. The argument interprets *making the sure or risky choices as a sign of risk aversion or acceptance, respectively*. Examples of risk-acceptant choices were Britain and France intervening in Suez (Richardson, 1992), and Eisenhower lying about U-2 overflights (McDermott, 1997). A cautious decision was Eisenhower staying out of Suez (McDermott, 1997). Any attribution of risk aversion should imply a standard, and here it is the rejected course of action -- a decision maker who chose the other road would be showing the opposite risk attitude.

The logic appears solid: opting for the uncertainty seems like an obvious sign of risk acceptance. The problem is that utility theory and prospect theory do not support this conclusion -- their usages of the words "aversion" and "acceptance" are misleading. As

Boettcher has pointed out, within these theories any decision maker, risk-averse or acceptant, will choose the course with the higher expected utility (or in prospect theory, the higher weighted value). This may be the uncertain one, so the choice does not show risk acceptance. The misunderstanding is exploited by a simple examination question that reliably trips up students. A risk-averse person has two options, a sure outcome with utility 100, or a 50/50 gamble with expected utility 100. Which will be chosen? Students want to say that a risk averter will prefer the sure thing, but since the domain is utilities rather than payoffs, the correct answer is indifference. A safe choice can be used to identify someone's risk attitude only when the alternatives can be compared by their expected face values.

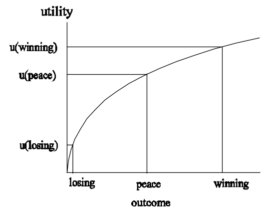
One manifestation of this misunderstanding in the prospect theory literature has been the continual the failure of a decision implies risk-acceptance. The bulk of the actions cited as risky have been ones that were later regretted. One might find an analysis of Kennedy's invasion in the Bay of Pigs, but not his blockade of Cuba during the missile crisis. If attributing a risk attitude requires a standard, here it is hindsight. By definition risky actions often fail, but the literature's uniformity of bad outcomes suggests that there has been a selection bias in that direction. They sometimes succeed, but then the analyst seems ready to interpret the leader as willing to act on a chance worth taking. This is a plausible explanation, but it might be just as true that the failed decisions were sensible given what the actor knew at the time. Taking failure is an indicator of rashness and risk acceptance works as a substitute for calculating expected face values, but it is not a valid one.

The idea that choosing a risky course implies risk acceptance appears in Bueno de Mesquita's revised expected utility theory of war (1985) and its development by Morrow (1987). A state can choose different policy positions, and to each is associated a danger of attack. A policy's "insecurity measure" is defined as the sum of the expected utilities that the other states would derive from starting a war against the state given its policy choice. The state's risk aversion is the closeness of its insecurity measure to the minimum value, normalized by the distance between the minimum and maximum. However, a state's policy choice may lie near the minimum simply because of good luck in the configuration of political forces, i.e., because it is the desirable course on various grounds. One cannot attribute the choice to risk attitude, and if the measure correlates with other variables, as this literature has indicated, the finding will not be properly interpreted and understood.

A second common approach is *suggesting that there is an appropriate interval scale, compared to which the decision maker is being risk-averse or acceptant, but not elaborating* *on how to measure the scale.* As an example of the international implications of prospect theory, Levy (1992a, 93) states that after suffering losses in territory, reputation or domestic support, leaders tend to take "excessive risks" to recover them. He adds that "'excessive' is defined with respect to predictions based on expected value." This implies that there is a objectively right way to combine the goals on the list into an expected value, separate from leaders' utilities, but how to do it is not specified. Even if one could objectively rank order the outcomes according to their promotion of the "national interest," that would not be enough, since attributing risk attitude requires an interval scale. Further, if more than one interval scale were possible, one would have to specify which one is being taken as the standard, since, as the investing-for-a-vacation example showed, risk attitudes depend on which scale is meant.

A related approach is to suggest that there are correct *utilities* for a decision, compared to which the person is being too safe or risky. Jervis (1992) presents a prospect theory interpretation of crisis instability, and suggests that a leader will tend to order a pre-emptive strike "[i]n cases in which the standard expected utility model would predict the actor to cut his losses." However, there is no standard theory for the leader's utilities. Many strategic analyses of war have appeared, but they do not give such calculations; they estimate weapons destroyed or lives lost, but these quantities are only part of a nation's considerations in choosing peace or war. Utilities reflect what the decision maker does; they do not give a proper risk-neutral way to behave on the brink of war.

To introduce risk aversion, Fearon (1995, 386) assumes that states fight over territory. This is unobjectionable on methodology grounds, since territory is a variable with a ratio scale, even stronger than an interval scale. However, it raises the question of how to interpret the model's conclusions about risk aversion and war in the general international situation, when no such scale is available.



A third approach to importing the economics definition involves *spacing the possible outcomes at equal intervals on the x-axis.* An example is Bueno de Mesquita's original expected utility theory of war (Figure 2), where a national leader can choose the status quo of peace, or go to war, possibly to win or lose.[10] If this equal spacing produces a concave utility function, then he terms the leader risk-averse. (The utility function is defined at only three points, but if the middle value lies above the line joining the other two, it can be properly called concave, as stated in footnote 2.) Morrow (1994) presents a modification that labels the middle outcome the "median," suggesting that in a decision with more than three possible outcomes, they should again be spaced equally. In effect, the method constitutes a rule for constructing an interval scale for a given decision. The problem, however, is the arbitrariness of the equal spacing. There is no meaningful empirical sense of "distance" in which the distance between losing a war and the status quo is equal to the distance between the status quo and winning. The arbitrariness can be shown by splitting one of the outcomes into different versions. In Figure 2, one could divide the outcome of losing the war into three: "losing badly," "losing" and "losing indecisively." This would alter the relative positions on the x-axis of the original three outcomes and change the median outcome to "losing indecisively." The latter has a lower utility than the current median, and if its utility is sufficiently low, the risk attitude will change from averse to acceptant. This consequence is unsatisfactory: a party's risk attitude should not depend on whether the analyst splits or groups certain outcomes.

[10] Bueno de Mesquita's graph puts peace on the left and has the curves going downwards -- the choice is purely a convention.

A fourth approach involves *introducing an interval-scale variable over which the leader has a preference, but one that is not the goal.* This category includes a variety of models, and two will be discussed briefly. In one by Morrow (1996), a state is rising in power and will rival the dominant state. The latter can compromise by making concessions, or resist with the danger of war, and Morrow relates risk aversion to the likelihood of war. The states are deriving continuous benefits from a situation drawn out over time, and a special feature is his way of calculating utilities in this context. At a given time, a state enjoys a certain input of benefit from its world position, and this instantaneous utility is integrated over time to get its total utility. If a state's utility function is such that a constant situation yields increments in benefits that are a decreasing concave function of the time over which the situation is experienced, then the state is termed risk-averse. Thus, what appears on the x-axis is duration, a ratio scale and stronger than an interval scale. A difficulty, however, is the model's use of integration, which conflicts with the concept of risk aversion. Standard risk aversion means that if two benefits are received simultaneously, their utility is less than the sum of the individual utilities -- the utility of getting $1 twice (i.e., u($2)), is less than twice that of gaining $1 (of 2u($1)). If this subadditivity applies to two benefits coming in at the same time, it is problematic to have the utility of getting $1 twice jump to the sum of the individual values when one of the dollars is delayed momentarily. However, this is what integrating over time (the continuous version of adding over time) implies.[11]

Another example of the fourth approach is Huth, Bennett and Gelpi's model (1992) of system uncertainty and war. They see a state as risk-averse if it tends to avoid choices without clear probability values in favour of those with specified probabilities. Such states then are showing preferences over probability distributions over probabilities -- they prefer the distributions that have low variance. The x-axis is probability, which is more than an interval scale. However this phenomenon is more often called ambiguity aversion, and is conceptually different from risk aversion. Someone might dislike choices with undefined probabilities but be drawn to unpredictable gambles whose probabilities are clear, and thus be ambiguity-averse but

[11] The problem is clearer in problems involving finite time and summation, where total utility is taken as the sum of utilities at each time: ^{t} $u(x_t)$, for t = 1, 2, . . . The factor is the time discount factor, and u is assumed concave to represent risk aversion (e.g., Garfinkel, 1991). Consider two options: (1) a 50% chance of getting $10 daily for four days and a 50% chance of nothing for four days; (2) for each of the four days independently, a 50% chance of $10 and a 50% chance of $0. A risk-averse person would choose the second, but the summation formula gives them equal utilities. Solutions have been discussed by Ayman, Hindy and Huang (1992), Huang, Hindy and Kreps (1992), Pratt (1995), and others.

risk-acceptant. Indeed Kuchen (1997) lists several experimental studies that found no correlation in their subjects between the degrees of risk and ambiguity aversion.

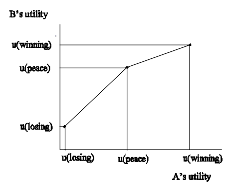
A fifth approach involves spatial models, where each leader has an ideal point over some policy space and has a preference that decreases with the distance of the true situation from the ideal. Risk aversion is introduced by postulating that each country's utility is a concave function of this distance (e.g., Morrow, 1986). These models do not specify an interval-scale measure for the space or for distance, but it can be shown that their assumption of single-peaked preferences in effect determines one almost exactly when sufficiently many decision makers with different ideal points are included (Coombs, 1954).

In summary, spatial models and certain others cannot be criticized on meaningfulness grounds, but they apply in a limited context. Most ways of circumventing the scale problem are unsuccessful.

4 Relative Risk Aversion: A Commodity-Free Definition

Several studies have used a sixth approach to attribute risk attitude, and this section shows that it leads to a solid concept. These involve *contrasting a decision maker's action with what others did in a similar situation, and identifying a party making the more cautious or risky choice as risk-averse or acceptant.* The standard of risk neutrality then becomes someone else's behaviour. A definition can be provided that supports this argument, but it requires some restatement of assumptions and conclusions, since it labels an actor as not risk-acceptant in the absolute sense, but as relatively more risk-acceptant than some other party.

The revised definition goes as follows. Suppose that A and B each face decisions. Their choices may have different utilities and probabilities over the possible outcomes, but the outcomes in one player's set can be paired with those in the other's. A plot is constructed placing each utility for A on the horizontal axis and B's utility for the matched outcome vertically (Figure 3). If the resulting points lie on an increasing straight line, the individuals are **equal in risk attitude**; if they form a one-to-one increasing concave function then B is **more risk-averse** than A; a one-to-one increasing convex function means that B is **more risk-acceptant** than A. ("One-to-one" is defined with respect to the set of each player's utility-values for the outcomes.) Any other result would mean that the risk attitudes are not comparable. The advantage of the definition is that it does not require an external interval-scale goal; the other party's utilities are put on the horizontal axis to play that role. The criterion of risk attitude is the same as before, concavity or convexity, but the difference with the regular definition is shown by the possibility that for a decision over money, one party might be more risk-averse than the other in the relative sense, but both are risk-seeking in the



usual sense.[12] If there are two outcomes for which one party is indifferent while the other has a preference, the parties cannot be compared for risk. To allow a one-to-one function, the set of outcomes should be small, or if it is large or infinite, it should have an ordered structure like amounts of money, that gives its different elements different desirabilities.

The definition requires that the outcomes be matched in a sensible way, "sensible" meaning not arbitrary or idiosyncratic, but reasonably likely to allow a generalization to a lawful regularity. For McDermott's case of the Suez Crisis, Britain's utility for intervening successfully could be associated with the United States' utility for the same action, and so on. There is no restriction that the matched pair of events be similar or even compatible. If two

[12] The connection between this construction and the degree of risk aversion r(x), as defined in footnote 10, has been noted by several authors (e.g., Ross, 1981; Roth and Rothblum, 1982). If B's r(x) is higher than A's for all x, then this curve, which is the function $u_A u_B^{-1}$, will be concave. This measure r(x) presumes an interval scale.

states face a possible war against each other, victory could be matched with victory and defeat with defeat. The events in such a pair cannot both occur, but the arrangement might still be a fruitful way to compare risk attitudes.

A concept's definition should have more than face appeal -- its consequences should also fit our intuitions. The simplest implication of the definition is that if A is more risk-averse than B, then B is more risk-seeking than A. Another rule is that relative risk aversion is transitive: if A is more risk-averse than B, and B more risk-averse than C, then A is more risk-averse than C. This is easy to prove from the definition of concavity. A third appropriate consequence is that the more risk-averse person behaves more conservatively, in a defined sense. Suppose that A and B each have the same choice between a specified sure outcome and a specified gamble, and that A chooses the sure outcome. Then if B is more risk-averse than A according to the definition, B also chooses the sure outcome. The corresponding statement can be made for a B-chooser who is more risk-acceptant than A: when A chooses the gamble, B also chooses the gamble. A related result applies to two persons facing the same choice of a two-outcome gamble or a sure outcome whose utility lies between the two utilities from the gamble. Each person will have a **critical probability** for the gamble, a minimum probability of winning that would be required before choosing the gamble. A relatively more risk-averse person will have a higher critical probability.[13]

A further test of adequacy is whether the concept is more or less constant over various contexts in the world. We do not have a special word for a winged giraffe because we do not need one, and if this concept of risk attitude does not correspond to some regularity, then it has no purpose. A body of research suggests that it is stable in this way, in effect that it represents a personality trait, given a sensible matching rule. The work comes from psychology and management science, and addresses the worry that more risk-acceptant people tend to get involved in crime, automobile accidents, teenage pregnancy, drug abuse, or bad business decisions. Kagan and Wallach (1965) presented college students with imaginary dilemmas facing a third party, and asked for the critical probability of success before they would recommend that the person take a chance. Karen could become a doctor and be sure of monetary success, or go to music school in hopes that she can do what she really loves. What minimum probability of success in music should she require? MacCrimmon and Wehrung

[13] For a two-outcome gamble, consider the ratio of gain over loss, where gain and loss are utility differences with respect to the sure thing. If the ratio is always greater for one person than the other, then the latter is more risk-averse by the definition.

(1986) presented business executives with fictitious memos from the boss, calling for a recommendation in a problem situation. The researchers also asked for reports of subjects' actual risk behaviours: How much insurance do you have? Have you ever left a job without having a new one to step into?[14] Kowert and Hermann (1997) report further studies of this kind.

These questions fit the present relative definition and not the standard one. There is no interval scale on the amount of happiness Karen experiences during a career in music. The question measures the relative risk aversion among the subjects by finding their different responses to the same choice, separating them according to whether their critical probabilities lie above or below that implied in the question. These researchers report an association between risk propensity and traits like openness, sensation-seeking, impulsiveness, age, and position in a company. Other authors, reviewed by MacCrimmon and Wehrung, have concluded that regular patterns of risk propensity emerge if risky situations are clustered by type. People act differently for physical or emotional or monetary danger, but show a regularity within each category. Stable results like these suggest that risk aversion in the relative sense is empirically important.

The relative notion of risk aversion seems to be what some international relations authors had in mind. McDermott contrasted British and French intervention in Suez with the American decision to stay out, and suggested that the former states were risk-acceptant, consistent with prospect theory's prediction. However, more risk-acceptant is different from absolutely risk-acceptant, and the conclusions have to be restated accordingly. Sometimes the comparison is within an individual, when an external event is seen as shifting the person's mental reference point. Farnham compared President Roosevelt's behaviours at different times: before a certain point he would not intervene in the Munich Crisis, then he switched his position. Berejekian (1997) applied this argument to the European Community move to accepting the Montreal Protocol for the protection of the ozone layer, and McDermott used it for President Carter, who came to allow the Shah of Iran to enter the United States. However, one can conclude only that these actors became relatively more risk-acceptant.

The relative concept of risk aversion is significant and applicable, then, and it is also somewhat closer to the everyday usage of the word "risk" than the economics definition. It fits within a common approach in international relations theory, simplified models of *a type* of decision, such as losing a war, winning, or staying at peace, on which different actors are

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[14] The authors also gave subjects choices involving money gambles.

compared. It also fits the idea of several authors cited above of comparing decision-makers with themselves at other times. However, it supports a relative conclusion about risk-aversion, not an absolute one. It connects the empirical findings not to concave utility functions or to prospect theory, but to psychological research on personality.

5 Absolute Risk Aversion: a Commodity-Free Definition

The second definition of risk aversion is quite different from the standard one. The idea was alluded to by di Finetti (1952) but fully stated by Richard (1975). There has been very little theoretical literature on it since (exceptions being Mosler, 1984, and Scarsini, 1985, 1988, 1991), and it has been applied in only a few contexts (e.g., Ingersoll, 1992). Consider two objects that you prefer to the status quo, for example, a sports car and a speedboat. You must choose between the following two gambles:

Receive both the car and boat (probability 1/2) or neither (probability 1/2),

or

Receive only the car (probability 1/2) or only the boat (probability 1/2)?

To make the choice, a first impulse might be to ponder whether the car or the boat is more desirable to you. However, whichever gamble you choose, you will receive the car with probability ½ and the boat with probability ½. Neither gamble gives you a better chance at your favorite item, so your relative preference for the car or boat should not matter. The issue is whether you want to go for both or neither, as in the first gamble, or be sure of getting at least one item, as in the second. Someone who is indifferent is defined as **risk-neutral,** someone who strictly prefers all or nothing is **risk-acceptant**, and someone who wants to be sure of one or the other is **risk-averse.** The standard of risk neutrality, then, involves making decisions purely on the two likelihoods of getting each item, without attaching a special value to getting at least one or getting both.

The definition is applicable when the individual's goal involves separate dimensions. (In the example, the dimensions are simply: car, no car and boat, no boat.) An important requirement is a prespecified ranking on certain combinations of the items, to signify that one involves "having objectively more" than the other, for instance, that acquiring the car is more than acquiring nothing, and getting both is more than getting the boat alone. Other pairs of outcomes are not pre-ranked, e.g., the car alone versus the boat alone. This ranking is necessary to determine which gamble to associate with risk aversion, and just as the standard definition attributes risk aversion to someone *with respect to* a certain interval scale, and the relative one compared parties *with respect to* a certain pairwise matching of outcomes, this one holds *with respect to* this ordering. The definition can easily be extended to several levels on a dimension, which here might mean different kind of cars and boats, or to more than two dimensions.[15]

A dimensional structure is common in foreign policy decisions -- national security is put as a question of guns or butter, and an alliance decision trades off support in one's own conflicts against entanglement in the wars of others. A state under a deterrence threat compares the benefits of pursuing its course against the punishment suffered in reaction. Farnham describes the Munich crisis as dilemma for Roosevelt between preserving the peace as an international goal and dealing with attitudes of isolationism domestically. Such situations are amenable to this concept of risk aversion.

Assuming that the individual is maximizing the expectation of u, this definition of risk aversion is equivalent to the following condition on the chooser's utilities:

$$\tfrac{1}{2}\, u(\text{nothing}) + \tfrac{1}{2}\, u(\text{car boat}) < \tfrac{1}{2}\, u(\text{car}) + \tfrac{1}{2}\, u(\text{boat})$$

or simply, $u(\text{nothing}) + u(\text{car boat}) < u(\text{car}) + u(\text{boat})$,

where x y means objects x and y being acquired jointly. To test for risk aversion, then, one has to know the party's utility function. This seems more restrictive than before where researchers attributed risk attitudes by observing choices, but one point of this paper has been that this was a chimera. Also, if the model put forward is well-specified, the revised definition can be applied within it. Morgan and Palmer (1997, 1998a,b), for example, have developed a "two-good" theory of foreign policy decisions, the goals being "preservaction" and "proaction," defined respectively as maintaining one's security and promoting one's other goals. The 1998b paper gives the decision maker an explicit utility function: for two goods in positive amounts Q_1 and Q_2, it is a product of powers: $u(Q_1, Q_2) = Q_1^a\, Q_2^b$, where a and b are positive. The present definition of absolute risk aversion looks at a lower and higher level on each dimension, corresponding to not having or having the car or boat, so to apply it here one chooses levels $Q_1 < Q_1'$ and $Q_2 < Q_2'$, and compares $u(Q_1, Q_2) + u(Q_1', Q_2')$ with

[15] A general form still would be a lattice, where higher up indicates additional goods in some objective sense. As the standard notion of risk aversion requires concavity, this one requires *submodularity:* u(x) + u(y)   u(x y) + u(x y), where x y and x y are the greatest lower and least upper bounds, respectively, of lattice elements x and y (Birkhoff, 1967).

$u(Q_1', Q_2) + u(Q_1, Q_2')$. If the latter is greater, the chooser is risk-averse. Substituting the given utility function, then, risk aversion requires $Q_1^a Q_2'^b + Q_1'^a Q_2^b > Q_1^a Q_2^b + Q_1'^a Q_2'^b$. In fact, the reverse inequality holds for all allowed values of the quantities and exponents, so the assumed utility function implies universal risk-acceptance.[16] Since the definition uses only the ordinal properties of the scales for the Q's, this conclusion holds for any way of measuring them that is a monotonic with the original. The implication that decision makers are always risk-acceptant goes against the available experimental evidence, as will be discussed, that they tend to be risk-averse for gains and risk-acceptant for losses.[17]

This definition and the standard one are of the absolute variety, and they have another common feature: they portray a risk-seeker as someone who goes for all or nothing. To see this, suppose that an individual might gain a Canadian dollar or a German mark or both, and thinks of these prospects as lying on separate dimensions, i.e., the person does not make the decision by translating one currency into an equivalent amount of the other and adding them together. Risk aversion in the present sense requires:

u($1CDN) + u(1DM) > u(nothing) + u($1CDN 1DM).

If it is assumed that the currencies are worth the same, and that the person comes to treat them as one entity, then decision is one-dimensional, and this requirement becomes:

2 u(1 unit) > u(nothing) + u(2 units).

Dividing by 2 reveals that this condition is the standard economic one where a risk-averse person chooses one unit for certain over an even chance at two units. In general, when both dimensions of the items possess interval scales and these are the same scale, the definitions coincide. The two definitions have a common element: in a risk-seeker's utility function for money, one dollar bill is synergistic with another, and the same is true for the utility function

[16] This can be shown by representing $Q_1'^a$ as $Q_1^a + k_1$ and $Q_2'^b$ as $Q_2^b + k_2$, with positive k_1, k_2, and expanding each side. The condition for risk aversion when there is an interval scale on the quantities is $^2u(Q_1, Q_2)/ Q_1 Q_2$ 0 (Richard, 1975). Applying this test here gives the derivative as $abQ_1^{a-1}Q_2^{b-1}$, which is always positive, implying that the parties are risk-acceptant.

[17] An alternative is the utility function 1 - exp($-aQ_1 - bQ_2$). Applying the derivative test of footnote 16, gives -ab exp($-aQ_1 - bQ_2$) which is negative, implying risk aversion in the commodity-free sense.

for the car and boat. The important difference is that this definition of absolute risk aversion needs no interval scale, and has nothing to do with convexity or concavity.[18]

Luce's Associative Model of Joint Receipts

In line with the principle cited earlier, the definition should correspond to some empirical regularity. A body of evidence has emerged under the name of "joint receipt" (Thaler, 1985; Luce, 1997; see also Luce, 1995, 1996; Cho, Luce and von Winterfeldt, 1994; Cho and Luce, 1995; Luce and Fishburn, 1991, 1995; Fishburn and Luce, 1995). This literature tries to derive a theory of how people evaluate items in combination and to find a scale for the items. As a student, would I rather get 2 B's, or an A and a C? Do I put my passport and wallet in the same pocket? Would I prefer to have two pieces of bad news on the same day or on different days? How about good news? Questions like these elicit predictable answers -- people want to hear pieces of bad news at once and keep good news separated. When the news is mixed but mostly bad, they want to separate the parts and keep the good news as a "silver lining."[19]

Much of the joint receipt experiments involve money and much of the theory makes essential use of its ratio scale. This does not help our purpose, but Luce and Fishburn have developed a version that is free of strong scales on the object of choice (Luce and Fishburn, 1991; Fishburn and Luce, 1995; Luce, 1997). For two items both preferred to the status quo, the following rule is suggested by some reasonable normative arguments and is also consistent with experimental evidence. A decision maker's preferences among joint receipts are represented by a function u satisfying:

$$u(x\ y) = u(x) + u(y) - u(x)\ u(y)/C.$$

[18] The definition is not connected with concavity, either in one or two dimensions, since if both dimensions had interval scales, one transform them with arbitrary monotonically increasing functions f(x) and g(y). This could change concavity of the utility function but not its risk aversion properties by this definition, which is independent of an underlying scale.

[19] Joint receipt is meant to define a utility scale as an alternative method to von Neumann and Morgenstern's constructing gambles. It is an approach that has generated scepticism from some theorists on the grounds that preferences will depend on the particular commodities considered. Suppose we asked subjects about left shoes and right shoes. We would expect everyone to be "risk-acceptant," when clearly their choices are determined by the context. The solution is to use reasonable discretion in choosing the context, as one would in applying any theory. For example, the regularities around adding weights on each side of a pan balance would disappear if one used objects that melted or evaporated. The model is not meant to apply to goods without an obvious physical complementarity.

Here x and y are not numbers but received items, like going on a trip or meeting a friend. The rule relates the utility for receiving two items jointly to the utilities of each singly. It is assumed that C is a positive constant, and u(e) = 0, where e is defined as the situation where neither item is acquired. The values of u in the formula will be positive since the items are preferred to e, and it will be bounded above by C. The combination of two items can be thought of as item in itself, and the model is called "associative" because it satisfies u(x (y z)) = u((x y) z). This means that in a system of three items, if two are combined into one, and this is subsequently combined with the third, then the order of the grouping does not matter.

For the joint receipt of two undesirable items, the u-values will be negative and follow another rule with another positive constant K:

$$u(x\ y) = u(x) + u(y) + u(x)\ u(y)/K.$$

This associative model can be combined with the present definition of absolute risk aversion. However, this requires some care. The condition for risk aversion involves the preference ordering for two 50/50 lotteries, and to predict the decision maker's choice, one might be tempted simply to sum the utility values weighted by .50. The difficulty is that the associative model as presented gives no grounds for doing this. Luce's decision-makers maximize utilities, but not necessarily *expected* utilities. However, the argument will go through, if made less directly. Suitable further axioms lead to a representation of the decision maker's choices by utilities multiplied by probability weights, which are functions of the given probabilities but may also depend on the preference rank of the associated outcome among the possible outcomes for a course of action, or they may depend on the reference point. The definition's test for risk aversion uses binary gambles with symmetrical probabilities, so the probability weights will be equal, and the definition's criterion for risk aversion will be:

$$u(x) + u(y) > u(e) + u(x\ y)$$

Substituting u(e) = 0 and Luce's formula for u(x y) in the positive case, gives

$$u(x) + u(y) > u(x) + u(y) - u(x)u(y)/C.$$

Since C and the u terms are positive, this condition is always satisfied. The reverse inequality holds for negative items, so in all, people can be described as risk-averse for gains and risk-seeking for losses. The form of prospect theory's conclusion is reproduced. One can check this prediction empirically by the kind of experiments familiar in prospect theory, where a reversal of preference is induced by manipulating the status quo point from getting nothing to car boat, and asking people how they would choose among prospects of the items being taken away.[20]

The model can generate predictions about when decisions will be faced, if there is a choice about timing. Individuals will prefer to receive gains at separate times, assuming the benefits from combining or separating them in time are respectively u(x y) and u(x) + u(y), since the latter is greater. The extension of the model to the joint receipt of a positive and negative good suggests that decision makers will separate a small gain from a larger loss (keep a silver lining), and combine a small loss with a larger gain. Thus, if a set of unpleasant or dangerous tasks has to be done now or later, a leader will tend to undertake them together. If bad news arrives, and there is a further distasteful decision that has to be faced sooner or later, there will be a tendency to take on the second one and get it over with.

Compared to the first revised definition, then, this one is closely connected to prospect theory. It fits well in Luce's model, which includes many prospect theory elements: dependence on a reference point, different risk attitudes for losses and gains, and possibly loss aversion. It raises an issue that is important in leaders' decision making but is not usually seen as connected to risk attitude: the timing of decisions that are seen as inevitable.

6 The Metaphor of Risk as a Substance

These definitions clarify the nature of risk. They eliminate the ephemeral element of the interval scale, and show it in its essence. The economics theory moves partway in this

[20] For outcomes mixing gains and losses the rules are

u(x y) = [Ku(x) + Cu(y)]/[K + u(y)] when x, x y e y, and

u(x y) = [Ku(x) + Cu(y)]/[C - u(x)] when x e x y, y.

(Here "x y" is interpreted as the decision maker preferring or being indifferent to x over y.) These rules are more complex, but are necessary to preserve associativity. Prospect theory's notion of loss aversion can be interpreted in Luce's model: suppose that x is the event of gaining a specified good and x' is the event of having it taken away. Since x x' = e, then u(x x') = 0, and either formula above gives Ku(x) + Cu(x') = 0, or u(x') = -K/C u(x). Loss aversion thus means K > C.

direction by making risk aversion an abstract property of the utility function. A person on the street might connect risky behaviour with physical danger -- bungee jumping or driving too fast -- but the economics definition regards these as surface aspects, and makes it a matter of one's utilities, whatever they may be for. To look at risk in a still purer form, these definitions eliminate the interval-scale commodity.

Risk attitude has become associated with a certain metaphor, in the sense that our way of talking and thinking about it tends to map it into a more familiar domain (Lakoff, 1987). The metaphor around risk treats it as an external substance that has a quantity and is divisible. Risk is "shared" or "transferred." We "bear" it or "manage" it. We are "attracted" to it or "avoid" it, as we might be to heavy metal music or anchovies on our pizza. Like anchovies, we talk of "taste" for risk. It is common for a substance metaphor to arise around an abstract pattern of behaviour, as when nations "acquire" prestige, people "contain" their anger, or "share" their love.

Risk-as-a-substance is a handy way of thinking, but it can be misleading. It promotes the thought pattern that this paper is arguing against. In the metaphorical way of thinking, risk is an autonomous entity that one can like or dislike. A course of action brings one risk and possibly the goal as well, as if these were two commodities. The metaphor conceals the fact that the decision is entirely in pursuit of the goal, and risk behaviour depends on how one measures that objective and its interaction with uncertainty. The present argument can be seen as involving a *reductio ad absurdum* of the metaphor. The investment example described earlier showed that someone could be risk-averse or risk-seeking depending on the scale used for the goal. Mapping this example into the metaphor suggests that I might prefer anchovies when the pizza is measured by diameter, but dislike them when it is measured by area. The fact that this makes no sense shows that the risk-as-a-substance metaphor does not really fit. These alternative definitions give ways to think about the subject without these misleading consequences.

7 Conclusions

Some international relations theory is aimed at specific policy questions, such as how to implement deterrence or whether larger powers should intervene in ethnic conflicts. Studies of risk aversion have had a different purpose, to develop a basic understanding of decision making, in hopes that insights from theory and laboratory experiments can be transferred up to states. The great interest in prospect theory comes from the hope that now the discipline has an account of decision making with empirically validity. The work has been useful, strengthening the connection of international relations with psychology, and introducing the order of a coherent theory. The subsequent frustration expressed by many writers may come from the fact that some of its aspects suit economics but not international relations. This paper tries to delineate just which parts can be imported and to argue for dropping the rest. The empirical research on joint receipts is small as yet and the theory tends to be mathematical and inaccessible, but the approach seems more promising, as it is less bound by inappropriate assumptions.

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