Risk Aversion, Intertemporal Substitution, and the Term Structure of Interest Rates

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Abstract

An equilibrium model of the term structure of interest rates is derived from a representative agent framework with recursive utility preferences. A key ingredient is a time-varying subjective discount factor, which is linked to the short-term rate of interest. The proposed model incorporates a vector-autoregression description of macroeconomic dynamics and links them to those of the term structure so that nominal bond yields are affine functions of observable state variables. The model is estimated and compared to both a restricted expected-utility version of the model and a reduced-form no-arbitrage model. We find that all three models can fit the term structure equally well, but that only the unrestricted non-expected utility model can empirically account for the hump-shaped pattern in the term structure of volatilities. Further, the non-expected utility model is the only one that accounts for the tent-shaped pattern and magnitude of coefficients from predictive regressions of excess bond returns on forward rates—documented by Cochrane and Piazzesi [2005. Bond Risk Premia. American Economic Review 95, 138–160]. The unrestricted equilibrium model fits the term structure and captures the important features of the yield curve with economically plausible values for the structural preference parameters.

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1 Introduction

Traditional models of the term structure of interest rates are formulated in continuous time and in an arbitrage-free framework. Bond yields are affine functions of a number of unobserved state variables that capture the uncertainty present in the economy. When three factors are specified, they are often interpreted as the level, slope, and curvature of the yield curve, following Litterman and Scheinkman (1991). Dai and Singleton (2003) and Piazzesi (2005) provide thorough surveys of this class of models.

Recently, several researchers have added observable macroeconomic variables to the latent factors to try to understand the channels through which the economy influences the term structure, and not simply describe or forecast the movements of the term structure. Ang and Piazzesi (2003) and Ang, Dong, and Piazzesi (2004) introduce measures of inflation and real activity as macroeconomic factors. The joint dynamics of these macro factors and the latent factors are captured by vector-autoregression (VAR) models, where identifying restrictions are based on the absence of arbitrage. More structural models have also been proposed to explore the dynamic interaction between the macroeconomy and the term structure.[2]

In these models based on the absence of arbitrage, risk premiums for the various sources of uncertainty are obtained by specifying time-varying prices of risk that transform the risk-factor volatilities into premiums. The prices of risk, however, are estimated directly from the data without accounting for the fact that investors' preferences and technology may impose some constraints between these prices.

In this paper, we price bonds in a representative agent framework with recursive utility preferences. The traditional power utility model is restrictive in that it makes the elasticity of intertemporal substitution (EIS) the reciprocal of the coefficient of relative risk aversion (CRRA). This means that if investors are extremely risk averse, then with power utility they must also be extremely unwilling to substitute intertemporally. An attractive feature of the recursive utility model is that it separates the CRRA from the EIS. Yet, as Campbell (1999) explains, recursive utility preferences are not enough to solve the equity premium puzzle,[3] since there is direct evidence for a low EIS in consumption. Gregory and Voss

[2]Models with more macroeconomic structure have been proposed recently by Hördahl, Tristani, and Vestin (2006), Rudebusch and Wu (2004), and Bekaert, Cho, and Moreno (2003). These models combine the affine arbitrage-free dynamics for yields with a New Keynesian macroeconomic model, which typically consists of a monetary policy reaction function, an output equation, and an inflation equation. Diebold, Rudebusch, and Aruoba (2006) propose a dynamic Nelson-Siegel empirical model of the term structure, complemented by a VAR model for real activity, inflation, and a monetary policy instrument.

[3]The equity premium puzzle of Mehra and Prescott (1985) is that the risk premium on equity is too high to be consistent with observed consumption behavior, unless investors are extremely risk averse.

(1991) further show that these preferences do not offer a solution to the bond premium puzzle either.[4] As both Campbell (1999) and Gregory and Voss (1991) state, it is not easy to construct an equilibrium model that captures the important features of asset prices with plausible values for the structural preference parameters.

Melino and Yang (2003) generalize the standard recursive utility framework by allowing the representative agent to display state-dependent preferences and show that such preferences can account for moments on equity and the risk-free rate. In order to explain the term structure of interest rates, however, we need only to allow for a time-varying subjective discount factor; CRRA and EIS remain time-invariant and can thus be deemed structural in our framework. Preferences with time-varying rates of time preference were introduced by Uzawa (1968), and have been extended by Epstein (1983, 1987). Those preferences specify the subjective discount factor as a function of consumption, so that the marginal utility of consumption in a given period can vary with consumption in other periods. A key ingredient of our model is that the time-varying subjective discount factor is linked to the short-term rate of interest. As in Obstfeld (1990), our model implies that consumption and asset prices depend on the short-term rate of interest. That link is also motivated by the central role played by the short rate in the determination of bond prices. Indeed, most models in the bond pricing literature find a way to introduce the short-term rate in the pricing kernel, including the popular bond pricing models of Vasicek (1977) and Cox, Ingersoll, and Ross (1985). Those models are special cases of a larger class of affine term structure models (Duffie and Kan 1996 and Dai and Singleton 1999), in which the pricing kernel is a function of multiple factors, in addition to the short rate.

Our model incorporates a VAR description of macroeconomic dynamics and links them to those of the term structure so that nominal bond yields are affine functions of observable state variables. We start by estimating a first-order VAR comprising the short-term rate of interest, the five-year term spread, a measure of the return on the market portfolio, the rate of inflation, and the rate of consumption growth. We use a sample of quarterly data from the third quarter of 1959 to the last quarter of 2004. Given the parameter estimates of the VAR, we can estimate the preference parameters and other crucial parameters for risk premiums by minimizing the least-square distance between the observed market yields and the model-implied yields.

For comparison purposes, we also estimate a restricted expected-utility version of our model and a reduced-form no-arbitrage model similar to that used by And and Piazzesi

[4]Backus, Gregory, and Zin (1989) and Donaldson, Johnsen, and Mehra (1990) document a bond premium puzzle: the representative agent model with power utility can account for the average risk premiums in holding-period bond returns and forward rates only with implausibly large values of the coefficient of relative risk aversion.

(2003) and Ang, Piazzesi, and Wei (2006). In the latter, the authors use an approach that is similar to the method just described, but in a no-arbitrage framework.[5] They first estimate the VAR and use the estimated parameters together with the no-arbitrage bond-yield formulas to estimate the prices of risk that minimize a distance between the theoretical yields and the observed yields. By using the same VAR for the macroeconomic variables, we are able to assess the relative contributions of the different modeling strategies. The comparison is especially interesting since both strategies specify bond prices as affine functions of the state variables.

We find that all three models can fit the term structure equally well. When we assess the in-sample fit, and compute variance decompositions and out-of-sample pricing errors, the three estimated models are found to perform similarly well. The estimated preference parameters are economically plausible: in the non-expected utility model, the CRRA is around 6 and the EIS is around 0.36.

Statistical tests of the expectations hypothesis conclude that bond risk premiums vary with the shape of the yield curve and that excess bond returns are indeed predictable. In particular, Cochrane and Piazzesi (2005) run predictive regressions of one-year excess returns on forward rates and find that the forecasts are highly significant. Cochrane and Piazzesi find a robust tent-shaped pattern of slope coefficients for all maturities, with regression R^2 around 35%. This violation of the expectations hypothesis extends the classic regressions of Fama and Bliss (1987) and Campbell and Shiller (1991). Fama and Bliss found that the spread between the n-year forward rate and the one-year yield predicts the one-year excess return of the n-year bond, with R^2 around 18%. Campbell and Shiller found similar results forecasting yield changes with yield spreads. Cochrane and Piazzesi's findings substantially strengthen that evidence against the expectations hypothesis. Most important, they show that the same linear combination of forward rates predicts bond returns at all maturities, while Fama and Bliss and Campbell and Shiller relate each bond's expected excess return to a different forward spread or yield spread.

When we analyze the risk premiums implied by each estimated model, we find that only the unrestricted version of our equilibrium model can account for the violations of the expectations hypothesis documented by Cochrane and Piazzesi (2005). The restricted expected-utility model and the reduced-form no-arbitrage model cannot account for the tent-shaped pattern and magnitude of coefficients from predictive regressions of excess bond returns on forward rates. The non-expected utility model also produces mean slope coefficients with a downward pattern across maturities and the actual coefficients are well

[5] Ang, Piazzesi, and Wei (2006) propose such a sequential estimation strategy. What we gain in flexibility by proceeding in such a sequential manner, we may lose in efficiency of the estimators. Joint estimation is possible, but will add a significant layer of complexity.

covered by the respective confidence intervals.

The recent paper by Piazzesi and Schneider (2006) is certainly the closest to our paper. They also derive equilibrium yield curves under recursive preferences, but their approach differs in several respects. First, they express the pricing kernel in terms of news about future consumption instead of a proxy return for the market portfolio as we do. They specify an exogenous state-space system for consumption growth and inflation and set values for the preference parameters in order to infer the equilibrium yields. We estimate the parameters of a VAR system including consumption growth and inflation and the preference parameters that rationalize the observed yields. In this sense, we follow more closely the approach that has been used in no-arbitrage models by Ang, Piazzesi and Wei (2006), for example. In this case, they extract the prices of risk that rationalize the observed yields.

Several authors have shown the limitations of the traditional consumption-based capital asset pricing model (CCAPM) with expected utility in representing the historical co-movements of consumption and returns on bonds. Campbell (1986a) explores the relation between bond risk premiums and the time-series properties of consumption in a similar model. He shows that positive serial correlation in consumption growth imparts a downward slope to the yield curve. Boudoukh (1993) considers a model with power utility, but where consumption growth and inflation are determined by a heteroskedastic VAR. Boudoukh finds that heteroskedasticity in consumption growth and inflation is not strong enough to generate the predictability of excess bond returns found in the data. In Piazzesi (2005), affine general-equilibrium models are specified with preference shocks that are related to state variables, as in Campbell (1986b) and Bekaert and Grenadier (2003).

Wachter (2006) also proposes a consumption-based model of the term structure of interest rates, where nominal bonds depend on past consumption growth through habit, and on expected inflation. This model is essentially the same as the habit model of Campbell and Cochrane (1999), but the sensitivity function of the surplus consumption to innovations in consumption is chosen so as to make the risk-free rate a linear function of the deviations of the surplus consumption from its mean. Moreover, Wachter calibrates her model so as to make the nominal risk-free rate in the model equal to the yield on a three-month bond at the mean value of surplus consumption.

The rest of this paper is organized as follows. Section 2 describes the equilibrium model with recursive utility preferences that will be used to price bonds. We also specify the dynamics of the macroeconomic variables that will influence the yields. Section 3 is dedicated to model estimation and evaluation. We specify the benchmark no-arbitrage model, the data sources, and the econometric method used to estimate the parameters and ultimately to compute the yields. We report the pricing errors for the various specifications as well as variance decompositions and out-of-sample forecasts. Section 4 presents the empirical implications for the term structure of volatilities and the analysis of risk premiums. Section 5 offers some conclusions.

2 Equilibrium Model

The recursive utility model suggested by Epstein and Zin (1989) and Weil (1989) allows for a constant Arrow-Pratt CRRA that can differ from the reciprocal of the EIS. In so doing, that framework provides a partial separation of attitudes toward risk from preferences over deterministic consumption paths. Melino and Yang (2003) generalize the standard recursive utility framework by allowing the representative agent to display state-dependent preferences and show that such preferences can account for moments on equity and the risk-free rate. In order to explain the term structure of interest rates, however, we need only to allow for a variable rate of time preference; CRRA and EIS remain time-invariant and can thus be deemed structural in our framework. As in the standard framework, we consider an infinitely lived representative agent who receives utility from the consumption of a single good. In any period t, current consumption is deterministic but future consumption is uncertain. The agent's lifetime utility is characterized by

$$U_t = (C_t^{\rho} + \beta_t \mu_t^{\rho})^{\frac{1}{\rho}}, \tag{1}$$

where $0 < \beta_t$ is the time-varying subjective discount factor and $\mu_t = E_t[\widetilde{U}_{t+1}]$ is a certainty equivalent of random future utility, $\widetilde{U}_{t+1}$, given the information available to the agent at time t. The way the agent forms the certainty equivalent of random future utility is based on risk preferences, which are assumed to be isoelastic; i.e., $\mu_t^{\alpha} = E_t[\widetilde{U}_{t+1}^{\alpha}]$. Melino and Yang show that, as in the standard recursive utility case, $\alpha \leq 1$ can be interpreted as a relative risk aversion parameter with the degree of risk aversion increasing as α falls ($1 - \alpha$ is the CRRA). The parameter ρ can be interpreted as reflecting substitution, since $1/(1 - \rho)$ is the EIS.

The stochastic discount factor (SDF) (or pricing kernel or, in equilibrium, the intertemporal marginal rate of substitution) used by the agent to discount future payoffs to determine current asset prices is expressed as

$$m_{t+1} = \beta_t^{\gamma} \left(\frac{C_{t+1}}{C_t} \right)^{\gamma(\rho-1)} (R_{t+1})^{\gamma-1}, \tag{2}$$

where R_{t+1} is the one-period gross rate of return on the market portfolio and $\gamma = \alpha/\rho$. Equation (2) shows that the SDF is a geometric weighted average of the rate of growth of consumption and the rate of return on the market portfolio. Market prices can then be expressed by the expected-value relation

$$p_t = E_t[m_{t+1} g_{t+1}], \tag{3}$$

where p_t is the asset price and g_{t+1} is the asset's future payoff. Note that the quantity in (2) is a strictly positive random variable that must satisfy (3).

The basic asset pricing equation can also be written as $1 = E_t[m_{t+1} r_{t+1}]$, where $r_{t+1} = g_{t+1}/p_t$ defines gross returns. Gross returns can be defined either in nominal or real terms; correspondingly, the SDF must then be expressed in nominal or real terms. In nominal terms, the SDF in (2) becomes

$$m^{\$}_{t+1} = \beta_t^{\gamma} \left(\frac{C_{t+1}}{C_t} \right)^{\gamma(\rho-1)} (R_{t+1})^{\gamma-1} \left(\frac{P_{t+1}}{P_t} \right)^{-1}, \tag{4}$$

where P_{t+1}/P_t is the gross rate of inflation between periods t and $t+1$; P_t is the nominal price index at time t. Let $r_t = \log R_t$ represent the logarithm of the return on the market portfolio, $\pi_t = \log P_t/P_{t-1}$ the rate of inflation, and $c_t = \log C_t/C_{t-1}$ the rate of consumption growth.

Preferences with time-varying rates of time preference were introduced by Uzawa (1968), and have been extended by Epstein (1983, 1987). Those preferences specify the subjective discount factor as a function of consumption, so that the marginal utility of consumption in a given period can vary with consumption in other periods. This type of preferences has been applied to problems in international trade by Calvo and Findlay (1978), Obstfeld (1981), Mendoza (1991), Uribe (1997), and Schmitt-Grohé (1998). A further study includes Bergman (1985), where the implications of such preferences for the CAPM are examined. Obstfeld (1990) presents a general class of recursive utility functions, where the rate of time preference is a function of the interest rate. In our nominal model, the subjective discount factor is linked to an exogenously determined risk-free rate of interest via the key restriction

$$\gamma \log \beta_t = -y_t^{(1)}, \tag{5}$$

where $y_t^{(1)}$ is the log yield on a one-quarter bond; i.e., one period is a quarter in our discrete-time yield curve model. Note that (5) implies that β_t^{γ} takes values between zero and one, since it equals the price of the one-quarter bond. As in Obstfeld (1990), our model with a variable rate of time preference implies that consumption and asset prices depend on a short-term rate of interest. The restriction in (5) might give the impression that the model will admit arbitrage opportunities. We will see that the SDF in (4) with (5) coupled with an affine specification ensures that the resulting bond prices remain arbitrage-free.

Further, if not more important, the restriction in (5) is motivated by the central role played by the short-term rate of interest in the determination of bond prices. Indeed, the short rate is a fundamental building block for yields of other maturities, which are just risk-adjusted averages of expected future short rates. Typically, bond pricing models are formulated as affine functions of a number of state variables that capture the uncertainty present in the economy. When three latent factors are specified, they are often interpreted as the level, slope, and curvature of the yield curve, following Litterman and Scheinkman (1991). At a quarterly frequency, the first principal component of yields accounts for 97.2% of the variation of yields and that first principal component has a −95.6% correlation with the short rate (Ang, Piazzesi, and Wei 2006). Obviously, a model misspecification of this quantity leads to considerable pricing errors. Therefore most models in the bond pricing literature find a way to introduce the short-term rate in the SDF. Two of the most popular bond pricing models are those by Vasicek (1977) and Cox, Ingersoll, and Ross (1985) (CIR). Each of these models has a single factor, typically associated with the short rate. For example, Bansal and Zhou (2002) introduce the short rate by assuming that the log return on the asset that delivers the consumption stream (in a standard consumption-based asset pricing model) follows a CIR process and by using the fact that the conditional mean of the SDF is equal to the price of the risk-free, one-period discount bond. The Vasicek and CIR model are special cases of a larger class of affine term structure models (Duffie and Kan 1996 and Dai and Singleton 2000) The SDF in these models is a function of multiple factors, in addition to the short rate. Ang, Piazzesi, and Wei (2006) use the short rate as a proxy for the latent level factor of the yield curve, which is then used with other observable factors to price bonds at longer maturities. Bekaert and Grenadier (2003) use moments of the nominal short rate to calibrate the moments of the latent level factor in various arbitrage-free and equilibrium models, while Wachter (2006) extends the external habit model of Campbell and Cochrane (1999) by making the interest rate a function of surplus consumption. The average level of the short interest rate in the model is set equal to its sample counterpart.

As in Ang, Piazzesi, and Wei (2006), our model is based entirely on observable factors, which we collect in a state vector X_t. Both macroeconomic variables and yield curve factors are included in the state vector. Ang, Piazzesi, and Wei (2006) argue that two yield curve factors are sufficient to model the dynamics of yields at the quarterly frequency. Following those authors, we use the short rate, $y_t^{(1)}$, to proxy for the level factor of the yield curve and the five-year term spread, $y_t^{(20)} - y_t^{(1)}$, to proxy for the slope factor of the yield curve. The term spread has a −86.5% correlation with the second principal component of yields. Adding the second principal component brings the percentage of yield-curve variation to 99.7%. Althought the term structure factors can explain the yield curve, that does not necessarily mean that they can explain risk premiums. Indeed, we will see that the macroeconomic factors and the restricted way in which they enter the SDF play a key role in explaining violations of the expectations hypothesis. The macroeconomic factors are collected along with two term structure factors in the state vector so that $X_t = (y_t^{(1)}, y_t^{(20)} - y_t^{(1)}, r_t, \pi_t, c_t)'$. The vector of state variables follows a first-order VAR process:

$$X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t, \tag{6}$$

where the errors are normally distributed with mean zero and $E[\varepsilon_t \varepsilon_t'] = \Sigma\Sigma'$. The logarithm of the nominal SDF can be written as

$$\log m_{t+1}^{\$} = -y_t^{(1)} + J X_{t+1}, \tag{7}$$

where $J = (0, 0, \gamma - 1, -1, \alpha - \gamma)$. We will see that an affine structure ensures the identification of the corresponding equations in the state VAR process, even though the vector J contains zeros.

The time-t price of a nominal bond that pays one dollar at time $t + n$ is determined by the recursive relation

$$P(t, n) = E_t \left[m_{t+1}^{\$} \times P(t + 1, n - 1) \right],$$

with the terminal condition $P(t + n, 0) = 1$. Note that when $n = 1$, the SDF in (7) will satisfy the usual relationship

$$r_t^f = 1/E_t[m_{t+1}^{\$}],$$

where r_t^f is the gross risk-free rate of interest. Bond prices are parameterized as exponential linear functions of the state vector so that

$$P(t, n) = \exp\left(A(n) + B(n)' \times X_t\right), \tag{8}$$

for a scalar $A(n)$ and a 5×1 vector $B(n)$ of coefficients that are functions of the time-to-maturity n. Solutions for those coefficients are based on the assumption that $m_{t+1}^{\$} \times P(t + 1, n - 1)$ is conditionally log-normal and the associated moments:

$$\begin{aligned}
E_t \left[\log m_{t+1}^{\$} \right] &= -y_t^{(1)} + J(\mu + \Phi X_t), \\
E_t \left[\log P(t + 1, n - 1) \right] &= A(n - 1) + B(n - 1)' \times (\mu + \Phi X_t), \\
Var_t \left[\log m_{t+1}^{\$} \right] &= J\Sigma\Sigma' J', \\
Var_t \left[\log P(t + 1, n - 1) \right] &= B(n - 1)' \times \Sigma\Sigma' \times B(n - 1), \\
Cov_t \left[\log m_{t+1}^{\$}, \log P(t + 1, n - 1) \right] &= B(n - 1)' \times \Sigma\Sigma' J'.
\end{aligned}$$

More precisely, bond prices are given by (8) with coefficients $A(n)$ and $B(n)'$ determined by the backward recursions

$$\begin{aligned} A(n+1) &= A(n) + [J + B(n)']\mu + \tfrac{1}{2}[J + B(n)']\Sigma\Sigma'[J' + B(n)], \\ B(n+1)' &= [J + B(n)']\Phi - e_1, \end{aligned} \tag{9}$$

where $e_1 = (1, 0, 0, 0, 0)$. The initial conditions are $A(1) = 0$ and $B(1)' = -e_1$. The difference equations in (9) that determine $A(n)$ and $B(n)'$ are derived by induction, exactly as in Ang and Piazzesi (2003).

The inclusion of the two term structure factors $y_t^{(1)}$ and $y_t^{(20)} - y_t^{(1)}$ in the state vector implies that the model prices the one- and twenty-quarter bonds without error. The first set of these internal consistency constraints is given by the initial conditions for the recursive definitions of the coefficients $A(n)$ and $B(n)$. The second set of constraints is

$$\begin{aligned} A(20) &= 0, \\ B(20) &= -20(e_1 + e_2), \end{aligned} \tag{10}$$

where e_i is a 5×1 vector of zeros with a 1 in the ith element. These constraints ensure that the twenty-quarter yield is the sum of the first two factors in X_t. The other yields are then functions of $y_t^{(1)}$ and $y_t^{(20)} - y_t^{(1)}$ and the other factors included in X_t. The yields not included as factors are thus subject to a sampling error.

The second equation of the backward recursions in (9) features the product $J\Phi$, which might give the impression that the short rate and the term spread cannot be identified via bond prices. The initial conditions, however, ensure the identification of the short-rate equation in the state VAR process. To see that the term spread is also identified, note that when the persistence matrix Φ admits an inverse, the factor loadings can be written as a forward recursion:

$$B(n)' = [B(n+1)' + e_1]\,\Phi^{-1} - J, \tag{11}$$

with the terminal conditions in (10). This recursion is mathematically equivalent to the one in (9) and makes clear that the term-spread equation is statistically identified.

The bond pricing equation in (8), along with the coefficients in (9), provides a characterization of the entire yield curve. In particular, it describes the joint dynamics of bond yields of various maturities and the vector of state variables. The model-implied yield on a continuously compounded n-period zero-coupon bond, $Y(t,n) = -\log P(t,n)/n$, is an affine function of the state vector:

$$Y(t,n) = -\frac{A(n)}{n} - \frac{B(n)'}{n} X_t. \tag{12}$$

From the bond pricing equation, the time-t model-implied forward rate which applies between times n and $n+s$ ($s \geq 1$), $F(t,n,s) = (\log P(t,n) - \log P(t,n+s))/s$, can be computed as

$$F(t,n,s) = -\frac{[A(n+s) - A(n)]}{s} - \frac{[B(n+s)' - B(n)']}{s} X_t, \tag{13}$$

and the short rate expected to prevail at time $t+n$ is given by

$$E_t[y_{t+n}^{(1)}] = e_1 \left[\sum_{i=1}^{n} \Phi^{n-i}\mu + \Phi^n X_t \right], \tag{14}$$

where Φ^0 is set equal to the 5×5 identity matrix.

The expressions in (13) and (14) show that in addition to the preference parameters in J, the persistence matrix Φ also plays a crucial role for the risk premiums. Indeed when $\Phi = 0$, we have $B(n) = -e_1'$, for all n, and $-[A(n+1) - A(n)] = -(J - e_1)\mu - \frac{1}{2}(J - e_1)\Sigma\Sigma'(J' - e_1')$. We further have $E_t[y_{t+n}^{(1)}] = e_1\mu$ in this i.i.d. case. So when there is no systematic risk because the variables in the state vector X_t are i.i.d., the expectations hypothesis holds since the forward rate equals the expected future short rate plus a constant risk premium—given by $-J - \frac{1}{2}(J - e_1)\Sigma\Sigma'(J' - e_1')$.

With the restriction $\alpha = \rho$, the model reduces to an expected utility model albeit except for the time-varying subjective discount factor. The expected utility version implies a separable time-additive preference structure, since the short-term rate of interest is exogenous; i.e., the subjective discount factor does not depend on consumption choices. In that case, the CRRA is the reciprocal of the EIS and the return on the market portfolio plays no contemporaneous role in the SDF. The next section presents and empirical assessment of the equilibrium model and the role played by the macroeconomic factors in explaining risk premiums.

3 Model Estimation and Evaluation

3.1 Benchmark model

The described equilibrium model links the dynamics of the term structure of interest rates to macroeconomic variables. Ang and Piazzesi (2003) also establish such a link through a reduced-form model of the term structure. For comparison, their approach is used here to derive a reduced-form bond pricing equation given the same specification of state variables used for the equilibrium-based model.

The approach assumes that the nominal SDF follows a conditionally log-normal process of the form

$$\log m_{t+1}^{\$} = -y_t^{(1)} - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}, \tag{15}$$

where λ_t are time-varying market prices of risk. The vector λ_t is parametrized as an affine process:

$$\lambda_t = \lambda_0 + \lambda_1 X_t, \tag{16}$$

so that λ_0 is 5×1 vector and λ_1 is a 5×5 matrix. Equations (15) and (16) relate shocks in the state VAR process to the SDF and therefore determine how those factor shocks affect all yields. The implied no-arbitrage bond yields are given by

$$Y^{na}(t,n) = -\frac{A^{na}(n)}{n} - \frac{B^{na}(n)'}{n}X_t. \tag{17}$$

where the coefficients $A^{na}(n)$ and $B^{na}(n)'$ are defined recursively by

$$\begin{aligned} A^{na}(n+1) &= A^{na}(n) + B^{na}(n)' \times (\mu - \Sigma\lambda_0) + \tfrac{1}{2}B^{na}(n)' \times \Sigma\Sigma' \times B^{na}(n), \\ B^{na}(n+1)' &= B^{na}(n)' \times (\Phi - \Sigma\lambda_1) - e_1, \end{aligned} \tag{18}$$

with $e_1 = (1,0,0,0,0)$ and initial conditions $A^{na}(1) = 0$ and $B^{na}(1)' = -e_1$, as before. See Ang and Piazzesi (2003) for additional details.

The definition of the SDF in (15) makes clear the role of λ_0 and λ_1 for risk premiums in the reduced-form model. When $\lambda_0 = 0$ and $\lambda_1 = 0$, there are no risk premiums and a local version of the pure expectations hypothesis holds. In this case, the price of an n-period bond is $P^{na}(t,n) = E_t[\exp(-\sum_{i=1}^{n} y_{t+i}^{(1)})]$, so that apart from some Jensen-inequality terms, long-term rates are simply the expected value of average future short-term rates. When $\lambda_1 = 0$, market prices of risk do not depend on Y_t and the risk premium is constant.

3.2 Data description

The macroeconomic fundamentals VAR model is estimated using data on U.S. nominal interest rates, equities, inflation, and real consumption. Although the raw data are available at the monthly frequency, we follow Campbell and Viceira (2001) and Wachter (2006) and construct a quarterly data set in order to reduce the influence of higher-frequency noise in inflation and short-term movements in interest rates. As Wachter states, higher-frequency interest-rate fluctuations would seem difficult to explain using an equilibrium model with macroeconomic variables.[6]

[6] Another important consideration is the computational cost involved. Indeed, the depth of recursions when computing (9) and (18) with monthly data prohibits a thorough exploration of the parameter space.

Real aggregate consumption is based on personal consumption expenditures on nondurables and services obtained from the Bureau of Economic Analysis. Per capita consumption is obtained by dividing the real aggregate consumption by the total population. The level of the market portfolio is proxied using a value-weighted index of stocks, including dividends, traded on the NYSE, AMEX, and NASDAQ markets obtained from the Center for Research in Security Prices (CRSP). For inflation, we use data on the Consumer Price Index (CPI) obtained from the Federal Reserve Bank of St. Louis. The level data on real per capita consumption, the stock index, and the CPI were aggregated up to the quarterly frequency by averaging the monthly observations. The return on the market portfolio, the rate of inflation, and the growth rate of consumption were then defined as the changes in the (log) values of the corresponding level data. The bond data consist of a set of monthly zero-coupon yields obtained from CRSP. These monthly yields were averaged to obtain quarterly yields on bonds with maturities of 1, 2, 4, 8, 12, 16, and 20 quarters. These data definitions ensure that the yields incorporate information about the rates of inflation, consumption growth, and market return throughout the quarter. The quarterly data set has 182 observations from 1959Q3 to 2004Q4.

Table 1 provides summary statistics of the yield data at the quarterly frequency. As usual, the yield curve slopes upward on average. Further, the standard deviation, skewness, and kurtosis tend to be higher for shorter bond maturities.

3.3 Estimation methodology

Following Ang, Piazzesi, and Wei (2006), we adopt a consistent two-step procedure to estimate the model parameters. For the reduced-form model, we estimate in a first step the VAR parameters μ, Φ, and Σ by least squares. In the second step, we estimate the parameters that determine the market prices of risk (λ_0 and λ_1) given the estimates of the VAR parameters from the first step.[7] This is done by solving the non-linear least-squares problem:

$$\min_{\{\lambda_0,\lambda_1\}} \sum_{T} \sum_{N} (y_t^{(n)} - Y^{na}(t,n))^2, \tag{19}$$

where $y_t^{(n)}$ is the market yield of an n-period bond at time t and $Y^{na}(t,n)$ is the corresponding model-implied yield; the first summation is over available time observations and the second summation is over the yields used to estimate the model. Minimization was done with the Nelder-Mead simplex algorithm, and once the optimum was found, the

[7] Of course, this step-by-step estimation methodology does not deliver the most statistically efficient estimates. On the other hand, its computational simplicity is a considerable advantage, especially when the models need to be updated on a regular basis.

covariance matrix was estimated using numerical derivatives of the non-linear regression function with respect to the vector of parameters.[8]

A similar two-step procedure is used to estimate the parameters of the equilibrium-based model, thereby ensuring a meaningful comparison across specifications. As explained above, the two preference parameters (γ and α) and the 5×5 matrix Φ play a crucial role for the equilibrium risk premiums. To emphasize this point, note that if the data were actually generated according to the local expectations hypothesis, we would expect to find statistically insignificant values of λ_0 and λ_1 in the reduced-form model. On the other hand, we would expect to find insignificant values of Φ if the equilibrium-based model was taken to the yields data under the expectations hypothesis. For this reason, estimation of the equilibrium-based model takes only the least-squares estimate of μ and Σ as given. The second step solves the non-linear least-squares problem with respect to the CRRA$= 1 - \alpha$, the EIS$= 1/(1 - \rho)$, where $\rho = \alpha/\gamma$, and the persistence matrix Φ, subject to the constraints in (10). In other words, we let the bond market data tell us whether risk premiums are time-varying.

3.4 Estimation results

Estimation results for the equilibrium model are reported in Table 2, along with 95% confidence interval for each parameter. The table reports results for both the non-expected utility case and the expected utility case where the CRRA is the reciprocal of the EIS. The point estimate for the CRRA in the unrestricted case is around 6, and is estimated quite precisely as seen from the narrow confidence interval. This value is roughly consistent with the GMM results of Epstein and Zin (1991), who found a low value of the CRRA close to one. Schwartz and Torous (1999) argue that empirical tests have difficulty disentangling the EIS from the CRRA because the data that is typically used, which includes returns on stocks and short-term bonds, do not capture the time dimension needed to accurately measure the EIS. Schwartz and Torous explain that the EIS deals with the willingness of investors to allocate consumption over time, and thus term structure data can better capture this temporal effect. Schwartz and Torous find a GMM point estimate for the CRRA of 5.65 in the recursive utility framework. These values of the CRRA around 6 fall in the range obtained by Malloy, Moskowitz, and Vissing-Jørgensen (2006) also in the recursive utility framework, but from micro-level household consumption data. These authors show

[8]As a further check, we also computed bootstrap standard errors by recursively generating data according to the VAR specification, then generating yield data from the bond pricing formulas, and finally estimating the model parameters using the simulated data. A bootstrap distribution was generated from 1000 replications of this (numerically intensive) procedure. The bootstrap confidence intervals were similar to those reported in Tables 2 and 4.

that the CRRA implied by the cross-sectional reward for long-run consumption risk of stockholders is around 8, and as low as 5 for the wealthiest third of stockholders with the largest equity holdings. Table 2 shows that the EIS is also estimated quite precisely, with a point estimate of 0.359. That value is also consistent with the findings of Epstein and Zin (1991) who found the EIS to be statistically less than 1. Schwartz and Torous (1999) report a point estimate of 0.226 for the EIS. There results corroborate the work of Hall (1988) and Campbell (1999), who conclude that the EIS is small and positive and statistically different from zero. It is interesting to note that, while the non-expected and expected utility cases have statistically different estimates of the CRRA, they nonetheless have similar estimates of the EIS. Figure 1 shows the subjective discount factor, β_t, implied by the non-expected utility model. The plot corresponds to the price of the one-quarter bond scaled by the estimated value of γ; see equation (5).

Looking at the estimates of Φ in the non-expected utility case, each of the state variables appears statistically significant as some element. The short rate and the term spread appear as their own significant predictors, which is not surprising given the persistent nature of those variables. The term spread, inflation, and consumption growth appear to be significant predictors of the market return in the third row of Φ. Inflation, in the fourth row, is explained by its own lag and consumption growth. Finally, consumption growth is explained by its own lag.

As in Piazzesi and Schneider (2006), inflation brings bad news for future consumption since inflation is negatively correlated with future consumption growth. However, the implied correlation between consumption growth and lagged consumption growth is negative (-0.409) while it is positive historically in the data (0.208; see Table 3). The interpretation of this result is challenging. Reasons could be that the implied dynamics are for a representative investor since substantial differences have been put forward between the consumption of stockholders and non-stockholders.[9] An intertemporal substitution effect could explain the negative consumption growth autocorrelation. Evidence of such forward-looking consumption-savings decisions by households is found in Nalewaik (2006). Using twenty years of microeconomic data from the Consumer Expenditure Surveys, he finds a large negative first-order autocorrelation for consumption growth.[10] Another reason could

[9]Mankiw and Zeldes (1991) proposed the idea that limited participation in asset markets matters for the relation between consumption and asset returns. They found large differences in relative risk aversion estimates between the stockholders and the non-stockholders, implied by different consumption processes for these two groups. Vissing-Jørgensen (2002) shows that estimates of the EIS also differ significantly between asset holders and non-asset holders.

[10]It is interesting to note that Otrok, Ravikumar, and Whiteman (2002) find that the autocorrelation of annual consumption growth is -0.26 over the period 1890–1930, and Chapman (2002) finds the autocorrelation to be -0.16 over the period 1890–1948. The consumption process used by Mehra and Prescott

be a misspecification of the consumption process. The representative investor may fear a regime with a very negative consumption growth that translates into a negative estimate for the coefficient of lagged consumption in the consumption growth equation because this bad regime is not accounted for in the model. Garcia, Luger, and Renault (2003) find evidence of such regime effects in the context of an equilibrium-based option pricing model. Interestingly, the signs and magnitudes of the coefficients in the market return equation are similar in the implied Φ matrix in Table 2 and the matrix estimated without the bond data in Table 3. That is also the case for the yield spread. The signs are also preserved in the short rate equation for inflation and consumption growth.

The estimate of Φ under expected utility exhibits a very different pattern. In that case, the only significant elements are the short rate and the term spread as their own predictors. Lagged values of the inflation rate, the return on the market portfolio, and consumption growth are nowhere significant. The fact that consumption makes no significant contribution provides yet more evidence against the consumption-based asset pricing model with power utility. It is interesting to note that the point estimates coefficients of lagged inflation and lagged consumption growth in both the inflation and the consumption equations are not different in the non-expected utility and the expected utility panels.

Table 4 reports the parameter estimates for the reduced-form model. The reported confidence intervals reveal that many of the parameter estimates have large standard errors, as is common in reduced-form factor models of the term structure. The market return is the only variable that appears significant in the average market price of risk, λ_0. This result is interesting since Ang and Piazzesi (2006) find that such unconditional means are hard to pin down in small samples owing to the persistent nature of bond yields. On the other hand, each of the state variables plays some significant role in determining the time variation of market prices of risk. The significance of every element in the third column of λ_1 corresponding to the market return is worth noticing.

Table 5 reports summary statistics of the in-sample absolute pricing errors (in basis points) for the various specifications. It is immediately clear that relaxing the expected utility constraint improves the fit of the equilibrium model. This result confirms that the market return plays a relatively important contemporaneous role in the pricing of bonds. The unrestricted equilibrium model fares well against the reduced-form model, as seen from the small differences in pricing errors. The maximal pricing error in Table 5 is only about 140 basis points, which occurs under the restricted equilibrium model with 4-quarter bonds. Despite the relative differences across models, the pricing errors in Table 5 show that the three specifications fit very well by any standard.

(1985) has an autocorrelation of -0.14. They base their parameter values on annual data covering the period 1889–1978.

3.5 Variance decompositions

The model-implied yields in equations (12) and (17) show that the effects of each state variable on the yield curve are determined by the factor loadings $B(n)$ and $B^{na}(n)$, respectively. Further, those equations identify the error in forecasting yields with the error in forecasting the VAR. Following And and Piazzesi (2003), the proportion of the forecast error attributable to each state variable can be computed from a standard variance decomposition of the VAR. Tables 6–8 show the relative contributions of each state variable to the mean squared forecast errors of bond yields, for various forecast horizons. Note that even though the market return plays no contemporaneous role in the SDF under expected utility, it is still a predictor of the other state variables in the VAR and hence still makes a contribution in forecasting future bond yields.

It is immediately clear upon comparing Tables 6–8 that the state variables make similar contributions in forecasting future bond yields across the three specifications. The contribution of the short rate decreases with both the maturity and the forecast horizon. On the other hand, the spread's contribution increases with the bond's maturity, but tends to decrease with the forecast horizon as the maturity increases. The contribution of the two yield curve factors across maturities follows by construction. Recall that the three models are constrained to price the shortest and the longest maturity bonds without error in-sample. The proportions of unconditional variance accounted for by the short rate and the term spread range from around 80% and 1.3%, for the 2-quarter yield, to about 70% and 10%, for the 16-quarter yield, respectively.

The proportions of forecast variance explained by the the market return, inflation, and consumption exhibit interesting patterns. The market return's contribution is about 2.5% across both the bond maturity and the forecast horizon. The contribution by the rate of inflation is increasing with the forecast horizon, and about constant across bond maturities. That proportion increases from about 2% to 10% as the forecast horizon increases from 4 quarters to very long horizons. The effect of consumption growth is also increasing in the forecast horizon, and slightly decreasing in bond maturity. The long-run contribution of consumption growth in forecasting bond yields is about 7.5%. It is interesting to note that the overall proportion of unconditional variance accounted for by the term structure factors is about 80% for all bond maturities, and the remaining 20% is accounted for by the market return, inflation, and consumption.

3.6 Out-of-sample forecasts

Duffee (2002) shows that traditional affine term structure models produce forecasts that are typically worse than forecasts produced by simply assuming that future yields are equal to current yields.[11] Duffee explains that the poor forecasting performance of those traditional models is due to the fact that the implied compensation for risk is a multiple of the variance of the state vector. This tight link makes it difficult to replicate some stylized facts of historical excess bond returns. Duffee concludes that for the purpose of forecasting, traditional affine term structure models are largely useless. Ang and Piazzesi (2003) remark that market prices of risk that are affine functions of both macroeconomic and latent factors, which were not considered by Duffee (2002), seem to improve the forecasts. Ang and Piazzesi conclude that adding macro factors to a given number of latent factors in an affine term structure model results in better forecasts, even outperforming the random walk model.

The equilibrium and reduced-form models, based entirely on observable factors, are compared to the benchmark random walk model in terms of their one-quarter-ahead predictions. For each quarter t, we estimate the VAR model and the term-structure models using data up to and including quarter t, and then forecast the next quarter's yield curve using the VAR's forecasts for period $t + 1$. Hence, we use only information available in period t when forming the forecasts for period $t + 1$. The forecasts from the benchmark random walk model are produced by simply assuming that future yields are equal to current yields. Given that we need at least 30 observations to estimate the reduced-form model, prediction abilities are compared over the period 1967Q1–2004Q4, resulting in 152 one-quarter-ahead forecasts.[12]

Table 9 reports summary statistics of the one-quarter-ahead absolute forecasts errors (in basis points). The first panel shows the results for the benchmark random walk model. The next two panels show the results for the non-expected and expected utility equilibrium models, respectively. The last panel shows the results for the reduced-form model. Note that, by construction, both versions of the equilibrium model and the reduced-form model have identical predictive abilities for 1- and 20-quarter yields.

As in Ang and Piazzesi (2003), we also find that the three term-structure models slightly outperform the random walk model in terms of mean absolute errors. There are also some noticeable differences in terms of the other moments. In particular, the maximum absolute forecast errors from the term-structure models tend to be smaller than those from the random walk. Further, among the term-structure models, the maximum absolute forecast errors from the non-expected utility model are smaller than those from the other models for maturities of 4, 8, 12, and 16 quarters. The fact that the three term-

[11]See Egorov, Hong, and Li (2006) for related evidence.

[12]We also repeated the forecast comparison starting at the sample mid-point to see if there were any effects from the choice of the initial estimation window. The results based on the resulting 91 observations were qualitatively similar to those reported in Table 9.

structure models only have slight differences when compared in terms of their predictive ability is perhaps not surprising given their similar variance decompositions.[13]

4 Empirical Implications

4.1 Volatilities of Yields and Yield Changes

Litterman, Scheinkman, and Weiss (1991) document a hump-shaped pattern in the term structure of unconditional volatilities of yields and yield changes. The top panel of Table 10 shows the volatilities of the actual market yields across maturities. Here the hump occurs at a maturity of two quarters: volatility is relatively lower for one-quarter bonds, peaks for two-quarter bonds, and then decreases monotonically as the maturity increases from four to twenty quarters. A similar pattern occurs in the term structure of unconditional volatilities of yield changes, shown in the top panel of Table 11.

Can any of the three model specifications reproduce the term structure of volatilities? To answer this question, we generated 1000 samples of artificial yields for each maturity of the same length as the actual data, under each model specification. This involved using the OLS estimates to recursively generate data for the state variables according to the VAR specification and then feeding those data into the bond-pricing formulas, evaluated at the point estimates in Tables 2 and 4, to generate the yields data.

The volatilities of the simulated yields and yield changes are reported in the bottom panels of Tables 10 and 11, respectively. The reported statistics are the mean values across the 1000 replications, along with asymmetric 95% confidence intervals constructed from the quantilies of the simulated distributions. For both yields and yield changes, the non-expected utility model succesfully reproduces the hump shaped pattern of volatilities across bond maturities. On the contrary, the expected utility and the reduced-form models do not reproduce the hump. Indeed, Tables 10 and 11 show a strictly decreasing term structure of volatilites for those two specifications.

4.2 Violations of the Expectations Hypothesis

Campbell-Shiller Regressions

According to the expectations hypothesis of the term structure of interest rates, long-term yields are the average of expected future short yields over the holding period of the long-term asset, plus a constant risk premium. This implies that current spreads between

[13]The out-of-sample forecast comparison was also extended to a four-quarter horizon. The relative differences across the three models were similar to those at the one-quarter horizon.

yields of different maturities predict future yield changes. Campbell and Shiller (1991) consider predictive regressions of the form

$$y_{t+1}^{(n-1)} - y_t^{(n)} = \beta_0^{(n)} + \beta_1^{(n)} \frac{1}{n-1} \left(y_t^{(n)} - y_t^{(1)} \right) + \varepsilon_{t+1}^{(n)}, \tag{20}$$

which should produce a slope coefficient of 1 under the expectations hypothesis. Campbell and Shiller find that the slope coefficient is less than 1 and decreasing in n.[14] Bansal and Zhou (2002) show that this predictability evidence can be explained by a term structure model, where the short rate and the market prices of risks are subject to regime shifts. More generally, Dai and Singleton (2002) and Duffee (2002) show that the Campbell-Shiller finding can be explained by reduced-form term structure models, provided that the market prices of risk take some flexible form so that the expected excess bond returns are positively correlated with the yield spread. Wachter (2006) shows that a consumption-based model of the term structure with market prices of risk generated by external habit can also explain the Campbell-Shiller finding. It should be noted that Wachter calibrates her model to the data.

The question we ask here is whether any of the general-purpose term structure models we consider can generate the required risk premiums for the specific set of parameter values that correctly fit the data. Table 12 shows the results for the regression in (20) with $n = 4, 8, 12, 16, 20$.[15] The top panel shows the slope coefficients and R^2's found in the actual data. As in previous studies, the slope coefficients are negative and decreasing with maturity.

The three lower panels of Table 12 show how closely the three models can mimic the pattern of slope coefficients. Following Bansal and Zhou (2002) and Wachter (2006), we generated 1000 samples of artificial yields, as described above for the term structure of volatilities. For each simulated sample, we ran the regression in (20) and computed the R^2. The three lower panels of Table 12 report the mean slope coefficients along with asymmetric 95% confidence intervals. The non-expected utility model produces mean slope coefficients with the downward pattern across maturity and the actual coefficients are well covered by the respective confidence intervals. In the expected utility case, however, the mean slope coefficients do not decrease monotonically with n, although the actual coefficients are covered by the respective confidence intervals. Perhaps a more serious problem revealed by Table 12 is the slope coefficient associated with $n = 4$ in the reduced-form model

[14] The observed violations of the expectations hypothesis could also be the result of monetary policies that adjust short rates in response to the slope of the yield curve (see McCallum 1994, Kugler 1997, and Gallmeyer, Hollifield, and Zin 2005).

[15] As usually done, the change $y_{t+1}^{(n)} - y_t^{(n)}$ is used instead of $y_{t+1}^{(n-1)} - y_t^{(n)}$, since $y_{t+1}^{(n-1)}$ is not available. Bekaert, Hodrick, and Marshall (1997) discuss the effects of this approximation.

(first column, bottom panel). In that case, the mean slope coefficient is positive and the actual slope coefficient of -0.603 is only marginally covered by the confidence interval $[-0.675, 1.747]$. This indicates that there might be a deeper problem with the implied risk premiums. We examine this further in predictive regressions of excess bond returns using forward rates.

Cochrane-Piazzesi Regressions

Another way to state the expectations hypothesis of the term structure of interest rates is that holding-period excess returns should not be predictable. Cochrane and Piazzesi (2005) consider the predictive regression of 4-quarter excess bond returns on the initial yield and forward rates:

$$rx_{t+4}^{(n)} = \beta_0^{(n)} + \beta_1^{(n)} y_t^{(4)} + \sum_{i=2}^{5} \beta_i^{(n)} f_t^{(4i)} + \varepsilon_{t+4}^{(n)}, \qquad n = 8, 12, 16, \text{ and } 20, \tag{21}$$

where $rx_{t+4}^{(n)} = p_{t+4}^{(n-4)} - p_t^n - y_t^{(4)}$ is the return (in excess of the 4-quarter bond yield) from buying an n-quarter bond at time t and selling it as an $(n-4)$-quarter bond at time $t+4$, and $f_t^n = p_t^{(n-4)} - p_t^{(n)}$ is the forward rate for loans between time $t+n-4$ and $t+n$; p_t^n is the log price of an n-year bond at time t. Note that time increments are in years. Cochrane and Piazzesi find a robust tent-shaped pattern of slope coefficients for all maturities, with regression R^2 values around 35%. This violation of the expectations hypothesis extends the classic regressions of Fama and Bliss (1987) and Campbell and Shiller (1991). Fama and Bliss found that the spread between the n-year forward rate and the one-year yield predicts the one-year excess return of the n-year bond, with R^2 about 18%. As mentioned above, Campbell and Shiller found similar results forecasting yield changes with yield spreads. Cochrane and Piazzesi's findings substantially strengthen that evidence against the expectations hypothesis. In particular, they show that the same linear combination of forward rates—the regressors in (21)—predicts bond returns at all maturities, while Fama and Bliss and Campbell and Shiller relate each bond's expected excess return to a different forward spread or yield spread.

The size of the predictability and nature of projection coefficients in regressions like (21) is quite puzzling and, as Bansal, Tauchen, and Zhou (2004) state, "constitutes a serious challenge to term structure models." Bansal, Tauchen, and Zhou account for the predictability evidence from the perspective of latent factor term structure models. They show that the regime-switching model of Bansal and Zhou (2002) can empirically account for these challenging features of the data, while affine specifications cannot. In this section, we ask whether the risk premiums generated by our model (based on observable factors) can also account for the tent-shaped predictability pattern. To preview the results, it is only the non-expected utility model that can do so. Both the expected utility version of the equilibrium model and the reduced-form model fail to account for these important features. An important note is that the question is not whether one can construct market prices of risk that generate the return regressions in an affine model. Cochrane and Piazzesi (2005) show exactly how that can be done. As with the Campbell-Shiller regressions, the question we ask is whether any of the term structure models we consider can generate the required risk premiums for the specific set of parameter values that correctly fit the data.

Estimation results for the regressions in (21) are reported in the top panel of Table 13. Consistent with the findings of Bansal, Tauchen, and Zhou (2004), we also found that the use of the five forward rates in (21) creates a near-perfect collinearity problem in our data set and, therefore, we concentrate on the regressions with $y_t^{(4)}$, $f_t^{(12)}$, and $f_t^{(20)}$ as regressors. The tent-shaped finding of Cochrane and Piazzesi (2005) is apparent in Figure 2, which plots the estimated regression coefficients. The regression R^2 reported in Table 13 further confirm their findings. The table shows that when the 8-quarter excess return is the regressand, the R^2 is around 34%, and that value reaches nearly 38% when the 16-quarter excess return appears as regressand.

The three lower panels of Table 13 show how closely the three models can mimic the tent-shaped pattern of regression coefficients. Following Bansal, Tauchen, and Zhou (2004), we generated 1000 samples of the same length as the actual data for each model. As with the Campbell-Shiller regressions, this involved using the OLS estimates to recursively generate data for the state variables according to the VAR specification and then feeding those data into the bond-pricing formulas, evaluated at the point estimates in Tables 2 and 4, to generate the yields data. For each simulated sample, we ran the regression in (21) and computed the R^2. The three lower panels of Table 13 report the mean regression coefficients along with asymmetric 95% confidence intervals. Figures 3, 4, and 5 plot the mean regression coefficients for the non-expected utility model, the expected utility model, and the reduced-form model, respectively. From those figures, it is immediately clear that only the non-expected utility model can empirically account for the tent-shaped pattern of coefficients from predictive regressions of excess bond returns on forward rates. Figure 4 shows that the expected utility model fails to capture the predictability of the 3- and 5-year forward rate for all excess returns. As Figure 5 shows, the reduced-form model fails even more so at capturing those predictability components. These shortcomings are further confirmed even when sampling error is accounted for. The confidence intervals in Table 13 show more formally the correspondence between the non-expected utility model and the actual data. In that case, all the actual coefficients are covered by the respective confidence intervals. On the contrary, the confidence intervals for both the expected utility model and the reduced-form model fail to cover several of the actual coefficients. In particular, all the coefficients associated with the 3- and 5-year forward rates ($\beta_3^{(n)}$ and $\beta_5^{(n)}$) are not covered by the respective confidence intervals derived under those two specifications.

4.3 Key Differences

Why are the implied risk premiums so different? A comparison of the coefficients in (9) with those in (18) provides some hints. Aside from the presence of the vector J in (9), the most notable difference between the two specifications is that the reduced-form model has $\mu - \Sigma\lambda_0$ and $\Phi - \Sigma\lambda_1$ in (18) instead of just μ and Φ, respectively, in (9). This means that the effects of μ and Φ on bond yields cannot be disentangled from that of Σ. Figure 6 plots the intercept and factor loadings for maturities ranging from 1 to 20 quarters, where the solid lines correspond to the non-expected utility model, the dashed lines to the expected utility model, and the dotted lines to the reduced-form model. By construction, the intercept and factor loadings are identical in value at the beginning and end points. Both the short rate and the term spread load in similar fashions across the three specifications. Using the non-expected utility model as a benchmark for comparisons, we see that the tight link between μ and Φ and Σ in the reduced-form specification leads to marked differences for the intercept terms (upper left plot), the market return loadings (middle right plot), the inflation loadings (lower left plot), and the consumption loadings (lower right plot). In those cases, we see a built-up effect as n increases. Consider next the expected and non-expected utility model. The obvious difference is that the return on the market portfolio plays no contemporaneous role in the SDF under the expected utility specification. The expected utility restriction appears most noticeably in terms of the consumption loadings, especially for longer bond maturities (lower right plot).

Another related and important difference between the non-expected utility model and the reduced-form one can be seen from an examination of the innovations to their respective (log) SDFs, $\log m_{t+1}^{\$} - E_t[\log m_{t+1}^{\$}]$. The time series of implied innovations for the non-expected utility model are shown in Figure 7 and those for the reduced-form model are shown in Figure 8; the two plots are shown on the same scale. A striking result is the difference between the volatilities of the innovations. Indeed, the reduced-form SDF innovations appears far more volatile than those of the non-expected utility model. This clearly illustrates why the parameter estimates for the reduced-form model (in Table 4) have large standard errors. It also explains the behavior of the reduced-form factor loadings.

The predictability results presented here can be related to those obtained by Bansal, Tauchen, and Zhou (2004) with term structure models based on latent factors. Those authors show that their preferred two-factor regime-switching specification captures business cycle movements between economic expansions and recessions, and that these transitions affect the term structure of interest rates. A recession usually means a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP. It is therefore not surprising that our equilibrium model featuring inflation and consumption—an important component of GDP—also justifies the size and nature of bond return predictability. What is more intriguing are the contrasts between the non-expected utility model and its restricted expected utility version and the reduced-form model.

5 Conclusion

We have proposed an equilibrium model of the term structure of interest rates based on a representative agent framework with recursive utility. The preference specification belongs to the class proposed by Melino and Yang (2003), which generalizes the standard recursive utility framework by allowing the representative agent to display state-dependent preferences. In order to explain the term structure of interest rates, we need only allow for a variable rate of time preference. The preference parameters associated with risk aversion and intertemporal substitution remain time-invariant and can thus be deemed structural in our framework. The key ingredient of our model is that the time-varying subjective discount factor is linked to the short-term rate of interest. Without that link, the model could not fit the term structure as well as reduced-form models. Our preference specification is coherent with the general class of recursive utility functions in Obstfeld (1981), where the rate of time preference is a function of the interest rate. A consequence of this type of specification is that consumption and asset prices depend on the short-term rate of interest. Our preference specification is also motivated by the central role played by the short-term interest rate in the determination of bond prices.

The proposed model incorporates a VAR description of macroeconomic dynamics and links them to those of the term structure so that nominal bond yields are affine functions of observable state variables. The vector of state variables comprises the short-term rate of interest, a yield spread, a measure of the return on the market portfolio, the rate of inflation, and the rate of consumption growth. The implied bond prices therefore account for the fact that investors' preferences impose some constraints between these prices, since identifying restriction are based on the first-order conditions that describe the representative investor's optimal consumption and portfolio plan. We estimate the model and compare it to both an expected utility version of the model and a reduced-form no-arbitrage model. Each model is based on the same VAR description of macroeconomic dynamics, but links them to bond yields in different ways. The expected utility model restricts the CRRA to the reciprocal of the EIS so that the market return plays no contemporaneous role in the SDF. Risk premiums in the reduced-form model are obtained by specifying time-varying prices of risk that transform the risk-factor volatilities into premiums. Identifying restrictions in that case are based only on the absence of arbitrage.

Our empirical assessment reveals that all three models can fit the term structure of interest rates equally well. Variance decompositions show that the state variables make very similar contributions in forecasting future bond yields across the three specifications. A noteworthy result is that the overall proportion of unconditional variance accounted for by the two term structure factors is about 80% for all bond maturities, and the remaining 20% is accounted for by the return on the market portfolio, the rate of inflation, and the rate of consumption growth. An out-of-sample forecast exercise shows that the three term-structure models have only slight differences when compared in terms of their predictive abilities.

The value added by the new model appears in the implied risk premiums. We find that only the non-expected utility model can empirically account for the tent-shaped pattern and magnitude of coefficients from predictive regressions of excess bond returns on forward rates—documented by Cochrane and Piazzesi (2005). This is an important result since the equilibrium model ties the predictable variation in excess bond returns to underlying macroeconomic fundamentals. The equilibrium model fits the term structure and captures the important features of bond risk premiums with economically plausible values for the structural preference parameters. The results emphasize the importance of both non-expected utility preferences and the variable rate of time preference for explaining violations of the expectations hypothesis.

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Table 1. Summary Statistics of Yield Data

	Maturity in quarters						
	1	2	4	8	12	16	20
Mean	0.056	0.059	0.061	0.063	0.065	0.066	0.067
Std. deviation	0.028	0.028	0.027	0.027	0.026	0.025	0.025
Skewness	1.010	0.962	0.826	0.824	0.852	0.871	0.875
Kurtosis	4.474	4.298	3.886	3.712	3.664	3.597	3.478
Min	0.009	0.010	0.011	0.014	0.017	0.022	0.025
Max	0.151	0.159	0.155	0.154	0.151	0.150	0.145

Note: The quarterly data set has 182 observations from 1959Q3 to 2004Q4.

Table 2. Parameter Estimates: Equilibrium Model

Non-expected utility case					
Preference parameters					
CRRA	6.057				
	[5.393, 6.722]				
EIS	0.359				
	[0.297, 0.421]				
Persistence matrix Φ					
Short rate	0.975	0.263	-0.024	0.101	0.284
	[0.346, 1.604]	[-0.355, 0.881]	[-0.576, 0.527]	[-0.498, 0.699]	[-0.298, 0.866]
Spread	0.021	0.804	0.028	-0.133	-0.337
	[-0.704, 0.747]	[0.120, 1.488]	[-0.742, 0.798]	[-0.835, 0.568]	[-1.103, 0.428]
Market return	0.136	1.067	0.126	-0.625	-1.526
	[-0.512, 0.785]	[0.346, 1.788]	[-0.542, 0.795]	[-1.215, -0.040]	[-2.130, -0.923]
Inflation	-0.020	0.402	0.042	0.641	0.372
	[-0.749, 0.708]	[-0.289, 1.092]	[-0.616, 0.700]	[0.017, 1.267]	[-0.247, 0.992]
Consumption	0.046	0.184	0.025	-0.242	-0.409
	[-0.291, 0.383]	[-0.228, 0.596]	[-0.283, 0.334]	[-0.578, 0.093]	[-0.718, -0.101]
Expected utility case					
Preference parameters					
CRRA	2.747				
	[1.880, 3.614]				
EIS	0.364				
	[0.249, 0.478]				
Persistence matrix Φ					
Short rate	1.007	0.182	0.009	-0.006	-0.063
	[0.457, 1.556]	[-0.352, 0.717]	[-0.421, 0.440]	[-0.560, 0.548]	[-0.686, 0.558]
Spread	-0.043	0.881	-0.038	-0.110	-0.396
	[-1.069, 0.981]	[0.006, 1.756]	[-0.796, 0.720]	[-1.007, 0.786]	[-1.463, 0.671]
Inflation	0.608	0.316	0.402	0.404	0.515
	[-0.326, 1.543]	[-0.588, 1.222]	[-0.530, 1.335]	[-0.573, 1.382]	[-0.474, 1.506]
Consumption	-0.226	-0.121	-0.156	-0.271	-0.449
	[-1.097, 0.645]	[-1.109, 0.867]	[-1.133, 0.819]	[-1.088, 0.545]	[-1.357, 0.459]

Notes: CRRA denotes the coefficient of relative risk aversion, EIS the elasticity of intertemporal substitution. The expected utility model restricts the CRRA to the reciprocal of the EIS so that the market return plays no contemporaneous role in the SDF. The numbers in square brackets are symmetric 95% confidence intervals. In the restricted case, the confidence limits for the EIS were found by the delta method.

Table 3. VAR Estimation Results

Persistence matrix Φ					
Short rate	0.918	0.043	0.011	0.323	0.407
	[0.859, 0.978]	[-0.086, 0.172]	[-0.007, 0.029]	[0.091, 0.554]	[0.139, 0.675]
Spread	0.025	0.842	0.004	-0.117	-0.267
	[-0.016, 0.068]	[0.750, 0.934]	[-0.009, 0.017]	[-0.282, 0.048]	[-0.458, -0.076]
Market return	0.096	0.865	0.175	-0.274	-1.628
	[-0.393, 0.586]	[-0.195, 1.927]	[0.025, 0.326]	[-2.182, 1.633]	[-3.835, 0.578]
Inflation	0.024	-0.078	-0.002	0.779	0.231
	[-0.003, 0.052]	[-0.139, -0.018]	[-0.010, 0.007]	[0.671, 0.888]	[0.105, 0.356]
Consumption	-0.008	0.056	0.009	-0.102	0.208
	[-0.040, 0.024]	[-0.013, 0.125]	[-0.001, 0.019]	[-0.227, 0.023]	[-0.063, 0.353]

Notes: The entries are the estimation results from the macroeconomic data. The numbers in square brackets are symmetric 95% confidence intervals.

Table 4. Parameter Estimates: Reduced-form Model

	λ_0	λ_1				
Short rate	-0.194	-12.340	-12.378	1.053	32.508	14.225
	[-3.451, 3.063]	[-14.857, -9.823]	[-15.821, -8.935]	[0.534, 1.573]	[29.025, 35.990]	[9.387, 19.062]
Spread	3.569	2.825	-13.052	12.491	6.406	-5.336
	[-4.013, 11.152]	[-4.089, 9.741]	[-20.963, -5.141]	[5.029, 19.951]	[-1.767, 14.580]	[-13.630, 2.958]
Market return	18.465	7.236	3.831	36.799	14.925	7.398
	[11.032, 25.897]	[-1.454, 15.928]	[-5.264, 12.926]	[28.188, 45.410]	[6.212, 23.638]	[-2.829, 17.627]
Inflation	3.425	-5.155	-30.101	10.808	3.845	14.739
	[-5.709, 12.559]	[-13.344, 3.032]	[-41.409, -18.791]	[1.353, 20.264]	[-6.883, 14.575]	[3.738, 25.741]
Consumption	-5.741	-4.640	-92.542	-16.065	-0.639	0.067
	[-14.795, 3.312]	[-12.479, 3.199]	[-102.075, -83.009]	[-24.614, -7.517]	[-9.417, 8.139]	[-9.072, 9.207]

Note: The numbers in square brackets are symmetric 95% confidence intervals.

Table 5. In-Sample Absolute Pricing Errors (Basis Points)

	Maturity in quarters				
	2	4	8	12	16
Equilibrium model					
Non-expected utility case					
Mean	13.20	19.94	16.36	11.91	7.45
Std. dev.	12.50	17.03	13.70	9.39	6.23
Min	0.02	0.36	0.09	0.01	0.12
Max	72.15	104.88	71.51	47.19	37.74
Expected utility case					
Mean	25.74	28.07	25.03	18.61	13.66
Std. dev.	18.38	22.14	19.47	14.75	11.85
Min	0.25	0.31	0.25	0.01	0.11
Max	87.70	139.46	112.66	70.98	64.55
Reduced-form model					
Mean	14.55	20.56	16.93	13.72	12.36
Std. dev.	12.26	17.38	13.19	10.68	9.34
Min	0.42	0.34	0.05	0.16	0.34
Max	75.22	107.98	70.31	53.61	47.79

Note: The absolute pricing errors are calculated over the 182 quarterly observations for each of the 5 maturities that are not assumed to be priced without any sampling error. The 1- and 20-quarter yields are priced without error. The expected utility model restricts the CRRA to the reciprocal of the EIS.

Table 6. Variance Decompositions: Non-Expected Utility Model

	Forecast horizon (quarters)				
	4	8	20	40	∞
2-quarter yield					
Short rate	90.29	84.76	80.31	79.40	79.29
Spread	0.79	1.24	1.35	1.30	1.29
Market return	1.87	2.19	2.20	2.18	2.18
Inflation	2.33	5.41	8.41	9.07	9.15
Consumption	4.72	6.40	7.73	8.05	8.09
4-quarter yield					
Short rate	87.30	82.12	78.25	77.51	77.42
Spread	3.39	3.46	2.90	2.63	2.59
Market return	2.24	2.49	2.41	2.36	2.35
Inflation	2.60	5.85	8.93	9.60	9.68
Consumption	4.47	6.08	7.51	7.90	7.96
8-quarter yield					
Short rate	78.36	75.42	73.92	73.78	73.77
Spread	12.83	10.32	6.96	5.96	5.84
Market return	2.65	2.86	2.64	2.55	2.54
Inflation	2.87	6.36	9.56	10.24	10.32
Consumption	3.29	5.04	6.92	7.47	7.53
12-quarter yield					
Short rate	70.57	70.24	71.01	71.40	71.45
Spread	21.21	15.90	9.93	8.32	8.13
Market return	2.87	3.05	2.76	2.64	2.62
Inflation	2.85	6.45	9.75	10.45	10.53
Consumption	2.50	4.36	6.55	7.19	7.27
16-quarter yield					
Short rate	65.52	67.00	69.31	70.03	70.12
Spread	26.63	19.42	11.72	9.72	9.48
Market return	2.97	3.13	2.81	2.68	2.66
Inflation	2.79	6.44	9.81	10.52	10.60
Consumption	2.09	4.01	6.35	7.05	7.14

Note: The entries are the percentage contribution of the ith factor to the h-step-ahead forecast of the bond yield.

Table 7. Variance Decompositions: Expected Utility Model

	Forecast horizon (quarters)				
	4	8	20	40	∞
2-quarter yield					
Short rate	90.50	84.97	80.43	79.49	79.38
Spread	0.63	1.10	1.26	1.22	1.21
Market return	1.79	2.16	2.18	2.17	2.16
Inflation	2.09	5.20	8.29	8.97	9.05
Consumption	4.99	6.57	7.84	8.15	8.20
4-quarter yield					
Short rate	89.16	83.55	79.25	78.40	78.30
Spread	1.96	2.29	2.11	1.96	1.93
Market return	2.06	2.36	2.32	2.29	2.28
Inflation	2.45	5.66	8.74	9.41	9.49
Consumption	4.37	6.14	7.58	7.94	8.00
8-quarter yield					
Short rate	83.79	79.18	76.18	75.69	75.64
Spread	7.41	6.46	4.73	4.15	4.07
Market return	2.12	2.51	2.44	2.38	2.38
Inflation	2.87	6.28	9.40	10.07	10.14
Consumption	3.81	5.57	7.25	7.71	7.77
12-quarter yield					
Short rate	74.93	73.16	72.63	72.71	72.73
Spread	16.22	12.43	8.05	6.83	6.67
Market return	2.33	2.72	2.58	2.49	2.48
Inflation	3.11	6.64	9.80	10.47	10.54
Consumption	3.41	5.05	6.94	7.50	7.58
16-quarter yield					
Short rate	64.83	66.82	69.26	69.99	70.08
Spread	26.34	18.91	11.38	9.45	9.22
Market return	2.45	2.86	2.67	2.56	2.55
Inflation	3.18	6.78	10.00	10.67	10.75
Consumption	3.20	4.63	6.69	7.33	7.40

Note: The entries are the percentage contribution of the ith factor to the h-step-ahead forecast of the bond yield. The expected utility model restricts the CRRA to the reciprocal of the EIS. The market return is still a predictor of the other state variables in the VAR and hence still makes a contribution in forecasting future bond yields.

Table 8. Variance Decompositions: Reduced-form Model

	Forecast horizon (quarters)				
	4	8	20	40	∞
2-quarter yield					
Short rate	90.00	84.54	80.19	79.30	79.19
Spread	0.77	1.21	1.32	1.27	1.26
Market return	1.92	2.21	2.21	2.19	2.18
Inflation	2.50	5.56	8.49	9.14	9.22
Consumption	4.81	6.48	7.79	8.10	8.15
4-quarter yield					
Short rate	86.91	81.90	78.13	77.41	77.33
Spread	3.28	3.42	2.88	2.61	2.57
Market return	2.35	2.56	2.44	2.39	2.38
Inflation	2.65	5.89	8.94	9.61	9.68
Consumption	4.81	6.23	7.61	7.98	8.04
8-quarter yield					
Short rate	77.33	74.85	73.63	73.54	73.53
Spread	12.74	10.29	6.96	5.98	5.86
Market return	2.79	2.92	2.67	2.58	2.56
Inflation	3.05	6.48	9.60	10.27	10.35
Consumption	4.09	5.46	7.14	7.63	7.70
12-quarter yield					
Short rate	68.00	68.69	70.21	70.74	70.80
Spread	22.16	16.52	10.29	8.63	8.43
Market return	3.00	3.09	2.78	2.66	2.64
Inflation	3.39	6.85	9.94	10.60	10.67
Consumption	3.45	4.85	6.78	7.37	7.46
16-quarter yield					
Short rate	60.68	64.07	67.79	68.80	68.92
Spread	29.34	21.15	12.66	10.51	10.24
Market return	3.31	3.28	2.88	2.74	2.72
Inflation	3.61	7.04	10.11	10.75	10.83
Consumption	3.06	4.46	6.56	7.20	7.29

Note: The entries are the percentage contribution of the ith factor to the h-step-ahead forecast of the bond yield.

Table 9. Out-of-Sample Absolute Pricing Errors (Basis Points)

	Maturity in quarters						
	1	2	4	8	12	16	20
Random walk model							
Mean	57.99	60.24	62.05	58.01	54.01	50.91	47.78
Std. dev.	71.39	69.71	61.69	52.66	45.09	41.11	38.45
Min	0.16	0.04	1.50	1.06	1.50	2.57	1.23
Max	492.16	506.20	458.94	367.10	291.80	239.83	197.20
Equilibrium model							
Non-expected utility case							
Mean	54.55	59.10	61.29	55.91	49.29	46.31	43.30
Std. dev.	67.92	65.64	58.16	50.91	45.34	41.94	38.04
Min	0.92	0.28	1.59	0.02	0.13	0.37	0.34
Max	475.71	479.81	439.58	337.51	291.60	264.35	238.89
Expected utility case							
Mean	54.55	58.67	61.13	54.97	48.56	45.20	43.30
Std. dev.	67.92	64.82	57.26	49.11	42.80	39.84	38.04
Min	0.92	0.21	0.94	0.51	0.50	0.56	0.34
Max	475.71	469.32	447.59	362.00	311.48	278.61	238.89
Reduced-form model							
Mean	54.55	59.28	61.59	54.86	48.14	44.69	43.30
Std. dev.	67.92	64.97	58.27	50.50	44.05	40.71	38.04
Min	0.92	0.07	0.33	0.43	0.03	0.87	0.34
Max	475.71	467.03	450.73	365.34	313.89	275.36	238.89

Note: The entries are one-quarter-ahead absolute forecast errors. The expected utility model restricts the CRRA to the reciprocal of the EIS. The forecasts from the benchmark random walk model are produced by simply assuming that future yields are equal to current yields. By construction, both versions of the equilibrium model and the reduced-form model have identical predictive abilities for 1- and 20-quarter yields.

Table 10. Volatilities of yields

n	1	2	4	8	12	16	20
Actual data							
	2.799	2.830	2.776	2.704	2.613	2.563	2.524
Equilibrium model							
Non-expected utility case							
	2.501	2.535	2.462	2.332	2.272	2.239	2.198
	[1.651, 3.704]	[1.652, 3.768]	[1.567, 3.736]	[1.434, 3.606]	[1.394, 3.557]	[1.368, 3.524]	[1.332, 3.486]
Expected utility case							
	2.497	2.415	2.343	2.265	2.239	2.242	2.204
	[1.604, 3.683]	[1.550, 3.571]	[1.466, 3.480]	[1.350, 3.412]	[1.321, 3.405]	[1.316, 3.441]	[1.299, 3.407]
Reduced-form model							
	2.490	2.481	2.429	2.349	2.296	2.248	2.197
	[1.646, 3.642]	[1.623, 3.653]	[1.553, 3.617]	[1.458, 3.524]	[1.409, 3.461]	[1.382, 3.395]	[1.330, 3.308]

Note: The top panel reports the standard deviation of percentage yields. The next 3 panels show the same statistics implied by the 3 model specifications. The reported statistics are the mean values across 1000 bootstrap replications. The numbers in square brackets are asymmetric 95% confidence intervals constructed from the quantiles of the bootstrap distribution.

Table 11. Volatilities of yield changes

n	1	2	4	8	12	16	20
Actual data							
	0.850	0.852	0.809	0.725	0.651	0.605	0.567
Equilibrium model							
Non-expected utility case							
	0.860	0.864	0.786	0.662	0.613	0.594	0.575
	[0.771, 0.955]	[0.773, 0.954]	[0.705, 0.868]	[0.594, 0.733]	[0.553, 0.679]	[0.534, 0.657]	[0.518, 0.637]
Expected utility case							
	0.861	0.847	0.770	0.664	0.611	0.596	0.574
	[0.775, 0.944]	[0.760, 0.934]	[0.693, 0.849]	[0.599, 0.734]	[0.551, 0.674]	[0.532, 0.659]	[0.513, 0.632]
Reduced-form model							
	0.863	0.841	0.773	0.678	0.641	0.636	0.573
	[0.771, 0.952]	[0.752, 0.930]	[0.693, 0.852]	[0.611, 0.751]	[0.575, 0.706]	[0.572, 0.702]	[0.516, 0.631]

Note: The top panel reports the standard deviation of percentage yield changes. The next 3 panels show the same statistics implied by the 3 model specifications. The reported statistics are the mean values across 1000 bootstrap replications. The numbers in square brackets are asymmetric 95% confidence intervals constructed from the quantiles of the bootstrap distribution.

Table 12. Predictability of Yield Changes using Yield Spreads

n	4	8	12	16	20
Actual data					
$\beta_1^{(n)}$	-0.603	-1.019	-1.438	-1.685	-1.839
R^2	0.009	0.017	0.027	0.032	0.033
Equilibrium model					
Non-expected utility case					
$\beta_1^{(n)}$	-0.135	-0.482	-0.963	-1.396	-1.873
	[-1.258, 1.139]	[-1.549, 0.767]	[-2.179, 0.401]	[-2.793, 0.115]	[-3.391, -0.252]
R^2	0.006	0.011	0.020	0.027	0.037
	[0, 0.029]	[0, 0.050]	[0, 0.075]	[0, 0.083]	[0, 0.105]
Expected utility case					
$\beta_1^{(n)}$	-0.937	-0.747	-1.341	-2.061	-1.896
	[-1.823, 0.006]	[-1.891, 0.409]	[-2.615, -0.116]	[-3.467, -0.718]	[-3.451, -0.340]
R^2	0.017	0.012	0.027	0.050	0.038
	[0, 0.056]	[0, 0.048]	[0, 0.084]	[0.004, 0.126]	[0.001, 0.108]
Reduced-form model					
$\beta_1^{(n)}$	0.452	-0.357	-0.970	-1.649	-1.901
	[-0.675, 1.747]	[-1.497, 0.805]	[-2.295, 0.259]	[-3.214, -0.283]	[-3.551, -0.379]
R^2	0.008	0.008	0.019	0.032	0.038
	[0, 0.038]	[0, 0.042]	[0, 0.072]	[0, 0.096]	[0.002, 0.112]

Note: The top panel reports the estimated slope coefficients and R^2's in the predictive regression of yield changes on yield spreads in (20). The next 3 panels show the same statistics implied by the 3 model specifications. The reported coefficients are the mean values across 1000 bootstrap replications. The numbers in square brackets are asymmetric 95% confidence intervals constructed from the quantiles of the bootstrap distribution. Values less than 10^{-3} are reported as zero.

Table 13. Predictability of Excess Returns using Forward Rates

n	$\beta_0^{(n)}$	$\beta_1^{(n)}$	$\beta_3^{(n)}$	$\beta_5^{(n)}$	R^2
Actual data					
8	-0.060	-0.900	2.159	-1.045	0.335
12	-0.089	-1.823	4.408	-2.287	0.362
16	-0.128	-2.631	6.132	-3.101	0.378
20	-0.168	-3.216	7.139	-3.438	0.366
Equilibrium model					
Non-expected utility case					
8	-0.057	-1.354	3.291	-1.737	0.257
	[-0.139, -0.005]	[-2.349, -0.258]	[0.168, 6.072]	[-3.712, 0.411]	[0.080, 0.446]
12	-0.105	-2.547	6.197	-3.316	0.268
	[-0.246, -0.014]	[-4.305, -0.694]	[0.845, 11.117]	[-6.770, 0.414]	[0.096, 0.455]
16	-0.151	-3.616	8.866	-4.792	0.265
	[-0.352, -0.024]	[-6.107, -1.098]	[1.343, 15.896]	[-9.622, 0.471]	[0.100, 0.446]
20	-0.214	-4.630	10.446	-5.210	0.287
	[-0.482, -0.048]	[-7.803, -1.379]	[0.797, 19.549]	[-11.551, 1.564]	[0.113, 0.482]
Expected utility case					
8	-0.051	-0.458	0.972	-0.303	0.210
	[-0.122, -0.004]	[-0.916, 0.026]	[0.345, 1.556]	[-0.624, 0.046]	[0.071, 0.380]
12	-0.102	-1.139	2.137	-0.609	0.260
	[-0.225, -0.017]	[-1.982, -0.257]	[1.004, 3.214]	[-1.181, 0.005]	[0.106, 0.442]
16	-0.146	-1.953	3.405	-0.918	0.297
	[-0.324, -0.031]	[-3.115, -0.686]	[1.784, 4.908]	[-1.729, -0.060]	[0.136, 0.485]
20	-0.188	-2.385	3.197	-0.190	0.270
	[-0.414, -0.040]	[-3.836, -0.739]	[1.162, 5.107]	[-1.237, 0.907]	[0.099, 0.466]
Reduced-form model					
8	-0.083	-0.498	0.586	0.204	0.241
	[-0.151, -0.031]	[-0.901, -0.032]	[-0.041, 1.187]	[-0.261, 0.687]	[0.074, 0.420]
12	-0.132	-1.045	1.255	0.267	0.246
	[-0.261, -0.040]	[-1.760, -0.241]	[0.039, 2.351]	[-0.569, 1.124]	[0.077, 0.418]
16	-0.184	-1.953	3.405	-0.918	0.297
	[-0.376, -0.057]	[-2.633, -0.474]	[0.399, 3.640]	[-1.055, 1.334]	[0.079, 0.415]
20	-0.262	-2.032	1.700	1.163	0.263
	[-0.514, -0.102]	[-3.293, -0.560]	[-0.424, 3.672]	[-0.422, 2.664]	[0.088, 0.435]

Note: The top panel reports the estimated coefficients and R^2's in the predictive regression of excess bond returns on forward rates in (21). The next 3 panels show the same statistics implied by the 3 model specifications. The reported coefficients are the mean values across 1000 bootstrap replications. The numbers in square brackets are asymmetric 95% confidence intervals constructed from the quantiles of the bootstrap distribution.

Figure 1. Subjective discount factor implied by the non-expected utility model.

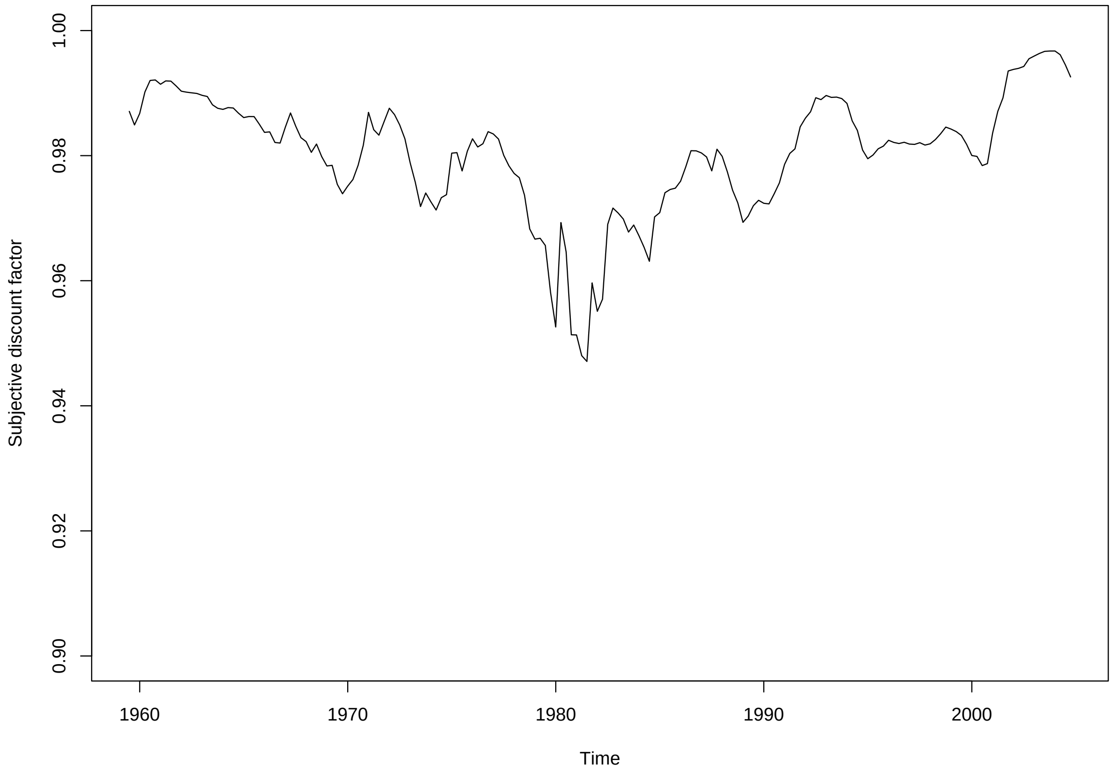

Figure 2. Predictability regression coefficients in the observed market data.

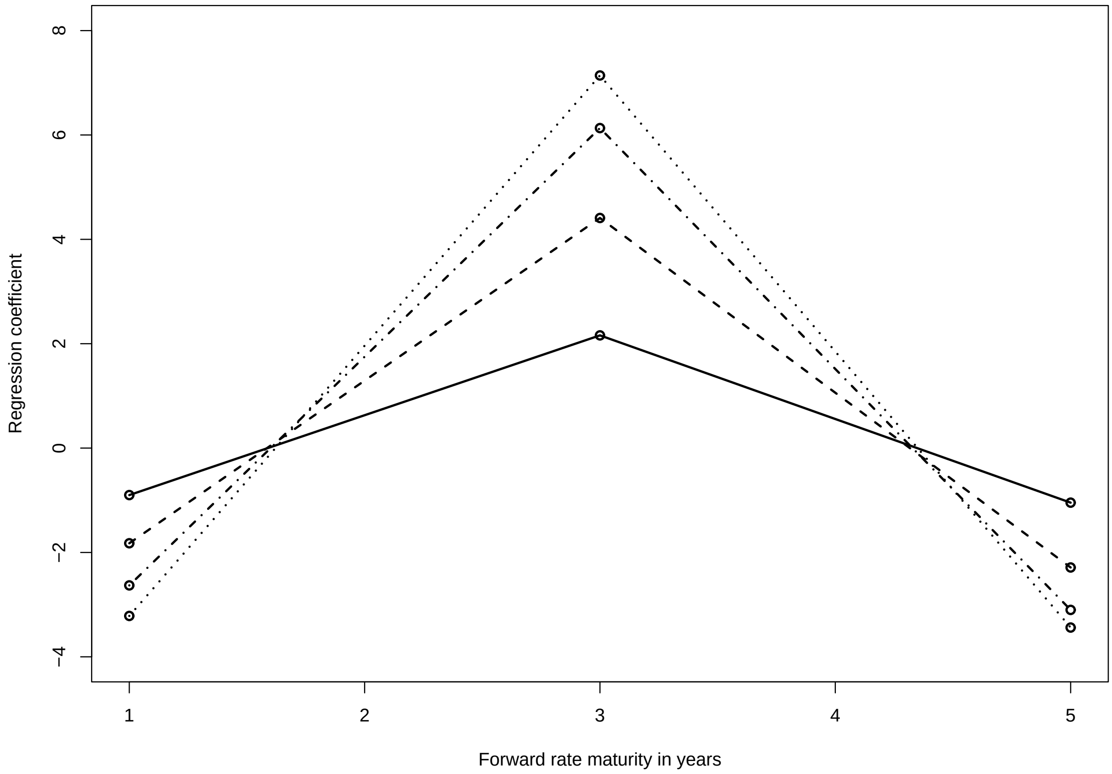

Figure 3. Predictability regression coefficients implied by the non-expected utility model.

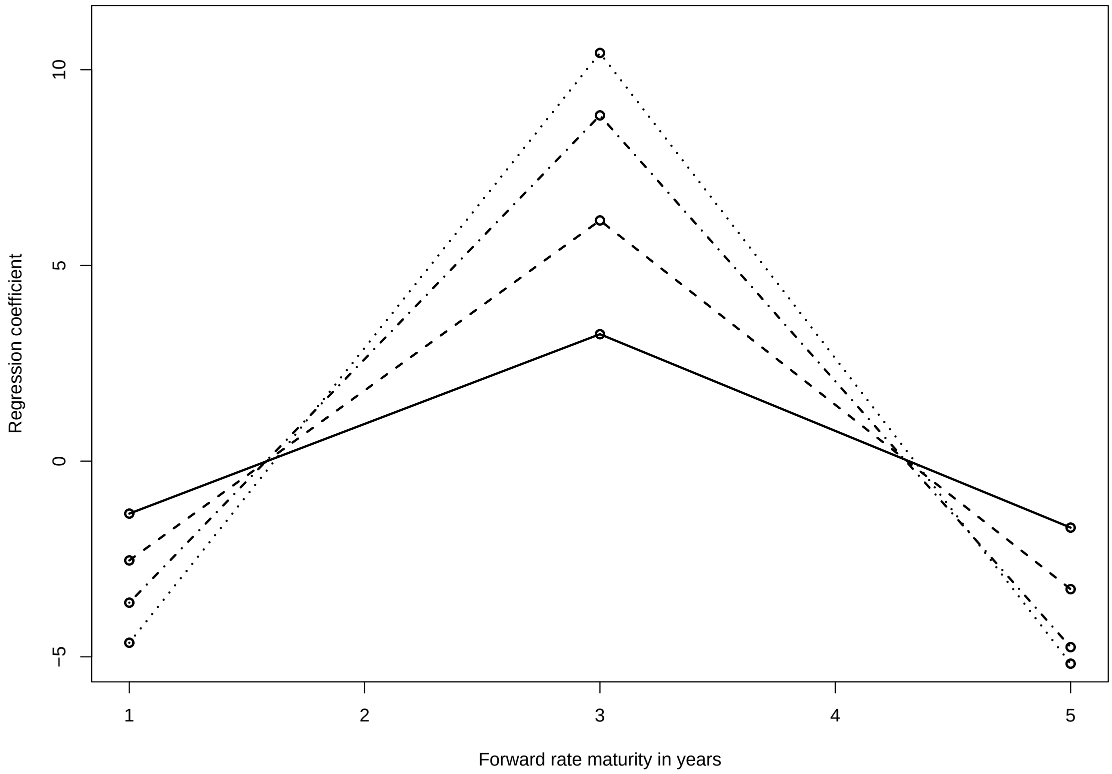

Figure 4. Predictability regression coefficients implied by the expected utility model.

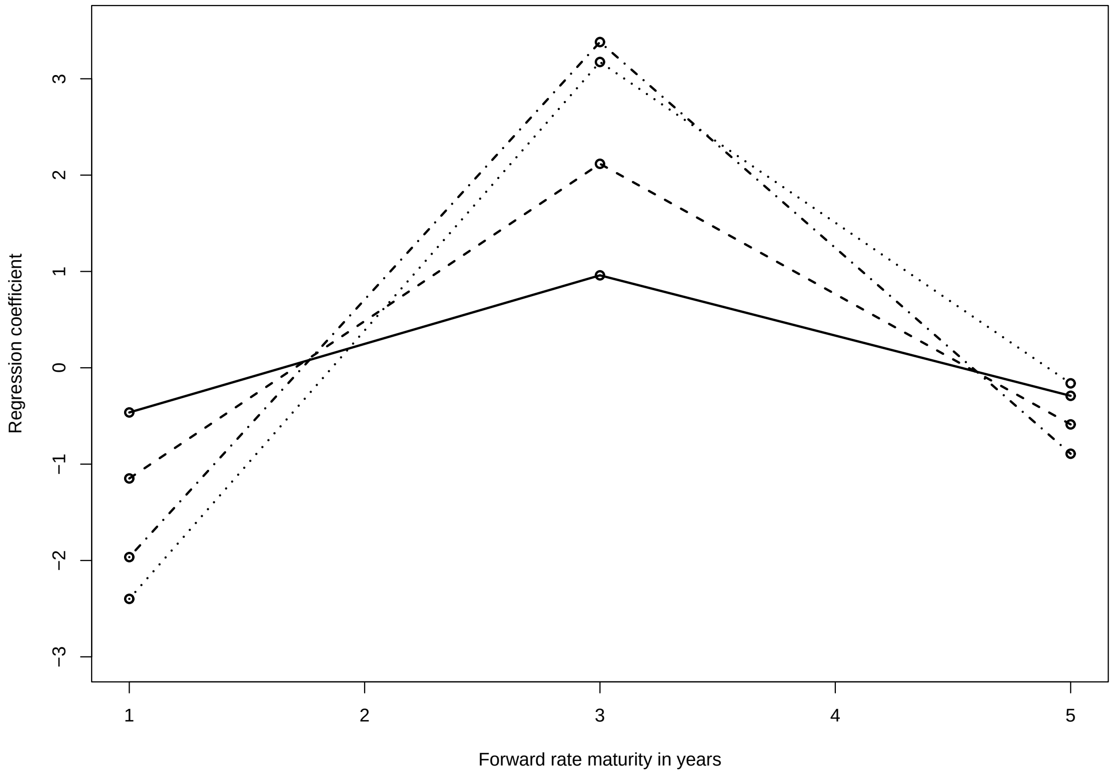

Figure 5. Predictability regression coefficients implied by the reduced-form model.

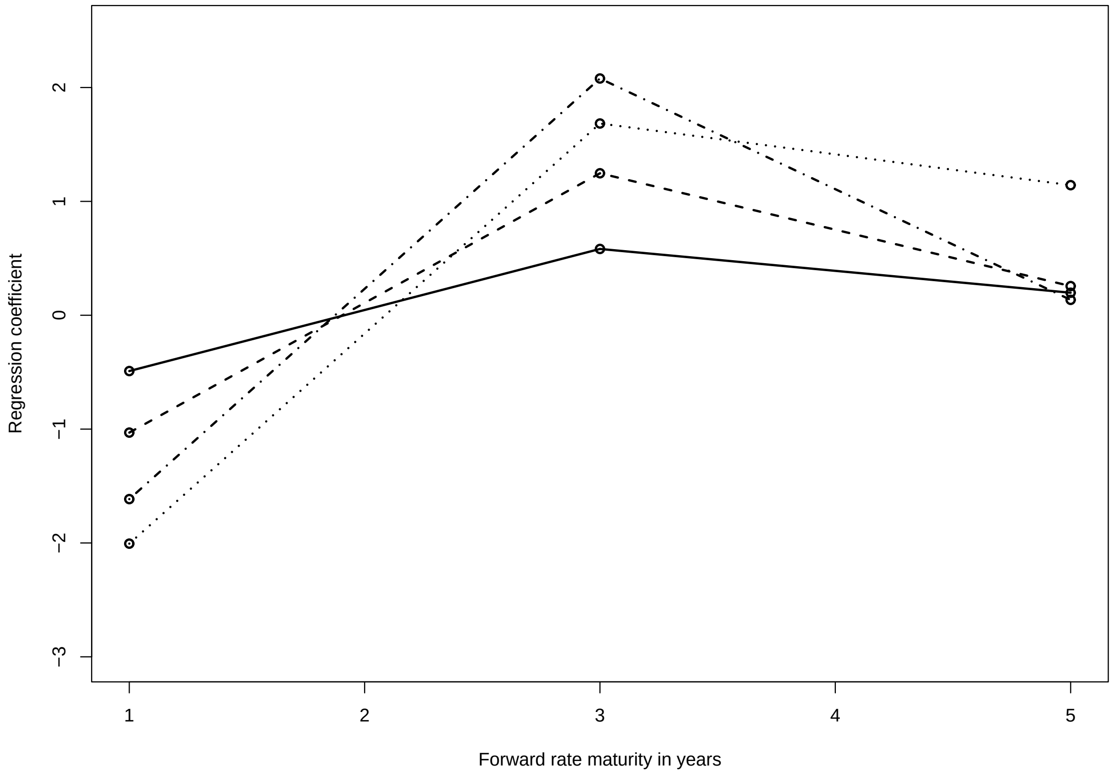

Figure 6. Intercept and factor loadings: the solid lines correspond to the non-expected utility model, the dashed lines to the expected utility model, and the dotted lines to the reduced-form model.

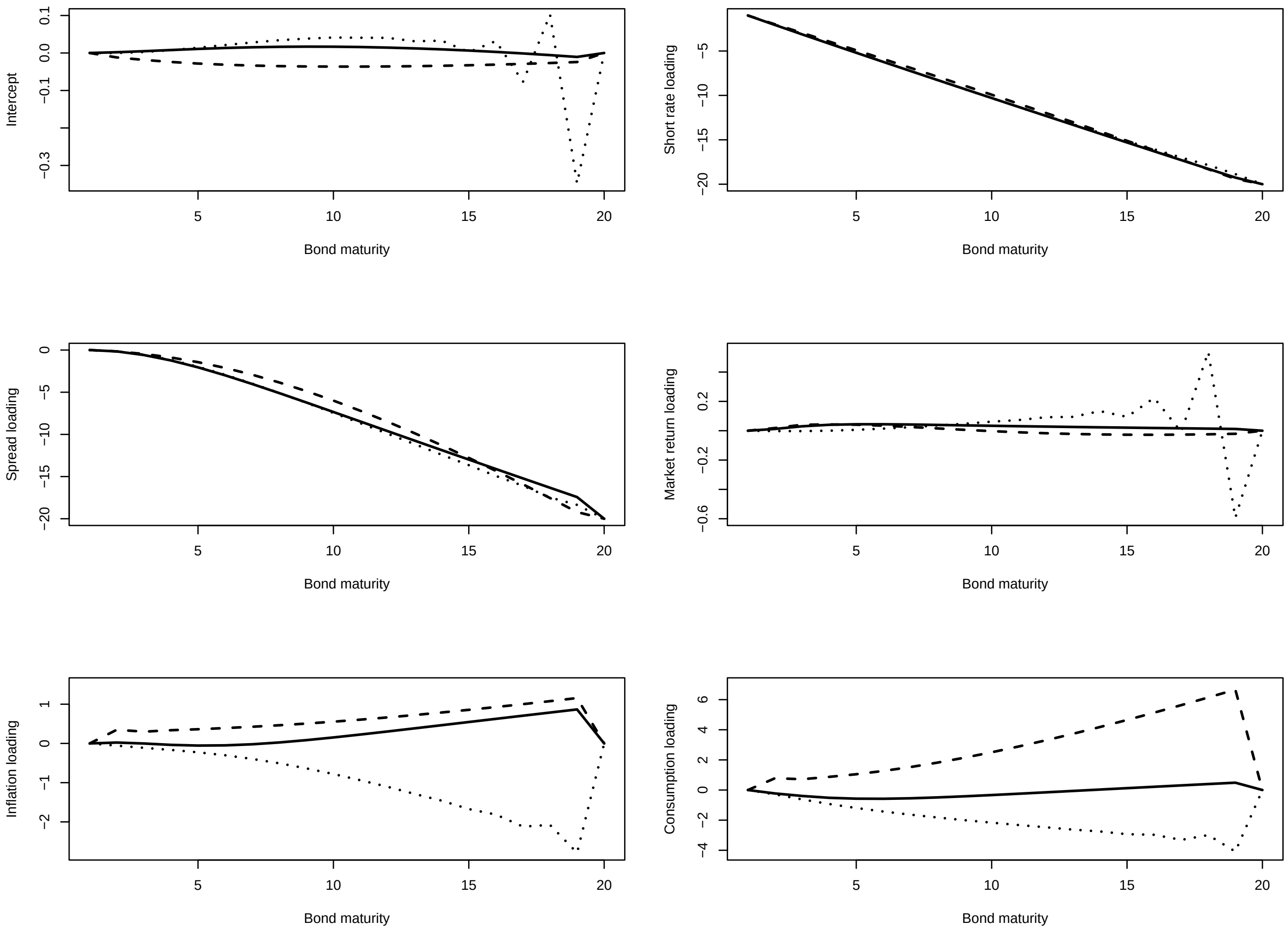

Figure 7. Innovation to the SDF of the non-expected utility model.

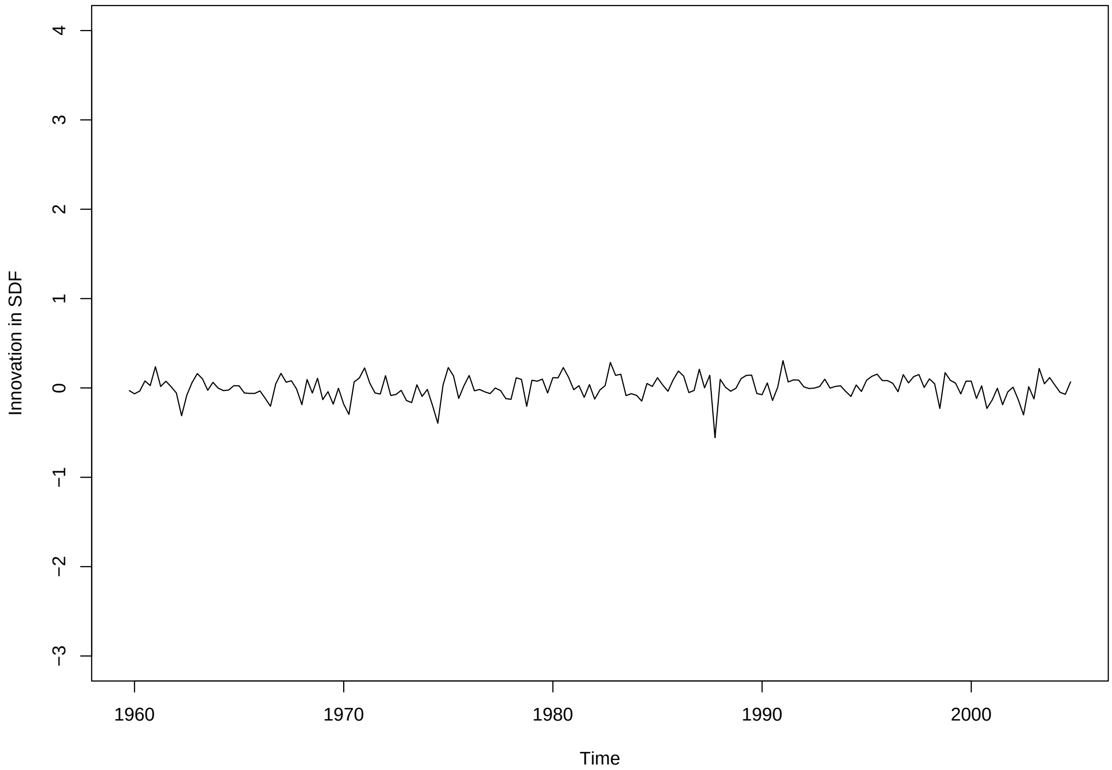

Figure 8. Innovation to the SDF of the reduced-form model.