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Risk Evaluation in Failure Mode and Effects Analysis Based on D Numbers Theory

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Abstract:
Failure mode and effects analysis (FMEA) is a useful technology for identifying the potential faults or errors in system, and simultaneously preventing them from occurring. In FMEA, risk evaluation is a vital procedure. Many methods are proposed to address this issue but they have some deficiencies, such as the complex calculation and two adjacent evaluation ratings being considered to be mutually exclusive. Aiming at these problems, in this paper, A novel method to risk evaluation based on D numbers theory is proposed. In the proposed method, for one thing, the assessments of each failure mode are aggregated through D numbers theory. For another, the combination usage of risk priority number (RPN) and the risk coefficient newly defined not only achieve less computation complexity compared with other methods, but also overcome the shortcomings of classical RPN. Furthermore, a numerical example is illustrated to demonstrate the effectiveness and superiority of the proposed method.

Keywords: failure mode and effects analysis, Dempster-Shafer evidence theory, D numbers, risk evaluation, aggregate assessment.

1 Introduction

Failure mode and effects analysis (FMEA) is an efficient technology for identifying potential faults, problems, risk, and errors from the system, procedure, and service. It improves the reliability by preventing these faults, problems, risks, and errors from occurring. Risk evaluation is a pivotal procedure in FMEA. Nowadays, FMEA is developed so fast that it is extensively applied in many fields, such as medical care [3,29,32,49,60], society [51], environmental protection [1,8], financial service [65], industry [2,24,69,78], and so on.

A traditional method for risk evaluation in FMEA is the classical risk priority number (RPN), which is obtained by multiplying the grades of the occurrence assessment, severity assessment, and detection assessment. Therefore, how to aggregate the assessment information of these three risk factors is a significant issue, especially when it comes to the information with uncertainty. Focusing on this problem, many math models such as fuzzy sets [14, 16, 17, 79], R numbers [53], D numbers [20], Z numbers [21,26,27,39,76] and evidence theory [10,55], have been applied to the real applications [37, 50]. In [28], Kim et al. present a general model to explain the functional relationship among the three factors, and use the model to discuss the unique role of each factor for comparing the risk of different failure modes. In [77], a new method to risk evaluation based on Dempster-Shafer evidence theory is proposed. Some other evidential FEMA are presented recently [5].

Nevertheless, although classical RPN is easy to use because of its succinct form, it is still criticized for its weaknesses. For example, the experts usually give the assessment with uncertainty or fuzzy information, but classical RPN is not appropriated to treat the fuzzy assessment. Furthermore, different combination of risk factors might acquire the same RPN, however, the potential risk might be totally different so that they might have different priorities. With the aim of overcoming these weaknesses, many methods, such as Chin's method [9], are proposed. However, existing methods almost is not only too complex to calculation, but also do not take the non-exclusiveness between two adjacent rankings into account. Actually, because of the subjectivity of the experts, two adjacent estimation scales are supposed to be not exclusive mutually. In order to solve the problems, a novel method based on D number theory [13] is proposed in this paper. On the one hand, the assessments for each failure mode are aggregated through constructing and combining D numbers because two propositions are allowed to be non-exclusive in D numbers theory. On the other hand, RPN is applied in the proposed method so as to reduce the computation complexity, simultaneously, novel compute mode to RPN and risk coefficient present in this paper is capable to get rid of the weaknesses of the classical RPN. Last but not the least, an illustrative example is used to show the effectiveness and superiority of the proposed method.

The remainder of this paper is organized as follows. Key concepts and previous theories are reviewed in short in Section 2. In Section 3, A novel risk evaluation in failure mode and effects analysis based on D numbers theory is proposed. To demonstrate the effectiveness and superiority of the proposed method, a numerical example is illustrated in Section 4. Last but not the least, A brief conclusion is drawn in Section 5.

2 Preliminaries

2.1 Dempster-Shafer evidence theory

The real application is inevitable to deal with uncertainty [4, 6, 45–47]. Dempster-Shafer evidence theory (D-S theory) [10, 55], is a significant theory to handle uncertainty information [11]. Compared with Bayesian theory, it needs weaker conditions so that it is often deemed as an extension of the Bayesian theory. D-S theory is widely applicated in many fields, such as decision making [6,22,30,44,71], pattern recognition [40,41,43,73], evidential reasoning [15,42,74,83–85], risk and reliability [52, 54], information fusion [59, 61, 75], uncertainty modelling [19, 25, 58] and conflict management [36, 68, 81].

Definition 1. Let $\Theta = \{H_1,\ H_2,\ \cdots,\ H_N\}$ be a finite nonempty set, which consist of N mutually exclusive elements. Let $P(\Theta)$ be the power set of Θ, which is composed of 2^N elements. The basic probability assignment (BPA) function is defined as a mapping of the power set $P(\Theta)$ to a number between 0 to 1, that is $m : P\left(\Theta\right) \rightarrow [0, 1]$, and which satisfies the following conditions:

$$m\left(\emptyset\right) = 0; \tag{1}$$

$$\sum_{A \in P(\Theta)} m\left(A\right) = 1. \tag{2}$$

The mass m(A) represents how strongly the evidence supports to A.

Definition 2. Let $m_1,\ m_2$ be two BPAs defined on the frame of discernment Θ. The Dempster's combination rule, denoted by $m = m_1 \bigoplus m_2$, is defined as follows:

$$m\left(A\right) = \begin{cases} \frac{1}{k-1}\sum_{B\cap C=A} m_1(B)m_2(C), & A \neq \emptyset \\ 0, & A = \emptyset \end{cases} \tag{3}$$

with

$$K = \sum_{B\cap C=\emptyset} m_1(B)m_2(C) \tag{4}$$

where K is a normalization constant which reflects the conflict of two bodies of evidence.

Actually, $0 \leq K \leq 1$. $K = 0$ shows the absence of conflict between two bodies of evidence. While $K = 1$ shows complete conflict between m_1 and m_2. Besides, when $K = 1$, the Dempster's combination rule is not any longer applicable, a possible explanation is open world assumption [32, 33]. In order to make decision in terms of the BPA, a method called pignistic probability transformation is present in [57], which derive a distribution of probabilities from the BPA. The pignistic probability transformation function is defined as follows:

Definition 3. Let m be a BPA on the frame of discernment Θ, a pignistic probability transformation function $BetP_m:\ \Theta \longrightarrow [0,1]$ associated to m is defined by

$$BetP_m\left(x\right) = \sum_{x\in A,\ A\in\Theta} \frac{1}{|A|}\frac{m(A)}{1-m(\emptyset)} \tag{5}$$

where $m\left(\emptyset\right) \neq 1$ and $|A|$ is the cardinality of proposition A.

2.2 Fuzzy set theory

Fuzzy sets were proposed independently by Zadeh [79] in 1965 as an extension of the classical notion of set. Fuzzy set theory is widely applied in many fields [23, 33, 72]. It reflects the stay of the object and its fuzzy concept as a fuzzy set. Then, it sets up the appropriate membership functions through fuzzy set about operation and transform, and analyzes the fuzzy object based on the fuzzy mathematics. In the objective world, there are many fuzzy phenomena. For example, when evaluating a person's appearance, people usually use linguistic variables whose values are represented by words or sentences in a natural or artificial language, such as "very pretty", "pretty", "general", "ugly", and "very ugly".

Definition 4. Denote L as the universe of discourse, a fuzzy set A is described by a membership function μ_A satisfying

$$\mu_A:\ L \longrightarrow [0,1]$$

where $\mu_A(x)$ is called the membership degree of $x \in L$ belonging to fuzzy set A.

For $L = \{x_1,\ \ldots, x_i,\ \ldots,\ x_n\}$, the fuzzy set (A, μ_A) is represent by

$$\frac{\mu_A\left(x_1\right)}{x_1}, \ldots, \frac{\mu_A\left(x_i\right)}{x_i},\ \ldots, \frac{\mu_A\left(x_n\right)}{x_n}.$$

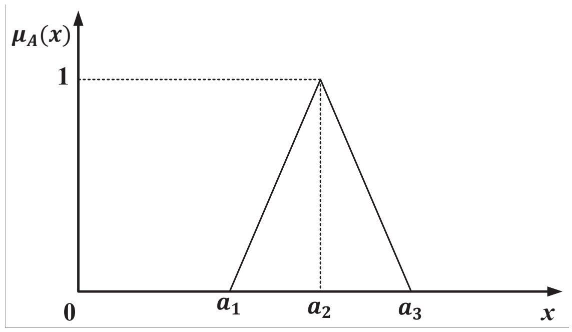

Figure 1: Graphical presentation of the triangular fuzzy number

It is obvious that a fuzzy set is characterized entirely by its membership function. When $\mu_A(x)$ get value from $\{0, 1\}$, fuzzy set A degrade into a classical set. A is a fuzzy subset of the real number R, and its membership function satisfies

$$\mu_A(x) : R \longrightarrow [0, 1]$$

where x is real number and there exists an element x_0 such that $\mu_A = 1$. Triangular fuzzy numbers are the most extensively applied fuzzy numbers. A triangular fuzzy number is usually expressed as $A = (a_1, a_2, a_3)$, as graphically shown in Figure 1, which has the following membership function

$$\mu_A(x) = \begin{cases} 0, & x < a_1 \\ \frac{x-a_1}{a_2-a_1}, & a1 \leq x \leq a2 \\ \frac{a3-x}{a3-a2}, & a_2 \leq x \leq a_3 \\ 0, & x > a_3 \end{cases} \tag{6}$$

where $a_1 < a_2 < a_3$.

In practice, fuzzy numbers are bound up with linguistic variables to describe the fuzzy evolution to objects.

2.3 Risk priority number

The real systems are too complicated to be modelled. Risk priority number (RPN) is a traditional and typical method to model and evaluate risk in FMEA. RPN is calculated by multiplying the grades of occurrence assessment (O), severity assessment (S), and detection assessment (D). That is

$$\mathrm{RPN} = \mathrm{O} \times \mathrm{S} \times \mathrm{D} \tag{7}$$

where O stands for the probability of occurrence of failure mode, S refers to the severity of failure mode and D refers to the probability of failure being detected. The three risk factors are evaluated by FMEA experts using a 1 to 10 numeric scale. Besides, occurrence assessment is expressed in Table 1. The larger RPN is, the more important degree it is supposed to be assigned, referring to the failure mode should be more priority to be corrected. Although this method is easy to use because of its sententious form, traditional RPN is still criticized for its weaknesses. For example, traditional RPN does not take the weights of three risk factors into consideration. Besides, different combination of risk factors might acquire the same RPN, however, the potential risk might be totally different so that they might have different priorities.

2.4 D numbers

D number, is a useful tool to model uncertain information, which overcomes the shortcomings of Dempster-Sharfer theory [20]. Nowadays, D number is widely used in many fields such as decision making [7, 19, 31], risk assessment [18, 29, 35, 67], reliability analysis [70, 80], data fusion [8, 62, 66]. It is defined as follows:

Definition 5. Let Ω be a finite nonempty set, D number is a mapping $D: \ \Omega \longrightarrow [0,1]$, such that

$$\sum_{B \subseteq \Omega} D(B) \leq 1 \text{ and } D(\emptyset) = 0 \tag{8}$$

where $\emptyset$ is the empty set and B is a subset of Ω.

If $D(B) = 1$, the information is regarded to be complete while $D(B) < 1$, it is considered to be incomplete. Most importantly, different from D-S theory, D number does not request the elements of set Ω to be mutually exclusive. In order to express the non-exclusiveness in Ω, a fuzzy membership function is used to measure the exclusive/non-exclusive degree [82].

Definition 6. Let A_i and A_j be two non-empty elements in 2^Ω, the non-exclusive degree between A_i and A_j is characterized by a fuzzy membership function $u_{\neg E}$ as follows:

$$u_{\neg E} : 2^\Omega \times 2^\Omega \longrightarrow [0,1] \tag{9}$$

with

$$u_{\neg E}(A_i, A_j) = \begin{cases} 1, & A_i \cap A_j \neq \emptyset \\ p, & p \in [0,1], \ A_i \cap A_j = \emptyset \end{cases} \tag{10}$$

besides,

$$u_{\neg E}(A_i, A_j) = \max_{x \in A_i, \ y \in A_j} \{u_{\neg E}(x, y)\} \tag{11}$$

Let u_E be the exclusive degree between A_i and A_j, then $u_E = 1 - u_{\neg E}$.

An illustrative example is given as follows to express the calculation of the non-exclusive degree.

Table 1: Assessment rankings for occurrence in FMEA

Ranking	Probability of Occurrence	Possible Failure Rate
10	Extremely high: failure almost inevitable	$\geq 1/2$
9	Very high	1/3
8	Repeated failures	1/8
7	High	1/20
6	Moderately high	1/80
5	Moderate	1/400
4	Relatively low	1/2000
3	Low	1/15000
2	Remote	1/150000
1	Nearly impossible	$\leq 1/150000$

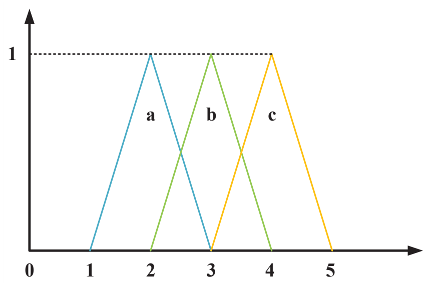

Figure 2: Graphically presentation of fuzzy variables in Table 2

Example 7. *Suppose there is a non-empty set $\Omega = \{a, b, c\}$, where a, b, c are three fuzzy variables represented by triangular fuzzy numbers given in Table 2 and shown in Figure 2. The non-exclusiveness degree can be calculated as follows [12]:*

$$u_{\neg E}(A, B) = \frac{Area_{A\cap B}}{Area_A + Area_B - Area_{A\cap B}} \tag{12}$$

where the areas of fuzzy numbers A and B are characterized by $Area_A$ and $Area_B$, and the area of the overlap of A and B is $Area_{A\cap B}$. Thus, according to Eq.12, we have

$$\begin{aligned}
u_{\neg E}(a, b) &= \frac{Area_{a\cap b}}{Area_a + Area_b - Area_{a\cap b}} = \frac{0.25}{1 + 1 - 0.25} \approx 0.1429 \\
u_{\neg E}(b, c) &= \frac{Area_{b\cap c}}{Area_b + Area_c - Area_{b\cap c}} = \frac{0.25}{1 + 1 - 0.25} \approx 0.1429 \\
u_{\neg E}(a, b, c) &= \max\{u_{\neg E}(a, b), u_{\neg E}(b, c)\} = \max\{0.1429, 0.1429\} = 0.1429 \\
u_{\neg E}(a, \{a, b\}) &= \max\{u_{\neg E}(a, a), u_{\neg E}(a, b)\} = \max\{1, 0.1429\} = 1
\end{aligned}$$

Similar with Dempster combination rule, the combination rule is discussed under two cases: complete information and incomplete information.

Definition 8. (complete information). Let D_1 and D_2 be two numbers over Ω with $\sum_{A\subseteq\Omega} D_1(A) = 1$ *and* $\sum_{A\subseteq\Omega} D_2(A) = 1$, the combination of D_1 and D_2, indicated by $D = D_1 \bigodot D_2$, is defined by

$$D(A) = \begin{cases} 0, & A = \emptyset \\ \frac{1}{1-K_D}\Big(\sum\limits_{B\cap C=A} u_{\neg E}(B, C) D_1(B) D_2(C) + \\ \sum\limits_{B\cup C=A, B\cap C=\emptyset} u_{\neg E}(B, C) D_1(B) D_2(C)\Big), & A \neq \emptyset \end{cases} \tag{13}$$

with

$$K_D = \sum_{B\cap C=\emptyset} (1 - u_{\neg E}(B, C)) D_1(B) D_2(C) \tag{14}$$

Table 2: The fuzzy number of fuzzy variables

Fuzzy Variables	Fuzzy Numbers
a	(1, 2, 3)
b	(2, 3, 4)
c	(3, 4, 5)

It is worth mentioning that this combination rule can be degenerated to the classical Dempster's rule if $u_{\neg E} = 0$ for any $B \cap C = \emptyset$. a numerical example is used to illustrate the combination of two D numbers under the complete information situation as follows:

Example 9. *There are two D numbers over* $\Omega = \{a, b\}$*:*

$$D_1(a) = 0.4, \qquad D_1(b) = 0.6$$
$$D_2(a) = 0.9, \qquad D_2(b) = 0.1$$

And assume $u_{\neg E}(a, b) = 0.1429$*. Thus, we have* $D\prime = D_1 \bigodot D_2$ *that*

$$\begin{aligned} K_{D\prime} &= (1 - 0.1429) \times [0.4 \times 0.1 + 0.6 \times 0.9] = 0.4971 \\ D\prime(a) &= \frac{1}{1 - 0.4971} \times 1 \times 0.4 \times 0.9 = 0.7159 \\ D\prime(b) &= \frac{1}{1 - 0.4971} \times 1 \times 0.6 \times 0.1 = 0.1193 \\ D\prime(a, b) &= \frac{1}{1 - 0.4971} \times 0.1429 \times [0.4 \times 0.1 + 0.6 \times 0.9] = 0.1648 \end{aligned}$$

Definition 10. (incomplete information). Let D_1 and D_2 be two numbers over Ω with $\sum_{A \subseteq \Omega} D_1(A) < 1$ *and*
$\sum_{A \subseteq \Omega} D_2(A) < 1$, the combination of D_1 and D_2, defined by $D = D_1 \bigodot D_2$ and calculated as follows:

$$D(A) = \begin{cases} 0, & A = \emptyset \\ f(Q_1, Q_2) \frac{D_t(A)}{\sum_{B \subseteq \Omega} D_t(B)}, & A \neq \emptyset \end{cases} \tag{15}$$

with

$$D_t(A) = \sum_{B \cap C = A} u_{\neg E}(B, C) D_1(B) D_2(C) + \sum_{B \cup C = A, B \cap C = \emptyset} u_{\neg E}(B, C) D_1(B) D_2(C), \forall\ A \in \Omega$$

and

$$Q_1 = \sum_{A \subseteq \Omega} D_1(A), \quad Q_2 = \sum_{A \subseteq \Omega} D_2(A) \tag{16}$$

where $f(Q_1, Q_2)$ is a function satisfying $0 \leq f(Q_1, Q_2) \leq \max\{Q_1, Q_2\}$, $f(Q_1, Q_2) = 1$ if $Q_1 = 1$ and $Q_2 = 1$.

Next, a simple example is used to illustrate the combination process of D numbers according to Definition 10.

Example 11. *There are two D numbers over* $\Omega = \{a, b\}$*:*

$$D_1(a) = 0.7, \qquad D_1(b) = 0.2$$
$$D_2(a) = 0.5, \qquad D_2(b) = 0.3$$

Assume $u_{\neg E}(a,b) = 0.1429$*, and let* $f(Q_1,Q_2) = Q_1 \times Q_2$*. Thus, we have* $D = D_1 \odot D_2$ *that*

$$\begin{aligned} f(Q_1,Q_2) &= (0.7+0.2)\times(0.5+0.3) = 0.72 \\ D_t(a) &= 0.7\times 0.5 = 0.35 \\ D_t(b) &= 0.2\times 0.3 = 0.06 \\ D_t(a,b) &= 0.1429\times(0.7\times 0.3+0.2\times 0.5) \approx 0.044 \end{aligned}$$

Thus,

$$\begin{aligned} \sum_{B\subseteq\Omega} D_1(B) &= 0.35+0.06+0.044 = 0.454 \\ D(a) &= f(Q_1,Q_2)\frac{D_t(a)}{\sum\limits_{B\subseteq\Omega} D_1(B)} \approx 0.555 \\ D(b) &= f(Q_1,Q_2)\frac{D_t(b)}{\sum\limits_{B\subseteq\Omega} D_1(B)} \approx 0.095 \\ D(a,b) &= f(Q_1,Q_2)\frac{D_t(a,b)}{\sum\limits_{B\subseteq\Omega} D_1(B)} \approx 0.070 \end{aligned}$$

If there are n D numbers expressed as $D_1,\ D_2,\ \cdots,\ D_n$, whose weights are $w_1, w_2,\ \cdots,\ w_n$, satisfying $\sum_{i=1}^{n} w_i = 1$. At first, the average D number among $D_1, D_2,\ \cdots, D_n$ is defined as

$$\bar{D}(A) = \sum_{i=1}^{n} w_i D_i(A), \forall A \subseteq \Omega. \tag{17}$$

Then, the result of combination $D_1, D_2,\ \cdots, D_n$ is acquired by combining average D number $\bar{D}$ with itself $n-1$ times.

$$D = \bar{D} \odot \bar{D} \odot \cdots \odot \bar{D} \tag{18}$$

where $\odot$ is the combination rule given in Definition 8 and Definition 10.

3 Proposed method

Failure mode and analysis (FMEA) is an efficient technology to identify and remove potential faults, errors and risk from systems. In FMEA, risk evaluation is a significant procession. Traditionally, risk priority number (RPR) is used to evaluate risk, which is calculated by multiplying the grades of occurrence assessment (O), severity assessment (S), and detection assessment (D). Although classical RPN is easy to use for its concise form, it is still criticized for its weaknesses. For example, traditional RPN does not take the weights of three risk factors into consideration. Besides, different combination of risk factors might acquire the same RPN, however, the potential risk might be totally different so that they might have different priorities. So far, many methods are presented. Nevertheless, most of them not only have rather complex algorithm, but also do not consider the non-exclusiveness between two rankings in assessment.

With the aim of solving these problems, in this paper, a novel method to risk evaluation based on D numbers theory is proposed. First of all, each assessment rating is treated as a fuzzy variable which is represented as a fuzzy number. Then, non-exclusive degree between two ratings can be calculated. Next, for each failure mode, the assessments of experts are aggregated by D numbers theory. The assessments are treated as D numbers and then are combined by using D numbers combination rule. Furthermore, RPNs are calculated for ranking the failure mode. Last but not the least, when coming to the same RPN of some failure modes, a variable, named risk coefficient, is defined to rank the failure modes with the same RPN. The large risk coefficient is, the more important degree it is supposed to be assigned, referring to the failure mode should be more priority to be corrected.

Step 1. Make sure the triangular fuzzy numbers for each rankings in assessment and calculate the non-exclusive degrees by 12.

Step 2. The assessments of experts are regarded as D numbers, that is, for each failure mode, the assessments of each expert is constructed as a D number. Therefore, the aggregation of experts' assessment of each failure mode by combining the corresponding D numbers. If information is complete, the equations in Definition 8 are used to the combination. If the information is incomplete, the equations in Definition 10 are applied to the combination. What is more, if different experts have different weights, it is supposed to put the 17 into use.

Step 3. For the results of Step 2, use 5 to calculate pignistic probability transformation (PPT).

Step 4. Calculate RPN. Firstly, calculate the mathematical expectation of each assessment. Then, use Eq. (7) to calculate the RPN of each failure mode.

Step 5. Calculate risk coefficient.

Definition 12. Let s be the standard deviation of the ratings of three assessments, defined as risk coefficient.

As mentioned above, different combination of risk factors might acquire the same RPN, however, the potential risk might be totally different. In this paper, risk coefficient, the standard deviation of the ratings of three assessments, is used to evaluate such kinds of failure modes. As a matter of fact, this method is reasonable. For example, there are two failure modes, the assessments of which are shown in Table 3.

Table 3: The assessments of three risk factors

	occurrence assessment	severity assessment	detection assessment
FM1	1	10	6
FM2	2	6	5

Apparently, two failure modes have the same RPN. However, as is shown in Table 3, compared with FM2, FM1 has the better grade in occurrence assessment but performs worse in severity and detection assessment. Therefore, FM1 should be more priority to be corrected. When observing and analyzing the data, it is not difficult to find that the distribution of FM2's data is more concentrated, which has smaller standard deviation. Besides, it is worth mentioning that standard deviation is usually used to measure the risk in financial field. Thus, it is reasonable that the standard deviation of the ratings of three assessments is applied to measure the risk in risk evaluation.

Step 6. Rank the failure modes though RPN. The larger RPN is, the more priority failure mode is supposed to be corrected. If the failure modes have the same RPN, their rankings depend on risk coefficient defined in Definition 12. The failure modes with the lager risk coefficient are assumed to be more significant and should be given higher priorities.

Table 4: The results of the risk evaluation

Item	Rating of risk factor								
	Expert 1			Expert 2			Expert 3		
	O	S	D	O	S	D	O	S	D
1	3:40% 4:60%	7	2	3:90% 4:10%	7	2	3:80% 4:20%	7	2
2	2	8	4	2	8:70% 9:30%	4	2	8	4
3	1	10	3	1	10	3	1	10	3
4	1	6:80% 7:20%	3	1	6	3:70% 2:30%	1	6	3
5	1	3	2:50% 1:50%	1	3	1:70% 2:30%	1	3:60% 2:40%	1
6	2	6	5	2	6	5	2	6	5
7	1	7	3	1	7	3	1	7	3
8	3	5:60% 6:40%	1	3	5:80% 6:20%	1	3	5:80% 7:20%	1
9	2:90% 1:10%	10:60% 9:40%	4	2:75% 1:25%	10:90% 9:10%	4	2:80% 1:20%	10:90% 9:10%	4
10	1	10	6	1	10	6	1	10	6
11	1	10	5	1	10	5	1	10	5
12	1	10	6:60% 5:40%	1	10	5:80% 4:20%	1	10	6:70% 5:30%
13	1	10	5:80% 4:20%	1	10	5	1	10	5
14	1	10	6	1	10	6:80% 7:20%	1	10	6
15	2	7:95% 6:5%	3	2	7	3	2	7	3:70% 4:30%
16	2:90% 1:10%	4	3	2:75% 1:25%	4	3	2:80% 1:20%	4	3:80% 2:20%
17	2	5:90% 6:10%	3	2	5:90% 6:10%	3	2	5:60% 6:40%	3

4 Numerical example

In order to demonstrate the effectiveness and superiority of the proposed method, a numerical example in [77] is solved in this section. Supposing there are three experts who evaluate 17 failure modes and identify the ratings of the three risk factors. The assessment results are expressed in Table 4. In this illustrative example, the weights of experts and risk factors are supposed to be equal to 1. Taking the failure mode 1 for example, the detailed computing is expressed as follows:

Step 1. The ratings are represented by triangular fuzzy numbers listed in Table 5 and shown in Figure 3. From Table 4, the occurrence assessments of three experts are $(3:40\%, 4:60\%)$, $(3:90\%, 4:10\%)$ and $(3:80\%, 4:20\%)$. Therefore, according to the Eq. (12), the non-

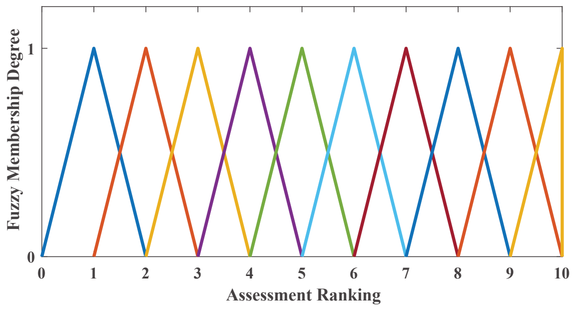

Figure 3: Graphically presentation of fuzzy variables in Table 5

Table 5: The fuzzy numbers of the assessment ratings

Assessment Ratings	Fuzzy Numbers
1	(0, 1, 2)
2	(1, 2, 3)
3	(2, 3, 4)
4	(3, 4, 5)
5	(4, 5, 6)
6	(5, 6, 7)
7	(6, 7, 8)
8	(7, 8, 9)
9	(8, 9, 10)
10	(9, 10, 10)

exclusiveness degrees are calculated as follows:

$$u_{\neg E}(3,3) = 1, \qquad u_{\neg E}(4,4) = 1, \qquad u_{\neg E}(3,\{3,4\}) = 1$$

$$u_{\neg E}(4,\{3,4\}) = 1, \qquad u_{\neg E}(3,4) = \frac{0.25}{1+1-0.25} \approx 0.1429$$

Step 2. Construct D numbers and combine D numbers. According to Section 3, corresponding to the evaluations of three experts, three D numbers are constructed as follows:

$$D_1(3) = 0.4, \qquad D_1(4) = 0.6$$
$$D_2(3) = 0.9, \qquad D_2(4) = 0.1$$
$$D_3(3) = 0.8, \qquad D_3(4) = 0.2$$

Then, because these D numbers have complete information, the combination among D_1, D_2 and D_3 is calculated by 10-14, that is $D_1 \bigodot D_2 \bigodot D_3$.

At first, calculate $D' = D_1 \bigodot D_2$:

$$K_{D'} = (1-0.1429) \times [0.4 \times 0.1 + 0.6 \times 0.9] = 0.4971$$

$$D'(3) = \frac{1}{1-0.4971} \times 1 \times 0.4 \times 0.9 = 0.7159$$

$$D'(4) = \frac{1}{1-0.4971} \times 1 \times 0.6 \times 0.1 = 0.1193$$

$$D'(3,4) = \frac{1}{1-0.4971} \times 0.1429 \times [0.4 \times 0.1 + 0.6 \times 0.9] = 0.1648$$

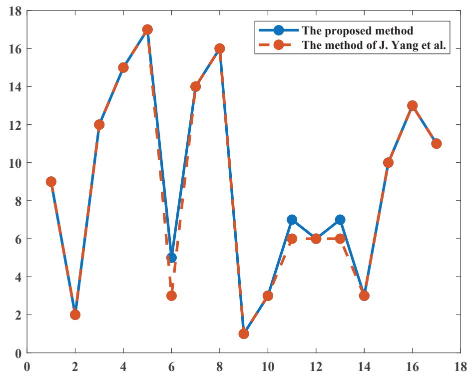

Figure 4: The risk priority of the failure modes using two methods

Next, calculate $D = D' \bigodot D_3$:

$$K_D = (1 - 0.1429) \times [0.7159 \times 0.2 + 0.1193 \times 0.8] = 0.2045$$
$$D(3) = \frac{1}{1 - 0.2045} \times [1 \times 0.7159 \times 0.8 + 1 \times 0.1648 \times 0.8] = 0.8857$$
$$D(4) = \frac{1}{1 - 0.2045} \times [1 \times 0.1193 \times 0.2 + 1 \times 0.1648 \times 0.2] = 0.0714$$
$$D(3,4) = \frac{1}{1 - 0.2045} \times 0.1429 \times [0.7159 \times 0.2 + 0.1193 \times 0.8] = 0.0429$$

Step 3. Calculate pignistic probability transformation (PPT) of the result of Step 2 by 5

$$BetP(3) = 0.8857 + \frac{0.0429}{2} \approx 0.9071 BetP(4) = 0.8857 + \frac{0.0429}{2} \approx 0.0929$$

Step 4. The mathematical expectation of the occurrence assessment is $3 \times 0.9071 + 4 \times 0.0929 = 3.0929$. Besides, three experts have the same evaluation to the severity assessment (S) and detection assessment (D) of failure mode 1, what is more, the evaluation are real numbers, 7 and 2, without any uncertainty. Thus, in these two assessments, the results through Step 1 to 4 are still 7 and 2.

Step 5. Calculate the RPN of failure mode 1 (RPN_1) by 7.

$$RPN_1 = O_1 \times S_1 \times D_1 = 3.0929 \times 7 \times 2 = 43.3006$$

Step 6. Calculate the risk coefficient of the failure mode 1.

$$\bar{x}_1 = \frac{3.0929 + 7 + 2}{3} \approx 4.0310$$

$$s_1 = \sqrt{\frac{1}{3} \cdot [(3.0929 - 4.0310)^2 + (7 - 4.0310)^2 + (2 - 4.0310)^2]} \approx 2.6287$$

The data of other failure modes are treated through Step 1 to 6 as mentioned above. Then, according to the RPNs and risk coefficients, the risk priorities of the failure modes are obtained, which are shown in Table 6. Meanwhile, in order to demonstrate the effectiveness of the proposed method, the results are compared with that of the J. Yang et al.'s method [77].

As shown in Table 6, the results of two methods are similar. Apart from failure mode 6, 11 and 13, other failure modes have the same risk priority rankings in both two methods. In addition, it is indicated that the five of highest risk priority rankings are failure mode 9, 2, 10,

Table 6: The results of the risk evaluation

Failure mode	RPN	Risk coefficient	The rankings of the proposed method	The rankings of the J. Yang et al.'s method
1	43.3006	2.6287	9	9
2	64	3.0551	2	2
3	30	4.7258	12	12
4	18	2.5166	15	15
5	3.0726	1.1478	17	17
6	60	2.0817	5	3
7	21	3.0551	14	14
8	15.0657	2.0110	16	16
9	78.10103376	4.1593	1	1
10	60	4.5092	3	3
11	50	4.5092	7	6
12	51.452	4.5047	6	6
13	50	4.5092	7	6
14	60	4.5092	3	3
15	42	2.6458	10	10
16	23.5152	1.0203	13	13
17	30.4038	1.5643	11	11

14, and 6, which refers that these 5 faults are most likely to occur. Furthermore, in both two methods, failure mode 16, 7, 4, 8, and 5 have the five of lowest priorities, indicating that these 5 failures are almost impossible to happen.

Figure 4 shows the comparison of risk priorities of two methods, in which the ranking is on the abscissa axis while the failure mode is on the vertical axis. As shown in Figure 4, two curves have the similar trend, which indicates that the proposed method is as effective as J. Yang et al.'s method.

Nevertheless, the results also reflect the different evaluations using two methods, which precisely demonstrates the superiority of the proposed method. As seen in Figure 4, using J. Yang et al.'s method, failure mode 11, 12, 13 have the same risk priority, while failure mode 12 has the larger risk priority compared with failure mode 11 and failure mode 13 in the proposed method. Because the highest rating of failure mode 12 in detection assessment is 6, which is higher than that of failure mode 11, 13 in detection assessment. Thus, failure mode 12 is obviously supposed to have higher risk priority compare with failure mode 11 and failure mode 13. What is more, in J. Yang et al.'s method, failure mode 6, 10, 14 have the same risk priority, but in the proposed method, failure mode 6 has the lower risk priority compared with failure mode 10 and failure mode 14. Although these failure modes have the same RPN, compared with failure mode 10 and failure mode 14, failure mode 6 has two lower risk ratings in severity assessment and detection assessment and merely a lager rating in occurrence assessment. Therefore, failure mode 6 is supposed to have the lower risk priority compared with failure mode 10 and failure mode 14.

5 Conclusion

In this paper, a novel method to risk evaluation in failure mode and effects analysis based on D numbers theory is proposed. In the proposed method, the application of the D numbers not only aggregates the fuzzy assessment in risk evaluation, but also takes the non-exclusiveness into account. Besides, the shortcomings of RPN are overcome successfully. Furthermore, the numerical example has demonstrated that the proposed method achieves less computation complexity compared with most existing method to risk evaluation in FMEA. In conclusion, the proposed method is an advanced and efficient method to risk evaluation in FMEA.

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Conflict of Interest

The authors declare no conflict of interest.

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