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RISK, INFLATION, AND THE STOCK MARKET

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ABSTRACT

Most explanations for the decline in share values over the past two decades have focused on the concurrent increase in inflation. This paper considers an alternative explanation: a substantial increase in the riskiness of capital investments. We show that the variance of firms' real gross marginal return on capital has increased significantly, increasing the relative riskiness of investors' returns on equity, and that this can explain a large part of the market decline. We also assess the effects of increases in the mean and variance of the inflation rate, and a decline in firms' expected return on capital.

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1. Introduction

From January 1965 to December 1981 the New York Stock Exchange Index declined by about 68 percent in real terms. Including dividends, the average real return as measured by this index was close to zero. Most explanations of this performance focus on the concurrent increase in the average rate of inflation.¹ For example, Modigliani and Cohn (1979) suggested that investors systematically confuse real and nominal discount rates when valuing equity,² Fama (1981) associates higher inflation rates with changes in real variables that reduce the return on capital, and Feldstein (1981a) argues that increased inflation reduces share prices because of the interaction of inflation with the tax system.

Feldstein (1981a,b) and Summers (1981a,b) claim that this last effect can explain a large fraction of the decline in share prices. The main sources of the effect are the "historic cost" method of depreciation and the taxation of nominal capital gains, both of which cause the net return from stock to fall when inflation rises. However, inflation also reduces the real value of the firm's debt, and reduces the net real return on bonds. The size and direction of the overall effect has been debated, and it clearly depends on the values of tax and other parameters.³ I will argue that increases in expected inflation -- together with concurrent increases in the variance of inflation -- should have had a small and possibly positive effect on share values.

Malkiel (1979) suggested another reason for the decline in share prices: changes occurred in the U.S. economy during the 1970's that substantially increased the riskiness of capital investments.⁴ This paper elaborates on and supports Malkiel's suggestion. It shows that the variance of the firm's real gross marginal return on capital has increased significantly since 1965, that this has increased the relative riskiness of investors' net real returns from holding stocks, and that this in turn can explain a large fraction of the market decline.

The volatility of stock returns has indeed increased, as illustrated by Figure 1, which shows the variance of the total monthly nominal return on the New York Stock Exchange Index, exponentially smoothed around a linear trend line ($\tilde{\sigma}_t^2 = .1\sigma_t^2 + .9\tilde{\sigma}_{t-1}^2$), for 1950-1981. (The computation of the sample variance is discussed in Section 3.) Also shown is the linear trend line $\bar{\sigma}_t^2 = .000764 + 4.369 \times 10^{-6}t$, which was fitted to the unsmoothed data. Observe that the variance has fluctuated widely, but has roughly doubled over the past twenty years. If shares are rationally valued, this reflects an increase in the variance of firms' gross marginal return on capital, and/or an increase in the variance of inflation.

Volatility in the firm's gross marginal return on capital comes from the stochastic nature of the instantaneous marginal product of capital (e.g. crop harvests, worker productivity, and physical depreciation all have random components), and from the capital gains and losses caused by unforeseen events that alter the expected future flow of marginal revenue product from existing capital (e.g. the effects of unanticipated regulatory change, exchange rate fluctuations that alter the competitive positions of goods produced abroad, etc.) Because the capital gains and losses are largely unrealized, the firm's gross marginal return on capital cannot be measured directly. However, its variance can be estimated indirectly from stock market data (assuming rational share valuation). As we will see, that variance has grown significantly, in a way consistent with Malkiel's suggestion that the business environment has become much more uncertain.

Increases in the expected rate of inflation have been accompanied by increases in the variance of that rate, and this can also affect the variance of stockholders' returns. First, inflation affects net real returns directly through the tax system, so that volatility of inflation causes volatility in these returns. Second, there is a well known negative correlation between unanticipated inflation and stock returns.⁵ We do not explain that correlation; as Fama (1981) has shown, it may in part be an indirect one occurring through

correlations with real economic variables. However, it implies a negative correlation between unanticipated inflation and the gross marginal return on capital, so that volatility of inflation will be associated with volatility in that gross return.⁶

On the other hand, an increased volatility of inflation also increases the riskiness of nominal bonds. The relative size of the effect again depends on tax rates and other parameters, but we will see that overall it makes bonds relatively riskier, and should therefore increase share values.

The next section of this paper shows how investors' net real returns on equity and bonds depend on taxes, inflation, and the gross marginal return on capital. The specification of those returns extends Feldstein's (1980b) model so that risk is treated explicitly. In Section 3 we discuss the data and parameter values, estimate the variance of the gross marginal return on capital and its covariance with inflation, and examine their behavior over time. In Section 4 a simple optimal portfolio model is used to relate changes in the price of equity to changes in the mean and variance of inflation, and the mean and variance of the marginal return on capital. Section 5 shows how changes in these means and variances over the past two decades can explain a good part of the behavior of share values.

Before proceeding, the main argument of this paper can be illustrated with two simple regressions. Summers (1981a) used a "rolling ARIMA" forecast to generate an expected inflation series, $\Delta\pi^e(t)$, and then (using quarterly data for 1958-78) regressed the real excess returns on the NYSE Index on the change in the expected inflation, $\Delta\pi^e(t)$. He obtained a negative coefficient for $\Delta\pi^e$, supporting his argument that increases in inflation cause decreases in share values.

I computed a similar series for $\pi^e(t)$, using annual averages of a rolling ARIMA forecast of monthly data.⁷ The corresponding OLS regression, for the

period 1958-81, is shown below (t- statistics in parentheses):

$$ER = .00335 - 3.615\Delta\pi^e$$

(1.19) (-2.29)

$$R^2 = .193$$

$$SER = .0137$$

$$D.W. = 1.93$$

As in Summers' regression, the coefficient of $\Delta\pi^e$ is negative and significant. But now let us add another explanatory variable, the change in the variance of stock returns, $\Delta\sigma_S^2$:

$$ER = .00258 + 1.481\Delta\pi^e - 8.936\Delta\sigma_S^2$$

(1.12) (0.76) (-3.48)

$$R^2 = .488$$

$$SER = .0112$$

$$D.W. = 1.85$$

Observe that the coefficient of $\Delta\sigma_S^2$ is negative and highly significant, while the coefficient of $\Delta\pi^e$ is now insignificantly different from zero.⁸ This simple regression suggests that increased risk and not increased inflation caused share values to decline. The analysis that follows explores that possibility.

2. Asset Returns

For simplicity, portfolio choice in this paper is limited to two assets, stocks and nominal bonds. I treat the rate of inflation as stochastic, so that the real returns on both of these assets are risky. Trading is assumed to take place continuously and with negligible transactions costs, and asset returns are described as continuous-time stochastic processes. As we will see, this provides a convenient framework for analyzing the effects of risk. In this section we derive and discuss expressions for investors' real after-tax asset returns. All of the parameters and symbols introduced here and throughout the paper are summarized in Appendix A.

A. The Return on Bonds

We describe inflation and bond returns as in Fischer (1975). The price level follows a geometric random walk, so that the instantaneous rate of inflation is given by

$$dP/P = \pi dt + \sigma_1 dz_1 \tag{1}$$

where $dz_1 = \varepsilon_1(t)\sqrt{dt}$, with $\varepsilon_1(t)$ a serially uncorrelated and normally distributed random variable with zero mean and unit variance, i.e. $z_1(t)$ is a Wiener process. Thus over an interval dt , expected inflation is πdt and its variance is $\sigma_1^2 dt$.

Bonds are short-term, and yield a (guaranteed) gross nominal rate of return R . We can view this return as an increase in the nominal price of a bond, i.e.

$$dP_B/P_B = Rdt \quad (2)$$

The gross real return on the bond is therefore:⁹

$$d(P_B/P)/(P_B/P) = (R - \pi + \sigma_1^2)dt - \sigma_1 dz_1 \quad (3)$$

Interest payments are taxed as income, so the investor's net nominal return on bonds is $(1-\theta)Rdt$, where θ is the personal income tax rate. The net real return on bonds over an interval dt is therefore:

$$\begin{aligned} \xi_b &= [(1-\theta)R - \pi + \sigma_1^2]dt - \sigma_1 dz_1 \\ &= r_b dt - \sigma_1 dz_1 \end{aligned} \quad (4)$$

This characterization of bond returns contains the simplifying assumption that stochastic changes in the price level are serially uncorrelated.¹⁰ If inflation actually followed (1), the real return on long-term bonds would be no riskier than that on short-term bonds.¹¹ In reality stochastic changes in the price level are autocorrelated, so that long-term bonds are indeed riskier. Our model could be expanded by adding long-term bonds as a third asset and allowing for autocorrelation in price changes, but the added complication would buy little in the way of additional insight, and would not qualitatively change any of the basic results.

B. The Return on Stocks

To derive an expression for investors' net real return on stocks, we begin with a description of the firm's gross marginal return on capital. Following

Feldstein (1980b), we then introduce the effects of inflation and taxes.

Over a short interval of time dt , the gross real return to the firm from holding a marginal unit of capital will consist of two components: the instantaneous marginal product of the unit, and the instantaneous change in the present value of the expected future flow of marginal product. This second component is just a capital gain or loss. However, it will generally be an unrealized capital gain or loss, so that the firm's gross marginal return on capital is not an accounting return.

Both components will be in part stochastic. The current marginal product of capital will have a stochastic element arising from random shocks in the production process: the weather in farming, random discovery rates in response to natural resource exploration, strikes, random week-to-week fluctuations in labor productivity, etc. Capital gains and losses are almost entirely stochastic, and occur when unforeseen events alter the expected value of the future flow of marginal product: for example, an OPEC oil shock that reduces the value of factories producing large cars while raising the value of drilling equipment, an exchange rate fluctuation that gives certain domestically produced goods a competitive advantage or disadvantage, a regulatory change that makes some existing capital obsolete or raises the cost of using it, etc.

On an aggregate basis, it is reasonable to assume that the stochastic part of the real gross marginal return on capital is normally distributed. We can then write that return as

$$m = \alpha dt + \sigma_2 dz_2 \quad (5)$$

where α is the expected return (largely the expected current marginal product). As Fama (1981) and others have stressed, this return is likely to be negatively correlated with the rate of inflation.¹² The magnitude and significance of this correlation will be addressed shortly; here we simply denote $E(dz_1 dz_2) = \rho dt$.

Although both the current and expected future marginal products of capital contribute to the stochastic term in eqn. (5), most of the variance is due to the capital gain component.¹³ Since these capital gains and losses are largely unrealized, they are not taxed directly. However they are taxed indirectly in that the corresponding future marginal products are taxed. We can therefore treat the corporate income tax as applying to both the deterministic and stochastic components of m .¹⁴ Letting τ_s be the statutory corporate income tax rate, and τ_e the effective corporate income tax rate ($\tau_e < \tau_s$ because of accelerated depreciation and the investment tax credit), and denoting corporate borrowing per unit of capital by b , the firm's net real return on capital in the absence of inflation is then $(1-\tau_e)\alpha dt - (1-\tau_s)bRdt + (1-\tau_e)\sigma_2 dz_2$.

Following Feldstein (1980b), we can adjust this net return for the effects of inflation. First, inflation reduces the real value of the firm's debt, so that the net after-tax cost of borrowing is $(1-\tau_s)bRdt - b(dP/P)$.¹⁵ Second, because the value of depreciation allowances is based on original or "historic" cost, inflation reduces the real value of depreciation and increases real taxable profits. We use Feldstein's linear approximation that a 1-percent increase in the price level reduces net profits per unit of capital by an amount λ . Then letting q denote the price of a share (representing a unit of capital), the firm's real net earnings per dollar of equity over an interval dt , ψ_s , is given by:

$$(1-b)q\psi_s = (1-\tau_e)\alpha dt - (1-\tau_s)bRdt + (1-\tau_e)\sigma_2 dz_2 + (b-\lambda)(dP/P) \quad (6)$$

Substituting eqn. (1) for dP/P , we then have:

$$(1-b)q\psi_s = [(1-\tau_e)\alpha - (1-\tau_s)bR + (b-\lambda)\pi]dt + (b-\lambda)\sigma_1 dz_1 + (1-\tau_e)\sigma_2 dz_2 \quad (7)$$

Now consider the after-tax return to investors. Let d be the fraction of net earnings paid out as dividends, and θ_c the effective tax rate on capital gains. Then in the absence of inflation, investors' net real return per dollar

of equity would be $\psi_s [(1-\theta)d + (1-\theta_c)(1-d)q]$. Inflation creates nominal capital gains at a rate $(dP/P)q$ per share, or $(dP/P)q(1-b)$ per unit of capital. Thus investors' net real return per dollar of equity, ξ_s is given by:

$$\xi_s = \psi_s [(1-\theta)d + (1-\theta_c)(1-d)q] - \theta_c (dP/P) \quad (8)$$

Again, we can substitute for dP/P , and for ψ_s . Letting

$$a = [(1-\theta)d + (1-\theta_c)(1-d)q]/(1-b)q \quad , \quad (9)$$

the net return is:

$$\begin{aligned} \xi_s &= \{a[(1-\tau_e)\alpha - (1-\tau_s)bR + (b-\lambda)\pi] - \theta_c \pi\}dt + [a(b-\lambda) - \theta_c]\sigma_1 dz_1 + a(1-\tau_e)\sigma_2 dz_2 \\ &= r_s dt + s_1 dz_1 + s_2 dz_2 \end{aligned} \quad (10)$$

C. Inflation and Asset Returns

Feldstein (1980a,b) has argued that increased inflation has reduced the expected real net return to investors from holding stocks, thereby depressing share values. As Friend and Hasbrouck (1982) have shown, that argument depends on tax rates and other parameter values. It also depends on the way the nominal interest rate R changes in response to changes in the expected inflation rate π . The conventional wisdom is $dR/d\pi = 1$, at least in long-run equilibrium. As Summers (1983) shows, in theory $dR/d\pi$ should be about 1.3 (if savings are interest inelastic) because of the taxation of nominal interest payments. Summers provides convincing evidence that $dR/d\pi$ has historically been much less than 1 -- at most about 0.6 -- even in the long run. I take the true value of $dR/d\pi$ to be an unresolved empirical question,¹⁶ and denote it by the parameter R_π . We can then examine how changes in π should affect share values for alternative values of R_π .

To see how inflation affects asset returns, numerical values are needed for the tax rates θ , θ_c , and τ_s , as well as the parameters b , λ , and d . All of the parameters are discussed in Appendix B, and reasonable values are: $\theta = .30$,

$\theta_c = .05$, $\tau_s = .48$, $b = .30$, $\lambda = .26$, and $d = .43$. Setting $q = 1$, we have $a = 1.2$.

As can be seen from eqns. (4) and (10), an increase in π reduces investors' expected return on equity as long as R_π is positive, but it also reduces the expected return on bonds as long as $R_\pi < 1.43$. What is relevant is the differential effect on expected stock returns versus bond returns. The parameter values above imply that

$$\frac{d}{d\pi} [E(\xi_s) - E(\xi_b)]/dt = - .887R_\pi + .998 \quad , \quad (11)$$

which is positive as long as $R_\pi > 1.13$. Since most estimates put R_π well below 1.13, it seems doubtful that increases in expected inflation depressed share values by differentially reducing the expected return on equity; in fact they could have worked to increase share values.

Asset demands also depend on the variances of these net returns. As shown shortly, σ_2^2 , the variance of the gross marginal return on capital m , has increased significantly over time, and this can explain much of the decline in share values. The variance of inflation, σ_1^2 , also increased in the 1970's. This increased the variance of bond returns, but to what extent did it contribute to the variance of investors' net return on equity? Observe from eqn. (10) that the variance of that return is:

$$(1/dt)\text{Var}(\xi_s) = [a(b-\lambda) - \theta_c]^2 \sigma_1^2 + a^2(1-\tau_e)^2 \sigma_2^2 + 2a[a(b-\lambda) - \theta_c](1-\tau_e)\sigma_1\sigma_2\rho \quad (12)$$

Using the parameter values from above, this becomes:

$$(1/dt)\text{Var}(\xi_s) = .000004\sigma_1^2 + .518\sigma_2^2 - .0029\rho\sigma_1\sigma_2 \quad (13)$$

Thus any increases in the variance of inflation would have had a negligible direct effect on the variance of the net real return on equity.¹⁷ Of course increases in σ_1^2 could have also affected share values by shifting the demand for bonds, but as shown in Section 4, the magnitude of any such effect is small.

3. Inflation and the Marginal Return on Capital Over Time

The variance of the real gross marginal return on capital, σ_2^2 , and its coefficient of correlation with inflation, ρ , cannot be observed directly. However they can be inferred from σ_s^2 , the variance of nominal stock returns, Ω_{sp} , the covariance of nominal stock returns with inflation, the inflation variance σ_1^2 , and the tax and financial parameters. Here we estimate these variances and covariance, infer values of σ_2^2 and ρ , and examine their behavior over time.

We use a crude method to estimate the mean and variance of the monthly inflation rate, and the monthly variance σ_s^2 and covariance Ω_{sp} . Assuming the true values of these parameters are slowly varying over time (i.e. are roughly constant over intervals up to a year, but may vary over periods of several years), we compute a moving 13-month centered sample mean, and sample variances and covariance. This yields estimates that are rough, but at least as accurate as available estimates of the various tax and financial parameters.

The mean inflation rate π is computed as a moving, 13-month centered sample mean, using the CPI as the price index:

$$\pi_t = \sum_{j=-6}^6 \Delta \log P_{t+j} / 13 \quad (14)$$

Similarly, a monthly sample variance is computed for σ_1^2 :

$$\sigma_1^2(t) = \sum_{j=-6}^6 (\Delta \log P_{t+j} - \pi_t)^2 / 12 \quad (15)$$

Trends in π_t and $\sigma_1^2(t)$ over the period 1953-81 are illustrated in Figures 2 and 3.¹⁸ Observe the clear upward trend in π from 1965 to 1981; the average annual inflation rate for 1953-68 was 2%, compared to 9% for 1973-81. Increases in the variance of inflation were roughly confined to the oil shocks and recessions of 1973-75 and 1979-82; σ_1^2 had an average 1953-68 value of 2.6×10^{-6} , and an average 1973-81 value of 6.9×10^{-6} .

Monthly total (nominal) returns data for the New York Stock Exchange Index were obtained from the CRISP tape. The sample variance σ_s^2 was computed using a

constant value of 0.71% for the monthly expected return:¹⁹

$$\sigma_s^2(t) = \sum_{j=-6}^6 (x_{t+j} - .0071)^2/12 \quad (16)$$

where x_t is the logarithmic return at time t . (See Figure 1.) Finally, the covariance Ω_{sp} is computed as:

$$\Omega_{sp}(t) = \sum_{j=-6}^6 (x_{t+j} - .0071)(\Delta \log P_{t+j} - \pi_t)/12 \quad (17)$$

To infer values of σ_2^2 and ρ from σ_s^2 , Ω_{sp} , σ_1^2 , and the various parameters, note that nominal net earnings per dollar of equity, ψ_s^n , are given by:

$$\psi_s^n = \psi_s + dP/P + \psi_s(dP/P) \quad (18)$$

Substituting eqn. (1) for dP/P and eqn. (7) for ψ_s yields:

$$\begin{aligned} (1-b)q\psi_s^n = & [(1-\tau_e)\alpha - (1-\tau_s)bR + (b-\lambda)\pi + (1-b)q\pi + (b-\lambda)\sigma_1^2 + (1-\tau_e)\rho\sigma_1\sigma_2]dt \\ & + [(b-\lambda) + (1-b)q]\sigma_1 dz_1 + (1-\tau_e)\sigma_2 dz_2 \end{aligned} \quad (19)$$

we will assume that shares are rationally valued,²⁰ so that $\sigma_s^2 = (1/dt)\text{Var}(\psi_s^n)$, and $\Omega_{sp} = (1/dt)\text{Covar}(\psi_s^n, dP/P)$. Thus,

$$\sigma_s^2 = \{(1-\tau_e)^2\sigma_2^2 + [(b-\lambda) + (1-b)q]^2\sigma_1^2 + 2(1-\tau_e)[(b-\lambda) + (1-b)q]\rho\sigma_1\sigma_2\}/(1-b)^2q^2 \quad (20)$$

$$\text{and } \Omega_{sp} = \{[(b-\lambda) + (1-b)q]\sigma_1^2 + (1-\tau_e)\rho\sigma_1\sigma_2\}/(1-b)q \quad (21)$$

These equations are solved simultaneously for σ_2^2 and ρ , given values for the tax and financial parameters. Most of these latter parameters have been estimated by others, or can be roughly calculated in a straightforward way. Values for all of them are discussed in Appendix B, and are summarized in Appendix A.

The calculated series for σ_2^2 is shown in Figure 4, together with a fitted trend line.²¹ Observe that movements of σ_2^2 closely parallel those of σ_s^2 , with a clear positive trend beginning about 1960. The average value of σ_2^2 for 1953-68 was .0017, and for 1973-81 it was .0036, more than a doubling. σ_2^2 was sharply higher during the oil and agricultural price of shock of 1974 and recession of

1975, but a strong positive trend remains even if these years are excluded; its average value for 1976-81 was .0027, a large increase from the 1950's and 1960's.

It is also interesting to compare σ_2^2 with the variance of the marginal product of capital. Holland and Myers (1980) estimated the latter to be about .000576 on an annual basis, or 4.8×10^{-5} on a monthly basis. This is roughly two percent of our average estimates of σ_2^2 , confirming that most of the variance of the marginal return on capital is due to capital gains and losses.

The monthly estimates of the correlation coefficient ρ fluctuate considerably over time, and are shown exponentially smoothed ($\tilde{\rho}_t = .1\rho_t + .9\tilde{\rho}_{t-1}$) in Figure 5. Observe that ρ was not always negative. One might expect ρ to increase or turn positive during periods when economic fluctuations are driven by demand shocks. This is consistent with its behavior during the 1950's, and with a 1953-68 average value of -.092, as compared to a 1973-81 average value of -.244.

To summarize, over the past 15 or 20 years there have been large increases in π , σ_1^2 , and σ_2^2 , and a decrease in ρ . We should add to this that there is evidence from the research of others that α , the expected gross marginal return on capital, has declined from about .12 to about .10 at an annual rate. In order to determine the implications of these trends for share values, we need a model of asset demands.

4. Asset Demands and Share Values

Given eqns. (4) and (10) for the asset returns ξ_b and ξ_s , we can use the solution to the investor's consumption/portfolio problem to determine asset demands, and thereby determine how share values change in response to changes π , σ_1^2 , σ_2^2 , etc. To do this, we assume that future income streams are certain and can be borrowed against, and can therefore be capitalized in initial wealth. We also assume that investors have constant relative risk aversion utility of consumption C .²² The consumption/portfolio problem is then:

$$\max_{C, \beta} E_0 \int_0^{\infty} \frac{1}{1-\gamma} (C^{1-\gamma} + \gamma - 2) e^{-\delta t} dt \quad (22)$$

subject to

$$dW = [\beta(r_s - r_b)W + r_b W - C]dt + \beta[s_2 dz_2 + (\sigma_1 + s_1) dz_1]W - \sigma_1 W dz_1 \quad (23)$$

where W is wealth, β is the fraction of wealth invested in stocks, $\gamma > 0$ is the index of relative risk aversion, and r_s , r_b , s_1 and s_2 are defined in eqns. (4) and (10).

This is similar to the consumption/portfolio problem in Merton (1971), except that both assets are risky. The solution is (see Appendix C):

$$\beta^* = (r_s - r_b) / \gamma \Sigma_{12}^2 + \sigma_1 \Sigma / \Sigma_{12}^2 \quad (24)$$

$$\begin{aligned} \text{where } \Sigma_{12}^2 &\equiv s_2^2 + 2\rho s_2(\sigma_1 + s_1) + (\sigma_1 + s_1)^2 \\ &= (1/dt) \text{Var}(\xi_s - \xi_b) \end{aligned} \quad (25)$$

$$\begin{aligned} \text{and } \sigma_1 \Sigma &\equiv \sigma_1(\sigma_1 + s_1 + \rho s_2) \\ &= (1/dt) \text{Var}(\xi_b) - (1/dt) \text{Covar}(\xi_s, \xi_b) \end{aligned} \quad (26)$$

Observe that we can also write the portfolio rule (24) in terms of the deviation from equal shares ($\beta = 1/2$):

$$\beta^* = \frac{1}{2} + \frac{E(\xi_s - \xi_b) - \gamma [\text{Var}(\xi_s) - \text{Var}(\xi_b)]}{\gamma \text{Var}(\xi_s - \xi_b)} \quad (27)$$

Thus the share of wealth held in stocks depends in an intuitively appealing way on the relative expectations and variances of the returns. The amount by which that share exceeds one-half is proportional to the difference in the expected returns less the difference in the variances of the returns (adjusted by the index of risk aversion).²³ Finally, note that this portfolio rule is also the one that maximizes the weighted sum $E(\xi_p) - \frac{1}{2} \gamma \text{Var}(\xi_p)$, where $\xi_p = \beta \xi_s + (1-\beta) \xi_b$ is the portfolio return.

To determine the effects of changes in π , σ_1^2 , σ_2^2 , etc. on share values, we use a partial equilibrium framework and assume a fixed quantity of stock s , and fixed aggregate wealth. Let ϕ denote some parameter of interest. Remember that $\beta^* = \beta^*(q, \phi)$, where q is the share price, so that

$$\frac{dq}{d\phi} = \frac{W}{s} \frac{d\beta^*}{d\phi} = \frac{q}{\beta^*} \left(\frac{\partial \beta^*}{\partial \phi} + \frac{\partial \beta^*}{\partial q} \frac{dq}{d\phi} \right)$$

or

$$d \log q / d\phi = \frac{\partial \beta^* / \partial \phi}{\beta^* - q(\partial \beta^* / \partial q)} \quad (28)$$

Formulas for $d \log q / d\pi$, $d \log q / d\sigma_1^2$, $d \log q / d\alpha$, $d \log q / d\sigma_2^2$, and $d \log q / d\gamma$ are given in Appendix D. The numerical values of these derivatives will of course depend on the values of π , σ_1^2 , σ_2^2 , etc., as well as the tax and financial parameters. Here we calculate values for the derivatives using the following average values for the 1965-81 time period: $\bar{\pi} = .00539$, $\bar{R} = .00625$, $\bar{\sigma}_1^2 = 4.6 \times 10^{-6}$, $\bar{\sigma}_2^2 = .0030$, $\bar{\rho} = -.22$, $\bar{\alpha} = .00858$. (Note that these are monthly means and variances.) The values of the tax and other parameters are those listed in the summary table of Appendix A, and discussed in Appendix B.²⁴

The derivatives (28) are calculated around a base value of β^* . We take β^* to be the ratio of the value of equity to the value of equity plus debt, both long- and short-term. (To keep the model simple we are assuming separability of such assets as human capital, housing, land, money, etc., and ignoring the risk differentials across various debt instruments.) The derivatives also depend on the value of γ , the index of relative risk aversion. Taking the other parameter values as given, one can choose γ so that the calculated value of β^* is equal to 0.67, its average for the 1965-81 period as given by the National Balance Sheets. That value of γ is 5.8, which may appear large, but is consistent with Friend and Blume's (1975) estimates showing γ to be "in excess of two," and Grossman and Shiller's (1981) finding that γ appears to be about 4. We take the true value of γ to be an unresolved empirical question, and calculate

numerical values of the derivatives for alternative values of γ . These are shown in Table 1.

Observe that the sign of $d\log q/d\pi$ depends on the value of R_π . Changes in π affect only expected returns and not their variances or covariance, so from eqn.

(11) $d\log q/d\pi > (<) 0$ if $R_\pi < (>) 1.13$. The numbers in Table 1 indicate that changes in π should have a small effect on share values -- unless R_π is around .6 or less, as Summers (1983) findings indicate may be the case. If $R_\pi = 1.0$, an increase in the expected annual inflation rate from 5% to 10% ($\Delta\pi = .0039$) implies a 7% increase in q if $\gamma = 5$, and a 10% increase if $\gamma = 3$. However, if $R_\pi = .6$, the result is a 30% increase in q if $\gamma = 5$, and a 44% increase if $\gamma = 3$.

The sign of $d\log q/d\sigma_1^2$ depends on γ (see eqn. (D.2) in the Appendix). For our parameter values, $d\log q/d\sigma_1^2 > (<) 0$ if $\gamma > (<) .987$. An increase in σ_1^2 increases the relative riskiness of bonds, but it also increases their expected return (see eqn. (4)). If γ is small enough, the second effect increases the demand for bonds more than the first reduces it, so that q falls. Since γ is probably greater than one and possibly greater than four, we would expect $d\log q/d\sigma_1^2 > 0$. However, this derivative is small given the average size of σ_1^2 . In 1973-75 σ_1^2 roughly quadrupled from a pre-OPEC average value of about 2×10^{-6} . (See Figure 3.) This ($\Delta\sigma_1^2 = 6 \times 10^{-6}$) implies only a 0.4% increase in q if $\gamma = 5$ and a 0.3% increase if $\gamma = 3$.

As we have noted, there is some evidence that α , the expected real gross marginal return on capital, declined somewhat during the 1970's, perhaps from .12 to .10 at an annual rate.²⁵ This decline in α ($\Delta\alpha = .0015$) would imply an 18% decline in q if $\gamma = 5$, and a 26% decline if $\gamma = 3$. This is clearly a large effect, so even a perceived (as opposed to actual) decline in α could explain a significant amount of the market's performance.

As we saw in the preceding section, there is evidence that over the past two decades σ_2^2 has more than doubled from an average 1953-68 value of about

TABLE 1 - Values of Derivatives at Point of Means

r	$\frac{d \log q}{d \pi}$			$\frac{d \log q}{d \sigma_1^2}$	$\frac{d \log q}{d \alpha}$	$\frac{d \log q}{d \sigma_2^2}$	$\frac{d \log q}{d \gamma}$
	$R_{\pi}=1.3$	$R_{\pi}=1.0$	$R_{\pi}=0.6$				
.2	-115.0	82	345	-591	533	-51	-22.3
.5	-94.3	67.2	283	-299	436	-105	-7.29
1	-72.4	51.6	217	6	335	-161	-2.80
2	-49.4	35.3	148	326	229	-220	-0.96
3	-37.5	26.8	113	493	174	-251	-0.48
4	-30.3	21.6	90.7	594	140	-270	-0.29
5	-25.3	18.1	75.9	663	117	-282	-0.20
6	-21.8	15.5	65.3	712	101	-292	-0.14
7	-19.1	13.6	57.3	750	89	-298	-0.11
8	-17.0	12.1	51.1	779	79	-303	-0.08
10	-14.0	10.0	41.9	822	65	-312	-0.05
∞	0	0	0	1017	0	-348	0

.0017 (monthly) to an average 1973-81 value of .0036. This increase in σ_2^2 would imply a 54% decline in q if $\gamma = 5$, a 48% decline if $\gamma = 3$, and a 31% decline if $\gamma = 1$. If σ_2^2 (or investors' estimates of σ_2^2) indeed doubled, this would explain a large part of the market's decline for any reasonable value of γ .

Finally, observe that an increase in γ would also have a large negative effect on share values. For example, an increase in γ from 4 to 5 would imply a 29% decline in q. However, we have no evidence that γ has changed over time one way or the other, so our focus in the next section is only on changes in π , σ_1^2 , α , and σ_2^2 .

5. Explaining the Decline in Share Values

In Section 3 we observed differing patterns of change in π , α , σ_1^2 , and σ_2^2 , but in all cases there have been reasonably clear shifts from the period 1953-68 to the period 1973-81. To summarize those shifts, π increased from a 1953-68 average value of 2% annually to a 1973-81 average value of 9% annually ($\Delta\pi = .0056$ on a monthly basis), α fell from about a 12% annual rate to about a 10% annual rate ($\Delta\alpha = -.0015$), and the average monthly variances σ_1^2 and σ_2^2 increased from 2.6×10^{-6} to 6.9×10^{-6} and .0017 to .0036, respectively. Table 2 and Figure 6 show the effects of these changes on share values, individually and in combination, as a function of γ , and assuming $R_\pi = 1.0$.

Observe that the relative importance of changes in π , α , and σ_2^2 depends on the index of risk aversion γ . For very small values of γ , changes in π and α have large effects on q, but if γ exceeds 2 or 3, the change in σ_2^2 clearly dominates. The model developed in this paper suggests a value of γ around 5 or 6, and the work of Friend and Blume (1975) and Grossman and Shiller (1981) support values that are at least in excess of 2.

The increases that occurred in σ_1^2 should have had a negligible direct effect on share values. Of course we have ignored the fact that unanticipated price changes are autocorrelated, which in the 1970's caused a pronounced increase in the riskiness of longer-term bonds. For example, for 1973-81 the variance of

TABLE 2- Changes in Share Values
(Percentage change, assumes $R_{\pi} = 1.0$)

γ	$\Delta\sigma_2^2 = .0019$	$\Delta\sigma_1^2 = 4.3 \times 10^{-6}$	$\Delta\pi = .0056$	$\Delta\alpha = -.0015$	Δq_{TOTAL}
.2	- 9.7	-0.3	45.9	-80.0	-44.1
.5	-20.0	-0.1	37.6	-65.4	-47.9
.8	-26.9	0.0	32.6	-56.3	-50.6
1	-30.6	0.0	28.9	-50.2	-51.9
2	-41.8	0.1	19.8	-34.4	-56.3
3	-47.7	0.2	15.0	-26.1	-58.6
4	-51.3	0.3	12.1	-21.0	-59.9
5	-53.6	0.3	10.1	-17.6	-60.8
6	-55.5	0.3	8.7	-15.2	-61.7
7	-56.6	0.3	7.6	-13.3	-62.0
8	-57.6	0.3	6.8	-11.9	-62.4
10	-59.3	0.4	5.6	- 9.8	-63.1
∞	-66.1	0.4	0	0	-69.9

the real return on 5-year government bonds was about 50 times as large as that on 1-month Treasury bills.²⁶ We can roughly (and conservatively) account for this in our model by scaling up σ_1^2 and σ_2^2 by a factor of 50. The value of $d \log q / d \sigma_1^2$ in Table 1 then falls by about a factor of 5 if $\gamma = 3$, and a factor of 3 if $\gamma = 5$ (the other derivatives remain virtually unchanged), so that the implied increase in q would still not exceed 5%. Thus even if most debt were longer-term, accounting for autocorrelation would not change this result.

If one accepts that $\gamma > 2$, then our results point to increased capital risk as the major cause of the decline in share values. If one believes instead that $\gamma < 2$, then the decline in the expected return α stands out as the major explanatory factor, although increased capital risk is still important. As for increases in expected inflation, this should have increased share values, unless R_{π} is larger than recent estimates indicate.

6. Concluding Remarks

We have found that much of the decline in share values can be attributed to the behavior of the gross marginal return on capital. Others have shown that the expectation of that return has fallen (although there is some controversy over how much), and this paper has shown that its variance has approximately doubled. The relative importance of these two changes depends on investors' index of risk aversion; if that index exceeds 2, increased risk is the dominating factor.

These results are based on a very simple model of asset returns, asset demands, and share price determination. Some of the model's limitations have already been mentioned: the reliance on rational share valuation for the estimation of σ_2^2 , the assumption of a deterministic income stream in the consumption/portfolio problem, and the inclusion of only two assets in investors' portfolios. Even more limiting is the use of a partial equilibrium framework, which ignores the fact that as q falls the capital stock will begin to fall, and the expected return α will begin to rise, pushing q back up. Accounting for this would reduce the magnitude of the derivatives in Table 1, at least for the longer term.

Another issue that requires more attention is how investors' perception of capital risk should be measured. We estimate capital risk from the sample variance of stock market returns, but the use of survey data might yield better estimates of investors' perceptions of that risk. For example, if investors believe that there is a non-negligible probability of economic catastrophe, capital would be quite risky, even if stock returns are not very volatile.

Finally, we have implicitly assumed that increases in the estimated values of σ_2^2 reflect actual increases in the riskiness of capital as an aggregate. Other interpretations are possible once we recognize that the capital stock is heterogenous. For example, suppose technological change caused the expected return on riskier capital (i.e. the shares of high-tech growth firms) to rise. The demand for these riskier stocks would then rise, increasing the variance of the returns on a value-weighted aggregate stock index, and possibly causing share values on average to rise.

APPENDIX

A. Summary of Parameters and Symbols

<u>Parameter</u>	<u>Definition</u>	<u>1965-81 Mean Value of Equation No.</u>
a	$[(1-\theta)d + (1-\theta_c)(1-d)q]/(1-b)q$ (identity)	1.20
b	Corporate borrowing per unit of capital	.30
d	Ratio of dividends to net earnings	.43
m	Firms' real gross marginal return on capital	eqn. (5)
q	Price of a share	1
R	Monthly nominal return on bonds (4-6 month commercial paper rate)	.00625
r_b	Expected net real monthly return on bonds	-.00101
r_s	Expected net real monthly return on stocks	.00500
s_1	$[a(b-\lambda) - \theta_c]\sigma_1$ (identity)	-4.3×10^{-6}
s_2	$a\sigma_2$ (identity)	.066
α	Expected real gross marginal return on capital (monthly)	.0086
β	Fraction of wealth in stocks	.67
γ	Index of relative risk aversion	
λ	Reduction in net profits per unit of capital from 1 percent increase in price level	.26
π	Expected monthly rate of inflation	.00539
ρ	Correlation coefficient: $E(dz_1 dz_2)/dt$	-.22
σ_1^2	Variance of monthly inflation rate	4.6×10^{-6}
σ_2^2	Monthly variance of real gross marginal return on capital	.0030
Σ_{12}^2	$(1/dt)\text{Var}(\xi_s - \xi_b)$.0015
$\sigma_1 \Sigma$	$(1/dt)\text{Var}(\xi_b) - (1/dt)\text{Covar}(\xi_s, \xi_b)$	-1.40×10^{-5}
ψ_s	Firms' real net rate of earnings per dollar of equity	eqn. (7)
ψ_s^n	Firms' nominal net rate of earnings per dollar of equity	eqn. (18)
ξ_b, ξ_s	Investors' real net rates of return on bonds and equity	eqns. (4), (10)
θ	Personal tax rate on interest and dividends	.30
θ_c	Effective tax rate on capital gains	.05
τ_s	Statutory corporate income tax rate	.48
τ_e	Effective corporate income tax rate	.40

B. Tax and Financial Parameters

The relevant tax parameters are the statutory and effective corporate tax rates τ_s and τ_e (the latter accounts for accelerated depreciation and the investment tax credit), the marginal personal tax rate θ , and the effective tax rate on capital gains θ_c . Over 1960-82, the statutory corporate tax rate has varied from a high of .528 in 1968-69 to a low of .46 since 1979. We use a constant average value of .48. Because the effects of inflation are explicitly included in our model, to avoid double counting we must remove those effect when computing the effective tax rate τ_e . To do this we utilize Feldstein and Summers' (1979) estimates of the excess tax due to inflation. As can be seen in Table 3, the effective rate τ_e has fluctuated, but has had an average value of .41 for 1962-77. Given the lower values for 1975-77, we use a constant value of .40 in the model. Feldstein and Summers (1979) also estimate the effective tax rate on capital gains and find it to be about .05, the value that we use. Finally, we use a value of .30 for the personal tax rate θ .

A rough value for b , corporate borrowing per unit of capital, can be computed by dividing the total debt of non-financial corporations by the total value of their capital, using data from the National Balance Sheets. That ratio was .26 to .28 during 1962-66, but from 1967 to 1981 it remained between .29 and .31. We use a constant average value of .30 for b . Similarly, a rough value for d , the fraction of net earnings paid out as dividends, can be computed by dividing aggregate dividends by aggregate after-tax profits, using data from the Survey of Current Business. That ratio has declined from about .48 in 1962 to about .34 in 1980; we use a constant average value of .43. Finally, a value is needed for λ , which measures the reduction in net profits per unit of capital resulting from a 1% increase in the price level. We use a value of 0.26, as estimated by Feldstein (1980b).

One parameter remains, and it is an important one: α , the expected real gross marginal return on capital. There has been some debate over whether α has

TABLE 3- Computation of "Zero-Inflation" Effective Tax Rate τ_e

(Non-Financial Corporations)

Year	(a) Profits with IVA,CCA	(b) Interest (RbK)	(c) (a)+(b) αK	(d) Tax Liab. T	(e) Excess Tax Due to Inflation	(f) (d)-(e) + τ_s (b)	(g) (f)÷(c) τ_e
1962	45.6	4.5	50.1	20.6	2.4	20.5	.41
1963	51.2	4.8	56.0	22.8	2.1	23.2	.41
1964	57.7	5.3	63.0	24.0	2.0	24.7	.39
1965	67.7	6.1	73.8	27.2	1.8	28.3	.38
1966	72.2	7.4	79.6	29.5	2.0	31.1	.39
1967	68.8	8.7	77.5	27.7	2.4	29.5	.38
1968	73.3	10.1	83.4	33.4	3.2	35.5	.43
1969	67.5	13.1	80.6	33.1	4.1	35.9	.45
1970	52.7	17.0	69.7	27.0	4.8	30.6	.44
1971	62.1	18.0	80.1	29.8	5.5	32.9	.41
1972	72.7	19.1	91.8	33.6	6.0	36.8	.40
1973	78.6	23.0	101.6	40.0	6.9	44.1	.43
1974	63.6	29.6	93.2	42.0	10.3	45.9	.49
1975	86.1	30.8	116.9	41.2	15.4	40.6	.35
1976	107.3	29.5	136.8	52.6	17.3	49.5	.36
1977	126.3	33.2	159.5	59.4	19.1	56.2	.35

Sources: Survey of Current Business - (Table 1.13) - (a), (b), (d)
 Feldstein and Summers, National Tax Journal, 1979 - (e)

been falling during the past decade or two, and the evidence is mixed, particularly if α is measured on a cyclically adjusted basis. Using the cyclically adjusted estimates of α made by Feldstein, Poterba, and Dicks-Mireaux (1981), the average value for 1962-79 is .11 annual (.00858 monthly). Those estimates suggest that α has declined from .12 to .10, and we calculate the effect of such a decline on share values.

C. Optimal Consumption/Portfolio Rule

To solve the consumption/portfolio problems of eqn. (22) and (23), write the value function $V(W)$:

$$V = \max_{C, \beta} E_t \int_t^{\infty} \frac{1}{1-\gamma} (C^{1-\gamma} + \gamma - 2) e^{-\delta(\tau-t)} d\tau, \quad (C.1)$$

and note that it must satisfy

$$\delta V = \max_{C, \beta} \{ C^{1-\gamma} / (1-\gamma) + [\beta(r_s - r_b)W + r_b W - C] V_W + (\frac{1}{2} \beta^2 \Sigma_{12}^2 + \frac{1}{2} \sigma_1^2 - \beta \sigma_1 \Sigma) W^2 V_{WW} \} \quad (C.2)$$

where Σ_{12}^2 and Σ are defined by eqns. (25) and (26). The first-order conditions are:

$$C^* = V_W^{-1/\gamma} \quad (C.3)$$

$$\text{and } \beta^* = - (r_s - r_b) V_W / \Sigma_{12}^2 W V_{WW} + \sigma_1 \Sigma / \Sigma_{12}^2 \quad (C.4)$$

Now substitute (C.3) and (C.4) into (C.2) and solve the resulting differential equation for V to yield:

$$V(W) = A^{-\gamma} W^{1-\gamma} / (1-\gamma) \quad (C.5)$$

where

$$A = \frac{1-\gamma}{\gamma} \left[\frac{\delta}{1-\gamma} - r_b - (r_s - r_b)^2 / 2 \Sigma_{12}^2 \gamma - (r_s - r_b) \sigma_1 \Sigma / \Sigma_{12}^2 + \frac{1}{2} \gamma \sigma_1^2 (1 - \Sigma^2 / \Sigma_{12}^2) \right] \quad (C.6)$$

Substituting (C.5) into (C.3) and (C.4) gives $C^*(W) = AW$, and eqn. (24) for β^* .

D. Calculating Changes in Share Values

Based on the partial equilibrium relationship (28), one can calculate how the share price q changes in response to changes in π , σ_1^2 , α , σ_2^2 , or γ , as in Table 1. Note that β^* is given by eqn. (24), Σ_{12}^2 and Σ by eqns. (25) and (26), and r_b , r_s , s_1 , and s_2 by eqns. (4) and (10). The relevant derivatives are:

$$d\log q/d\pi = B[a(b-\lambda) - a(1-\tau_s)bR_\pi + (1-\theta_c) - (1-\theta)R_\pi]/\gamma\Sigma_{12}^2 \quad (D.1)$$

$$d\log q/d\sigma_1^2 = B[\eta(1-\beta a(1-\tau_e)\rho\sigma_2/\sigma_1) - 1/\gamma + a(1-\tau_e)\rho\sigma_2/2\sigma_1 - \beta\eta^2]\Sigma_{12}^2 \quad (D.2)$$

$$d\log q/d\alpha = Ba(1-\tau_e)/\gamma\Sigma_{12}^2 \quad (D.3)$$

$$d\log q/d\sigma_2^2 = B[a(1-\tau_e)\rho\sigma_1/2\sigma_2 - \beta a^2(1-\tau_e)^2 - \beta a(1-\tau_e)\rho\sigma_1/\sigma_2]/\Sigma_{12}^2 \quad (D.4)$$

$$d\log q/d\gamma = - Br/\gamma^2\Sigma_{12}^2 \quad (D.5)$$

where $\eta = a(b-\lambda) + 1 - \theta_c$, and

$$B = q\gamma(1-b)\Sigma_{12}^2 / \{q\beta\gamma(1-b)\Sigma_{12}^2 + (1-\theta)d[(1-\tau_e)\alpha - (1-\tau_s)bR + (b-\lambda)(\pi+\gamma\sigma_1^2) + (1-\tau_e)\gamma\rho\sigma_1\sigma_2 - (1-\tau_e)^2 2\beta a\sigma_2^2 - 2\beta(a(b-\lambda) + \eta)(1-\tau_e)\rho\sigma_1\sigma_2 + (b-\lambda)\eta\sigma_1^2]\} \quad (D.6)$$

FOOTNOTES

1. Of course we could argue that no explanation is needed, i.e. the performance of the market was simply an "unlucky" realization of a stochastic process.
2. It seems hard to believe that such a confusion would persist, particularly during a decade of high inflation. Also, as Summers (1981a) points out, such confusion should also have lead to declines in prices of owner-occupied housing.
3. See Friend and Hasbrouck (1982) and Feldstein (1982). Hendershott (1981) shows the effect is reduced considerably if debt and equity yields are made endogenous.
4. Changes cited by Malkiel include the return of severe recessions (viewed as a thing of the past during the 1960's), a higher and more variable inflation rate (treated explicitly in this paper), and an increase in both the extent and unpredictability of government regulation.
5. See Bodie (1976), Nelson (1976), and Fama and Schwert (1977).
6. There are good reasons to expect this. For example, supply shocks create unanticipated inflation and at the same time reduce the current and expected future marginal products of capital. Also, as Parks (1978) has shown, unanticipated inflation increases the dispersion of relative prices. This increases the dispersion of profits across firms (increasing risk for each firm), and if adjustment costs are significant, reduces expected profits overall. Related is Friedman's (1977) suggestion that unanticipated inflation reduces economic efficiency by magnifying the distortions caused by regulation and long-term contracting, and reducing the signal-to-noise ratio in the messages transmitted by relative prices.
7. At the end of each year an ARIMA(4,0,0) model is estimated using monthly CPI data for the preceding 10 years, and is used to forecast the inflation rate for the next 12 months. I calculate π^e each year as an average over those 12 months. Note that ER and π^e are both measured as monthly rates.
8. The results are qualitatively the same if my estimated series for the variance of the marginal return on capital is used instead of σ_s^2 . The data are described in Section 3. Black (1976) has shown earlier that stock returns tend to be contemporaneously negatively correlated with changes in price volatility.

Equation (3) is obtained by use of Ito's Lemma. Fischer (1975) derives eqn. (3), and also provides a brief introduction to Ito processes such as (1), and Ito's Lemma and its use. Observe that the greater the variance σ_1^2 of the inflation rate, the greater the expected real return on the bond. This is just a consequence of Jensen's inequality; the bond's real price P_B/P is a convex function of P .

I also assume the nominal interest rate is non-stochastic, so all of the risk from holding bonds (apart from default risk) comes from uncertainty over inflation. As Fama (1975) has shown, this assumption is roughly consistent with the historical data.

Note that a pure discount bond that pays \$1 at time T has a present value at time t of

$$\exp \left[- \int_t^T (R - \pi + \sigma_1^2) dt + \int_t^T \sigma_1 dz_1 \right],$$

and therefore a real instantaneous return of $(R - \pi + \sigma_1^2) dt - \sigma_1 dz_1$, as in eqn. (3) for the short-term bond.

Note that this is apart from the effects of inflation on investors' net real return that are brought about by the tax system, as explained by Feldstein (1980a,b).

In Section 3 we show that the variance of the current marginal product of capital only accounts for about two percent of the variance of m .

This is an approximation, first because losses may more than offset taxable profits, and second because depreciation allowances are calculated on an ex ante basis. For an analysis of the risk-shifting effects of the corporate income tax, see Bulow and Summers (1982).

I ignore non-interest bearing monetary assets, which are small relative to interest-bearing debt.

In addition to Summers (1983), see Fama (1975) and Nelson and Schwert (1977).

This is because $a(b-\lambda) \approx \theta_c$. Of course the variance of inflation could have had a significant indirect effect on the variance of stock returns by partially "explaining" increases in σ_2^2 , the variance of m . (See Footnote 6.) We account for inflation when estimating σ_2^2 in Section 3.

Inflation was extremely volatile during the Korean War, exceeding 8% during the first 6 months after the outbreak of the war in July 1950, and dropping to less than 1% in 1952.

19. This is the mean return for the period 1960-81, and is close to Merton's (1980) estimate of 0.87% obtained using data for 1926-78. In fact, the expected return on the market will change if corporate tax rates change, if π changes, or if the expected real gross marginal return on capital α changes. However, as Merton (1980) shows, even if we took this expected return to be zero, it would bias our estimates of σ_s^2 only slightly. Similarly, using a moving sample mean (as in the computation of σ_1^2) makes a negligible difference; for 1948-81, the monthly series for σ_s^2 computed in this way has a correlation coefficient of .981 with the corresponding series computed from eqn. (16). Finally, note that σ_s^2 should ideally be computed using daily data, but such data are not available before 1962.
20. As Shiller (1981) has shown, the validity of this assumption is questionable.
21. The trend line is:
$$\bar{\sigma}_2^2(t) = \underset{(3.31)}{7.51 \times 10^{-4}} + \underset{(8.56)}{6.88 \times 10^{-6}} t. \quad (R^2 = .157)$$
22. Friend and Blume (1975) provide empirical support for this assumption.
23. Because the real gross marginal product of capital is correlated with inflation, there is no feasible value of ρ that makes stocks and bonds perfect substitutes in this model. The assets are perfect substitutes of $\Sigma_{12}^2 = 0$, but this would require $\rho = [s_2^2 + (s_1 + \sigma_1)^2] / 2s_2(s_1 + \sigma_1) < -1$.
24. These values of ρ , σ_1^2 , σ_2^2 , R , π , α , and the tax and financial parameters imply an expected after-tax return to the firm $E(\psi_s)$, of .629% monthly, or 7.8% annually. This is well within the range of estimates of the after-tax real rate of return.
25. The evidence is mixed. See Feldstein and Summers (1977), and Feldstein, Poterba, and Dicks-Mireaux (1981). Holland and Myers (1980) shows a larger decline, from about 15% to 11%, but do not adjust for cyclical variation.
26. See Bodie, Kane, and McDonald (1983).

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Figure 1: Monthly Variance of Nominal Stock Returns (Exponentially Smoothed)

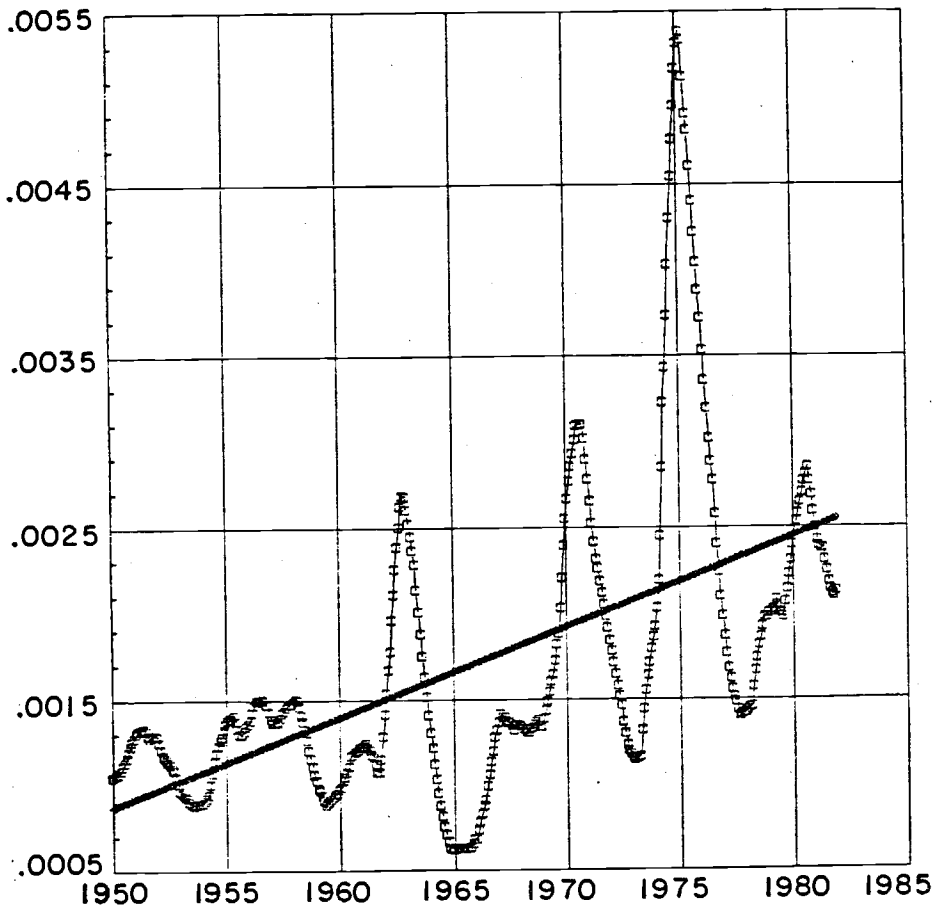


Figure 2: Mean Inflation Rate (Monthly)

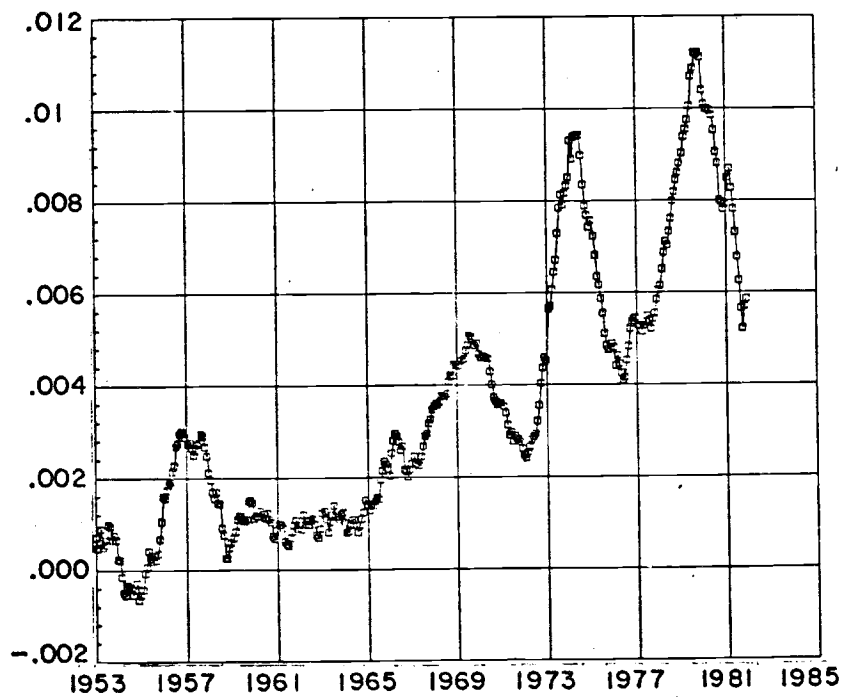


Figure 3: Monthly Variation of Inflation

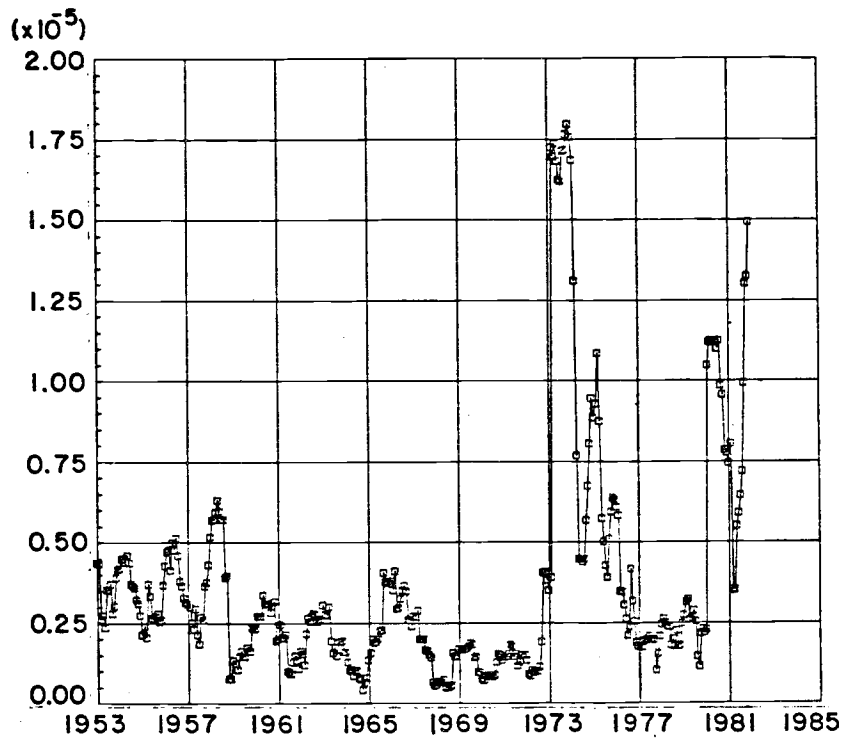


Figure 4: Monthly Variance of Marginal Return on Capital

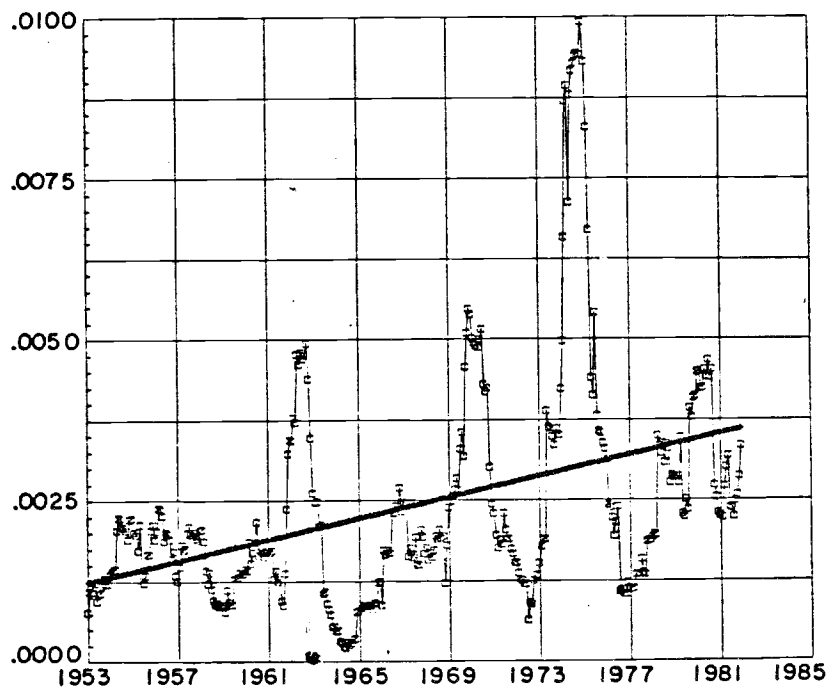


Figure 5: Correlation Coefficient ρ , Smoothed

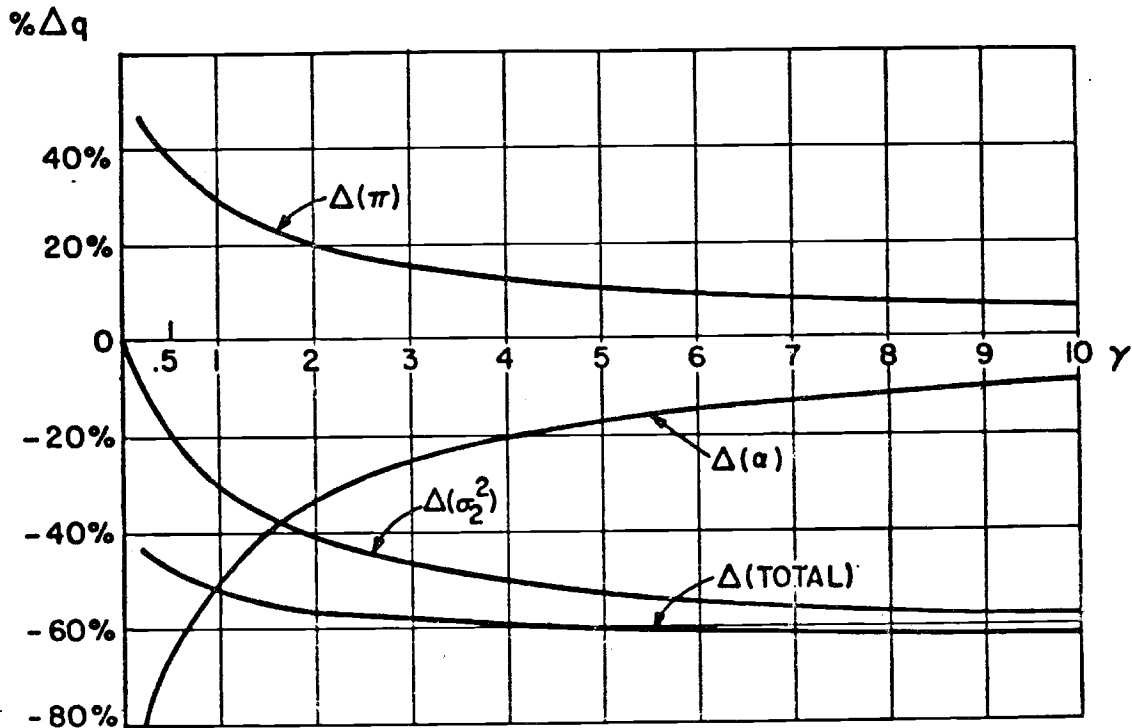
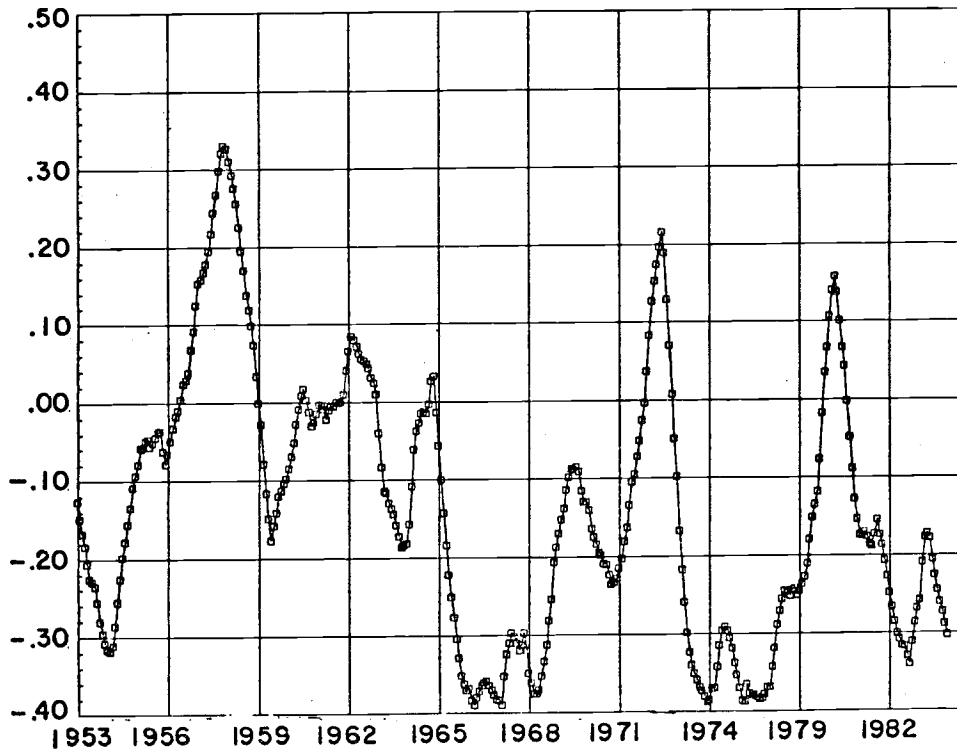


Figure 6: Contributions of Changes in π , α , and σ_2^2 to Change in Share Values