Risk Preferences Are Not Time Preferences*

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Abstract

Risk and time are intertwined. The present is known while the future is inherently risky. Discounted expected utility provides a simple, coherent structure for analyzing decisions in intertemporal, uncertain environments. Critical to such analysis is the notion that certain and uncertain utility are functionally interchangeable. We document an important and robust violation of discounted expected utility, which is essentially a violation of this interchangeability. In parameter estimations, certain utility is found to be almost linear while uncertain utility is found to be substantially more concave. These results have implications for discounted expected utility theory and decision theory in general. Applications are made to dynamic inconsistency, the uncertainty effect, the estimation of risk preferences and probability weighting.

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1 Introduction

> "...I viewed the principle of independence as incompatible with the preference for security in the neighbourhood of certainty...this led me to devise some counter-examples. One of them, formulated in 1952, has become famous as the 'Allais Paradox'. Today, it is as widespread as its real meaning is generally misunderstood." (Allais, 2008, p. 4-5)

Research on decision making under uncertainty has a long tradition. A core of tools designed to explore risky decisions has evolved, pinned down by the Savage (1954) axioms and the expected utility (EU) framework. There are, however, a number of well-documented departures from EU such as the Allais (1953) common consequence and common ratio paradoxes whose featured 'certainty effects' informed the development of prospect theory (PT) (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992). More recently, several authors have documented an 'uncertainty effect' (Gneezy et al., 2006; Simonsohn, 2009), incompatible with either PT or EU, where lotteries are valued lower than the certainty of their worst possible outcome.

An organizing principle behind these important violations of expected utility is that they seem to arise in situations where certainty and uncertainty are combined. Indeed this is exactly the desired demonstration of the Allais Paradox.[1] Allais (1953, p. 530) argued that when two options are far from certain, individuals act effectively as expected utility maximizers, while when one option is certain and another is uncertain a disproportionate preference for certainty prevails.[2]

In few decision environments is the mix of certainty and uncertainty more prevalent

[1] The common consequence paradox became known as the 'Allais paradox', and is presented prior to the common ratio paradox in Allais (1953).

[2] Allais' intuition has at least partially carried through to economic experiments. In their reviews of the experimental literature, Harless and Camerer (1994); Camerer and Ho (1994) note that violations of expected utility are less prevalent when all options are uncertain (i.e., on the interior of the Marschak-Machina triangle).

than intertemporal settings. The present is certain, while the future is inherently risky. The discounted expected utility (DEU) model is the standard approach to addressing decision-making in such contexts. Interestingly, there are relatively few noted violations of the expected utility aspect of the DEU model.[3]

We document an important violation of expected utility in an intertemporal setting. An implication of the standard DEU model is that intertemporal allocations should depend *only* on relative intertemporal risk. For example, if sooner consumption will be realized 50% of the time and later consumption will be realized 50% of the time, intertemporal allocations should be identical to a situation where all consumption is risk-free.

In an experiment with 80 undergraduate subjects at the University of California, San Diego, we implement Andreoni and Sprenger (2009) Convex Time Budgets (CTBs) under varying risk conditions. CTBs ask individuals to allocate a budget of experimental tokens to sooner and later payments. The relative value of sooner versus later tokens determines the gross interest rate. CTB allocation decisions are therefore equivalent to intertemporal optimization subject to a convex budget constraint. Andreoni and Sprenger (2009) show that preference parameters for both discounting and utility function curvature are easily estimable from CTB allocations.

We implement CTBs in two basic within-subject risk conditions: 1) A risk-free condition where all payments, both sooner and later, will be paid 100% of the time; and 2) a risky condition where, independently, sooner and later payments will be paid only 50% of the time. Under the standard DEU model, CTB allocations in the two

[3]Loewenstein and Thaler (1989) and Loewenstein and Prelec (1992) document a number of anomalies in the *discounting* aspect of discounted utility models. Machina (1989) demonstrates that non-EU preferences generate dynamic inconsistencies and Halevy (2008) shows that hyperbolic discounting can be reformulated in terms of non-EU probability weighting. The only evidence of intertemporal violations of EU known to the authors is Baucells and Heukamp (2009) and Gneezy et al. (2006) who show that temporal delay can generate an effect akin to the classic common ratio effect and that the uncertainty effect is present for hypothetical intertemporal decisions, respectively.

conditions should be identical. The pattern of results clearly violates DEU and is further inconsistent with non-EU concepts such as probability weighting (e.g., Tversky and Fox, 1995). In estimations of utility parameters, aggregate discounting is found to be around 30% per year, close to the findings of Andreoni and Sprenger (2009), and is virtually identical in both conditions. Interestingly, subjects exhibit almost linear preferences in the *risk-free* first condition, but substantial utility function curvature in the *risky* second condition.

A foundational assumption in the construction of the DEU model is the assumption that utility is continuous in probability. Continuity in probability implies that certain and uncertain utility are functionally identical.[4] We term this 'interchangeability'. The importance of interchangeability is clear: it implies that time-dated consumption is evaluated using the same utility function whether this consumption is risky or risk-free. The DEU violation we identify is more clearly viewed as a violation of interchangeability. Our results suggest a real difference between the utility functions that govern the evaluation of certain and uncertain consumption.[5]

To explore interchangeability in greater detail, we examine four additional experimental conditions with differential risk. In the first two conditions one payment, either sooner or later, is paid 50% of the time while the other is paid only 40% of the time. Allais argued that in these situations, far from certainty, individuals should behave approximately as expected utility maximizers. Indeed they do. In two further conditions, one payment is certain while the other is paid only 80% of the time. We demonstrate a disproportionate preference for certain payments that is inconsistent with interchange-

[4]Continuity is defined over bundles and states that given any three bundles in the domain of outcomes with a preference ordering $x_1 \succeq x_2 \succeq x_3$, there exists a probability, $p \in [0, 1]$, such that $x_2 \sim p \circ x_1 + (1 - p) \circ x_3$. If no such p exists, then utility is discontinuous in probability. If certain and uncertain utility are different then it is possible to find a set of three bundles for which there will exist no probability mixture satisfying the definition of continuity.

[5]Other violations of continuity exist in decision theory. For example, quasi-hyperbolic discounting is an example of preferences that are discontinuous over time. In Section 5, we demonstrate that quasi-hyperbolic discounting can arise from our observed violation of interchangeability.

ability, but can be readily resolved if certain and uncertain consumption are evaluated using different preference parameters. The observed effects are closely in line with the desired demonstration of the Allais paradox.

Our results have substantial implications for both experimental research on time and risk preferences and theoretical developments based on the DEU model. Specific applications of our results can be made to: hyperbolic discounting; the existence of an uncertainty effect; the measurement of risk preferences; and the identification of probability weighting. First, much attention has been given to dynamic inconsistencies such as quasi-hyperbolic discounting. We demonstrate that the quasi-hyperbolic pattern of discounting can be generated by differential assessment of certain and uncertain consumption. Second, the existence of an uncertainty effect is impossible in both EU and PT.[6] However, if certain and uncertain consumption are evaluated with different utility parameters, the uncertainty effect is no longer anomalous. Third, in the experimental measurement of risk preferences, subjects are often asked to give certainty equivalents for uncertain lotteries. Such methodology frequently generates extreme measures of risk aversion at odds with standard EU theory (Rabin, 2000). Our results suggest that we could potentially resolve this issue by allowing for differential assessment of certain and uncertain consumption. Fourth, probability weighting phenomena are generally identified from certainty equivalents experiments similar to those employed to measure risk preferences.[7] Our results indicate that differences between certain and uncertain utility can generate probability weighting phenomena.

The paper proceeds as follows: Section 2 presents a conceptual development of discounted expected utility, building to a testable hypothesis of decision making in

[6]In fact the uncertainty effect will be at odds with any utility theory satisfying a betweenness property.

[7]For example, in Tversky and Fox (1995), subjects were asked to provide the certainty equivalent x of a lottery with empirical probability p and payout y. Assuming a power utility function, $U(x) = x^{\alpha}$, with $\alpha = 0.88$ obtained by Tversky and Kahneman (1992), the authors then back out the probability weight, $\pi(p)$ as the the value that solves $x^{\alpha} = \pi(p)y^{\alpha}$.

certain and uncertain situations. Section 3 describes our experimental design. Section 4 presents results and Section 5 discusses the above-mentioned applications and concludes.

2 Conceptual Background

The continuity-in-probability of utility frequently assumed in decision theory implies that individuals assess certain consumption identically to uncertain consumption. Given some utility function for certain consumption, $u(\cdot)$, and a utility function for uncertain consumption, $v(\cdot)$, assumed to be separable and linearly additive over probabilistic states, interchangeability states that:

$$u(\cdot) \equiv v(\cdot).$$

When decisions are intertemporal and utility is time separable, this gives rise to the standard DEU model:

$$U = u(c_t) + \sum_{k=0}^{T} \delta^k E[u(c_{t+k})]$$

where present consumption is certain while future consumption is both discounted and uncertain. The expectation, $E[\cdot]$, is taken via a standard linear-in-probabilities weighting over N states: $E[u(c_{t+k})] = \sum_{s=1}^{N} p_s u(c_{t+k,s}) = \sum_{s=1}^{N} p_s v(c_{t+k,s})$. If all consumption is certain, the expectation disappears:

$$\tilde{U} = u(c_t) + \sum_{k=0}^{T} \delta^k u(c_{t+k})$$

If consumption at time t will be realized only with probability p_1 while later consumption will be realized with probability p_2, utility is:

$$\tilde{\tilde{U}} = p_1 u(c_t) + \sum_{k=0}^{T} p_2 \delta^k u(c_{t+k}) + Z$$

Where Z represents a sum of discounted and linear probability-weighted $u(0)$ terms.

In this framework, we consider two risky prospects temporally separated by k periods. Let the first prospect yield c_t with probability p_1 and zero otherwise. Let the second prospect yield c_{t+k} with probability p_2 and zero otherwise. We term this a *risky situation* and denote it with (p_1, p_2). Under the standard construction, the utility of the risky situation is:

$$p_1 u(c_t) + p_2 \delta^k u(c_{t+k}) + ((1 - p_1) + (1 - p_2)\delta^k) u(0)$$

Suppose an individual maximizes utility of the risky situation subject to the future value budget constraint

$$(1 + r)c_t + c_{t+k} = m$$

yielding the marginal condition:

$$\frac{p_1 u'(c_t)}{p_2 \delta^k u'(c_{t+k})} = (1 + r)$$

or

$$\frac{u'(c_t)}{\delta^k u'(c_{t+k})} = (1 + r)\frac{p_2}{p_1}$$

The tangency condition, in combination with the budget constraint, generally yields solution functions of the form:

$$c_t^*(p_1, p_2, k, 1 + r, m)$$

A key observation in this construction is that intertemporal allocations will depend *only* on the relative risk, p_2/p_1, and not on p_2 or p_1 separately. If $p_2/p_1 = 1$, then behavior should be identical to a risk-free situation. This is a critical and testable implication of the DEU model.[8]

Hypothesis: For a given a risky situation (p_1, p_2), if $p_2/p_1 = 1$, then $c_t^*(p_1, p_2, k, 1+r, m) = c_t^*(1, 1, k, 1+r, m) \; \forall \; k, (1+r), m$.

It is important to understand the degree to which this hypothesis hinges upon interchangeability. If $u(\cdot) \neq v(\cdot)$, then there is no reason to expect $c_t^*(p_1, p_2, k, 1+r, m) = c_t^*(1, 1, k, 1+r, m)$ when $p_2/p_1 = 1$. This is because the marginal conditions in the two situations will generally be satisfied at different allocation levels.[9]

In our later exposition it will be notationally convenient to use θ to indicate the *risk adjusted gross interest rate*:

$$\theta = (1+r)\frac{p_2}{p_1}$$

such that the tangency can be written as:

$$\frac{u'(c_t)}{\delta^k u'(c_{t+k})} = \theta$$

Provided that $u'(\cdot) > 0, u''(\cdot) < 0$, c_t^* will be decreasing in p_2/p_1 and $1+r$. As such, c_t^* will also be decreasing in θ.

[8] Note that restricting discounting to be exponential is an unnecessary simplification. Discounting could take a general form $D(t,k)$ and the implication would be maintained.

[9] In the risky situation the marginal condition will be $v'(c_t)/\delta^k v'(c_{t+k}) = (1+r)p_2/p_1 = (1+r)$, while in the risk-free situation the condition will be: $u'(c_t')/\delta^k u'(c_{t+k}') = (1+r)$. And $c_t' = c_t; c_{t+k}' = c_{t+k}$ only if the marginal utility functions $u'(\cdot)$ and $v'(\cdot)$ are equal. Though this may occur without $u(\cdot) = v(\cdot)$, it generally will not.

3 Experimental Design

In order to explore the evaluation of certain and uncertain intertemporal consumption, an experiment using Andreoni and Sprenger (2009) Convex Time Budgets under varying risk conditions was conducted at the Univeristy of California, San Diego in April of 2009. In each CTB decision, subjects were given a budget of experimental tokens to be allocated across a sooner payment, paid at time t, and a later payment, paid at time $t + k,\ k > 0$. Two basic CTB environments consisting of 7 allocation decisions each were implemented under six different risk conditions. This generated a total of 84 experimental decisions for each subject.

3.1 CTB Design Features

- *Choice of t and k:* Sooner payments in each decision were always seven days from the experiment date ($t = 7$ days). The design choice of using a 'front-end-delay' was made to avoid any direct impact of immediacy on decisions and to help eliminate differential transactions costs across sooner and later payments.[10] In one of the basic CTB environments, later payments were delayed 28 days ($k = 28$ days) and in the other, later payments were delayed 56 days ($k = 56$ days).

  The choice of t and k combinations was determined by the academic calendar. Payment dates were set to avoid holidays, school vacation days and final examination week. Payments were scheduled to arrive on the same day of the week (i.e., t and k both multiples of 7), to avoid differential week-day effects.

- *Token Budgets and Interest Rates:* In each CTB decision, subjects were given a token budget of 100 tokens. Tokens allocated to the sooner experimental payment

[10] See below for the recruitment and payment efforts that allowed sooner payments to be implemented in the same manner as later payments. For discussions of front-end-delays in time preference experiments see Coller and Williams (1999); Harrison et al. (2005).

had a value of ρ_t while tokens allocated to the later experimental payment had a value of ρ_{t+k}. In all cases, ρ_{t+k} was \$.20 per token and ρ_t varied from \$.20 to \$.14 per token. Note that $\rho_{t+k}/\rho_t = (1 + r)$, the gross interest rate over k days, and $(1 + r)^{1/k} - 1$ gives the standardized daily *net* interest rate. Daily net interest rates in the experiment varied considerably across the basic budgets, from 0 to 1.3 percent, implying annual interest rates of between 0 and 2100 percent (compounded quarterly). Table 1 shows the token values, gross interest rates, standardized daily interest rates and corresponding annual interest rates for the basic CTB budgets.

- *Risk Conditions* The basic CTB decisions described above were implemented in a total of six risk conditions.

  1. *100%-100% Condition*: All payments, both sooner and later were made 100% of the time, $(p_1, p_2) = (1, 1)$.
  2. *50%-50% Condition*: All payments, both sooner and later were made 50% of the time, $(p_1, p_2) = (0.5, 0.5)$.
  3. *50%-40% Condition*: Sooner payments were made 50% of the time. Later payments were made 40% of the time, $(p_1, p_2) = (0.5, 0.4)$.
  4. *40%-50% Condition*: Sooner payments were made 40% of the time. Later payments were made 50% of the time, $(p_1, p_2) = (0.4, 0.5)$.
  5. *100%-80% Condition*: Sooner payments were made 100% of the time. Later payments were made 80% of the time, $(p_1, p_2) = (1, 0.8)$.
  6. *80%-100% Condition*: Sooner payments were made 80% of the time. Later payments were made 100% of the time, $(p_1, p_2) = (0.8, 1)$.

For each payment involving uncertainty, a ten-sided die was rolled at the end of the experiment to determine whether the payment would be sent or not. The risk conditions were chosen for several reasons. The first two situations allow for a test of interchangeability as behavior in conditions 1 and 2 should be identical. If there are utility parameters that govern uncertain situations, then behavior in conditions 3 and 4 should be well predicted by behavior in condition 2. If not, then behavior in conditions 3 and 4 should be identical to behavior in conditions 5 and 6.[11] Condition 5 is of particular interest because it follows the pattern of risk seen in real-life intertemporal decisions and at times, inadvertently, created in laboratory settings. The sooner payment is certain while the future payment is risky.

3.2 Implementation and Protocol

One of the most challenging aspects of implementing any time discounting study is making all choices equivalent except for their timing. That is, transactions costs associated with receiving payments, including physical costs and confidence, must be equalized across all time periods. We took several unique steps in our subject recruitment process and our payment procedure in order to equate transaction costs over time.

3.2.1 Recruitment

In order to participate in the experiment, subjects were required to live on campus. All campus residents are provided with an individual mailbox at their dormitory. Students frequently use these mailboxes as all postal service mail and university organized intra-campus mail are received at this mailbox. Each mailbox is locked and individuals

[11]Conditions 3 and 5 share the common ratio $p_2/p_1 = 1.25$ and conditions 4 and 6 share the common ratio $p_2/p_1 = 0.8$.

have keyed access 24 hours per day. We recruited 80 undergraduate freshman and sophomores who lived on campus and so had campus mailboxes.

3.2.2 Experimental Payments

Using the campus mailboxes allowed us to equate physical transaction costs across sooner and later payments. All payments, both sooner and later, were placed in subjects' campus mailboxes. Subjects were fully informed of the method of payment.[12]

Several other measures were also taken to equate transaction costs. Upon beginning the experiment, subjects were told that they would receive a \$10 minimum payment for participating. This \$10 was to be received in two payments: \$5 sooner and \$5 later. All experimental earnings were added to these \$5 minimum payments, such that subjects would receive at least \$5 sooner and at least \$5 later. Two blank envelopes were provided. After receiving directions about the two minimum payments, subjects were asked to address the envelopes to themselves at their campus mailbox. At the end of the experiment, subjects were asked to write their payment amounts and dates on the inside flap of each envelope such that they would see the amounts written in their own handwriting when payments arrived.

One choice for each subject was chosen for payment by drawing a numbered card at random. All experimental payments were made by personal check from Professor James Andreoni drawn on an account at the university credit union.[13] Individuals were informed that they could cash their checks (if they so desired) at the university credit union. They were also given the business card of Professor James Andreoni and told

[12] See Section A.2 for the information provided to subjects.

[13] Payment choice was guided by a separate survey of $N = 249$ undergraduate economics students eliciting payment preferences. Personal checks from Professor Andreoni, Amazon.com gift cards, PayPal transfers and the university stored value system TritonCash were each compared to cash payments. Subjects were asked if they would prefer a twenty dollar payment made via each payment method or \$$X$ cash, where X was varied from 19 to 10. Personal check payments were found to have the highest cash equivalent value.

to immediately report any problems in receiving timely payment.

3.2.3 Protocol

The experiment was done with paper and pencil. Upon entering the lab subjects were read an introduction with detailed information on the payment process and a sample decision with different payment dates, token values and payment risks than those used in the experiment.[14] Subjects were informed that they would work through 6 decision tasks. Each task consisted of 14 CTB decisions: seven with $t = 7,\ k = 28$ on one sheet and seven with $t = 7,\ k = 56$ on a second sheet. Each decision sheet had a calendar on the left hand side, highlighting the experiment date (in yellow), the sooner payment date (in green) and the later payment date (in blue). This allowed subjects to visualize the payment dates and delay lengths.

Figure 1 shows a sample decision sheet. Identical instructions were read at the beginning of each task providing payment dates and the chance of being paid for each decision.[15] Subjects were provided with a calculator and a calculation sheet transforming tokens to payments amounts at various token values.

Four sessions were conducted over two days. Two orders of risk conditions were implemented to examine order effects: 1) (p_1, p_2) = (1, 1), (1, 0.8), (0.8, 1), (0.5, 0.5), (0.5, 0.4), (0.4, 0.5); and 2) (p_1, p_2) = (0.5, 0.5), (0.5, 0.4), (0.4, 0.5), (1, 1), (1, 0.8), (0.8, 1).[16]

[14] See Appendix A.3 for introductory text, instructions and examples.

[15] See Appendix Section A.3 for a sample of the task instructions.

[16] Each day consisted of an early session (12 pm) and a late session (2 pm). The early session on the first day and the late session on the second day share a common order as do the late session on the first day and the early session on the second day. There are no identifiable order or session effects in the data (see below).

4 Results

The results are presented in two broad sections. First, we examine behavior in the two basic situations: 1) $(p_1, p_2) = (1, 1)$ and 2) $(p_1, p_2) = (0.5, 0.5)$. We document a critical violation of the DEU model and show that the pattern of results is generally incompatible with various probability weighting concepts. In estimates of utility parameters, we show clear differences between the utility functions for certain and uncertain consumption. Second, we explore behavior in two further contexts: one where all payments are uncertain, but there is differential risk; and another where one payment is certain while the other is uncertain. We demonstrate a pattern of behavior consistent with the notion that individuals behave as expected utility maximizers away from certainty but exhibit a disproportionate preference for certainty when it is available.

4.1 Behavior Under Certainty and Uncertainty

The development of Section 2 provides a testable hypothesis for behavior across certain and uncertain intertemporal settings. For a risky situation, (p_1, p_2), if $p_2/p_1 = 1$ then behavior should be identical to a similarly dated risk-free situation, at all gross interest rates, $1 + r$, and all delay lengths, k.[17] Figure 2 graphs aggregate behavior for the situations: 1) $(p_1, p_2) = (1, 1)$ and 2) $(p_1, p_2) = (0.5, 0.5)$ across the experimentally varied gross interest rates and delay lengths. The mean earlier choice of c_t is graphed along with error bars corresponding to 95 percent confidence intervals (+/− 1.96 standard errors).

Though, under the DEU model, behavior should be identical across the two conditions, we find strong evidence to the contrary. At the lowest gross interest rate, $1 + r = 1$, subjects choose smaller allocations of c_t when $(p_1, p_2) = (0.5, 0.5)$. At all

[17] We ignore m because the experimental budget was held constant across all choices.

higher interest rates, subjects choose larger allocations of c_t when $(p_1, p_2) = (0.5, 0.5)$.[18] In a hypothesis test of equality across the two risky situations, the overall difference is found to be highly significant: $F_{14,2212} = 15.66,\ p < .001$.[19]

The results presented in Figure 2 are surprising from a classical decision theory perspective and are in clear violation of discounted expected utility. Revealed preference behavior is found to be significantly different across risk-free and risky situations. Though this is suggestive evidence against interchangeability, there may be alternative explanations. Principal among these alternatives is prospect theory and, in particular, the existence of probability weighting (e.g., Tversky and Fox, 1995).

Probability weighting generally states that individuals 'edit' probabilities internally via a weighting function, $\pi(p)$. $\pi(p)$ is monotonically increasing in the interval $[0, 1]$, but is S-shaped such that low probabilities are up-weighted and high probabilities are down-weighted. Standard probability weighting is unable to explain the phenomena observed in Figure 2. If $p_1 = p_2$, then $\pi(p_1) = \pi(p_2)$; $\pi(p_2)/\pi(p_1) = 1$ and behavior should again be identical to a risk-free situation.

Another potential explanation is that probabilities are weighted by their temporal proximity (see e.g., Halevy, 2008). Under this formulation, subjective probabilities are arrived at through some temporally dependent function $g(p, t) : [0, 1] \times \Re \rightarrow [0, 1]$ where t represents the time at which payments will be made. Provided freedom to pick the functional form of $g(\cdot)$ one could easily arrive at differences between the ratios

[18] This difference in allocations across conditions is obtained for all sessions and for all orders indicating no presence of order or day effects. Results available on request.

[19] Test statistic generated after analysis of variance with 2240 observations (28 per subject $\times$ 80 subjects) controlling for levels of interest rate (6 degrees of freedom), delay length (1 d.f), (interest rate) $\times$ (delay length) (6 d.f) and (risk condition) $\times$ (interest rate) $\times$ (delay length) (14 d.f). 2240 - 6 - 1 - 6 - 14 - 1(constant) = 2212 d.f. The F-test corresponds to testing the null hypotheses that the 14 (risk condition) $\times$ (interest rate) $\times$ (delay length) terms have zero explanatory power. ANOVA results available on request.

$g(1, t+k)/g(1, t)$ and $g(0.5, t+k)/g(0.5, t)$.[20]

These differences lead to a *new* risk adjusted interest rate similar to the θ defined in Section 2:

$$\tilde{\theta}_{p_1,p_2} \equiv \frac{g(p_2, t+k)}{g(p_1, t)}(1+r)$$

and note that either $\tilde{\theta}_{1,1} > \tilde{\theta}_{0.5,0.5}\ \forall\ (1+r)$ or $\tilde{\theta}_{1,1} < \tilde{\theta}_{0.5,0.5}\ \forall\ (1+r)$, depending on the form of $g(\cdot)$ chosen. Once one obtains a prediction as to the relationship between $\tilde{\theta}_{1,1}$ and $\tilde{\theta}_{0.5,0.5}$, it must hold for all gross interest rates.

Provided a concave utility function, c_t allocations should be decreasing in $\tilde{\theta}$. As such, one should never observe a switch in behavior where for one gross interest rate c_t allocations are higher when $(p_1, p_2) = (1, 1)$ and for another gross interest rate c_t allocations are higher when $(p_1, p_2) = (0.5, 0.5)$. This switch in behavior, which is observed in our data, is not consistent with temporally dependent probability weighting of the form proposed by Halevy (2008). Given the freedom granted in determining the function, $g(\cdot)$, even some hybrid of temporally dependent weighting and probability editing would be generally unable to generate this switch in behavior.

4.1.1 Estimating Risk-Dependent Preferences

The observed data in the cases of $(p_1, p_2) = (1, 1)$ and $(p_1, p_2) = (0.5, 0.5)$ are inconsistent with the interchangeability assumption of the DEU model and are difficult to reconcile with notions of probability weighting. Whereas allocations of c_t when

[20] Halevy (2008) gives the example of $g(p, t) = g(p^t)$ such that $g(0) = 0; g(1) = 1$. In this case:

$$\frac{g(1, t+k)}{g(1, t)} = \frac{g(1^{t+k})}{g(1^t)} = 1 \neq \frac{g(0.5, t+k)}{g(0.5, t)} = \frac{g(0.5^{t+k})}{g(0.5^t)}$$

provided $g(\cdot)$ does not take on identical values at 0.5^t and 0.5^{t+k}. If one further assumes $g(\cdot)$ is strictly monotonic and differentiable such that $g'(\cdot) > 0$, then

$$\frac{g(1, t+k)}{g(1, t)} = \frac{g(1^{t+k})}{g(1^t)} = 1 > \frac{g(0.5, t+k)}{g(0.5, t)} = \frac{g(0.5^{t+k})}{g(0.5^t)}$$

$(p_1, p_2) = (1, 1)$ vary substantially with the interest rate, the sensitivity of allocations to interest rates is lower when $(p_1, p_2) = (0.5, 0.5)$.

The sensitivity of intertemporal allocations to interest rates, that is the elasticity of intertemporal substitution, is generally determined by both time preferences and utility function curvature. Our experimental design allows us to identify and, given some structural assumptions, estimate both discounting and curvature. Following the methodology outlined in Andreoni and Sprenger (2009), we assume utility function:

$$u(c_t) = (c_t - \omega)^\alpha$$

where α represents utility function curvature and ω is a background parameter that could be interpreted as a Stone-Geary minimum parameter.[21] Under this formulation of the DEU model, the solution function c_t^* can be written as:

$$c_t^*(p_1, p_2, t, k, 1{+}r, m) = \frac{[1 - (\frac{p_2}{p_1}(1+r)\delta^k)^{\frac{1}{\alpha-1}}]}{[1 + (1+r)(\frac{p_2}{p_1}(1+r)\delta^k)^{\frac{1}{\alpha-1}}]}\omega + \frac{[(\frac{p_2}{p_1}(1+r)\delta^k)^{\frac{1}{\alpha-1}}]}{[1 + (1+r)(\frac{p_2}{p_1}(1+r)\delta^k)^{\frac{1}{\alpha-1}}]}m$$

or

$$c_t^*(\theta, t, k, 1+r, m) = \frac{[1 - (\theta\delta^k)^{\frac{1}{\alpha-1}}]}{[1 + (1+r)(\theta\delta^k)^{\frac{1}{\alpha-1}}]}\omega + \frac{[(\theta\delta^k)^{\frac{1}{\alpha-1}}]}{[1 + (1+r)(\theta\delta^k)^{\frac{1}{\alpha-1}}]}m \qquad (0)$$

We estimate the parameters of this function via non-linear least squares with standard errors clustered on the individual level to obtain $\hat{\alpha}$, $\hat{\delta}$ and $\hat{\omega}$. An estimate of the annual discount rate is generated as $1/\hat{\delta}^{365} - 1$, with corresponding standard error obtained via the delta method.

Table 2 presents discounting and curvature parameters estimated from the two

[21] Frequently in the time preference literature, the simplification $\omega = 0$ is imposed or ω is interpreted as *negative* background consumption and calculated from an external data source. In Andreoni and Sprenger (2009) we show the sensitivity of parameter estimates to these simplifications.

conditions $(p_1, p_2) = (1, 1)$ and $(p_1, p_2) = (0.5, 0.5)$. In column (1), we estimate a baseline model where discounting and curvature are restricted to be identical across the two risk conditions. The aggregate discount rate is estimated to be around 27% per year and aggregate curvature is estimated to be 0.98.

In column (2) we estimate separate discounting and curvature parameters for the two risk conditions. That is, we estimate a risk-free $u(\cdot)$ and a risky $v(\cdot)$. Discounting is found to be similar across the conditions at around 30% per year.[22] In the risk free condition, $(p_1, p_2) = (1, 1)$, we find almost linear utility while in the the risky condition, $(p_1, p_2) = (0.5, 0.5)$, we estimate utility to be markedly more concave. A similar result is observed in column (3) where discounting is restricted to be the same across risk conditions. Hypotheses of equal utility function curvature across conditions are rejected in both specifications: $F_{1,79} = 37.97$, $p < .001$; $F_{1,79} = 38.09$, $p < .001$, respectively. To illustrate how well these estimates fit the data, Figure 2 also displays solid lines corresponding to predicted behavior based on the parameters estimated in column (3). The general pattern of aggregate responses is well matched.[23]

Though discounting is estimated to be similar across conditions, substantial difference in curvature is estimated between $(p_1, p_2) = (1, 1)$ and $(p_1, p_2) = (0.5, 0.5)$. Figure 3 demonstrates the economic importance of this result, plotting the estimated two utility functions along with 95% confidence intervals of the estimates. While utility deviates only slightly from linear preferences when $(p_1, p_2) = (1, 1)$, the deviation is sizeable when $(p_1, p_2) = (0.5, 0.5)$, even over the monetary values used in the experiment. These results are suggestive evidence against the interchangeability assumption.

[22]For comparison, Andreoni and Sprenger (2009) find aggregate discount rate between 30-37% and aggregate curvature of around 0.92 in risk-free situations.

[23]Figure 2 additionally reports separate R^2 values for the two conditions: $R^2_{1,1} = 0.594$; $R^2_{0.5,0.5} = 0.761$, indicating that the solution function estimation approach does an adequate job of fitting the aggregate data. For comparison a simple linear regression of c_t on the levels of interest rates, delay lengths and their interaction in each condition would produce $\tilde{R}^2$ values of $\tilde{R}^2_{1,1} = 0.443$; $\tilde{R}^2_{0.5,0.5} = 0.346$. Results available on request.

Our estimation suggest that certain and uncertain payments are evaluated using different utility functions.

4.2 Behavior with Differential Risk

In this section we analyze behavior in conditions with differential risk. First, we examine conditions where all payments are uncertain but sooner and later payments differ in their level of risk. Second, we examine two hybrid conditions where one payment is certain while the other is uncertain.

4.2.1 Expected Utility Under Uncertainty

The individual's marginal condition under DEU establishes a tradeoff between relative risk, p_2/p_1, and the gross interest rate, $1+r$. This tradeoff is captured in the variable θ, the risk adjusted interest rate. As noted in Section 2, given a concave utility function, c_t allocations should be decreasing in both the relative risk and the gross interest rate. As such, c_t allocations should also be decreasing in θ. Note that this additionally implies that if θ is constant across situations, c_t allocations will be higher where the interest rate is lower.

Figure 4 presents aggregate behavior from three risky situtations: 2) $(p_1, p_2) = (0.5, 0.5)$ (in red, as before); 3) $(p_1, p_2) = (0.5, 0.4)$ (in green); and 4) $(p_1, p_2) = (0.4, 0.5)$ (in orange) over the experimentally varied values of θ and delay length. The mean earlier choice of c_t is graphed along with error bars corresponding to 95 percent confidence intervals ($+/-$ 1.96 standard errors). We also plot predicted behavior based on the aggregate responses in the $(p_1, p_2) = (0.5, 0.5)$ condition. That is, based on $\hat{\alpha}_{0.5,0.5}$, $\hat{\delta}$ and $\hat{\omega}$ estimated in Table 2, column (3), we predict out of sample behavior for the two conditions $(p_1, p_2) = (0.5, 0.4)$ and $(p_1, p_2) = (0.4, 0.5)$. These predictions are plotted as solid lines in green and orange.

We highlight two dimensions of Figure 4. First, the theoretical predictions are 1) that c_t should be declining in θ; and 2) that if two decisions have identical θ then c_t should be higher in the condition with the lower interest rate. These features are observed in the data. Allocations of c_t decline with θ and, where overlap of θ exists c_t is generally higher for lower gross interest rates.[24] Second, out of sample predictions match actual aggregate behavior. Indeed, the out of sample calculated R^2 values are high: 0.878 for $(p_1, p_2) = (0.5, 0.4)$ and 0.580 for $(p_1, p_2) = (0.4, 0.5)$.[25]

Figure 4 demonstrates that in situations where all payments are risky, utility parameters measured under uncertainty do well at describing behavior. That is, away from certainty, subjects act as expected utility maximizers, trading off relative risk and interest rates as predicted by the DEU model.

4.2.2 Differential Curvature: A Preference for Certainty

When all options are uncertain, individuals appear to recognize the trade-off between relative risk and interest rates. The results demonstrated in Figure 4 are in line with both Allais' intuition and the prior work on the identification of EU violations when all options are uncertain (see Harless and Camerer, 1994; Camerer and Ho, 1994). Interchangeability, however, requires that the same trade-offs between relative risk and interest rates be made when one option is certain. That is, interchangeability requires that behavior when $(p_1, p_2) = (0.5, 0.4)$ be identical to behavior when $(p_1, p_2) = (1, 0.8)$, and that behavior when $(p_1, p_2) = (0.4, 0.5)$ be identical to behavior when $(p_1, p_2) = (0.8, 1)$. These conditions share common ratios of p_2/p_1.

Figure 5 graphs behavior in these four conditions, demonstrating that allocations

[24] This pattern of allocations is obtained for all sessions and for all orders indicating no presence of order or day effects. Results available on request.

[25] By comparison, making similar out of sample predictions using utility estimates from $(p_1, p_2) = (1, 1)$ yields predictions that diverge dramatically from actual behavior (see Figure A1) and lowers R^2 values to 0.767 and 0.462, respectively. This suggests that accounting for differential utility function curvature in risky situations allows for an improvement of fit on the order of 15-25%.

when all payments are risky differ dramatically from allocations where some payments are certain.[26] Hypotheses of equality across conditions are rejected in both cases.[27] Subjects show a disproportionate preference for certainty when it is available. This result follows naturally from our Table 2 estimates, which show that utility function curvature is markedly more pronounced in uncertain situations relative to certain situations. Stated differently, the marginal utility of consumption is estimated to be higher under certainty. This higher marginal utility translates into a differential preference for certainty when it is available.

To explore the influence of combined certainty and uncertainty on experimental responses, Figure 6 plots aggregate behavior in three conditions: 1) $(p_1, p_2) = (1, 1)$ (in blue, as before); 5) $(p_1, p_2) = (1, 0.8)$ (in gray); and 6) $(p_1, p_2) = (0.8, 1)$ (in purple) over the experimentally varied values of θ and delay length. The mean earlier choice of c_t is graphed along with error bars corresponding to 95 percent confidence intervals (+/− 1.96 standard errors).

Under interchangeability, Figure 6 should be identical to Figure 4. Aggregate responses should be declining in θ and the gross interest rate, as before. Unlike the findings of Figure 4, c_t allocations are not decreasing in θ. Additionally, lower interest rates do not generally lead to higher c_t allocations when θ is equal across conditions.

The cross-over in allocations across the $(p_1, p_2) = (1, 0.8)$ and $(p_1, p_2) = (1, 1)$ conditions is particularly striking. When $\theta = 1$, c_t allocations are higher in the $(p_1, p_2) = (1, 1)$ condition, while at larger values of θ, c_t allocations are higher in the $(p_1, p_2) = (1, 0.8)$ condition. Such behavior is at odds with EU theory and cannot be explained by non-EU probability weighting.[28] Behavior in the $(p_1, p_2) = (0.8, 1)$

[26]This difference in allocations across conditions is obtained for all sessions and for all orders indicating no presence of order or day effects. Results available on request.

[27]For equality across $(p_1, p_2) = (0.5, 0.4)$ and $(p_1, p_2) = (1, 0.8)$, $F_{14,2212} = 14.60$, $p < .001$ and for equality across $(p_1, p_2) = (0.4, 0.5)$ and $(p_1, p_2) = (0.8, 1)$, $F_{14,2212} = 23.82$, $p < .001$

[28]The argument is identical to the one presented in Section 4.1.

condition seems to fit better with the $(p_1, p_2) = (1, 1)$ condition, however, allocations are generally quite low in this region, precluding strong inference.[29]

In Figure 6 we also plot predicted behavior based on the estimates of Table 2, column (3). The prediction is made under the assumption that certain consumption is evaluated using $\hat{\alpha}_{1,1}$ and uncertain consumption is evaluated using $\hat{\alpha}_{0.5,0.5}$. We predict out of sample for the conditions $(p_1, p_2) = (1, 0.8)$ and $(p_1, p_2) = (0.8, 1)$.[30] These predictions are plotted as solid lines in gray and purple.

Though the behavior illustrated in Figure 6 is at odds with interchangeability, its stylistic properties are easily explained if we allow uncertain and certain consumption to be governed by different utility functions. The solid lines show exactly this effect. The cross-over in behavior between the $(p_1, p_2) = (1, 0.8)$ and $(p_1, p_2) = (1, 1)$ conditions is predicted and the out of sample R^2 value of 0.854 for the $(p_1, p_2) = (1, 0.8)$ condition is notably high. Behavior when $(p_1, p_2) = (0.8, 1)$ is predicted to piece together with behavior when $(p_1, p_2) = (1, 1)$, though the out of sample R^2 value of 0.133 is notably low.

In sum, the data and corresponding estimations demonstrate that separate utilities govern the assessment of certain and uncertain consumption. Uncertain utility is able to predict behavior well in uncertain situations, where subjects act effectively as expected utility maximizers. However, subjects exhibit a preference for certainty when it is available. This behavior follows naturally from the finding that certain consumption has lower utility function curvature and so higher marginal utility than uncertain consumption. Indeed in hybrid situations where some payments are certain and others are not, this difference in utility parameters is able to explain behavior that is at odds with both standard DEU and PT theories. Finding differences between certain and uncer-

[29] This pattern of allocations is obtained for all sessions and for all orders indicating no presence of order or day effects. Results available on request.

[30] One does not arrive at an analytic solution function for c_t^* in these hybrid cases. Instead c_t is solved for as the root of a polynomial function. See Appendix A.4 for the solution procedure.

tain utility parameters has broad applications in decision theory. In our discussion, we sketch several applications.

5 Discussion

Intertemporal decision-making involves a combination of certainty and uncertainty. The present is known while the future is inherently risky. Though expected utility (EU) violations are frequently found in decision environments combining risk and certainty, there are few known violations of the EU aspect of discounted expected utility. In an experiment with Andreoni and Sprenger (2009) Convex Time Budgets under varying risk conditions, we document an important violation of discounted expected utility. The violation we document is more closely a violation of what we term interchangeability, or the notion that certain and uncertain consumption are assessed using identical utility parameters.

Our findings indicate that certain and uncertain consumption are evaluated very differently. Substantially less utility function curvature is associated with certain consumption relative to uncertain consumption. Additionally, individuals behave approximately as expected utility maximizers in uncertain situations, but exhibit a disproportionate preference for certainty when it is available. We interpret our findings as being consistent with both prior research on expected utility violations and the intuition of the Allais Paradox (Allais, 1953).

Demonstrating a difference between certain and uncertain utility has substantial impacts for decision theory. We highlight applications in four domains: 1) quasi-hyperbolic discounting; 2) the 'uncertainty effect'; 3) the measurement of risk preferences; and 4) the identification of probability weighting.

First, dynamic inconsistencies such as quasi-hyperbolic discounting are frequently documented (for a review, see Frederick et al., 2002). Recently, the hallmark of dynamic inconsistency, diminishing impatience through time, has been argued to be generated by differential risk on present and future payments (for psychological evidence, see Keren and Roelofsma, 1995; Weber and Chapman, 2005). Halevy (2008) argues that differential risk leads to dynamic inconsistency because individuals have a temporally dependent probability weighting function that is convex (see Section 4.1 for details). Our results suggest that one need not call on a complex probability weighting function to explain the phenomenon. If individuals exhibit a disproportionate preference for certainty when it is available, then present consumption will be disproportionately favored over future consumption. When all consumption is uncertain, this effect disappears, generating dynamic inconsistency.

Second, the 'uncertainty effect' (Gneezy et al., 2006; Simonsohn, 2009) of valuing a lottery lower than the certainty of its worst possible outcome is at odds with a number of utility theories, including both expected utility and prospect theory. Our results provide a simple resolution to this EU violation. If uncertain and certain consumption are assessed with different utility parameters, then the uncertainty effect is a comparison of two values: the expected utility of an uncertain gamble and the certain utility of its worst outcome. If, as we find, uncertain utility is more concave than certain utility one could well expect a gamble to be valued lower than its worst possible outcome. For example, consider the standard uncertainty effect, comparing a 50-50 lottery paying \$50 or \$100 to the certainty of \$50. Let certain consumption be evaluated with CRRA utility and a curvature parameter of 0.99 and let the lottery options be evaluated under expected CRRA utility with a curvature parameter of 0.88, as found in our estimates. The utility of the lottery is given as $U_L = 0.5 \times 50^{0.88} + 0.5 \times 100^{0.88} = 44.41$. The utility of the certain \$50 is given as $U_C = 50^{0.99} = 48.08$, demonstrating the uncertainty effect.

Third, risk preferences are frequently measured using certainty equivalence techniques. Such methodology frequently generates extreme measures of risk aversion at odds with standard EU theory via a calibration theorem (Rabin, 2000). A standard CRRA curvature parameter finding in such low stakes experiments is between 0.5 and 0.6. Our results suggest that at least part of the issue in these findings is the differential assessment of certain and uncertain consumption. Consider asking an individual to provide the certainty equivalent of a 50-50 lottery paying out \$50 or \$0. Let certain and uncertain consumption be evaluated as before. Normalizing $v(0) = 0$, we have: $CE^{0.99} = 0.5 \times 50^{0.88}$, yielding a certainty equivalent of $CE = 16.07$. If we assumed a single curvature parameter, a, and found the a that rationalizes $16.07^a = 0.5 \times 50^a$, we would solve for $a = 0.61$. As such, differential curvature for certain and uncertain consumption may help to explain the extremely high levels of risk aversion obtained in certainty equivalent experiments.

Fourth, experiments demonstrating prospect theory probability weighting also use certainty equivalence techniques (see Tversky and Fox, 1995). Following a similar logic to above, one can assume a curvature value, for example $a = 0.88$ (as in Tversky and Fox, 1995), and examine the probability weight $\pi(p)$ that rationalizes $CE^{0.88} = \pi(p) \times 50^{0.88}$ at various probabilities. For example, at $p = 0.95$ under our parameter values, we would obtain $CE = 30.73$ and a corresponding probability weight of $\pi(0.95) = 0.652$, demonstrating down-weighting of high probability events. And at $p = 0.01$, we would obtain $CE = 0.31$ and a probability weight of $\pi(0.01) = 0.013$, demonstrating a slight up-weighting of low probability events. Though this is far from the results obtained in probability weighting experiments, it suggests that probability weighting of objective probabilistic events may be conflated with differential utility for certain and uncertain consumption.

These brief applications of our central findings are provocative, but, of course, are not definitive. Future research should attempt to work through these issues in both intertemporal and static decision contexts as well as examine welfare effects and policy implications of differential utility over certain and uncertain consumption.

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Table 1: Basic Convex Time Budget Decisions

t (start date)	k (delay)	Token Budget	ρ_t	ρ_{t+k}	$(1+r)$	Daily Rate (%)	Annual Rate (%)
7	28	100	0.2	0.2	1.00	0	0
7	28	100	0.19	0.2	1.05	0.0018	85.7
7	28	100	0.18	0.2	1.11	0.0038	226.3
7	28	100	0.17	0.2	1.18	0.0058	449.7
7	28	100	0.16	0.2	1.25	0.0080	796.0
7	28	100	0.15	0.2	1.33	0.0103	1323.4
7	28	100	0.14	0.2	1.43	0.0128	2116.6
7	56	100	0.2	0.2	1.00	0	0
7	56	100	0.19	0.2	1.05	0.0009	37.9
7	56	100	0.18	0.2	1.11	0.0019	88.6
7	56	100	0.17	0.2	1.18	0.0029	156.2
7	56	100	0.16	0.2	1.25	0.0040	246.5
7	56	100	0.15	0.2	1.33	0.0052	366.9
7	56	100	0.14	0.2	1.43	0.0064	528.0

Table 2: Discounting and Curvature Parameter Estimates

	(1)	(2)	(3)
$\hat{\alpha}$	0.982		
	(0.002)		
$\hat{\alpha}_{(1,1)}$		0.988	0.988
		(0.002)	(0.002)
$\hat{\alpha}_{(0.5,0.5)}$		0.885	0.883
		(0.017)	(0.017)
Annual Rate	0.274		0.284
	(0.035)		(0.037)
Annual Rate$_{(1,1)}$		0.282	
		(0.036)	
Annual Rate$_{(0.5,0.5)}$		0.315	
		(0.088)	
$\hat{\omega}$	3.608	2.417	2.414
	(0.339)	(0.418)	(0.418)
R^2	0.642	0.673	0.673
N	2240	2240	2240
Clusters	80	80	80

Notes: NLS solution function estimators. Subscripts refer to (p_1, p_2) condition. Column (1) imposes the IA, $u(\cdot) = v(\cdot)$. Column (2) allows different curvature and different discounting in each (p_1, p_2) condition. Column (3) allows only different curvature in each each (p_1, p_2) condition. Annual discount rate calculates as $(1/\hat{\delta})^{365} - 1$, standard errors calculated via the delta method.

Figure 1: Sample Decision Sheet

2009 Calendar

S	M	T	W	Th	F	S
			April			
			1 ☆	2	3	4
5	6	7	8 ★	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		
			May			
					1	2
3	4	5	6 ★	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						
			June			
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

IN EACH ROW ALLOCATE 100 TOKENS BETWEEN

PAYMENT A (1 week from today) **AND** **PAYMENT B** (4 weeks later)

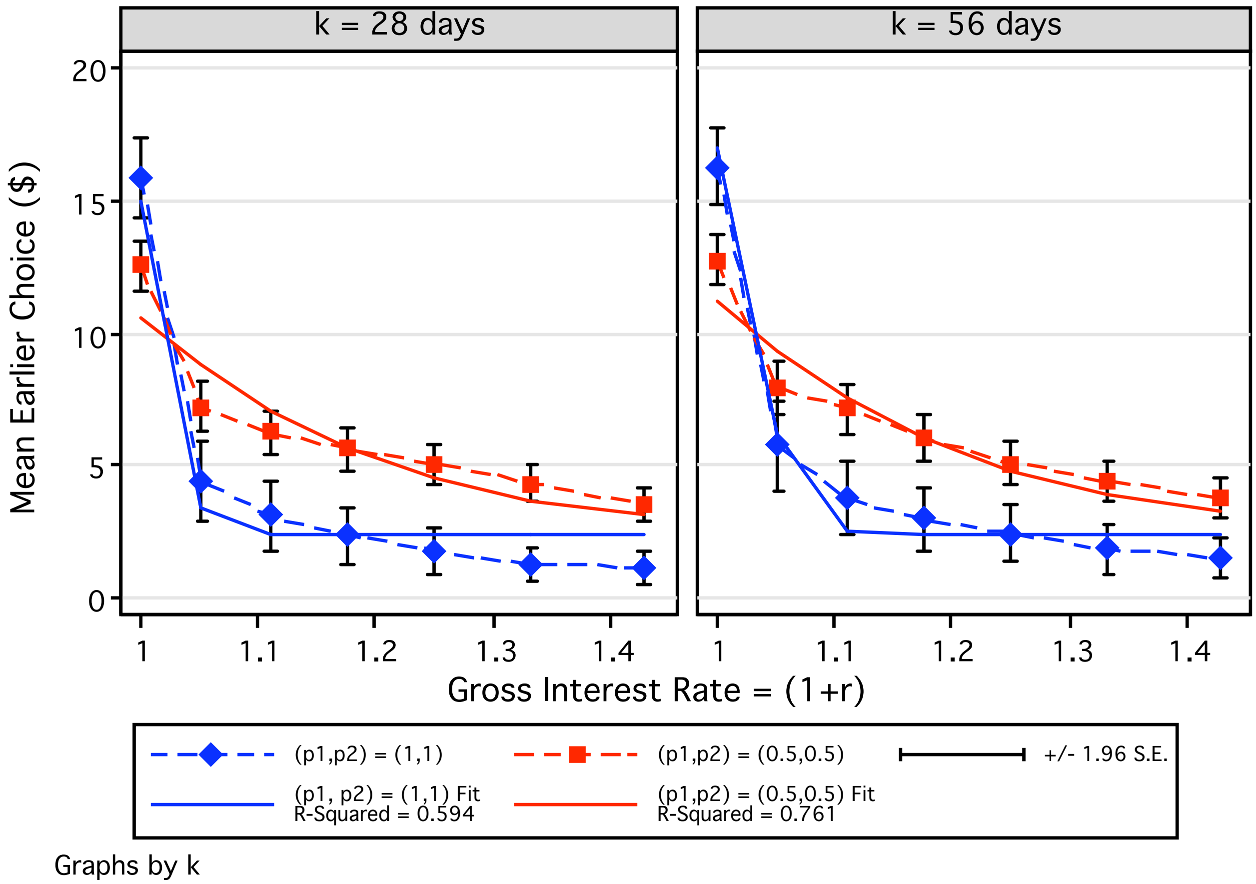
Date A: April 8, 2009 — **Chance A Sent: 40%**

Date B: May 6, 2009 — **Chance B Sent: 50%**

No.	A Tokens		Rate A $ per token	Date A	&	B Tokens		Rate B $ per token	Date B
1.	_______	tokens at	$.20 each	on April 8	&	_______	tokens at	$.20 each	on May 6
2.	_______	tokens at	$.19 each	on April 8	&	_______	tokens at	$.20 each	on May 6
3.	_______	tokens at	$.18 each	on April 8	&	_______	tokens at	$.20 each	on May 6
4.	_______	tokens at	$.17 each	on April 8	&	_______	tokens at	$.20 each	on May 6
5.	_______	tokens at	$.16 each	on April 8	&	_______	tokens at	$.20 each	on May 6
6.	_______	tokens at	$.15 each	on April 8	&	_______	tokens at	$.20 each	on May 6
7.	_______	tokens at	$.14 each	on April 8	&	_______	tokens at	$.20 each	on May 6

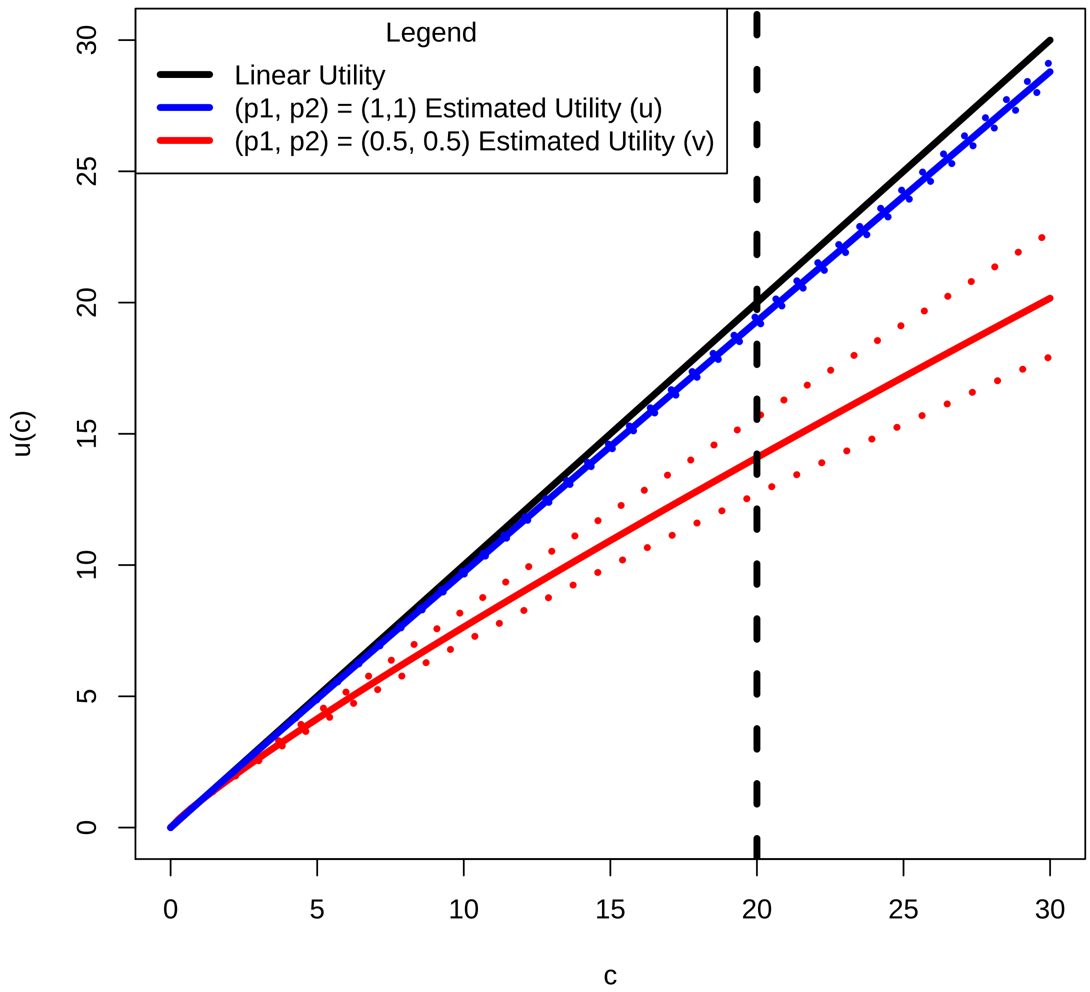
PLEASE MAKE SURE A + B TOKENS = 100 IN EACH ROW!

Figure 2: Aggregate Behavior Under Certainty and Uncertainty



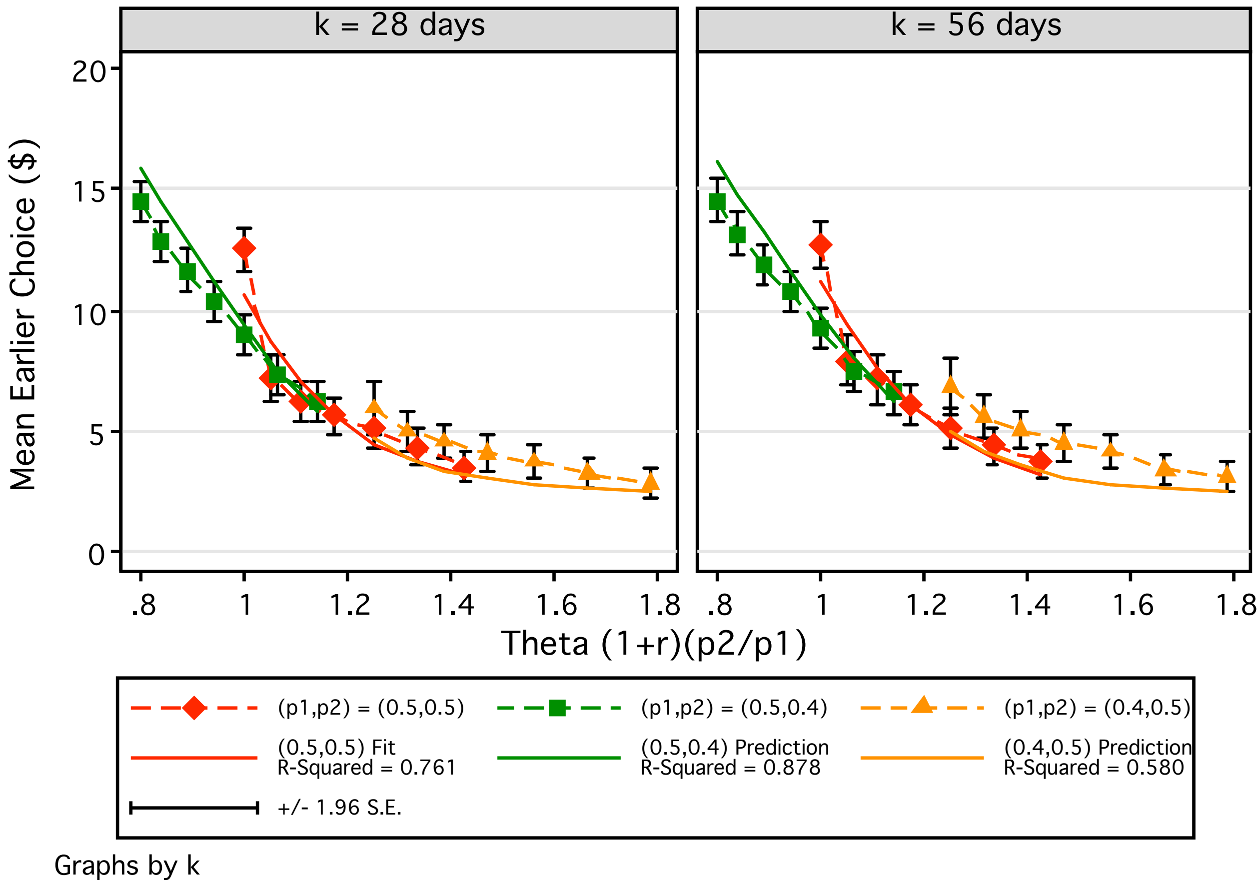
Note: The figure presents aggregate behavior for $N = 80$ subjects under two conditions: 1) $(p_1, p_2) = (1, 1)$, i.e. no risk, in blue; and 2) $(p_1, p_2) = (0.5, 0.5)$, i.e. 50% chance sooner payment would be sent *and* 50% chance later payment would be sent, in red. $t = 7$ *days* in all cases, $k \in \{28, 56\}$ *days*. Error bars represent 95% confidence intervals, taken as $+/-1.96$ standard errors of the mean. Test of H_0 : Equality across conditions 1 and 2: $F_{14,2212} = 15.66$, $p < .001$.

Figure 3: Estimated Utility Function Curvature Under Certainty and Uncertainty



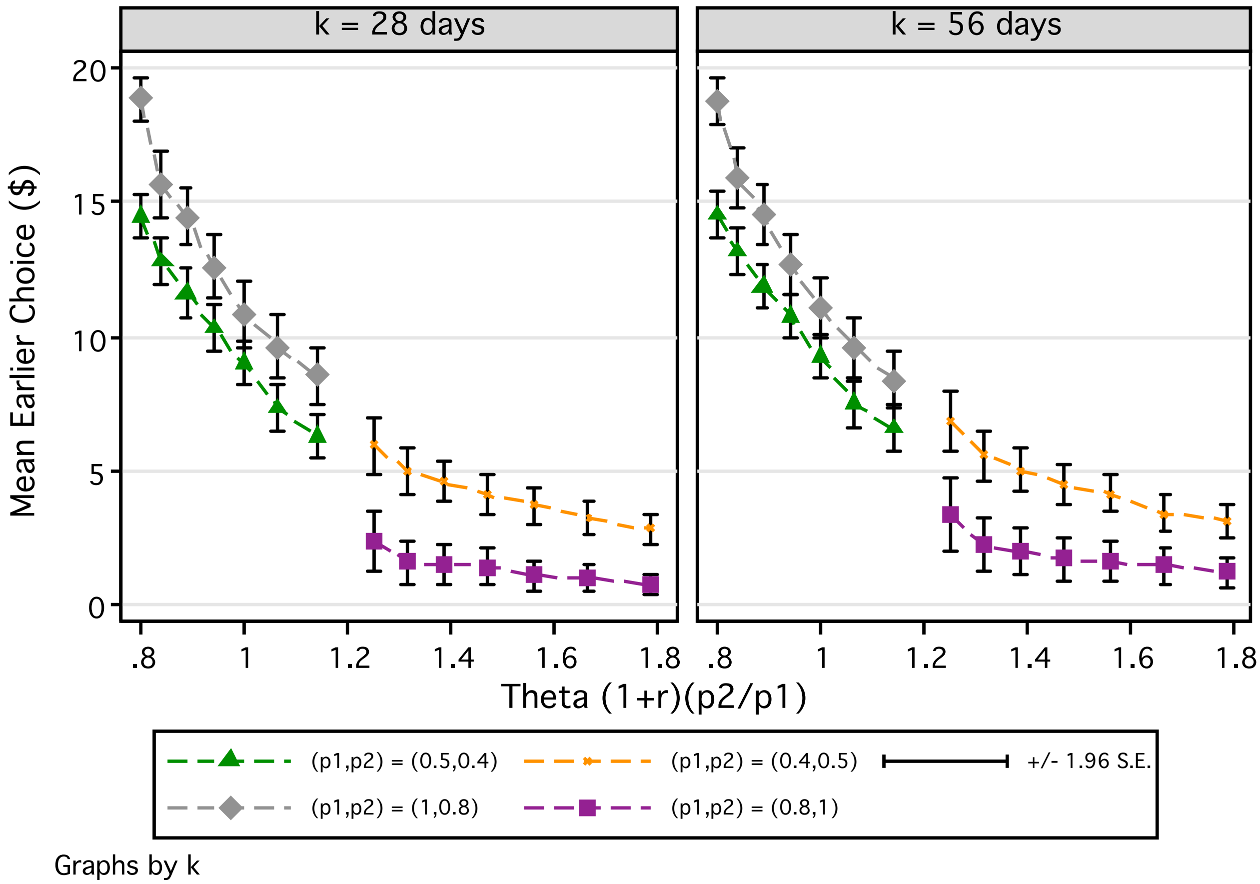
Note: The figure presents estimated utility functions (corresponding to the estimates of Table 2, column (5): $c^{\hat{\alpha}}$. Dotted lines represent 95% confidence intervals. $c = 20$ corresponds to the value of later payments in the experiment.

Figure 4: Aggregate Behavior Under Uncertainty



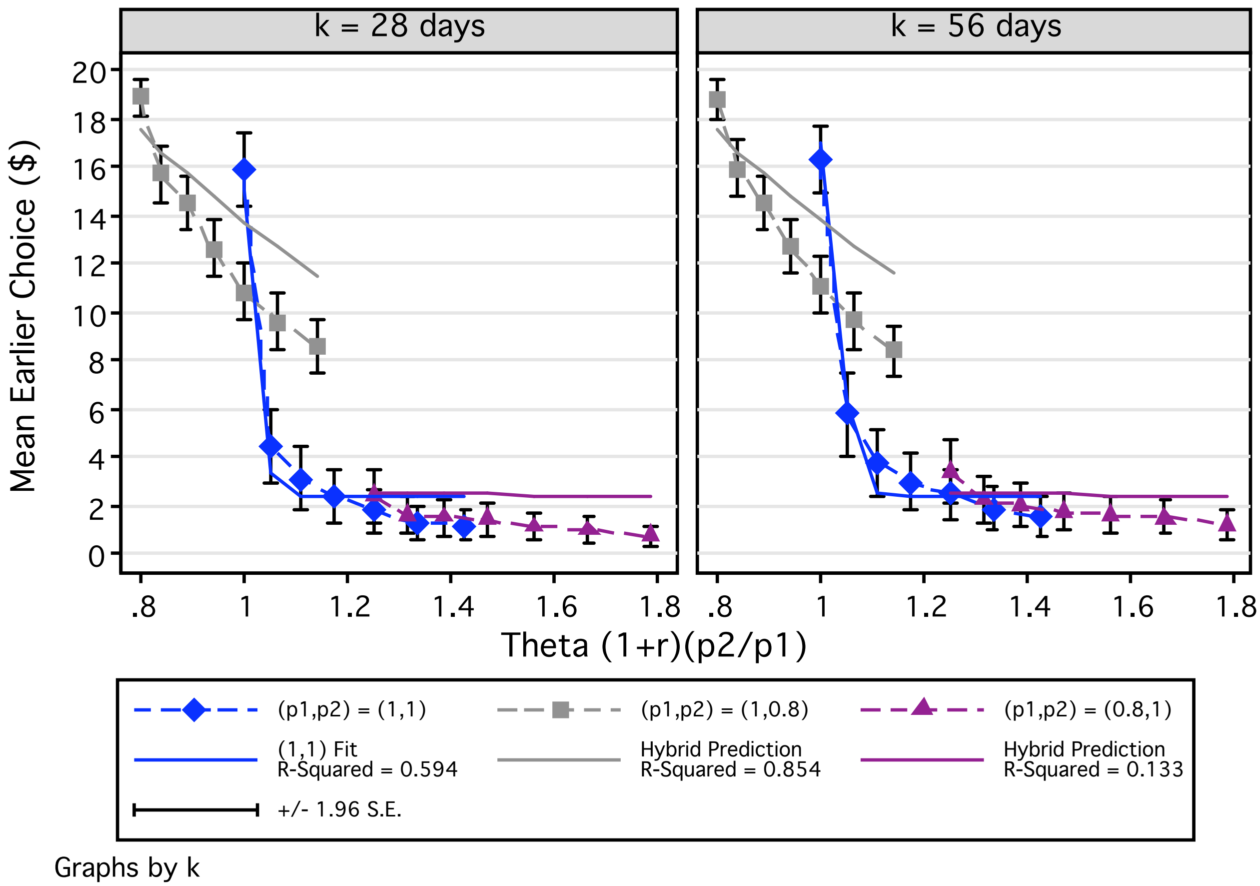
Note: The figure presents aggregate behavior for $N = 80$ subjects under three conditions: 1) $(p_1, p_2) = (0.5, 0.5)$, i.e. equal risk, in red; 2) $(p_1, p_2) = (0.5, 0.4)$, i.e. more risk later, in green; and 3) $(p_1, p_2) = (0.4, 0.5)$, i.e. more risk sooner, in orange. Error bars represent 95% confidence intervals, taken as $+/- 1.96$ standard errors of the mean. Solid lines correspond to predicted behavior using utility estimates from $(p_1, p_2) = (0.5, 0.5)$ as estimated in Table 2, column (3).

Figure 5: A Disproportionate Preference for Certainty



Note: The figure presents aggregate behavior for $N = 80$ subjects under four conditions: 3) $(p_1, p_2) = (0.5, 0.4)$ 4) $(p_1, p_2) = (0.4, 0.5)$, 5) $(p_1, p_2) = (1, 0.8)$ and 6)$(p_1, p_2) = (0.8, 1)$, Error bars represent 95% confidence intervals, taken as $+/- 1.96$ standard errors of the mean. Conditions 3 and 5 share a common ratio as do conditions 4 and 6. Test of H_0 : Equality across conditions 3 and 5: $F_{14,2212} = 14.60,\ p < .001$. Test of H_0 : Equality across conditions 4 and 6: $F_{14,2212} = 23.82,\ p < .001$.

Figure 6: Aggregate Behavior Under In Certain / Uncertain Situations

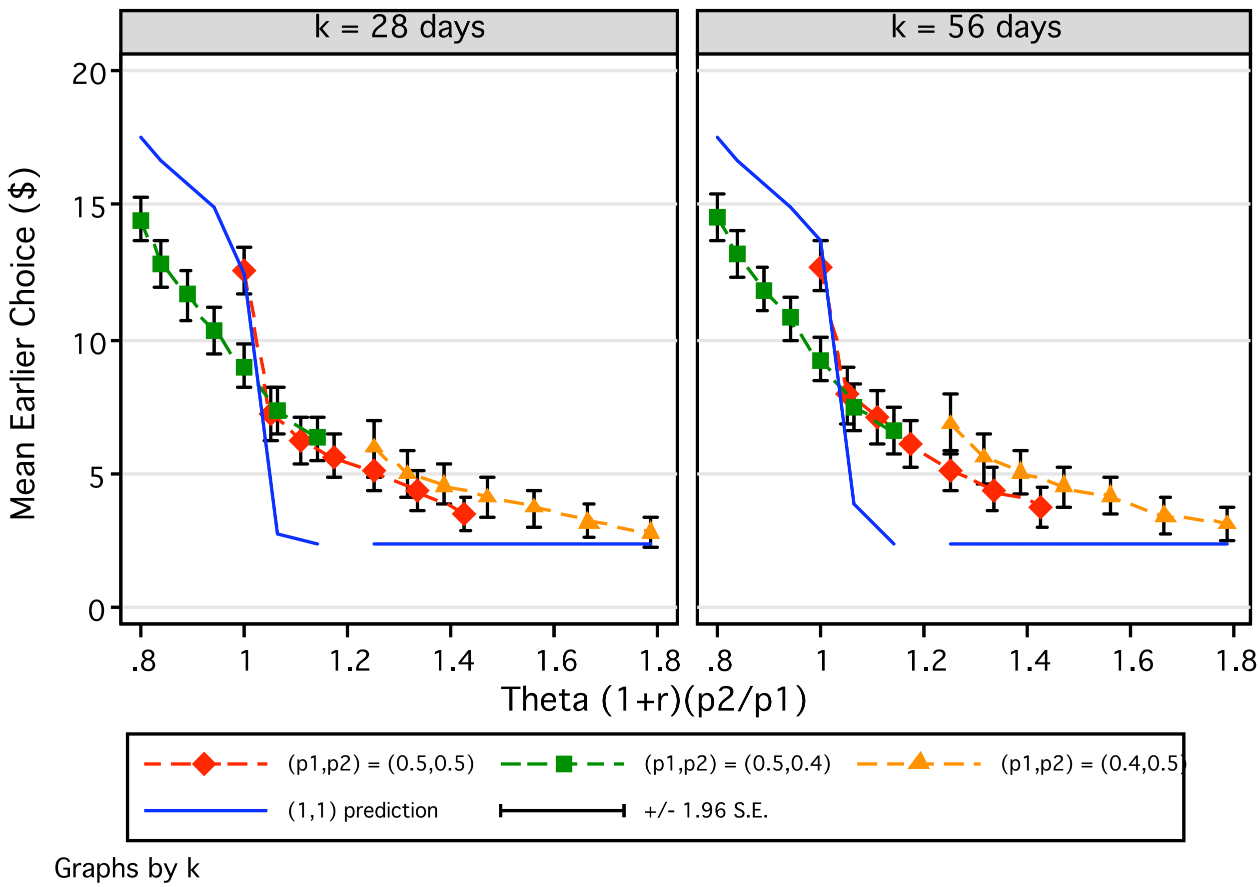


Note: The figure presents aggregate behavior for $N = 80$ subjects under three conditions: 1) $(p_1, p_2) = (1, 1)$, i.e. equal risk, in blue; 2) $(p_1, p_2) = (1, 0.8)$, i.e. more risk later, in green; and 3) $(p_1, p_2) = (0.8, 1)$, i.e. more risk sooner, in orange. Error bars represent 95% confidence intervals, taken as $+/-1.96$ standard errors of the mean. 'Hybrid Prediction' lines correspond to predicted behavior using utility estimates from $(p_1, p_2) = (0.5, 0.5)$ for uncertain payments and $(p_1, p_2) = (1, 1)$ for certain payments as estimated in Table 2, column (3).

A Appendix

A.1 Appendix Figures and Tables

Figure A1: Aggregate Behavior Under Uncertainty with Predictions Based on Certainty



Note: The figure presents aggregate behavior for $N = 80$ subjects under three conditions: 1) $(p_1, p_2) = (0.5, 0.5)$, i.e. equal risk, in red; 2) $(p_1, p_2) = (0.5, 0.4)$, i.e. more risk later, in green; and 3) $(p_1, p_2) = (0.4, 0.5)$, i.e. more risk sooner, in orange. Error bars represent 95% confidence intervals, taken as $+/-1.96$ standard errors of the mean. Blue solid lines correspond to predicted behavior using utility estimates from $(p_1, p_2) = (1, 1)$ as estimated in Table 2, column (3).

A.2 Welcome Text

Welcome and thank you for participating.

Eligibility for this study: To be in this study, you need to meet these criteria. You must have a campus mailing address of the form:

YOUR NAME
9450 GILMAN DR 92(MAILBOX NUMBER)
LA JOLLA CA 92092-(MAILBOX NUMBER)

Your mailbox must be a valid way for you to receive mail from now through the end of the Spring Quarter.

You must be willing to provide your name, campus mail box, email address, and student PID. This information will only be seen by Professor Andreoni and his assistants. After payment has been sent, this information will be destroyed. Your identity will not be a part of any subsequent data analysis.

You must be willing to receive your payment for this study by check, written to you by Professor James Andreoni, Director of the UCSD Economics Laboratory. The checks will be drawn on the USE Credit Union on campus. You may deposit or cash your check wherever you like. If you wish, you can cash your checks for free at the USE Credit Union any weekday from 9:00 am to 5:00 pm with valid identification (drivers license, passport, etc.).

The checks will be delivered to you at your campus mailbox at a date to be determined by your decisions in this study, and by chance. The latest you could receive payment is the last week of classes in the Spring Quarter.

If you do not meet all of these criteria, please inform us of this now.

A.3 Instruction and Examples Script

Earning Money:

To begin, you will be given a \$10 minimum payment. You will receive this payment in two payments of \$5 each. The two \$5 minimum payments will come to you at two different times. These times will be determined in the way described below. Whatever you earn from the study today will be added to these minimum payments.

In this study, you will make 84 choices over how to allocate money between two points in time, one time is 'earlier' and one is 'later'. Both the earlier and later times will vary across decisions. This means you could be receiving payments as early as one week from today, and as late as the last week of classes in the Spring Quarter, or possibly other dates in between.

It is important to note that the payments in this study involve chance. There is a chance that your earlier payment, your later payment or both will not be sent at all. For each decision, you will be fully informed of the chance involved for the sooner and later payments. Whether or not your payments will be sent will be determined at the END of the experiment today. If, by chance, one of your payments is not sent, you will receive only the \$5 minimum payment.

Once all 84 decisions have been made, we will randomly select one of the 84 decisions as the decision-that-counts. This will be done in three stages. First, we will pick a number from 1 to 84 at random to determine which is the decision-that-counts and the corresponding sooner and later payment dates. Then we will pick a second number at random from 1 to 10 to determine if the sooner payment will be sent. Then we will pick a third number at random from 1 to 10 to determine if the later payment will be sent. We will use the decision-that-counts to determine your actual earnings. Note, since all decisions are equally likely to be chosen, you should make each decision as if it will be the decision-that-counts. When calculating your earnings from the decision-that-counts, we will add to your earnings the two \$5 minimum payments. Thus, you will always get paid at least \$5 at the chosen earlier time, and at least \$5 at the chosen later time.

IMPORTANT: All payments you receive will arrive to your campus mailbox. On the scheduled day of payment, a check will be placed for delivery in campus mail services by Professor Andreoni and his assistants. Campus mail services guarantees delivery of 100% of your payments by the following day.

As a reminder to you, the day before you are scheduled to receive one of your payments, we will send you an e-mail notifying you that the payment is coming. On your table is a business card for Professor Andreoni with his contact information. Please keep this in a safe place. If one of your payments is not received you should immediately contact Professor Andreoni, and we will hand-deliver payment to you.

Your Identity:

In order to receive payment, we will need to collect the following pieces of information from you: name, campus mail box, email address, and student PID. This information will only be seen by Professor Andreoni and his assistants. After all payments have been sent, this information will be destroyed. Your identity will not be a part of subsequent data analysis.

On your desk are two envelopes: one for the sooner payment and one for the later payment. Please take the time now to address them to yourself at your campus mail box.

How it Works:

In each decision you are asked to divide 100 tokens between two payments at two different dates: Payment A (which is sooner) and Payment B (which is later). Tokens will be exchanged for money. The tokens you allocate to Payment B (later) will always be worth at least as much as the tokens you allocate to Payment A (sooner). The process is best described by an example. Please examine the sample sheet in you packet marked SAMPLE.

The sample sheet provided is similar to the type of decision sheet you will fill out in the study. The sample sheet shows the choice to allocate 100 tokens between Payment A on April 17th and Payment B on May 1st. Note that today's date is highlighted in yellow on the calendar on the left hand side. The earlier date (April 17th) is marked in green and the later date (May 1st) is marked in blue. The earlier and later dates will always be marked green and blue in each decision you make. The dates are also indicated in the table on the right.

In this decision, each token you allocate to April 17th is worth $0.10, while each token you allocate to May 1st is worth $0.15. So, if you allocate all 100 tokens to April 17th, you would earn 100x$0.10 = $10 (+ $5 minimum payment) on this date and nothing on May 1st (+ $5 minimum payment). If you allocate all 100 tokens to May 1st, you would earn 100x$0.15 = $15 (+ $5 minimum payment) on this date and nothing on April 17th (+ $5 minimum payment). You may also choose to allocate some tokens to the earlier date and some to the later date. For instance, if you allocate 62 tokens to April 17th and 38 tokens to May 1st, then on April 17th you would earn 62x$0.10 = $6.20 (+ $5 minimum payment) and on May 1st you would earn 38x$0.15 = $5.70 (+ $5 minimum payment). In your packet is a Payoff Table showing some of the token-dollar exchange at all relevant token exchange rates.

REMINDER: Please make sure that the total tokens you allocate between Payment A and Payment B sum to exactly 100 tokens. Feel free to use the calculator provided in making your allocations and making sure your total tokens add to exactly 100 in each row.

Chance of Receiving Payments:

Each decision sheet also lists the chances that each payment is sent. In this example there is a 70% chance that Payment A will actually be sent and a 30% chance that Payment B will actually be sent. In each decision we will inform you of the chance that the payments will be sent. If this decision were chosen as the decision-that-counts we would determine the actual payments by throwing two ten sided die, one for Payment A and one for Payment B.

EXAMPLE: Let's consider the person who chose to allocate 62 tokens to April 17th and 38 tokens to May 1st. If this were the decision-that-counts we would then throw a ten-sided die for Payment A. If the die landed on 1,2,3,4,5,6,or 7, the person's Payment A would be sent and she would receive \$6.20 (+ \$5 minimum payment) on April 17th. If the die landed 8,9, or 10, the payment would not be sent and she would receive only the \$5 minimum payment on April 17th. Then we would throw a second ten-sided die for Payment B. If the die landed 1,2, or 3, the person's Payment B would be sent and she would receive \$5.70 (+ \$5 minimum payment) on May 1st. If the die landed 4,5,6,7,8,9, or 10, the payment would not be sent and she would receive only the \$5 minimum payment on May 1st.

Things to Remember:

- You will always be allocating exactly 100 tokens.
- Tokens you allocate to Payment A (sooner) and Payment B (later) will be exchanged for money at different rates. The tokens you allocate to Payment B will always be worth at least as much as those you allocate to Payment A.

- Payment A and Payment B will have varying degrees of chance. You will be fully informed of the chances.
- On each decision sheet you will be asked 7 questions. For each decision you will allocate 100 tokens. Allocate exactly 100 tokens for each decision row, no more, no less.
- At the end of the study a random number will be drawn to determine which is the decision-that-counts. Because each question is equally likely, you should treat each decision as if it were the one that determines your payments. Two more random numbers will be drawn by throwing two ten sided die to determine whether or not the payments you chose will actually be sent.
- You will get an e-mail reminder the day before your payment is scheduled to arrive.
- Your payment, by check, will be sent by campus mail to the mailbox number you provide.
- Campus mail guarantees 100% on-time delivery.
- You have received the business card for Professor James Andreoni. Keep this card in a safe place and contact Prof. Andreoni immediately if one of your payments is not received.

A.4 Solving Numerically for Out of Sample c_t Predictions

We consider the case where certain and uncertain consumption are evaluated with different preference parameters. That is $u(\cdot) \neq v(\cdot)$. We assume CRRA utility in each case $u(c_t) = (c_t - \omega)^\alpha$ and $v(c_t) = (c_t - \omega)^\beta$ with $\alpha \neq \beta$. ω can be thought of as a Stone-Geary minimum parameter.

Let $p_1 = 1$ and $p_2 < 1$ such that sooner consumption is certain and later consumption is uncertain. The individual's optimization problem is:

$$max_{c_t, c_{t+k}} p_1(c_t - \omega)^\alpha + p_2\delta^k(c_{t+k} - \omega)^\beta \ s.t. \ (1+r)c_t + c_{t+k} = m$$

Yielding the marginal condition:

$$\frac{p_1\alpha(c_t - \omega)^{\alpha-1}}{p_2\delta^k \cdot \beta(c_{t+k} - \omega)^{\beta-1}} = (1+r)$$

$$\frac{(c_t - \omega)^{\alpha-1}}{(c_{t+k} - \omega)^{\beta-1}} = (1+r)(\frac{p_2}{p_1})(\frac{\beta}{\alpha})\delta^k$$

Raise everything to the $\frac{1}{\beta-1}$ power

$$\frac{(c_t - \omega)^{\frac{\alpha-1}{\beta-1}}}{c_{t+k} - \omega} = [(1+r)(\frac{p_2}{p_1})(\frac{\beta}{\alpha})\delta^k]^{\frac{1}{\beta-1}}$$

Substitute in the budget constraint:

$$\frac{(c_t - \omega)^{\frac{\alpha-1}{\beta-1}}}{m - (1+r)c_t - \omega} = [(1+r)(\frac{p_2}{p_1})(\frac{\beta}{\alpha})\delta^k]^{\frac{1}{\beta-1}}$$

$$(c_t - \omega)^{\frac{\alpha-1}{\beta-1}} = [(1+r)(\frac{p_2}{p_1})(\frac{\beta}{\alpha})\delta^k]^{\frac{1}{\beta-1}}[m - (1+r)c_t - \omega]$$

Define $A \equiv [(1+r)(\frac{p_2}{p_1})(\frac{\beta}{\alpha})\delta^k]^{\frac{1}{\beta-1}}$ and $B \equiv \frac{\alpha-1}{\beta-1}$

$$(c_t - \omega)^B = A[m - (1+r)c_t - \omega]$$

$$(c_t - \omega)^B = Am - A(1+r)c_t - A\omega]$$

$$(c_t - \omega)^B + A(1+r)c_t + A\omega - Am = 0$$

Provided estimates for $\alpha, \beta, \delta, \omega$ as obtained in Table 2, A and B are known constants. The numerical root to the above B^{th} order polynomial for a given $p_1, p_2, 1+r$ and k will be the predicted value of c_t in the situation. Many algorithms exist for obtaining such function roots. This is the methodology for obtaining out of sample predicted values in Figure 6 and is easily applied to situations where both payments are uncertain or both payments are certain.