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Risk Premiums and Macroeconomic Dynamics in a Heterogeneous Agent Model*

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Abstract

We analyze financial risk premiums and real economic dynamics in a DSGE model with three types of agents - shareholders, bondholders and workers - that differ in participation in the capital market and in terms of risk aversion. Aggregate productivity and distribution risk are shared among these agents via the bond market and via an efficient labor contract. The result is a combination of volatile returns to capital and a highly cyclical consumption process for the shareholders, which are two important ingredients for generating high and countercyclical risk premiums. These risk premiums are consistent with a strong propagation mechanism through an elastic supply of labor, rigid real wages and a countercyclical labor share. We discuss the implications for the real and nominal component of the risk premium on equity and bonds. We show how these premiums react to changes in the volatility of the shocks, as experienced during the great moderation. We also analyze the effects of changes in monetary policy behavior and the resulting inflation dynamics.

JEL codes: E32, E44, G12

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*The views expressed herein do not necessarily reflect those of the National Bank of Belgium, the Federal Reserve Bank of Dallas or the Federal Reserve System.

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1 Introduction

Time variation in risk premiums is generally considered an important driving process for asset price fluctuations and therefore may also contribute to real fluctuations in the economy. Economic models typically have a hard time reproducing the observed risk premiums and real statistics simultaneously. The need for such a consistent model is high. For instance, it would make it possible to extract the information contained in asset prices about future growth and inflation expectations of private investors by controlling for the implied risk premiums. At the same time, a model that can jointly match financial and real statistics would have strong empirical validity. The standard DSGE model with endogenous capital and labor has problems generating sufficiently large premiums and realistic real statistics because investors have various channels through which they can smooth consumption. Various solutions have been suggested in the literature to overcome this problem within the standard representative agent model. Recent examples include, among others, Lettau and Uhlig (2000) who evaluate the potential role of habit formation, Boldrin et al. (2001) suggest frictions in the labor allocation between sectors, Uhlig (2007) proposes real wage rigidity as a possible solution.

In this paper, we follow Guvenen (2008), Danthine and Donaldson (2002) and Danthine et al. (2006), and focus on the role of heterogeneous capital market participation across agents. This setup implies a number of interesting features that can facilitate the joint explanation of real and financial statistics. First, in such a setup, it is no longer aggregate consumption that drives the pricing kernel of asset prices. There is a well documented literature that suggests that the consumption of wealthy agents, that hold the majority of the capital stock, is more volatile than aggregate consumption. Second, in a context of heterogeneous agents, the valuation of the capital stock is not only determined by aggregate risk, but also by distribution risk. The volatile and highly procyclical nature of profits can potentially contribute significantly to the explanation of the equity risk premium and can help to differentiate between stock and bond risk premiums. The risk sharing between heterogeneous agents does not only affect the pricing of the claims on future profits but also offers the natural context to explain the observed acyclical behavior of real wages and the countercyclical behavior of the wage share. Third, an explanation of the risk premium based on heterogeneous capital market participation across agents has important empirical implications for the financial behavior of the different agents, for instance in terms of wealth accumulation and the resulting wealth distribution. Therefore, this approach has the advantage that the underlying assumptions can be validated more easily compared to alternative explanations which are often based on non-observable features of the utility functions (another popular solution to the equity premium in the context of a representative agent model). We implement such a heterogeneous agent structure in an otherwise standard DSGE model with monopolistic competitive firms, nominal rigidities and a monetary policy reaction function.

As a first contribution, we show that this model, driven by a combination of aggregate productivity and distribution shocks, is able to generate significant risk premiums. The model also produces realistic statistics for real aggregate volatilities and correlations. In particular, the optimal labor contract, motivated by risk sharing considerations, explains the observed rigidity and low volatility in the real wage, as well as the countercyclical wage share. Nominal rigidities, optimal wage contracts and stochastic distribution risk -which takes up possible shifts in the relative bargaining power of workers and firms- deliver a high volatility in profits, returns to equity and price-dividend ratios. This high volatility in the returns from capital, combined with the high concentration of capital market participation, results in a concentration of risk and a high consumption volatility for the shareholders. We analyze how the different features of the model contribute to generating such significant risk premiums.

Second, the joint focus on a variety of assets (nominal and real bonds, as well as stocks) imposes additional discipline in building the model. For one, reproducing observed differences in returns to stocks and bonds has implications for the degree of flexibility one has in modelling the agents' stochastic discount factor. Moreover, the macroeconomic fluctuations that underlie the various risk premiums are model-consistent. Put differently, the general equilibrium framework adopted ensures a joint explanation, without relying on, e.g., reduced form macroeconomic dynamics to mimic risk premiums. In addition to internal consistency, this allows for the possibility of feedback effects from risk premiums to the real economy. We also analyze the role of monetary policy in the model. The way in which the central bank responds to inflation is, obviously, of great importance to relative variations in real and nominal risk premiums, as well as the macroeconomy. In this sense, our paper complements papers studying bond and equity premiums in representative agent models with sticky prices and monetary policy. Examples of the latter approach include Hördahl et al. (2007), De Paoli et al. (2007), Ravenna and Seppälä (2007) and Rudebusch and Swanson (2008).

The third contribution of the paper is applied in nature. We estimate the series of productivity and distribution shocks for the period 1947-2007 and feed them into the model. Taking into account the limited stochastic structure of this exercise, the resulting time variation in risk premiums compares well to available proxies and estimates in the literature. We perform predictive regressions for stocks and bonds to further understand the success and limitations of the model. Finally, we provide some evidence on the potential effects of the Great Moderation on the variety of risk premiums.

In section 2, we present the model and its calibration. Section 3 documents the main financial and real statistics implied by the model, and compares them with analogue statistics in the data and implied statistics of the representative agent version of the model. The specific role of the utility function and the different risk sharing arrangements in the model are explained. In section 4, the different components of the risk premiums are analyzed in more detail. The difference between the equity and bond premium is discussed as well as the real and the nominal component of the premium. Section 5 presents the results on the implied time variation in the risk premiums, and shows how such variation affects the behavior of the price-dividend ratio and the yield spread. In section 6, we illustrate how the risk premiums depend on the stochastic structure by showing the potential impact of the Great Moderation. Finally, we document how the risk premiums and their nominal component, in particular, interact with the assumptions about monetary policy and inflation rigidity. Our analysis is based on simulation experiments with the first, second and third order approximation of the non-linear model using the Dynare and Dynare++ toolbox.

2 The Model

We start from a general setup which considers three types of agents. A first group of agents consists of the standard portfolio investors that allocate their wealth between stocks and bonds. These agents act as the marginal investors that clear the bond and stock markets. Therefore, their stochastic discount factor will determine the pricing of the corresponding risks. Motivated by empirical evidence, we assume that the portfolio managers are characterized by a lower risk aversion than the other agents in the economy. We refer to these agents as type 1 agents. A second group of agents (type 2) participates in the capital market by buying bonds. Their bondholdings depend on their desire to smooth consumption as well as their precautionary savings, and determine the wealth accumulation of these agents. Finally, a third group of agents (type 3), the workers, does not participate in the capital market and consumes immediately its income from labor. In order to smooth their marginal utility, these agents are completely dependent on the labor contract which provides the only opportunity for them to share their income risk with the other agents in the economy, in particular with the shareholders as owners of the firms. In a context of continued labor-firm relations, the optimal labor contract guarantees a predetermined relation between the marginal utility of the workers and the marginal utility of the shareholder of the firms. More risk averse type 2 and type 3 agents will try to transfer some of the aggregate risk towards the shareholders, either via savings in the bond market or via the wage contract. In exchange the shareholders will require a higher return.

This general setting allows us to review specific cases that have been considered previously in the literature. When the economy is vacated by shareholders alone, the model is very similar to the standard representative agent model, analyzed in e.g. Uhlig (2007). This makes it easy to compare the outcomes of the general model with the representative agent version, and to review the implications of the various model assumptions within a more standard setup. More specifically, we discuss the important implications of alternative specifications of the utility function for both the financial and the real decisions. We show that it makes a major difference whether the utility function is assumed to be separable or non-separable between consumption and labor. In order to clarify these implications we consider three different utility specifications which are standard in the DSGE modelling work: the separable power utility function, the King, Plosser and Rebelo (1988) utility function and the Greenwood, Hercowitz and Huffman (1988) utility function as examples of a non-separable utility function.

Alternatively, when more than one type of household is present, the model encompasses a variety of asset pricing models with heterogeneous agents. For instance, when both shareholders and bondholders are present, our model is similar to that of Guvenen (2008). Alternatively, when the economy consists of shareholders as well as workers, our setup is very close to that of Danthine and Donaldson (2002). With all three agents present, our model has the flavor of agent heterogeneity as analyzed in Chien et al. (2007). In a three agent framework, risk sharing considerations are more complicated, and we will show how some of the mechanisms at work in a two agent models can work out differently in a broader setting. We incorporate the labor decision for shareholders and bondholders in all versions to maintain comparability over different models. Excluding the labor choice for these agents would make it easier to fit the asset pricing moments, as the labor choice offers shareholders another channel to smooth fluctuations in marginal utility.

2.1 Households

There are three different types of households in our model economy: shareholders, bondholders and workers. The types of households differ in the way they insure against macroeconomic risk and in the way they participate in the financial market. All agents maximize expected utility, which depends positively on consumption and negatively on the amount of labor supplied.

Type 1 Agents: Shareholders

Shareholders are freely able to invest both in stocks and bonds. They choose the amount of working hours ($N_{1,t}$) they supply at the prevailing spot market wage rate (W_t^s). The decision problem for these shareholders is thus:

$$\max E_t \sum_{j=0}^{\infty} \beta^t U_1 \left(C_{1,t+j}, N_{1,t+j}\right)$$

subject to the requirement:

$$C_{1,t} + \frac{P_t^B}{P_t} B_{1,t+1} + \frac{P_t^S}{P_t} S_{1,t+1} \leqslant \frac{B_{1,t}}{P_t} + S_{1,t} \frac{\left(P_t^S + D_t\right)}{P_t} + \frac{W_t^s}{P_t} N_{1,t} + \frac{\Gamma_t}{P_t}$$

In words, the shareholders' budget constraint states that their expenditures on consumption ($C_{1,t}$), bonds ($B_{1,t+1}$) and stocks ($S_{1,t+1}$), cannot exceed total income. The aggregate price level is denoted by P_t. Bonds are sold at a price P_t^B, while shares trade at price P_t^S. In addition to labor income ($W_t^s N_{1,t}$), shareholders obtain funds from previous bond holdings ($B_{1,t}$), from stock holdings ($S_{1,t} P_t^S$) and through dividend payments by the firms ($S_{1,t} D_t$) and the financial intermediary (Γ_t, see below). This maximization problem results in the standard FOC (see Appendix A). In particular, they mimic the well known conditions for consumption, labor and asset holdings in a standard representative agent model. The stochastic discount factor of the shareholders is also used to price long term nominal and real bonds.[1]

Type 2 Agents: Bondholders

Bondholders do not hold any shares in their portfolio. Bondholders also differ from shareholders in that their momentary utility function is characterized by a higher degree of risk aversion, but they are otherwise very similar. In particular, the type 2 agents also work at the spot wage and thus maximize:

$$\max E_t \sum_{j=0}^{\infty} \beta_2^t U_2 \left(C_{2,t+j}, N_{2,t+j}\right)$$

[1] We consider decaying coupon perpetuities as in Rudebusch and Swanson (2008).

subject to:

$$C_{2,t} + \frac{P_t^B B_{2,t+1}}{P_t} \frac{1}{\phi(B_{2,t+1})} \leqslant \frac{B_{2,t}}{P_t} + \frac{W_t^s}{P_t} N_{2,t}$$

Bondholders engage in bond accumulation via a financial intermediary. In doing so, they are subject to a portfolio cost $\phi(B_{2,t+1})$. We introduce such a cost for bond holdings so that the return on bonds will depend on the macro bond supply. The more bondholders save, the lower the return. The more debt they accumulate, the higher the cost. This cost is taken as given from the point of view of an individual bondholder. This mechanism is the same as in Benigno (2007) who uses it in a two-country model. The introduction of such an intermediation margin is necessary to avoid infinite bond holdings or borrowing. This assumption has similar consequences as the discrete constraints on bond positions as imposed in e.g. Guvenen (2008). The latter setup cannot be used when applying perturbation methods to solve the model, as we do below. The intermediation profits made by the financial intermediary Γ_t are rebated to the shareholders.

Type 3 Agents: Workers

The third type of agents also derive utility from consumption and labor, with felicity function $U_3(.)$.

$$\max E_t \sum_{j=0}^{\infty} \beta^t U_3 \left(C_{3,t+j}, N_{3,t+j}\right)$$

The main difference from the other types of agents is that workers do not participate in financial markets at all and cannot accumulate wealth.[2] As a result, these agents consume their entire labor income each period:

$$C_{3,t} = \frac{W_t^c}{P_t} N_{3,t}$$

This does not, however, mean that workers are completely unable to share their risk with other agents in the economy. Rather to the contrary, similar to Danthine and Donaldson (2002), we assume that workers engage in long-term labor contracts with the firms.[3] The workers earn the contract wage (W_t^c) that corresponds to an optimal risk sharing arrangement with the shareholders of the firm, and in exchange they deliver the efficient labor input to

[2] In fact, this feature follows endogenously from the fact that these agents are able to sign optimal risk sharing contracts with the firm owners. Given these contracts, workers will be indifferent to participate in the capital market or not. In order to get a well defined solution, we need to make an additional assumption on their financial behavior. But the results of the model are unlikely to be affected if we would take an alternative assumption on this. In that sense, the model should not be in conflict with the empirically observed financial wealth that is held by these agents.

[3] We also considered an alternative setup, in which workers engage in one period contracts between workers and firms (See Boldrin and Horvath (1995) for more details). These contracts equalize the expected marginal utility of both parties to their relative bargaining power. The implications for the labor supply are very similar. To save space we do not report these results in the paper.

the firms. The contract is summarized by (see Appendix A for the complete derivation):

$$
\begin{aligned}
U^C_{1,t} &= \frac{(1-v_t)}{v_t} U^C_{3,t} \\
\frac{W^s_t}{P_t} &= -\frac{U^N_{3,t}}{U^C_{3,t}}
\end{aligned}
\tag{1}
$$

The contract wage guarantees an optimal risk sharing between workers and shareholders on a period-by-period basis for the given realization of the exogenous bargaining weight v_t. The steady state level of v is chosen such that the income distribution resulting from the contract is similar to the outcome under spot labor markets. With a fixed value of v, the contract provides optimal insurance against aggregate risk and reproduces the same outcome as the exchange of contingent securities (constant relative marginal utilities), at given wealth distribution. We will consider v as a time-varying process, driven by exogenous shocks to the bargaining power. We will refer to these shocks as distribution risk where:

$$\log(v_t) = (1-\rho_v)\log(\overline{v}) + \rho_v \log(v_{t-1}) + \varepsilon^v_t$$

The efficient contract wage has only distributive effects, and does not create any allocative distortion. This means that workers will supply labor up to the point where their marginal rate of substitution between labor and consumption is equal to the spot wage. We will refer to the difference between the spot and the contract wage as the wage insurance premium.

Utility Function

In the benchmark version of the model, we use the Greenwood, Hercowitz and Huffman (1988, henceforth GHH) utility specification for all three agents:

$$GHH : U_i\left(C_{i,t}, N_{i,t}\right) = \frac{(C_{i,t} - \psi_i N^{\phi}_{i,t})^{1-\sigma_i}}{1-\sigma_i} \tag{2}$$

We assume that σ_i, which we will refer to as the risk aversion with respect to consumption, differs between shareholders and the two other types of agents. The agents that participate freely in the financial market are assumed to be less risk averse. Our utility function imposes the exact inverse relation between the risk aversion and the intertemporal elasticity of substitution (IRS): shareholders are assumed to be characterized by a higher IRS. This assumption is in line with the empirical evidence on heterogeneity across different agents (e.g. Vissing-Jørgensen (2002)).

In the comparison of the heterogeneous agent model with a representative agent model, we also consider two alternative utility functions to illustrate the problem of standard models to generate significant risk premiums and to explain how the choice of the utility function influences our results. In these applications, the standard separable utility function and the King, Plosser and Rebelo preferences (1988, henceforth KPR) are specified as:[4]

[4] Our choice for these utility functions is motivated by the following considerations. First, they allow us to assess the effect of non-separability rigorously, as we will document below. Second, the KPR specification is consistent with a balanced growth path, which is a desirable feature for future extensions of the model,

$$SEP: U_t = \frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{N_t^{\phi}}{\phi} \tag{3}$$

$$KPR: U_t = \frac{(C_t(1-\psi N_t^{\phi}))^{1-\sigma}}{1-\sigma} \tag{4}$$

The final consumption good is defined as an aggregate basket over a continuum of differentiated goods. The same aggregator is assumed for all demand components:

$$C_t = \left[\int_0^1 c_t(i)^{1-\frac{1}{\varepsilon}} di\right]^{\frac{1}{1-1/\varepsilon}} \tag{5}$$

2.2 Firms

Firms maximize the present value of the dividend stream using the shareholders' stochastic discount factor.

$$\max E_t \left[\sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} D_{t+j}(i)\right]$$

with

$$D_t(i) = \left[\begin{array}{c} \left(\frac{P_t(i)}{P_t}\right) Y_t(i) - \frac{W_t^s}{P_t} N_{1,t}(i) - \frac{W_t^s}{P_t} N_{2,t}(i) - \frac{W_t^c}{P_t} N_{3,t}(i) \\ -PAC_t(i) - I_t(i) + \frac{\left(P_t^{B_f} B_{f,t}(i) - B_{f,t-1}(i)\right)}{P_t} \end{array}\right]$$

subject to:

$$\begin{aligned} Y_t(i) &= Z_t K_t(i)^{\theta} N_t(i)^{(1-\theta)} - \phi \\ Y_t(i) &= \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t \\ K_{t+1}(i) &= (1-\delta) K_t(i) + G\left(\frac{I_t(i)}{K_t(i)}\right) K_t(i) \\ N_t(i) &= N_{1,t}(i) + N_{2,t}(i) + N_{3,t}(i) \end{aligned}$$

Dividends are defined as total sales income minus the wage bill (spot wage plus insurance component), minus the price adjustment costs, minus investment expenditures, and plus the net receipts from debt financing. Note that the insurance the firms provide to the workers

and for taking the model to the data more rigorously. Third, one can interpret the GHH utility function as one limit case of Jaimovich and Rebelo (2007, henceforth JR) preferences, with KPR utility being on the other end of the spectrum. JR preferences are specified as $U_t = \frac{(C_t - \psi N_t^{\phi} X_t)^{1-\sigma}}{1-\sigma}$ where $X_t = C_t^{\gamma} X_{t-1}^{(1-\gamma)}$, with $0 < \gamma < 1$. The interpretation of GHH preferences as an extreme case of JR preferences, with $\gamma \to 0$, responds to the critique that GHH preferences are inconsistent with a balanced growth path. It implies that the wealth effect on labor supply is realized only very slowly over time, but in the long run this wealth effect exactly offsets the wage effect on labor supply. The cases we checked with JR utility, typically gave intermediate results between KPR ($\gamma = 0$) and GHH ($\gamma = 1$), so we restrict our analysis to these two limit cases.

does not affect the allocation decisions of the firm. Firms thus take a static labor demand decision for the remaining labor inputs which are hired at the spot labor market, and an intertemporal investment and price setting decision. In these optimization decisions, the monopolistically competitive firms face a demand schedule which is a negative function of their relative price and a Cobb-Douglas production function that contains a fixed cost which exactly offsets the mark-up consistent with the assumption of free entry. Firms are assumed to follow a simple debt policy: $B_{f,t}/P_t = \mu K_{t+1}$. Finally, the adjustment costs for capital are formulated as[5]

$$G = a_1 \left(\frac{I}{K}\right)^{(1-1/\xi)} + a_2$$

and the adjustment cost for prices are specified as a function of the change in prices, relative to the steady state inflation rate

$$PAC_t(i) = \frac{\chi^p}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - \overline{\pi}\right)^2$$

Firms are affected by standard productivity shocks Z_t, where:

$$\log(Z_t) = (1-\rho_z)\log(\overline{Z}) + \rho_z \log(Z_{t-1}) + \varepsilon_t^z.$$

The innovations to the productivity process and the distribution process are allowed to be correlated as discussed in the calibration below.

2.3 Equilibrium

Goods Market Clearing Condition:

$$Y_t = C_{1,t} + C_{2,t} + C_{3,t} + I_t + \frac{\chi^p}{2}(\pi_t - \overline{\pi})^2 + \overline{Y}\varepsilon_t^G \qquad (6)$$

When we add exogenous demand shocks ε_t^G to the model, these are assumed to follow an autoregressive process that appears as a simple additive term in the goods market clearing equation.

Bond Market Clearing Condition:

Given that there is no government debt in our model, the bond positions of bond and shareholders must add up to the debt issued by the firms:

$$B_{1,t} + B_{2,t} = B_{f,t}$$

All debt is in the form of one period discount bonds. Long term bonds are in zero net supply, and the stochastic discount factor of the shareholders is used to price these bonds.

Equity Market Clearing Condition:

In equilibrium the shareholders will own the entire net present value of the firm P_t^s. Therefore S_t, the share of the firm that the shareholders own, must be equal to 1 in equilibrium.

$$S_{1t} = S_t = 1 \qquad (7)$$

[5] We also investigated the impact of alternative investment adjustment costs but the result are not reported here.

Labor Market Clearing Condition:

$$N_{1,t} + N_{2,t} + N_{3,t} = N_t$$

Monetary Policy:

Monetary policy is assumed to follow a simple inflation targeting rule that contains some inertia in the policy reaction:

$$(1 + R_t) = \left[(1 + \overline{R})\overline{\pi} \left(\frac{\pi_t}{\overline{\pi}} \right)^{r_\pi} \right]^{(1-r_\rho)} (1 + R_{t-1})^{r_\rho} \exp(\varepsilon_t^m)$$

where we impose that the real rate $\overline{R}$ is in line with the equilibrium rate determined in the rest of the economy.[6] In one application, we will also allow for an autoregressive monetary policy shock.

2.4 Calibration

The model is calibrated using the parameter values in Table 1.

Table 1: Calibration of the Parameters

β	δ	θ	ξ	χ^P	ε	ϕ'_B	μ	σ_1	σ_2	σ_3	ϕ	r_z	r_ρ	$\overline{\pi}$
0.99	0.02	0.30	0.5	5	5	5x10^{-5}	0.3	4	10	10	1.75	3.0	0.90	0.0075

The discount factor (β) is set at 0.99. The depreciation rate (δ) is 2% per quarter. The capital adjustment costs are a function of the change in the capital stock with an elasticity (ξ) set at 0.50. The distribution parameter in the Cobb-Douglas production function (θ) is equal to (0.30). The values for these parameters are standard in the literature. The price adjustment costs (χ^P) are set at 5, which corresponds with a relatively small adjustment cost and therefore a high coefficient of the marginal cost in the linearized Phillips curve which is purely forward looking. This adjustment cost is chosen to generate a realistic volatility for inflation in the model. The elasticity of substitution in the demand aggregator is assumed to be 5, so that the mark-up is 25%. Under the assumption of zero excess profits, this implies a fixed cost in the production function which amounts to 25% of output as well. The financial intermediation costs that bondholders face are a linear function of their bond holdings with a small sensitivity of 0.00005. This guarantees that the effective interest rate that bondholders face will never deviate more than 12.5% from the market interest rate in the benchmark model. This parameter also generates a realistic wealth distribution in the benchmark model. Firm debt is assumed to be 30% of the capital stock.

More important for our application are the functional form and the parameters of the utility function. Under the benchmark specification with GHH preferences, we assume a risk aversion with respect to consumption (σ) of 10 for the workers and the bondholders, and 4 for the shareholders. The Frisch elasticity of labor supply with respect to wages is assumed

[6] In a stochastic setting, this rate will adjust to the risk premium present in the equilibrium real short rate.

to be equal to 1.5 for all agents (ϕ = 1.75), and ψ is chosen so that hours worked for all agents is the same and scaled at 1.

The monetary policy rule in the benchmark model is characterized by a strong inflation reaction (r_π = 3) and a relatively high degree of inertia (r_ρ =0.9). These parameters are chosen to obtain a reasonable match for the volatility of the short real rate and inflation in the benchmark model. The steady state inflation rate ($\overline{\pi}$) is equal to 0.75% on a quarterly basis corresponding with the average historical inflation rate.

The shares of the population are fixed so that workers make up 60%, bondholders 30% and shareholders 10% of the total population. The fraction of the workers should reflect the share of the population which is engaged in a labor contract with the firms. The remaining 40% can be thought of as the self-employed or entrepreneurs who do not benefit from a standard labor contract, but earn a spot wage that reflects their marginal productivity (corrected for the marginal cost).

We estimate the stochastic processes for productivity and distribution risk based on US data over the period 1947-2007. The parameters characterizing these shock processes are given in Table 2. The autocorrelation in the TFP process is estimated at 0.97 with a standard deviation for the Solow residual of 0.75%. The exogenous process (v_t) which determines the bargaining power of workers and firms in the contract wage, is also estimated based on the optimal contract condition (Equation (1)). In the estimation of this shock, we evaluate the marginal utilities of both parties in the contract by approximating the consumption of the workers by the total wage bill and the consumption of the shareholders by GDP minus the wage bill and investment. Both agents are assumed to work the same numbers of hours. The resulting process for the distribution shock has an autocorrelation of 0.92 with a st.dev. of 0.20. The innovation in TFP and the distribution process have a negative correlation of -0.5. We refer to this correlation as the distributive effect of the productivity shock, which is not captured by the endogenous dynamics of the model (see Rios-Rull and Santaeulalia-Llopis (2008) for a discussion of the distributive consequences of TFP shocks).

Table 2: Calibration of the Stochastic Structure

σ_z	ρ_z	σ_ν	ρ_ν	$\rho_{z,\nu}$	σ_g	ρ_g	σ_m	ρ_m
0.0075	0.97	0.20	0.92	-0.50	0.006	0.95	0.15	0.5

When we use the historical realizations for these two exogenous processes in the simulation of the model (see section 5), we are able to explain 0.85% of the growth rate in GDP and 0.93% of the change in the wage share. So, with this stochastic structure, we capture a large fraction of the observed macroeconomic volatility. The volatility of both innovations displays a remarkable decline over time and in particular around 1984. For the pre- and post 1984 subsamples, we observe a 50% decline in the st.dev. of the Solow residual (from 0.88 to 0.44) and 20% decline in the distribution shock (from 0.22 to 0.18).

The benchmark model that is presented below, contains only these two sources of shocks. In one experiment, we also consider two additional demand shocks: a public spending shock that affects the aggregate demand equation (with autocorrelation 0.95 and standard error of 0.6% expressed as a percentage of GDP) and an interest rate shock (with autocorrelation 0.5 and a st.error of 0.15 which corresponds to 60 basis points in the annual rate).

3 Dynamic Properties of the Model

First, we discuss the overall statistics of the model both for the real, the nominal and the financial side of the economy and we compare these results to the data statistics. We focus on the model with two types of shocks or risks in the economy: aggregate productivity shocks and distribution shocks. In order to illustrate the contribution of the heterogeneous agent assumption, we compare the results with the outcomes of a representative agent model, in which only type 1 agents are present. In this way, we show how the heterogeneous agent structure helps to overcome the shortcomings of the representative agent model. Second, we illustrate the dynamics of the model by discussing the transmission mechanism of the shocks through the economy in more detail. Third, we document the important role of the utility function that is retained in our model. Fourth, we turn to the analysis of the risk sharing arrangements in the model. We discuss successively the risk sharing between bondholders and shareholders (similar to the set up retained in Guvenen (2008)) and the risk sharing between workers and shareholders (similar to the setup retained in Danthine and Donaldson (2002)). We analyze the relative importance of both types of risk sharing in the model by looking at the impact on the shareholder budget constraint over the cycle. Finally, we discuss the specific role of distribution risk and add demand shocks to make the stochastic structure more complete.

3.1 Overall Statistics of the Model

Table 3 and 4 summarize the overall statistics of the model. We compare these results with a representative agent version in which only shareholders (type 1 agents) are present in the economy but all other parameters remain the same (the distribution risk is not active in that economy and risk aversion is kept at 4). We should not expect the model to fit the data moments exactly: in reality more shocks are present and additional nominal and real frictions will affect the transmission of the shocks. So the objective is not to fit the data in all dimensions, but rather to illustrate that the model with heterogeneous agents improves significantly on the representative agent case in the desired direction compared to the data. The results in Table 3 and 4 are based on the unconditional moments of a second order approximation of the model.[7]

The benchmark model with three types of heterogeneous agents fits the data well both in terms of financial and real variables. The model is able to generate an important risk premium both for equity ($EP = 4.77$) and for the holding period return on a 10-year bond (BP =1.99). For the bond, this excess holding period return corresponds with a yield spread ($y - R^n$) of 1.56. The model generates a risk free real rate (R^f) of 1.20% with a standard deviation of 3.50. The volatility of the return to equity is 20.18, which yields a Sharpe ratio (SR) of 0.24. These statistics are close to the observed post-war average. We slightly underestimate the equity risk premium and the SR. However, as mentioned before, this analysis considers only two sources of risk (for instance, the risk premium on equity increases to 5.61 if we also consider demand shocks). Moreover, as we discuss later

[7] We cross-checked the consistency of these results with the statistics based on a second order approximation using the moments of a first order approximate solution of the model and assuming lognormally distributed variables as in Uhlig (2007).

Table 3: Financial Statistics

	SR	EP	BP	$y^f - R^n$	R^f	σ_{R^f}	σ_{R^S}	σ_π
Benchmark Model	0.24	4.77	1.99	1.56	1.20	3.50	20.18	3.00
Data								
1947-2007	0.39	6.11	-	1.34	1.19	2.84	15.50	2.62
1926-2007	0.26	5.85	-	-	0.60	5.27	22.35	5.28
Representative agent								
SEP	0.03	0.09	0.04	-0.03	4.00	0.62	3.22	0.52
KPR	0.07	0.53	0.24	0.11	3.80	1.32	7.68	1.20
GHH	0.14	1.90	0.74	0.52	3.09	2.10	13.90	1.60
Heterogeneous agents								
productivity only	0.21	3.44	1.38	1.10	1.93	2.62	16.81	1.88
distribution only	0.06	0.31	0.14	0.04	3.89	1.48	5.59	1.60
demand extra	0.26	5.61	2.52	1.96	0.58	4.20	21.59	4.40
T1+T2	0.19	3.11	1.29	1.01	2.18	2.62	17.80	1.88
T1+T3	0.26	5.74	2.42	1.88	0.68	4.06	36.2	3.68

Note: For the period 1926:1-1998:4 we use the dataset of Campbell (2003). For the period 1999:1-2007:4 we use the United States MSCI from Datastream to calculate the equity statistics. To calculate the bond statistics we use the FED Funds rate and the ten year bond from the BIS. The standard deviation of the annualized interest and inflation rate is computed as 400 times the quarterly model concept. The standard deviation of the equity return is computed as that of a compounded annual return. The standard deviation of the annualized equity return is 200 times the quarterly model concept.

on, there has been a great moderation in the volatilities of the underlying shocks, and it is not unlikely that one might underestimate the average risk premium by working with the average standard deviation of the shocks.

In terms of the real statistics, the model generates an aggregate output volatility that is slightly higher than observed in the data (1.86% against 1.70%). On the other hand, hours worked is less volatile compared to the data (0.82 versus 1.34), but the model reproduces the high correlation between total hours and output. The consumption volatility is too high (1.59 versus 1.17), while investment is not sufficiently volatile (3.31 versus 4.94). Both demand components also display an excessive correlation with output. These results indicate a significant shortcoming of the model. Decreasing the capital adjustment costs does not substantially improve these statistics because it would strongly reduce the risk premiums in the model. Another explanation is the very persistent nature of the two exogenous shocks that we consider. Lowering the persistence of these shocks improves the relative volatility and lowers the correlation with output, but this goes along with a cost in terms of the risk premium. Results might improve if we would allow for more, less persistent sources of risk in the model. For instance, the introduction of investment-specific technology shocks, which is suggested in some recent papers as an alternative important source of volatility, could increase the volatility of investment and reduce the variation and the procyclical nature of consumption. Real wages in the model are somewhat smoother than in the data (0.63 versus 0.78). While the correlation with output is moderate, it still overpredicts the wage-output correlation in the data (0.55 versus 0.09). The wage share is relatively volatile, countercyclical

Table 4: Macroeconomic Statistics

	σ_Y	σ_I	$\rho_{I,Y}$	σ_C	$\rho_{C,Y}$	σ_N	$\rho_{N,Y}$	σ_W	$\rho_{W,Y}$	$\sigma_{WN/Y}$	$\rho_{WN/Y,Y}$
Benchmark Model	1.86	3.31	0.93	1.59	0.98	0.82	0.94	0.63	0.55	2.37	-0.28
Data											
1947-2007	1.70	4.94	0.76	1.17	0.79	1.34	0.87	0.78	0.09	2.34	-0.19
Representative agent											
SEP	0.48	0.60	1.00	0.46	1.00	0.84	-0.99	1.15	1.00	0.31	-0.50
KPR	1.13	1.37	1.00	1.07	1.00	0.13	-0.80	0.91	0.99	0.66	-0.43
GHH	1.92	2.25	1.00	1.84	1.00	0.82	0.99	0.61	0.99	1.39	-0.32
Heterogeneous agents											
Productivity only	1.90	2.66	1.00	1.72	1.00	0.81	0.98	0.58	1.00	1.92	-0.25
Distribution only	0.27	1.07	-0.50	0.52	0.89	0.28	0.99	0.63	0.71	0.86	0.40
Demand extra	1.93	3.77	0.78	1.71	0.93	0.97	0.90	0.66	0.54	2.37	-0.27
T1+T2	1.90	2.66	1.00	1.72	1.00	0.81	0.98	0.60	0.98	1.66	-0.27
T1+T3	1.86	3.70	0.86	1.59	0.93	0.86	0.91	0.74	0.41	2.68	-0.27

Note: The data come from the FRED database at the St-louis Fed and the BLS. All real variables have been detrended with the Hodrick-Prescott filter except for the wage share. The output correlation of the wage share is the correlation between HP-filtered output and the unfiltered wage share.

and close to the behavior in the data. We consider this last result as a very important and positive feature of the benchmark model, since the income distribution plays a crucial role for the volatility of the capital returns and therefore also for the pricing of the underlying assets. Note also that the model does a reasonable job in fitting the volatility of inflation in the data.

The benchmark model endogenously generates the following wealth distribution: 85.5% of financial assets are held by the top 10% of the population that consists of shareholders, 14.5% of financial wealth is held by the next 20% of the population that is represented by the bondholders, and 0% of the wealth is held by the workers. The shareholdings are -by definition- concentrated in the first group. This distribution implies a concentration of wealth and stock market participation that is very similar to the one typically measured in the US wealth distribution (see e.g. Wolff, 2006).[8]

The representative agent (RA) model is not able to fit the data as well both on the financial and the real side. However, our results also illustrate that the results of the representative agent case depend strongly on the specification of the utility function. The failure of the RA model to explain the observed risk premiums in a model with endogenous capital and labor decisions, as documented widely in the literature (see, e.g., Jermann (1998), Lettau and Uhlig (2000), Boldrin et al. (2001), Uhlig (2007), ...), is most obvious if we look at the more standard setup of a separable utility function (separable in consumption and labor). With this utility function and a risk aversion of 4, the labor supply is used intensively to smooth the consumption stream. The endogenous labor decision implies that labor supply is strongly countercyclical so that the overall volatility in the economy is reduced. With a

[8] Over the last two decades, there has been an increasing share of the population that participates in the stock market. An interesting extension of the paper would be to analyze the implications of this increase in stock market participation for the risk premium.

smooth consumption stream and a fairly low volatility in the return to capital, there is no longer a reason to require high risk premiums.

A non-separable utility function goes a long way to overcome this problem even in a representative agent context. This is especially the case with GHH preferences, where the labor supply condition is no longer affected by an income/wealth effect. In that model, too, the volatility of the return to capital increases considerably. This is mainly due to the assumption of monopolistic competition and sticky prices, which makes the mark-ups highly time-varying and the implied profits highly procyclical, at least in a model that is driven by supply shocks. In addition, the presence of fixed costs, which are calibrated to offset the mark-up in steady state, increases the operational leverage of the firms and increases the volatility of the return to capital. This result is also illustrated by the countercyclical behavior of the wage share implied by this model. However, compared to the benchmark model, the RA-GHH model yields a higher stochastic real rate and less volatility in the return on stocks. If we allow for a higher risk aversion for the representative agent, the results for the financial statistics improve, but problems remain in terms of explaining the a-cyclical nature of wages and the countercyclicality of the wage share.

3.2 Impulse Response Function of the Shocks

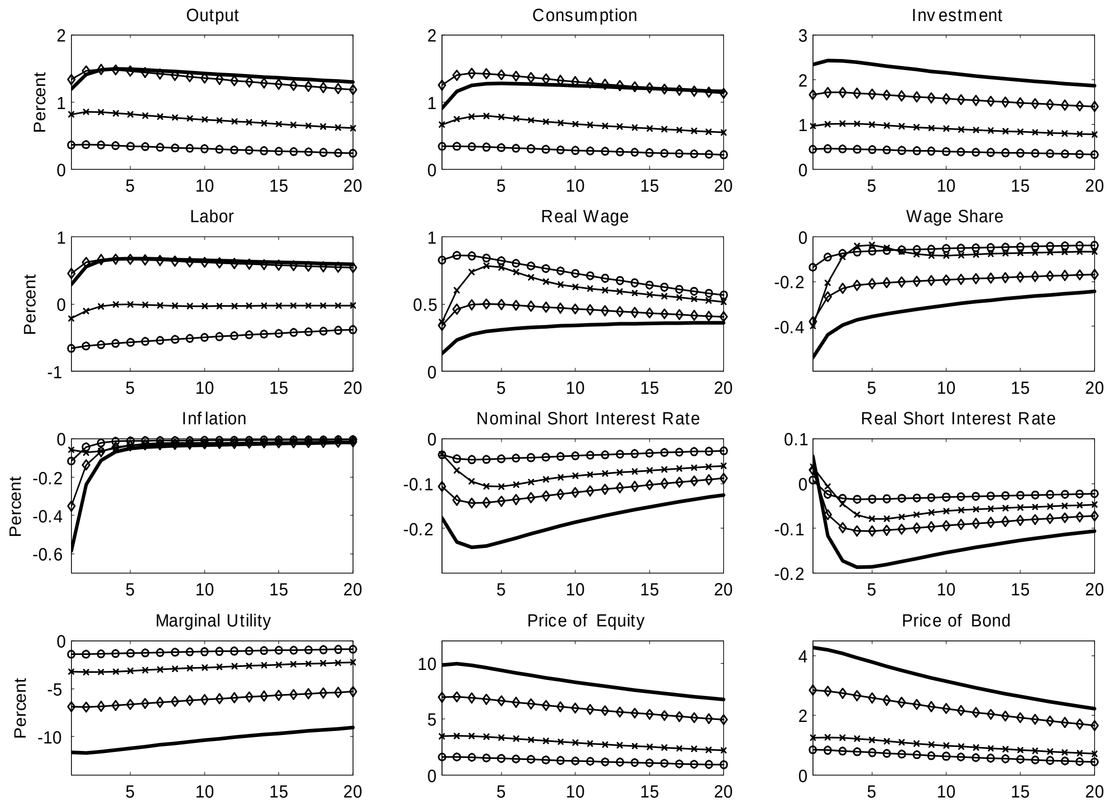
In order to demonstrate the transmission mechanisms of the model, Figure 1 reports the impulse response functions (IRF) for the benchmark three-agent model and the RA model with three types of utility functions. The graph concentrates on aggregate variables, except for the marginal utility which refers to the shareholders in case of the heterogeneous agent model.

First of all, the benchmark model produces a fairly strong propagation of the productivity shock (which has a standard error of 0.75) on total output. The relative response of investment is significantly stronger than the impact on consumption. Consumption reacts modestly in the short run and the reaction displays a hump-shaped profile. This behavior is related to the very modest reaction in the real wage in the short run. As a consequence the wage share drops strongly on impact.[9] The assumed correlation between the productivity and the distribution shock enhances this drop in the wage share, as the bargaining power of workers declines following a positive productivity shock. The large drop in the wage share explains why inflation declines relatively strongly. The price adjustment costs imply that the mark-up goes up significantly in the short run. All this implies a very volatile and procyclical behavior of profits. With standard price adjustment costs, inflation is purely forward looking and jumps on impact. Monetary policy accommodates the productivity shock by lowering the short rate. The policy rule generates a gradual but persistent reaction of the short rate.

In terms of asset prices, we observe a strong increase in equity and bond prices. This strong reaction is driven by the large variations in the marginal utility of the investors on the one hand, and the high volatility in profits and the implied dividend stream, on the other hand.

[9] This type of behavior of the wage share has also been illustrated in the data by Rios-Rull and Santaeulalia-Llopis (2008), although we do not observe the sign reversal in the wage share that was pointed out in that paper.

Figure 1: Impulse Response to a (one std. err.) Productivity Shock



Note: The bold line is the response of the benchmark model. The line with diamonds is the representative agent model with GHH preferences. The crossed line is the representative agent model with KPR preferences. The line with circles is the representative agent model with a separable utility function.

The results for the representative agent models depend strongly on the specification of the utility function. With a separable utility function, we obtain the standard results of the literature: labor supply is used to smooth consumption and reduces the risk premium. This happens at the cost of reducing the overall volatility in the economy. The productivity shocks are completely offset by adjustments in the labor supply through the wealth effects: with a high productivity, labor supply is decreased through a strong wealth effect. By working less, agents try to smooth the effect on consumption. The non-separable utility functions are affected a lot less by this wealth effect, and under GHH preferences, the reaction of aggregate hours worked and aggregate consumption is very similar to the reaction in the benchmark model. Note, however, that the marginal utility of the shareholders in the benchmark model is more volatile than in the RA model.

The labor share is also countercyclical in the representative agent model because of the time-varying mark-up and the presence of fixed costs which both help to generate countercyclical wage shares. In booms the mark-up will be higher and the excess profits lower the wage share in total output, whereas the fixed costs also reduce the wage share as output goes up. However, the representative agent models with spot labor markets are not able to generate the same countercyclical wage shares as we observe in the benchmark heterogeneous model with labor contracts. Higher volatility in marginal utility and capital income explain the stronger reaction of asset prices in the heterogeneous agent model.

3.3 Role of the Utility Function

The role of the utility function deserves some detailed discussion because it is crucial for the joint dynamics of the real and the financial decisions in the model. Its importance derives from the fact that marginal utility enters both in the stochastic discount factor for asset pricing and in the labor supply decision. To focus ideas, let us start from the case of a representative agent with a CRRA utility function and separability between consumption and labor.[10] The equity premium (EP_t^A) can be written as:

$$\begin{aligned} EP_t^A &= -\rho_{rs,\Delta\lambda}\sigma_{rs}\sigma_{\Delta\lambda} \\ &= \sigma\rho_{rs,\Delta c}\sigma_{rs}\sigma_{\Delta c} \end{aligned}$$

where σ and rs stand for the risk aversion and the return on stocks, $\Delta\lambda = \lambda_{t+1} - \lambda_t$, $\lambda_t = \frac{\partial U(C_t)}{\partial C_t} = U_t^C$, and σ_x and ρ_x denote the (conditional) standard deviation and correlation, respectively. The corresponding Sharpe ratio (SR_t^A) is:

$$\begin{aligned} SR_t^A &= -\rho_{rs,\Delta\lambda}\sigma_{\Delta\lambda} \\ &= \sigma\rho_{rs,\Delta c}\sigma_{\Delta c} \end{aligned}$$

The price of risk depends on the volatility of consumption growth, on the correlation between consumption and the return on capital, and on the degree of risk aversion.

The agent also has the opportunity to adjust her labor supply to smooth fluctuations in marginal utility. The linearized first order condition relates the supply of labor to real wages and consumption:

$$(\phi - 1)\hat{n}_t = \hat{w}_t - \sigma\hat{c}_t$$

Following a positive productivity shock, the increase in consumption, ceteris paribus, lowers the marginal utility of the real wage, which reduces labor supply. The reduction in working hours, in turn, mitigates the initial expansionary effects of the productivity shock. As a result, the rise in both marginal productivity (and thus the stock return rs) and consumption (and thus volatility of marginal utility $\sigma_{\Delta\lambda}$) will be smaller. Increasing the degree of risk aversion, which can contribute to the risk premium in a model with exogenous labor, will induce a more intensive utilization of the labor supply to smooth consumption.[11] The analytical expressions above reveal that there is no hope of improving asset pricing implications by allowing for endogenous labor in a separable utility framework. Moreover, in terms

[10] There are an enormous amount of variations on this basic representative agent framework, and our goal is not to provide a detailed overview (see e.g. Kocherlakota 1996). Rather, we here focus on the role of introducing the labor decision.

[11] Even in a model with exogenous labor, a higher degree of risk aversion is not an option for solving the equity premium puzzle as it generates the so-called risk free rate puzzle.

of macro implications, these results do not yield the observed positive correlation between labor inputs and output.

The introduction of non-separability between labor and consumption in the utility function strongly affects the model's macro and financial responses. The corresponding Sharpe ratio becomes (see e.g. Uhlig (2007) and Appendix B):

$$SR_t = \rho_{rs,\Delta c}\eta_{cc}\sigma_{\Delta c} - \rho_{rs,\Delta n}\eta_{cn,n}\sigma_{\Delta n} \tag{8}$$

where $\eta_{cc} = -\frac{U^{CC}*C}{U^C}$ is the relative risk aversion with respect to consumption. The cross-derivative term $\eta_{cn,n}$ measures the degree of non-separability in the utility function: $\eta_{cn,n} = \frac{U^{CN}*N}{U^C} > 0$ (complements) < 0 (substitutes).

The first term in (8) implies that the price of risk increases with the correlation between consumption and stock returns, the risk aversion, and the volatility of consumption growth. This is the traditional mechanism also at work in the representative agent model with exogenous labor and separable utility. Non-separability leads to an additional effect depending on the volatility of labor supply, the cross-derivative of marginal utility with respect to hours worked, and the correlation between hours worked and the return on equity. The sign of this term depends on the cross-derivative and the correlation.

On the macro side, the linearized FOC for labor supply can be written as:

$$(\eta_{cc} + \eta_{nc,c})\widehat{c}_t - (\eta_{nn} + \eta_{cn,n})\hat{n}_t = \widehat{w}_t \tag{9}$$

Equation (9) shows that the strength of the income and substitution effects on labor supply are also controlled by the cross-derivatives of the utility function to its respective arguments. This means that introducing non-separabilities in utility does not just buy some free parameters to scale up asset pricing moments, such as the Sharpe ratio. By contrast, we impose discipline on the exercise by examining a selection of both financial and macroeconomic moments. To aid intuition, Table 5 below summarizes the implied elasticities for each of the three preference specifications.

Table 5: Implied Elasticities for Utility Functions

	SEP	KPR	GHH
η_{cc}	σ	σ	$\sigma\frac{\phi}{\phi-1}$
$\eta_{cn,n}$	0	$(\sigma - 1)$	$\sigma\frac{\phi}{\phi-1}$
$\eta_{cc}+\eta_{nc,c}$	σ	1	0
$\eta_{nn}+\eta_{cn,n}$	$-(\phi - 1)$	$-\phi$	$-(\phi - 1)$

Note: This table assumes $C = \frac{W}{P}N$. SEP stands for separable utility function. KPR stands for King, Plosser and Rebelo (1988) utility function. GHH stands for Greenwood, Hercowitz and Huffman (1988) utility function.

The analytical expressions in the first two rows of the table help to evaluate the Sharpe ratio, while the lower two rows are crucial to determine the labor supply reaction. The cross-derivative terms affect both the SR in Equation (8) and the labor supply decision in Equation (9). Under KPR and GHH utility, consumption and labor are complements (see Table 5: $\eta_{cn,n} = (\sigma - 1) > 0$ under KPR and $\eta_{cn,n} = \sigma\frac{\phi}{\phi-1} > 0$ under GHH). In other words, agents will prefer positive comovement between consumption and labor, and the negative consumption/wealth effect on labor supply $(\eta_{cc} + \eta_{nc,c})$ is reduced to 1 under KPR preferences, while the effect drops to zero under GHH preferences. However, if hours worked become procyclical, so that the correlation $\rho_{rs,\Delta n}$ is positive, and the cross-derivative is positive, then the second term in the SR in Equation (8) has a negative effect on the price of risk. Hence, during a recession -when marginal utility is high due to low consumption- the reduction in hours worked will mitigate or even offset this increase in marginal utility.

In sum, non-separability can help by alleviating the strong income effects at work in the first order condition for labor supply under separable utility. This will reduce the strong countercyclical nature of employment, which reinforces the propagation of the productivity shock. However, because labor and consumption are complements, their comovement will also tend to stabilize marginal utility. This, in turn, limits the model's ability to generate significant risk premiums. Given that GHH preferences overcome -at least to some extent- the important problems of the separable utility function, we retain these preferences for all our versions of the heterogeneous agents model that we consider in this paper (see also Guvenen (2008)).

3.4 Role of the Risk Sharing Arrangements

In a model with heterogeneous agents and a complete market of contingent claims, one can easily show that the optimal risk sharing between agents results in a constant relative marginal utility:

$$\frac{U'_{c1,t}}{U'_{c2,t}} = \frac{U'_{c1,t+1}}{U'_{c2,t+1}} = \mu \tag{10}$$

where μ depends on the relative wealth of the two agents. In our setup, there are no contingent claims and we also assume that type 2 and type 3 agents have no access to the stock market. We consider two alternative risk sharing arrangements in this paper: the bond market and the labor contract.

The Bond Market

Type 2 agents' only recourse to smooth their marginal utility is the bond market. By selling and buying bonds to/from the shareholders, these two types of agents will try to achieve the equalization of their relative marginal utilities over time.[12] However, the bond market provides only an imperfect risk sharing device for the bondholders as they are confronted with an effective interest rate that will deviate from the market rate: the financial intermediation margin depends on their net wealth position and imposes the intertemporal consumption constraint on their consumption decision.

[12] Firms debt financing is assumed to be a constant fraction of the capital stock and this supply of bonds will determine the portfolio of the active shareholders. In a first order approximation of the model, bonds and equity are perfect substitutes and we do not need to consider the impact of this decision.

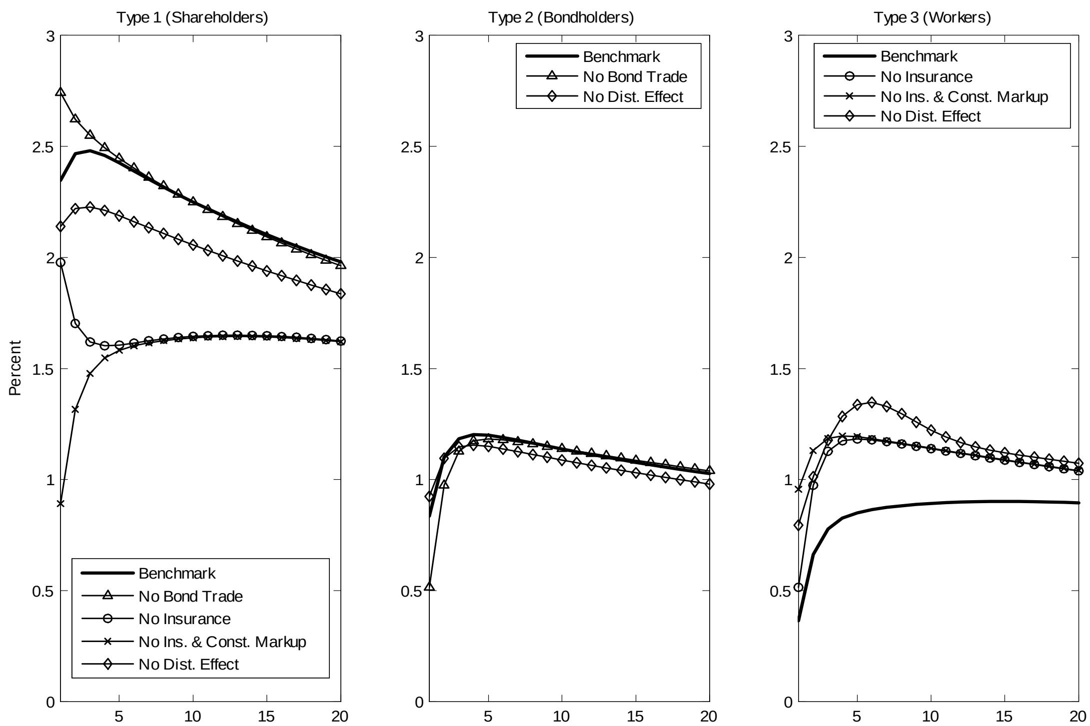
Figure 2 summarizes the consumption/savings decision of the bondholders in case of a positive productivity shock. This figure complements the evidence on the aggregate dynamics presented in Figure 1 with the evidence on the consumption and income responses for the different agents in the economy. As in Figure 1, the IR functions are based on a first order approximation of the model around a steady state in which the bondholders have zero net bond holdings. Higher current and expected labor income increases their wealth and drives up their consumption. In the model, the short run increase in real wage income is moderate as firms will lower their prices only gradually and the expansion of production is limited on impact as well. On the other hand, bondholders want to increase consumption more quickly in order to smooth the expected marginal utility. As a result, they will start to borrow in the bond market to finance consumption. Shareholders, who are the counterpart of this bond trade, experience a much stronger increase in their income as they benefit from the increased mark-up and other mechanisms in the model that generate a strongly procyclical profit share. Even after financing investment plans, firms have sufficient resources which they transfer to the shareholders in the form of higher dividend payments. Therefore, shareholders will be eager to provide the necessary funds to the bondholders, and the real interest rate, which clears the bond market, will decrease. The increase in the intermediation margin, which remains very small in magnitude, will prevent complete risk sharing. Note that this result is obtained despite the difference in the IRS of the two parties, which reinforces the smoothing motive of the bondholders relative to the shareholders.

From this discussion, it follows that the risk sharing through the bond market between type 1 and type 2 agents, allows the shareholders to smooth their consumption, which will tend to reduce the risk premium relative to an economy in which these agents are not allowed to trade in the bond market. This conclusion is opposite to the results obtained by Guvenen (2008). In that paper, the bond trade is mainly driven by the precautionary savings motive which is based on the countercyclical effective risk aversion of bondholders and which provides these agents with an additional motive to smooth their consumption volatility over the cycle. So in the model of Guvenen, bondholders will save after a positive productivity shock and this increases the consumption volatility of the shareholders. Our first order effect on the bond trade clearly dominates the precautionary savings motive which is central in the model of Guvenen.[13] The precautionary savings motive determines the positive bond position of the bondholders in the stochastic steady state which is important to evaluate the implications of the model for the wealth distribution in the economy. In the paper of Guvenen, the first order redistributive effects of a productivity shock are absent as the wage share is constant over the business cycle.

This conclusion about the role of bond trade for risk sharing does not imply that a model populated solely with T1 and T2 agents is unable to generate a significant risk premium. The outcomes of such a model are documented in Table 3 and 4. Compared to the RA model (both with GHH preferences), we see that the real macro variables remain very much the same, except for aggregate consumption, which becomes slightly more cyclical. Behind this aggregate consumption behavior, there is a more volatile consumption of shareholders and a smoother behavior for the more risk averse (or lower IRS) bondholders. Therefore, this model

[13] We will discuss the relative contribution of this mechanism later on, when we discuss the time-varying nature of the risk premium in a third order approximation of the model.

Figure 2: Consumption Dynamics for the Three Agents (one std. err. Productivity Shock)



Note: The impact of the insurance scheme, the bond trade and the mark-up are calculated, ceteris paribus, by adding (or subtracting depending on the agents) to consumption the size of the premium, savings and mark-up respectively, while keeping all other variables fixed as observed in the benchmark model.

still increases the risk premium because the shareholders bear most of the aggregate risk as they earn all the profits, and their income share is therefore highly procyclical. The bond trade provides only an imperfect risk sharing device and prevents an optimal distribution of risk as assumed in the RA model.

The Labor Contract

Type 3 agents have no access to the financial market, but they will design an optimal labor contract which allows for risk sharing.[14] If we assume that the worker-firm relation is a permanent relation, these contracts exactly reproduce the optimality condition for risk sharing (10) expressed above. The shareholders guarantee a consumption level to the workers that implies a constant relative marginal utility. The ratio between the two marginal utilities (μ) reflects the bargaining power of the two parties in the contract arrangement. In our steady state, we assume that the optimal contract will imply a wage and consumption level

[14] See Gomme and Greenwood (1995) and Boldrin and Horvath (1995) for a detailed discussion of a labor contracts between workers and entrepreneurs in an RBC model.

for the workers that is equivalent to the steady state outcome under the spot labor market.[15] This setup is also considered in Danthine and Donaldson (2002), who assume, however, that labor supply is exogenous.[16]

Relative to the outcome under a spot labor market, the contract wage contains an insurance premium through which workers exchange the risk with the marginal shareholder of the firms. Given the high risk aversion (or low IRS) of the workers, the contract wage guarantees them a smooth consumption stream. In exchange for the insurance provided by the firm, workers offer the required labor services to the firm. This optimal contract wage plays only a distributive role, and does not affect the allocation decision of the firms or the workers. The contract implies an efficient utilization of the labor supply which equalizes the marginal rate of substitution between labor and consumption of the worker to the spot wage, and the marginal product of labor (if there would be no mark-ups in the economy). The result is a countercyclical labor share and more volatile and highly procyclical profits.

Figure 2 shows the impact of the insurance premium on the workers' and shareholders' consumption for a positive productivity shock. The contract wage will increase less than the spot wage, meaning that the insurance premium received by the workers is negative which helps to stabilize their consumption. The contribution of the premium is relatively small on impact but increases in the periods following the shock. To further illustrate all the mechanisms at work, Figure 2 also contains the impact of the mark-up variation on the consumption streams. With sticky prices, the mark-up increases considerably on impact and drives a large wedge between the spot wage and the marginal productivity of labor. Therefore, the spot wage and the corresponding consumption of the workers would be substantially more volatile in a version with flexible prices, fixed mark-ups and no insurance mechanism. The opposite holds of course for the shareholders, who receive the short run excess profits, and so their consumption volatility would be much less outspoken under flexible prices. These redistributive effects of price stickiness dominate the contribution of the contract wage in the short run but the impact dies out relatively quickly as firms adjust their prices and mark-ups are restored at the equilibrium level.

Figure 2 (line with diamonds) also illustrates the contribution of the imposed correlation between productivity and distribution risk on the consumption dynamics. Recall that this correlation reinforces the decline in the wage share following the positive productivity shock, in line with the observed evidence in the data. It is obvious that shareholder consumption would have been less volatile in the absence of this additional distributive effect.[17]

[15] We could also assume that workers have some extra bargaining power, which would result in a consumption level above the spot market outcome. This would imply a lower level of dividends and consumption for the shareholders and at the same time a higher volatility in their dividends/consumption stream (implying a higher risk premium).

[16] An alternative assumption to the permanent relations is that the worker-firm relation takes the form of one-period contract: this contract will guarantee an expected relative marginal utility level to the workers. If workers have no bargaining outcome, then this expected relative utility will be equal to the expected outcome in the spot market. Again, if workers have some bargaining power the wage will guarantee some extra relative to the market outcome. This contract setup is similar to the one considered in Boldrin and Horvath (1995).

[17] One additional issue in this context is the choice of the bargaining weight (μ) in the contract. If we assume that workers have some bargaining power, and are able to negotiate a wage that is above the outcome in the spot market, then the average profit rate in the economy is lowered so that the volatility of profits,

In sum, the contract wage, the costly price adjustment and the redistributive effects of a productivity shock all contribute to smoothing workers' labor income and consumption. In doing so, they also exacerbate the cyclical reaction of shareholders' consumption. Therefore, these three mechanisms help to increase the risk premium in the model.

In Table 3 and 4, we also show the outcomes for a model that contains only type 1 and type 3 agents, which make up respectively 10% and 90% of the population. From the above discussion, it becomes clear why such a model generates a larger risk premium than in the model with three types of agents. By excluding the bondholders and closing the bond market trade, shareholders are no longer able to share their excess aggregate and distributive risk with the bondholders. Therefore, this risk will be more concentrated at the relatively small group of shareholders, who will require a high premium in exchange. The high premium reflects both the high volatility in the return to capital and the high volatility in the marginal utility of the marginal investor, and both determinants of the premium will be highly correlated: the high premium goes together with a high price of risk as reflected in the SR.

Risk Sharing and the Volatility of Shareholders' Consumption

The relative contribution of the bond market and the labor contract in the total amount of risk sharing can be illustrated by decomposing the variance of the shareholders consumption (see Guvenen for a similar decomposition) into three determinants: income from labor and capital (interest and dividend income excluding the wage insurance premium), the dividend income derived from the wage contract arrangement and the bond trade with the bondholders.

$$C_1 = Other\ Capital\ and\ Labor\ Revenue(OR) + Insurance(Ins) + Bondsavings(Bs)$$

Table 6 summarizes the contribution of the risk sharing arrangements on the volatility of shareholder consumption growth. The variation in income from capital and labor is clearly the dominant source of the consumption volatility of the shareholders, and its variance exceeds even the variance of consumption changes (the relative variance is 109.3%). The premium related to the labor contract reinforces the variability by 29.4% and its contribution is slightly positively correlated to the other income resources. The variability in the bond trade is small in absolute magnitude as the variance of these flows makes up only 6.4% of the consumption variance, though this contribution is strongly negatively correlated with the two other sources of income (apparent from covariance terms). In total, the bond trade allows shareholders to smooth their consumption variance by 41.8%.

Overall this table might give the impression that the heterogeneous agents framework, and in particular the additional risk sharing arrangements that we consider, tends to moderate the overall risk for the shareholders. This conclusion is misleading, however: the results only indicate that the shareholders' consumption volatility is lower in the benchmark model relative to a situation in which there is no bond trade at all and bondholders are completely liquidity constrained.

It is clear that the relative risk aversion of the agents plays a crucial role for both risk sharing devices. If the risk aversion of all agents is equal, then the risk sharing through the

that results from the risk sharing, increases.

Table 6: Shareholder Consumption Volatility

	Benchmark model
$var(\Delta C_1)$	0.141
Contribution from:	
$var(\Delta OR)$	109.3%
$var(\Delta Ins)$	29.4%
$var(\Delta Bs)$	6.3%
$2 * covar(\Delta OR, \Delta Ins)$	3.2%
$2 * covar(\Delta OR, \Delta Bs)$	-33.8%
$2 * covar(\Delta Ins, \Delta Bs)$	-14.3%

bond/fund market or through the labor market will reproduce an outcome similar to that of the representative agent economy. Small differences may arise from wealth distribution effects, in particular on labor supplies, and from financial costs. With a higher risk aversion (lower IRS) for type 2 and type 3 agents relative to the type 1 agents, their desire to smooth consumption increases and as a consequence available funds for the shareholders' consumption will tend to become more volatile and procyclical. The differences in risk aversion increase both the Sharpe ratio and the required risk premium on stocks. Our assumption on the relative risk aversion is confirmed in the data. For instance, Vissing-Jørgensen (2002) provides evidence of lower elasticities of intertemporal substitution (or higher risk aversion) for non-shareholders. This risk aversion cannot be estimated directly from the first order condition for stocks as these agents do not participate in that market. Wachter and Yogo (2007) propose a justification for the negative relation between risk aversion and wealth based on a non-homothetic function of two types of consumption goods (basic and luxury goods) and show how such a model also explains the observed positive relation between the share of risky assets in the portfolio and wealth.

3.5 Distribution Risk and Demand Shocks

The benchmark model contains both productivity and distribution risk and these two types of shocks are correlated as was discussed in the calibration section. In the previous discussion, we focused our presentation on the role of the productivity shock. Distribution risk further increases the risk premium and is also helpful to better match the variability in the wage share and the moderate cyclicality of wages. The heterogeneous agent model offers the natural context for introducing this type of uncertainty. Tables 3 and 4 also document the specific contribution of each of the two shocks when considered as two independent shocks.

The stochastic process (σ_v= 0.20 and ρ_v= 0.92) that was estimated for the distribution shock generates a moderate volatility in the real wage and the wage share (st.dev. of 0.63 and 0.86 in the HP-filtered series). Recall that distribution shocks only change the distribution of output between workers and firms, but do not cause any misallocation of labor: workers still try to equalize the marginal rate of substitution between consumption and labor to the spot wage. Output and employment are therefore only influenced through the induced variation in aggregate demand and these effects are very moderate. Shareholders are able to smooth consumption by adjusting investment expenditures and their bond trade with bondholders, so that the effect on SR and EP remains very small: distribution risk alone is not able to produce a significant SR or any risk premium. The high correlation between profits and investment prospects implies that shareholders can smooth consumption without large fluctuations in the short rate.

On the other hand, if one considers productivity shocks as the only source of risk, the model does generate a significant risk premium of 3.44. However, this version also implies a perfect correlation between wages and output. When combining the two types of shocks with a negative correlation -as we do in the benchmark model-, the risk premium is further enhanced, wages are much less correlated with output and the wage share becomes more volatile.

Finally, we also consider a version of the model which incorporates two types of demand shocks: exogenous or fiscal demand shocks and monetary policy shocks. These shocks increase the volatility in the aggregate demand components, especially investment, and lower their correlation with output. The high volatility and output-correlation of consumption is not resolved. In line with the data, the volatility of hours worked also increases, and the correlation with output improves as well. The wage statistics are not altered much compared to the benchmark model. The extra volatility in the economy appears also in the risk premium on equity, which increases to 5.61%, and a higher price of risk (0.26). Overall, augmenting the model with a more complete stochastic structure, helps to approach the data statistics along some dimensions.

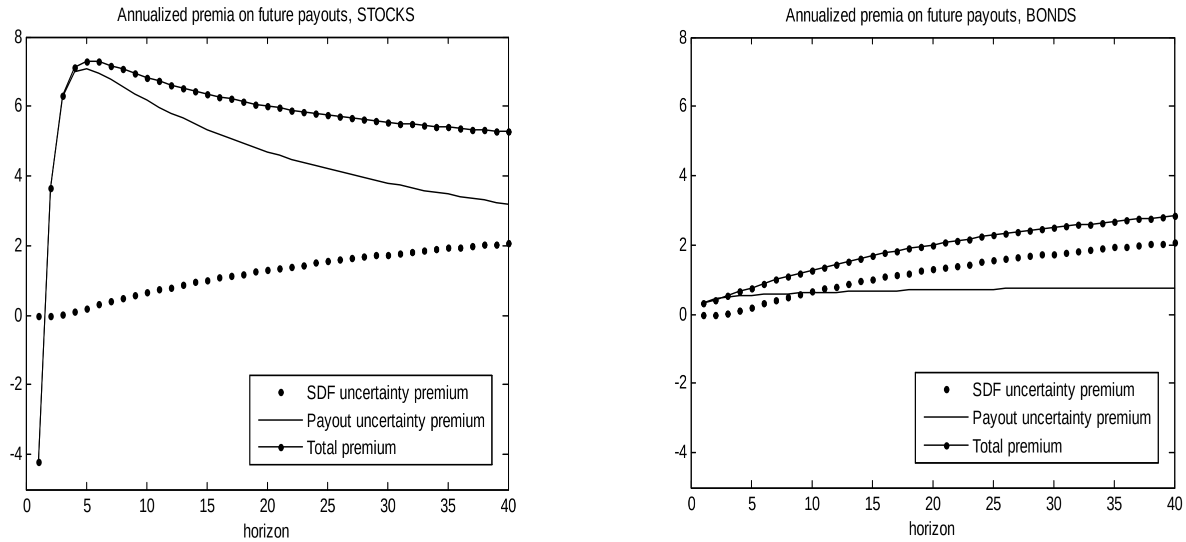
4 Decomposition of the Risk Premium for Equity and Bond Returns

In the benchmark model, the risk premium is significantly higher for equity than for bonds (4.77 for equity versus 1.99 for the excess holding period return on the 10 year bond). This difference is explained by the difference in the payout risk of both assets. In order to illustrate this, it is helpful to decompose the risk premium related to any future income stream (d_{t+k}) in its two components: the covariance between the expected stochastic discount factor and the marginal utility of the shareholder on the one hand, and the covariance between the future income stream and the marginal utility on the other (see Jermann (1998) for a more detailed discussion of this decomposition and Appendix C).

$$RP(d_{t+k}) = -cov(E_{t+1}\lambda_{t+k} - \lambda_{t+1}, \lambda_{t+1}) - cov(E_{t+1}d_{t+k}, \lambda_{t+1})$$

The returns on bond and equity investments can be written as weighted sums of their future income streams, and their risk premium can also be expressed as weighted averages of the corresponding covariance terms. The contribution of the first covariance term is common to stock and bonds and so any difference in their premiums has to be explained by differences in the payout risk. For a nominal bond, the payout uncertainty is only affected by the future inflation realizations, while for equity the payout risk is determined by the uncertainty in

Figure 3: Decomposition of the Risk Premium on Equity and Bonds



the dividend stream.[18]

Figure 3 provides more detail on the different components of the risk premium in the benchmark model. Figure 3a displays the two covariance terms that appear in the formula above, for the dividend stream at different horizons (expressed in quarters). The stochastic discount factor component increases smoothly with the horizon, while the payout premium is hump shaped reflecting the profile in the response of the dividend stream to the shocks. The payout risk is clearly dominant for the equity premium, which is a weighted average of these covariance terms. Figure 3b provides the same information for the bond premium. Here, the SDF uncertainty is relatively more important at least for longer horizon bonds. The payout or inflation premium is dominant for short horizon bonds, but for longer bonds makes up only one third of the total premium.

The dividend stream is affected by the operational risk, which was discussed in the previous sections, and by the financial risk that results from the debt financing of firms: the outstanding debt is assumed to remain a constant fraction of 30% of the capital stock. This financial leverage increases the volatility of dividends (after interest payments) and therefore also the risk premium on equity, while leaving the SR unaffected. In a model that disregards the financial leverage the premium on stocks is much smaller while the bond premium is unaffected (respectively 3.45 and 1.99). So the financial leverage adds 1.32% to the equity risk premium. On the other hand, financial leverage reduces the sensitivity of the real dividend stream to inflation fluctuations in our model: with productivity shocks, the inflation rate is higher during recessions, which reduces the real cost of debt and stabilizes the real dividend.

[18] For technical reasons, we can only calculate the real return for equity, while it is possible to work with nominal returns for bonds. So we are not able to discuss the nominal premium in equity returns.

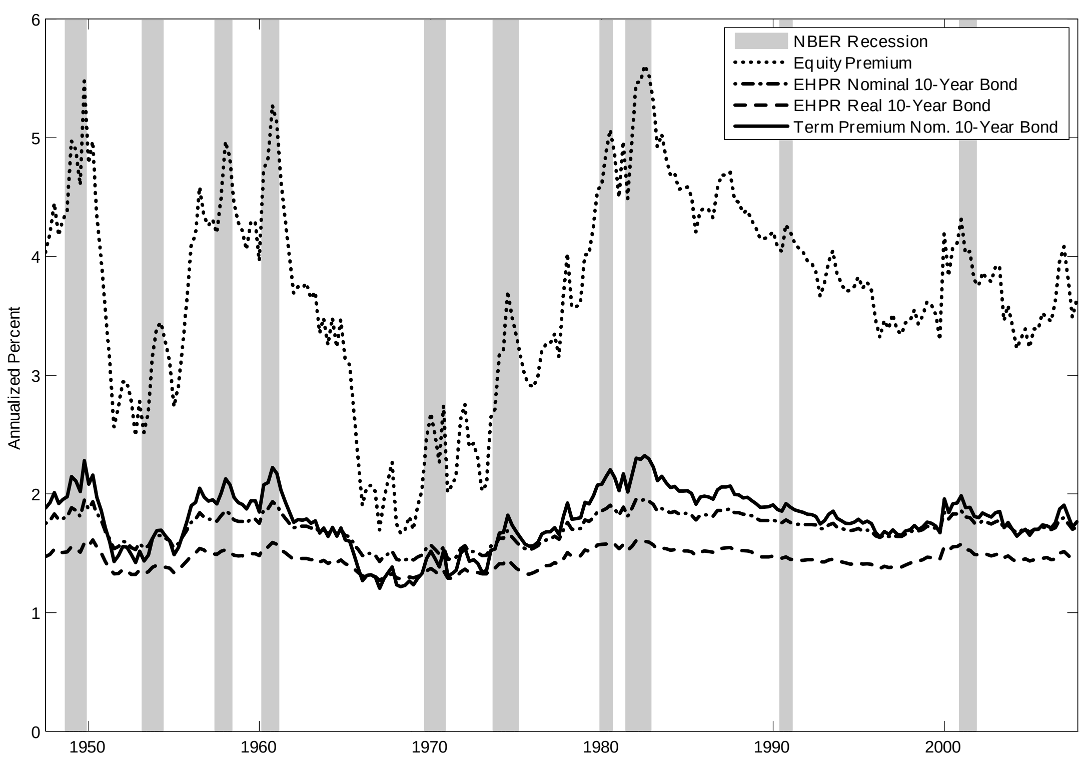
5 Time Variation in the Risk Premium

For analyzing average risk premiums, a second order approximation to the policy function is sufficient. Here, we are interested in time variation in the risk premium of the model, and thus need to use at least one order of approximation higher than a second order approximation. In this section we use a third order approximation to the policy function. We then simulate the model with the historical shocks for the productivity and the distribution risk over the period 1947-2007.

5.1 Cyclical Nature of Risk

Figure 4 shows the risk premiums generated by the model based on the historical series of productivity and distribution shocks. Table 7 measures the comovement of the various (expected and realized) premiums with the business cycle as well as their volatilities. Differences in statistics relative to the earlier calibration results arise for two main reasons: first, the present results are based on a higher (third) order approximation. Second, the results here are model outcomes based on the estimated historical shocks.

Figure 4: Time-Varying Risk Premiums



Note: The graph shows model implied expected one period returns for stocks, nominal and real bonds (10 year maturity). The term premium = $y_t^{40} - y_t^{40*}$, where y_t^{40*} is the risk neutral yield. The premiums are expressed in annualized percentages, i.e. quarterly model concepts are multiplied by 400.

Table 7: Time-Varying Risk Premiums

Measure of premium		Unconditional Std. Dev.	Output Corr.
Equity (excess, expected)	$E_t(r_{t+1}-r^f_{t+1})$	0.86	-0.91
Equity (excess, realized)	$r_{t+1}-r^f_{t+1}$	51.82	-0.29
Real Bond (excess, expected)	$E_t(hpr_{t+1})$	0.08	-0.78
Real Bond (excess, realized)	hpr_{t+1}	14.35	-0.10
Nominal Bond (excess, expected)	$E_t(hpr^{nom}_{t+1})$	0.12	-0.88
Nominal Bond (excess, realized)	hpr^{nom}_{t+1}	19.06	-0.14
Term Premium (realized)	$y^{40}_t-y^{40*}_t$	0.24	-0.88
Slope Yield Curve	$y^{40}_t-R^{nom}_t$	1.98	0.62

Figure 4 illustrates that the model generates quite some time variation in the equity and bond premium. There is no immediate counterpart in the data for these *expected* return series and related *conditional* moments. We compare our results to a number of proxies and results of contemporary models. For equity, Brav et al. (2003) and Söderlind (2008) analyze variations in ex ante measures of expected returns, based on analyst survey and options data. The former obtain an average expected return $E_t r_{t+1}$ of around 20% for the period 1975-2001. Söderlind's survey data suggest an average expected excess return $E_t(r_{t+1} - r^f_{t+1})$ of about 3.25% while the conditional volatility of returns $\sigma_t(r_{t+1})$ is around 16%. The average of the model expected risk premium in Figure 4 is close to that of Söderlind. The conditional standard deviation of the stock return $\sigma_t(r_{t+1})$ in the model, plotted in Figure 6, is slightly above 30% on average.

For bonds, the model produces an expected holding period return on nominal 10 year bonds of 1.71 (Figure 4). The degree of time variation in expected excess bond returns on bonds is limited (Table 7). Again, there are no immediate measures in the data to compare these to. Relative to estimated models such as Duffee (2002), Dewachter (2008) and Campbell et al. (2008) the magnitude of fluctuations in expected returns is small. By contrast, *realized* return average and variation are substantial in the model and compare well to -observable- measures of, e.g., Rudebusch and Swanson (2008) and Campbell et al. (2008). We also compute the term premium in the manner of Rudebusch and Swanson (2008), calculated as the yield difference between the 10 year bond and its risk neutral counterpart. We find that in the present model, this measure for the term premium is somewhat higher on average (1.77) and lower in volatility (0.24) than in their model (0.75 and 0.43, respectively). Finally, note that there is an ex ante premium for bearing the inflation risk of holding nominal bonds of about 25 basis points (Figure 4).

We now turn to the cyclical properties of these risk premiums. All the risk premiums are highly correlated and countercyclical: the correlation of each of them with linearly detrended output is around -0.8 to -0.9 (Table 7). Figure 4 shows how model risk premiums increase at the onset of recession periods, possibly with the exception of the 2001 recession which seems to be anticipated already one year in advance. The countercyclicality of risk premiums is consistent with both theory and evidence in the finance literature. Risk averse investors should require high returns when their marginal utility of consumption is high. Predictive regressions, for instance, also suggest expected returns are high in "bad" times.

Figure 5: Conditional Volatility of Shareholder Consumption Growth

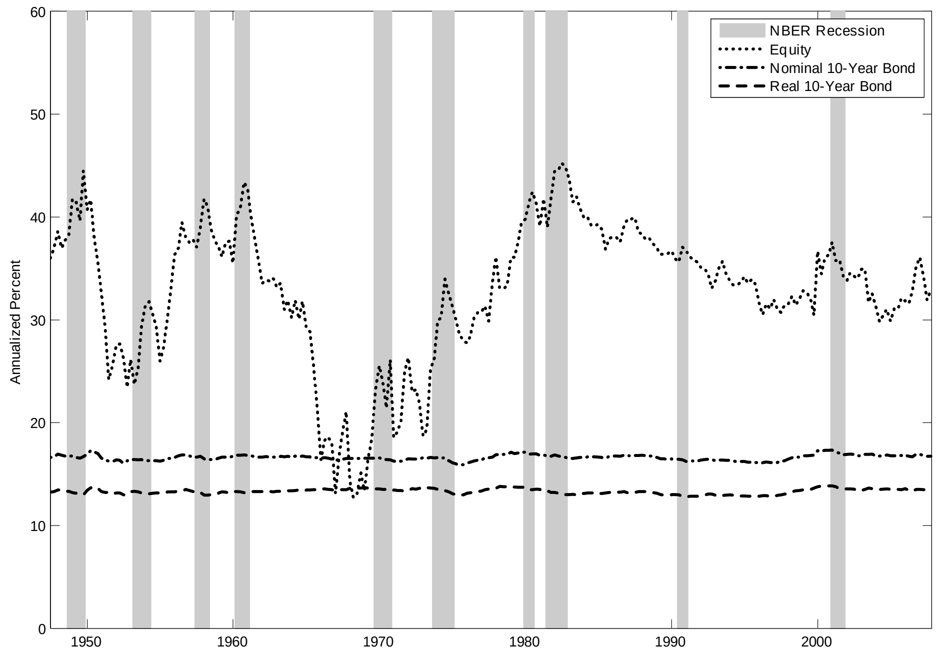
Note: The conditional volatility of consumption growth is the conditional standard deviation of consumption growth. The quarterly model concept is annualized by multiplying by 400.

It is noteworthy that the patterns of time variation in the ex ante equity return survey measures of Söderlind (2008) and, in particular, Brav. et al (2003) look very similar to that of Figure 4. The dynamics of the term premium, too, correspond with the general time-series profile detected in other studies of the term premium (as summarized in Rudebusch et al. (2007)). It seems to hold generally that, from the early eighties onward, the various measures indicate a gradual decrease.

It is useful to try to understand this time variability in the model a bit further. We know from the results in Section 4 that, on average, payout risk is the largest contributor to the equity premium. The bond premium is solely driven by SDF uncertainty, and is therefore substantially smaller. Similarly, time variability in risk premiums can work through two channels: the risk premium is the product of the price and the amount of risk, both of which can vary through time. Below, we provide variations in the different components of the risk premiums.

First, there is the time variation in the compensation for risk that the shareholders expect in return for bearing part of the aggregate risk. This corresponds to time variation in the price of risk, which is summarized in the Sharpe ratio. The Sharpe ratio itself is a function of the volatility of marginal utility, as well as its comovement with the asset's payoff. In the present model, the shareholders' effective risk aversion is virtually constant. The reason is that the shareholders are extremely wealthy (see the wealth distribution of

Figure 6: Conditional Volatility of Stock and Bond Returns



Note: The conditional volatility of asset returns is the conditional standard deviation of the returns. The quarterly model concept is annualized by multiplying by 400.

the three agents). As a result, time variation in the Sharpe ratio is fully driven by variation in the conditional volatility of consumption.[19] Figure 5 shows this conditional volatility of shareholder consumption growth and, in particular, its countercyclical nature. All risk premiums share this component. Hence, variation in consumption growth volatility is the driving force between the comovement of the various risk premiums of Figure 4.

Second, there is time variation in the amount of aggregate risk the shareholders bear across the cycle. The latter is summarized in the expected variability of the returns. Here lies the main explanation for the time variation and the overall magnitudes of risk premiums, as well as the relative differences in them between stocks and bonds. Figure 6 shows the conditional volatilities for the various assets in our model economy.

Clearly, the return volatility for stocks varies substantially over the business cycle. For bonds, this is much less the case. Moreover, in the baseline model, the compensation for inflation risk is almost constant over the cycle. Variations in the conduct of monetary policy will render this type of risk more important. Rudebusch and Swanson (2008) incorporate long run inflation risk by means of time variation in the inflation target of the central bank. Later, we consider related extensions.

[19] For the heterogeneous agent model with GHH preferences, the labor effect is quantitatively not very important.

5.2 Impact on Prediction Performance of P/D and Yield Spread

Predictability regressions for stock returns and bond returns are often used as tests for time-varying risk premiums. In this section we repeat the standard predictability regressions on the model-implied data in order to test whether our time-varying risk premiums can generate the same type of results as typically found in the data.

Stock Return Predictability

Stock returns in the data vary over the cycle, and seem to be forecastable by a variety of variables. Financial ratios are particularly successful in predicting future returns. Moreover, these financial ratios are tied to general business cycle conditions (see, e.g., Fama and French, 1989). The price-dividend ratio ($p_t - d_t$, where small case letters denote logs) is probably the variable that the finance literature has focused on the most.[20] In short, predictive regressions of the form

$$r_{t,t+h} = \alpha^r + \beta^r (p_t - d_t)_t + \varepsilon_t \tag{11}$$

typically find significantly negative estimates for β^r and high R^2, and both tend to increase (in absolute value) as the forecasting horizon h becomes longer. There is a large amount of evidence suggesting that one should be wary of the statistical properties of regressions like these, mostly due to the persistence of the right hand side variable. Cochrane (2008b) and Lettau and Van Nieuwerburgh (2008) are two recent examples that show how the basic finding of predictability remains despite many of the statistical artefacts.[21]

The left-hand panel of Table 8 replicates the above finding for the sample period 1947-2007. In the right-hand panel we perform the same regressions based on data generated by the model. Guvenen (2008) performs the same exercise. As in the latter study, the table shows that the present model, too, reproduces the main patterns found in predictive regressions. More so than in Guvenen (2008), however, the table suggests there is a significant amount of predictability in excess returns in the simulated data, which is consistent with the predictive regressions here and in the aforementioned studies.

Campbell-Shiller Regressions

The time variation in the risk premium for bonds is usually tested using the Campbell and Shiller (1991, CS) regressions:[22]

$$y_{n-1,t+1} - y_{n,t} = \alpha + \beta \frac{1}{n-1} (y_{n,t} - R_t^{nom}) + e_t \tag{12}$$

Under the null of the expectations hypothesis, i.e. of a constant term premium $\beta = 1$. In Table 9, we present the results of this regression applied to historical data for the ten-year bond yield, where the LHS of the regression is approximated by $(y_{n,t+1} - y_{n,t})$. The estimated beta coefficient is -3.8, which is significantly lower than the null hypothesis of $\beta = 1$. When

[20] For a more complete list of variables and papers studying them, we refer the reader to Lettau and Van Nieuwerburgh (2008).

[21] For a more comprehensive overview, see, e.g., Cochrane (2008a).

[22] There is another version of the test of the expectations hypothesis which uses forward rates instead of yields, see Fama and Bliss (1987). We do not take this approach here.

Table 8: Predictive Regressions: Stocks

	Data			Model		
Return Horizon	Beta	t-stat	R^2	Beta	t-stat	R^2
			Returns			
4	-0.124	-5.082	0.099	-0.180	-4.816	0.089
10	-0.288	-8.105	0.223	-0.399	-7.808	0.210
20	-0.560	-11.275	0.367	-0.779	-12.153	0.403
40	-1.338	-15.115	0.534	-1.235	-18.566	0.634
			Excess Returns			
4	-0.130	-5.352	0.109	-0.104	-2.684	0.030
10	-0.293	-8.450	0.238	-0.243	-4.250	0.073
20	-0.548	-11.786	0.388	-0.575	-7.135	0.189
40	-1.287	-17.047	0.594	-1.136	-11.718	0.401

Note: Regressions of log returns on log dividend yield. Horizon in quarters. Model dividend yield based on steady state level dividends. Data: see note Table 3.

testing this regression for the simulated data of the model, using the historical realizations of the two shocks, we follow Rudebusch and Swanson (2008), and adjust the above expression, which holds for a zero-coupon bond of maturity n, to hold for our decaying coupon perpetuity with duration n. The CS regression (12) becomes:

$$\log(p_t^n) - \log(p_{t+1}^n) = \alpha + \beta(y_t^n - R_t^{nom}) + e_t$$

where p_t^n the bond price, y_t^n its yield and R_t^{nom} the nominal short interest rate. Under the expectations hypothesis $\alpha = 0$ and $\beta = 1$, i.e. the coefficients in the above regression have the same interpretation as in the standard zero-coupon case. The estimated value for beta is 0.98 and not significantly different from one. Also the R^2 in the model is lower than in the data. Repeating the exercise for the real bonds delivers a lower estimate of 0.73, but still not significantly different from one.

Table 9: Predictive Regressions: Bonds

	Data			Model		
Horizon	Beta	t-stat	R^2	Beta	t-stat	R^2
			Nominal Yields			
40	-3.797	2.831	0.032	0.982	1.734	0.0124
			Real Yields			
40	-	-	-	0.735	1.352	0.007

Note: Regressions of yield change on spread for the data and minus log price change on spread for the model. Horizon in quarters. Data: see note Table 3.

The deviation from the expectation hypothesis (EH) is determined by the covariance of the spread with the excess holding period return on the bond relative to the variance of the spread. To understand why the model is not successful in generating larger deviations from the expectation hypothesis, this term can be further decomposed in the covariance term for the risk neutral spread and the covariance term for the term premium:

$$-\frac{cov(y_t^n - R_t^{nom}, ehpr_{t+1})}{var(y_t^n - R_t^{nom})} = -\frac{cov(y_t^{n*} - R_t^{nom}, ehpr_{t+1})}{var(y_t^n - R_t^{nom})} - \frac{cov(y_t^n - y_t^{n*}, ehpr_{t+1})}{var(y_t^n - R_t^{nom})}$$

$$-0.018 = 0.059 - 0.077$$

Three elements explain the model's inability to reproduce the CS puzzle. First, the model tends to overestimate the variance of the spread: the standard deviation in the data is 1.13 against 1.98 in the model. Given that this variance appears in the denominator, it makes it harder to generate large deviations from EH. Second, the covariance between the term premium, $y_t^n - y_t^{n*}$, and the next period return on the bond is small. This is a clear sign that the time variation in the bond risk premium is not sufficiently strong to generate large deviations from EH. Finally, the covariance between the risk neutral spread and the excess holding period return is negative, which offsets the contribution of the risk premium covariance on the estimated beta. The negative covariance in the spread and the holding period return is mainly related to the fact that the spread is procyclical in the model, while the (expected) holding period return is countercyclical. Our procyclical spread is certainly related to the fact that the model is only driven by two supply shocks, which tend to move inflation and the interest rate, and more so for the short rate, in the opposite direction of output. This is especially the case for the nominal spread because inflation is exclusively forward looking and jumps on impact, but less so for the real spread. In sum, to increase the deviation from the EH for the bond yields, the volatility in the bond risk premium should increase relative to the volatility in the spread, and the spread should be much less procyclical, in line with what is observed in the data.

6 Two Further Applications

6.1 Risk Premiums and the Great Moderation

In our model, risk premiums are consistent with the stochastic discount factor of the marginal investor and the behavior of the payouts on the assets. The average magnitude and the time variation in the risk premiums depend crucially on the volatility of these two determinants. These latter volatilities are themselves consistent with the overall volatility in the economy, and with the allocation of the macroeconomic risk and the implied behavior of the income distribution, in particular. The decline in the macroeconomic volatility since 1984 is by now widely documented in the literature. The strong decline in the volatility of the exogenous shocks in the model was discussed the calibration section, where we observed a 50% decline in the productivity shocks and a 20% decline in the distribution risk for the more recent period. Therefore, it is interesting to evaluate the implications of this reduction in overall macroeconomic risk for our financial risk premiums. In order to do so, we evaluate the implied risk premium of our model for two different parameterizations of the stochastic processes: a high volatility regime that corresponds with the pre-1984 subsample and a low volatility regime that corresponds with the post-1984 period. Table 10 summarizes these results and compares the outcomes with the empirically observed statistics for the financial and the real macroeconomic variables.

Table 10: Financial Statistics for High and Low Volatility Regime

	SR	EP	BP	$y^f - R^n$	R^f	σ_{R^f}	σ_{R^S}	σ_π
Benchmark Model	0.24	4.77	1.99	1.56	1.20	3.50	20.18	3.00
Data	0.39	6.11	-	1.34	1.19	2.84	15.50	2.62
pre-84 subsample	0.39	6.07	-	1.15	0.53	3.06	15.50	3.11
post-84 subsample	0.40	6.17	-	1.65	2.29	1.97	15.57	0.96
High volatility (pre-84)	0.28	6.43	2.67	2.12	0.25	4.02	23.40	3.40
Low volatility (post-84)	0.15	1.97	0.84	0.61	2.89	2.40	13.11	2.20

Table 11: Macroeconomic Statistics for High and Low Volatility Regime

	σ_Y	σ_I	$\rho_{I,Y}$	σ_C	$\rho_{C,Y}$	σ_N	$\rho_{N,Y}$	σ_W	$\rho_{W,Y}$	$\sigma_{WN/Y}$	$\rho_{WN/Y,Y}$
Benchmark Model	1.86	3.31	0.93	1.59	0.98	0.82	0.94	0.63	0.55	2.37	-0.28
Data	1.70	4.94	0.76	1.17	0.79	1.34	0.87	0.78	0.09	2.34	-0.19
pre-84 subsample	2.03	5.47	0.74	1.36	0.78	1.52	0.88	0.56	0.20	1.39	-0.40
post-84 subsample	0.92	3.79	0.85	0.75	0.85	0.97	0.85	1.03	-0.10	1.29	-0.08
High volatility (pre-84)	2.19	3.82	0.93	1.88	0.98	0.96	0.95	0.72	0.59	2.75	-0.29
Low volatility (post-84)	1.09	2.19	0.86	0.94	0.95	0.51	0.91	0.51	0.32	1.59	-0.27

The simulation results with the model show a 50% decline in volatility of aggregate output and consumption in line with the decline in aggregate productivity risk. Investment and employment volatilities decline slightly less, while the wage and the labor share volatility decline by only 30% and 40%. The risk premiums which are consistent with this macroeconomic moderation change even more. The equity premium drops with 70% from 6.43 for the pre-1984 period to 1.97 for the post-1984 period. The volatility of the return to equity decreases less than 50% and so does the corresponding SR. The excess return on bonds is also very sensitive and drops by more than 70%, while the implied yield spread drops from 2.12% to 0.61%.

So the model generates a huge impact of the Great Moderation on the financial risk premiums. Note, however, that these results show the impact of such a regime switch on the steady state outcomes. These results should not be compared directly with the historical realized returns over the two subperiods. the reason is that the transition dynamics will imply dynamics in the return that work in the opposite direction: in going from a high risk to a low risk regime, returns will temporarily increase during the transition period. The exercise presented here is also based on the hypothesis that the agents in the economy consider this drop in the average volatility since 1984 as a permanent drop. In reality, investors might be uncertain whether this decline is permanent, and may take into consideration the possibility that the high volatility regime will re-appear in the future. Such an exercise would require an evaluation based on a stochastic volatility process or a regime-switching setting.

In reality, the subjective perception of the macro economic uncertainty by the investors might also shift depending on the realization of the shocks or the state of the economy and this might also lead to sudden reversals in the required risk premiums. This type of additional uncertainty and volatility was absent in our analysis of the time variation in the risk premium, and the results in this section suggest that the implications of this additional source of variability can be relatively large in magnitude. Taking into account a stochastic model structure is therefore an extremely promising extension of the exercise performed here, but falls outside the scope of this paper.

6.2 Risk Premiums, Inflation Dynamics and Monetary Policy

The benchmark model retains a monetary policy rule that responds aggressively to inflation deviations from a fixed target, in order to have a reasonable fit for the volatility of inflation. With such a policy rule and with simple price adjustment costs, that generate a purely forward looking Phillips curve, it is difficult to capture the great inflation and the high persistence of inflation observed during the seventies and eighties. To reproduce this experience and to analyze its impact on the risk premium -especially on the nominal component- we here consider several alternative specifications for the monetary policy rule and for the price adjustment costs. In a first experiment, we decrease the long-run reaction coefficient to inflation in the policy rule (r^{π}) from 3 to 1.5. In a second variant, we model the perceived inflation target as a persistent process (ρ_{π} =0.995) that adjusts slowly to the actual inflation rate with a coefficient of 0.03, but we keep the variance of exogenous shocks to the perceived inflation target at zero:

$$\overline{\pi}_t = (1 - \rho_{\pi})\overline{\pi} + \rho_{\pi}\overline{\pi}_{t-1} + 0.03 * (\pi_t - \overline{\pi}_t) + \varepsilon_t^{\pi} \tag{13}$$

Both exercises imply a stronger and more persistent response of inflation to the endogenous inflation pressure in the model. In a third experiment, we introduce exogenous shocks to the perceived inflation process with a std. dev. $\sigma^{\pi} = 0.10$. Finally, we consider a case in which the inflation adjustment costs are smaller, such that inflation adjusts faster. The results of these experiments are presented in Table 12.

Table 12: Alternative Financial Statistics

	SR	EP	BP	$y^f - R^f$	R^f	σ_{R^f}	σ_{R^S}	σ_{π}
Benchmark Model	0.24	4.77	1.99	1.56	1.20	3.50	20.18	3.00
Data	0.39	6.11	-	1.34	1.19	2.84	15.50	2.62
Lower inflation response	0.23	4.46	3.00	2.17	1.31	3.55	19.24	5.92
Endog. inflation target	0.24	4.70	2.71	1.88	1.23	3.50	19.95	4.08
Exog. inflation target	0.24	4.77	1.94	1.25	1.21	3.52	20.19	4.98
No price adj.costs	0.25	5.24	2.06	1.61	0.96	3.82	21.29	4.24

Table 13: Alternative Macroeconomic Statistics

	σ_Y	σ_I	$\rho_{I,Y}$	σ_C	$\rho_{C,Y}$	σ_N	$\rho_{N,Y}$	σ_W	$\rho_{W,Y}$	$\sigma_{WN/Y}$	$\rho_{WN/Y,Y}$
Benchmark Model	1.86	3.31	0.93	1.59	0.98	0.82	0.94	0.63	0.55	2.37	-0.28
Data	1.70	4.94	0.76	1.17	0.79	1.34	0.87	0.78	0.09	2.34	-0.19
Lower inflation response	1.81	3.25	0.91	1.56	0.98	0.84	0.88	0.65	0.55	2.40	-0.27
Endog. inflation target	1.85	3.29	0.92	1.59	0.98	0.82	0.92	0.64	0.55	2.38	-0.28
Exog. inflation target	1.87	3.33	0.93	1.61	0.98	0.84	0.94	0.64	0.55	2.44	-0.28
No price adj.costs	2.03	3.45	0.96	1.73	0.99	0.93	1.00	0.61	0.59	2.34	-0.28

All four alternative specifications affect the risk premium on bonds, but the exact mechanism through which this happens differs from case to case. In the first scenario, monetary policy allows a larger and a more persistent inflation response to the fundamental shocks by responding less to the realized inflation rate. Investors adjust their inflation expectations accordingly and the bond price and the long term bond yield will also become more sensitive to the shocks. Bond prices increase following a positive productivity shock, so that they tend to comove strongly with the consumption of the bond- and shareholders. Therefore, these investors will require a higher risk premium to hold the bond, which appears also in a higher spread in the term structure. The increased risk premium is related to the increased uncertainty about future real payout risk of the bond, so it is mainly the inflation component of the premium that increases. In fact, under this policy rule, the contribution of the real term premium decreases, which also explains why the equity premium drops. The lower premium on equity is also explained by the fact that higher inflation reduces the real cost of the outstanding debt of the firms. This mechanism tends to stabilize the real dividends and the return to capital.

With the endogenously adjusting perceived inflation target, inflation and the policy reaction will not change that much relative to the benchmark model on impact, but expectations about long term inflation are becoming more sensitive to the shocks. As a consequence, nominal bond prices and yields will also be very sensitive to the shocks. So in this scenario, we find again an increase in the nominal premium, relative to the benchmark, while the real component is more or less unaffected. The results are very different for the exogenous target shocks. Inflation becomes more volatile, and as the target process is very persistent, the inflation expectations and therefore the bond yields and prices are also much more volatile, but this does not translate into a higher nominal premium. The reason for this result is that inflation target shocks, for instance a disinflation shock, tend to raise the real short rate in the short run. With nominal price rigidity, actual inflation will not immediately jump to the new inflation target, and monetary policy will keep the real interest rate temporarily high, which will induce a drop in aggregate demand and consumption. As a result, marginal utility and bond prices will be positively correlated in the short run, and bond prices offer the bondholders a hedge against this type of inflation risk. Therefore, exogenous target inflation shocks do not require a higher nominal premium, and they can result in a lower term spread (see also Gallmeyer et al. (2007)).

Lower price adjustment costs change the profile of the inflation process, as inflation will be much less persistent and prices adjust quicker to their desired levels. But the long term price level fluctuations will not change drastically and the payout risk for nominal bonds is therefore not significantly affected either. Less short run deviations in the real marginal costs tend to stabilize the profit/wage shares as well. But the faster adjustment of demand and production (which are no longer characterized by the hump shaped reaction and instead jump on impact) drives up the wedge between the spot wage and the desired smooth contract wage. The larger insurance component in the contract reinforces the cyclicality of the profit share, and substitutes for the variable mark-up as the main source for the variability in profits.

7 Conclusions

The objective of this research is to build a DSGE model that is able to fit well both on the real and the financial side of the economy. The analysis of this paper illustrates that a heterogeneous agent model can be a useful alternative to the standard representative agent model for modelling jointly the real and the financial side of the economy. Starting from a realistic classification of households in three groups, portfolio investors, bondholders and workers who differ from each other in terms of capital market participation and risk aversion, our model generates high risk premiums and reasonable dynamics for the intra- and intertemporal allocation decisions. There are two important ingredients for generating the high risk premiums. Firstly, the concentration of consumption risk in the group of shareholders results in high prices of risk. Secondly, the labor contract and the countercyclical mark-ups result in a high volatility of capital returns. The bond market allows shareholders to redistribute some of this risk to a larger group of bondholders, but the overall impact of this risk sharing mechanism remains moderate. This is due to differences in the risk aversion and because workers have no incentive to participate in this market given their efficient labor contract.

The model also generates a significant degree of countercyclical time variation in the risk premiums. The time variation in the equity premium is consistent with the empirically observed predictive power of price-dividend ratio's for future stock returns as well as more direct proxies of expected returns. The term premium on long bonds also displays some time variation but less than typically found in empirical models of the bond spread and insufficient to explain the predictive power of the spread for future excess returns on bonds. To overcome this problem, future analysis could introduce variations on utility functions (e.g. habit persistence or stochastic risk aversion) or stochastic volatility in macroeconomic risk to reinforce the variation in the price of risk, which is the main driver for bond premiums.

For future work, it would also be interesting to complete the stochastic structure of the model and to estimate the model on the data. Recent progress in higher order estimation methods suggests that such an exercise will become feasible soon. However, a complete higher order approximation of the model is probably not necessary given the limited size of the feedback effects from the risk premiums on the real economy. In order to increase these feedback effects, one could consider a model switching framework, in which the parameters of the model can also change in line with the stochastic risks that hit the economy. Alternatively, one could add financial frictions to the model so that the required risk premiums have first order effects on consumption and investment decisions. A joint fit of the financial and the real data will imply a strong validation test for the model. It remains to be seen how the heterogeneous agent setting performs in explaining the real variables relative to the representative agent models which are now standard in New-Keynesian monetary models (e.g. Smets and Wouters (2007)). In that respect, it is important to stress that the model is able to generate endogenously the observed real wage rigidity as the result from an efficient risk sharing arrangement between workers and firms. This view on wage rigidity was popular in the late seventies, and recently regained support from micro studies on wage dynamics and their reaction to transitory firm-specific shocks (Guiso et al. (2005)).

Furthermore, our model has important implications for the distribution of income and risks across different groups of agents. It would be interesting to develop the welfare implications of this model: how is the cost of business cycles and inflation allocated over the different agents? The implications of the model for the wealth distribution can also be helpful to refine the calibration of the model.

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A Derivation of the First Order Conditions for the Model

A.1 Households

Type 1 Agents: Shareholders

Shareholders maximize:

$$\max E_t \sum_{j=0}^{\infty} \beta^t U_1\left(C_{1,t+j}, N_{1,t+j}\right)$$

subject to the requirement:

$$C_{1,t} + \frac{P_t^B}{P_t} B_{1,t+1} + \frac{P_t^S}{P_t} S_{1,t+1} \leqslant \frac{B_{1,t}}{P_t} + S_{1,t} \frac{\left(P_t^S + D_t\right)}{P_t} + \frac{W_t^s}{P_t} N_{1,t} + \frac{\Gamma_t}{P_t}$$

First Order Conditions

$(\partial C_{1,t})$

$$\frac{\partial U_1\left(C_{1,t}, N_{1,t}\right)}{\partial C_{1,t}} - \lambda_{1,t} = 0$$

$(\partial N_{1,t})$

$$\frac{\partial U_1\left(C_{1,t}, N_{1,t}\right)}{\partial N_{1,t}} + \lambda_{1,t} \frac{W_t^s}{P_t} = 0$$

$(\partial B_{1,t+1})$

$$\beta E_t \left\{ \frac{\lambda_{1,t+1}}{\lambda_{1,t}} \frac{1}{P_t^B \pi_{t+1}} \right\} = \beta E_t \left\{ \frac{\lambda_{1,t+1}}{\lambda_{1,t}} R_{t+1}^f \frac{1}{\pi_{t+1}} \right\} = 1$$

$(\partial S_{1,t+1})$

$$\beta E_t \left\{ \frac{\lambda_{1,t+1}}{\lambda_{1,t}} \frac{\left(P_{t+1}^S + D_{t+1}\right)}{P_t^S} \frac{1}{\pi_{t+1}} \right\} = \beta E_t \left\{ \frac{\lambda_{1,t+1}}{\lambda_{1,t}} R_{t+1}^S \frac{1}{\pi_{t+1}} \right\} = 1$$

$(\partial \lambda_{1,t})$

$$C_{1,t} + \frac{P_t^B}{P_t} B_{1,t+1} + \frac{P_t^S}{P_t} S_{1,t+1} = \frac{B_{1,t}}{P_t} + S_{1,t} \frac{\left(P_t^S + D_t\right)}{P_t} + \frac{W_t^s}{P_t} N_{1,t} + \frac{\Gamma_t}{P_t}$$

In case the shareholders can also invest in N zero-coupon bonds , their budget constraint becomes

$$\begin{aligned} & C_{1,t} + \frac{P_t^B}{P_t} B_{1,t+1} + \sum_{n=1}^{N} P_t^{B,n} B_{1,t+1}^n + \frac{P_t^S}{P_t} S_{1,t+1} \\ = \; & \frac{B_{1,t}}{P_t} + \sum_{n=1}^{N} P_t^{B,n-1} B_{1,t}^n + S_{1,t} \frac{\left(P_t^S + D_t\right)}{P_t} + \frac{W_t^s}{P_t} N_{1,t} + \frac{\Gamma_t}{P_t} \end{aligned}$$

with the first order condition associated to $B^n_{1,t+1}$:

$\left(\partial B^n_{1,t+1}\right)$

$$P^{B,n}_t = \beta E_t \left[P^{B,n-1}_{t+1} \frac{\lambda_{1,t+1}}{\lambda_{1,t}\pi_{t+1}} \right]$$

Type 2 Agents: Bondholders

Bondholders maximize:

$$\max E_t \sum_{j=0}^{\infty} \beta_2^t U_2 \left(C_{2,t+j}, N_{2,t+j}\right)$$

subject to:

$$C_{2,t} + \frac{P^B_t B_{2,t+1}}{P_t} \frac{1}{\phi(B_{2,t+1})} \leqslant \frac{B_{2,t}}{P_t} + \frac{W^s_t}{P_t} N_{2,t}$$

First Order Conditions

$(\partial C_{2,t})$

$$\frac{\partial U_2 \left(C_{2,t}, N_{2,t}\right)}{\partial C_{2,t}} - \lambda_{2,t} = 0$$

$(\partial N_{2,t})$

$$\frac{\partial U_2 \left(C_{2,t}, N_{2,t}\right)}{\partial N_{2,t}} + \lambda_{2,t} \frac{W^s_t}{P_t} = 0$$

$(\partial B_{2,t+1})$

$$\beta E_t \left\{ \frac{\lambda_{2,t+1}}{\lambda_{2,t}} \phi(B_{2,t+1}) \left[\alpha^B \frac{1}{P^B_t} + (1-\alpha^B) \frac{P^S_{t+1} + D_{t+1}}{P^s_t} \right] \frac{1}{\pi_{t+1}} \right\} = 1$$

$(\partial \lambda_{2,t})$

$$C_{2,t} + \frac{1}{\alpha^B} \frac{P^B_t}{P_t} B_{2,t+1} \frac{1}{\phi(B_{t+1})} = B_{2,t} \frac{1}{P_t} + S_{2,t} \frac{\left(P^S_t + D_t\right)}{P_t} + \frac{W^s_t}{P_t} N_{2,t}$$

Type 3 Agents: Workers

Workers consume their wage income:

$$C_{3,t} = \frac{W^c_t}{P_t} N_{3,t}$$

A.2 Firms

Firms maximize the present value of the dividend stream using the shareholders' stochastic discount factor.

$$\max E_t \left[\sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} D_{t+j}(i) \right]$$

with

$$D_t(i) = \begin{bmatrix} (P_t(i)/P_t)Y_t(i) - W_t^s N_{1,t}(i) - W_t^s N_{2,t}(i) - W_t^c N_{3,t}(i) \\ -PAC_t(i) - I_t(i) + (P_t^{B_f} B_{f,t}(i) - B_{f,t-1}(i))/P_t \end{bmatrix}$$

subject to:

$$\begin{aligned}
Y_t(i) &= Z_t K_t(i)^{\theta} N_t(i)^{(1-\theta)} - \phi \\
Y_t(i) &= \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t \\
K_{t+1}(i) &= (1-\delta) K_t(i) + G\left(\frac{I_t(i)}{K_t(i)} \right) K_t(i) \\
N_t(i) &= N_{1,t}(i) + N_{2,t}(i) + N_{3,t}(i)
\end{aligned}$$

The adjustment costs for capital are formulated as:

$$G = a_1 \left(\frac{I}{K} \right)^{(1-1/\xi)} + a_2$$

and the adjustment cost for prices are specified as

$$PAC_t(i) = \frac{\chi^p}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - \overline{\pi} \right)^2$$

First Order Conditions After imposing that $P_t(i) = P_t$ for identical firms:

(∂I_t)

$$\kappa_t = G' \left(\frac{I_t}{K_t} \right)^{-1}$$

(∂K_{t+1})

$$\begin{aligned}
\kappa_t \;=\;& \beta \frac{\lambda_{t+1}}{\lambda_t} \left[rmc_{t+1} \theta Z_{t+1} K_{t+1}^{\theta-1} N_{t+1}^{(1-\theta)} + \kappa_{t+1} \left((1-\delta) + G\left(\frac{I_{t+1}}{K_{t+1}} \right) - G'\left(\frac{I_{t+1}}{K_{t+1}} \right) \frac{I_{t+1}}{K_{t+1}} \right) \right] \\
& + \mu \left(P_t^{B_f} - \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\pi_{t+1}} \right)
\end{aligned}$$

$(\partial P_t\,(i))$

$$\begin{aligned} 0 &= (1-\varepsilon)\,Y_t + \varepsilon * rmc_t Y_t \\ &\quad -\chi\,(\pi_t - \overline{\pi})\,\pi_t + \beta\frac{\lambda_{t+1}}{\lambda_t}\left[\chi\,(\pi_{t+1} - \overline{\pi})\,\pi_{t+1}\right] \end{aligned}$$

$(\partial \kappa_t)$

$$K_{t+1} = (1-\delta)K_t + G\left(\frac{I_t}{K_t}\right)K_t$$

(∂N_t)

$$\frac{W_t^s}{P_t} = rmc_t(1-\theta)Z_t K_t^{\theta} N_t{}^{-\theta}$$

The real marginal cost (rmc_t) equals the spot wage divided by marginal productivity of labor. Note that under no price/inflation adjustment costs the real marginal cost is constant:

$$\frac{\varepsilon - 1}{\varepsilon} = rmc_t$$

A.3 The Labor Contract

The wage contract solves:

$$\max E_t\left\{v_t U_1\,(C_{1,t}, N_{1,t}) + (1-v_t)U_3\,(C_{3,t}, N_{3,t})\right\}$$

subject to the budget constraint of the workers and the shareholders.
The FOC are:
(∂W^c)

$$\begin{aligned} \frac{\partial U_1\,(C_{1,t}, N_{1,t})}{\partial C_{1,t}} &= \frac{(1-v_t)}{v_t}\frac{\partial U_3\,(C_{3,t}, N_{3,t})}{\partial C_{3,t}} \\ U_{1,t}^C &= ds_t U_{3,t}^C \\ where\ ds_t &= \frac{(1-v_t)}{v_t} \end{aligned}$$

$(\partial N_{3,t})$

$$\begin{aligned} v_t\frac{\partial U_1\,(C_{1,t}, N_{1,t})}{\partial C_{1,t}}[rmc_t\frac{\partial F(K_t, N_t)}{\partial N_{3,t}} - W_t^c] + (1-v_t)\left\{\frac{\partial U_3\,(C_{3,t}, N_{3,t})}{\partial C_{3,t}}W_t^c + \frac{\partial U_3\,(C_{3,t}, N_{3,t})}{\partial N_{3,t}}\right\} &= 0 \\ v_t U_{1,t}^C[rmc_t(1-\theta)Z_t K_t^{\theta} N_t{}^{-\theta} - W_t^c] + (1-v_t)\left\{U_{3,t}^C W_t^c + U_{3,t}^N\right\} &= 0 \end{aligned}$$

Combing both first order conditions delivers:

$$U_{1,t}^C[rmc_t(1-\theta)Z_t K_t^{\theta} N_t{}^{-\theta}] + U_{3,t}^N = 0$$

$$\frac{W_t^s}{P_t} = -\frac{U_{3,t}^N}{U_{1,t}^C}$$

B Asset Pricing in Log-Linear Framework

A large body of literature on asset prices and macroeconomic dynamics uses first-order approximate solutions to derive asset prices and premiums. This literature started with Campbell (1994) and Jermann (1998). They assume that the variables are lognormally distributed, and that the first order approximation is good. The returns are derived from the Arrow-Lucas-Rubinstein asset pricing equation:

$$1 = E_t\left[\beta \frac{\Lambda_{t+1}}{\Lambda_t}\frac{R_{t+1}}{\Pi_{t+1}}\right]$$

with Λ and R the shadow value of wealth and the asset return, respectively.

B.1 Real Risk Free Rate

The real risk free rate is the real return on a (zero-) coupon one-period bond:

$$\begin{aligned}
1 &= E_t\left[\beta \frac{\Lambda_{t+1}}{\Lambda_t} R_{t+1}^{short}\right] \text{ with } R_{t+1}^{short} = \frac{1}{P_t^{short}} \\
0 &= \ln\beta + \ln\left(E_t\left[\exp\left(\triangle\ \lambda_{t+1} + r_{t+1}^{short}\right)\right]\right) \text{ with } \lambda = \ln\Lambda \text{ and } r^{short} = \ln R^{short} \\
0 &= \ln\beta + E_t\left[\triangle\ \lambda_{t+1} + r_{t+1}^{short}\right] + \frac{1}{2}\left[\sigma^2_{\triangle\lambda,t} + \sigma^2_{r^{short},t} + 2\rho_{\triangle\lambda,r^{short},t}\sigma_{\triangle\lambda,t}\sigma_{r^{short},t}\right] \\
\log E_t\left[R_{t+1}^{short}\right] &= -\ln\beta - E_t\left[\triangle\ \lambda_{t+1}\right] - \frac{1}{2}\left[\sigma^2_{\triangle\lambda,t}\right] \text{ because } \sigma_{r^{short},t} = 0
\end{aligned}$$

The unconditional expectation then becomes:

$$r^{short} = -\ln\beta - E\left[\triangle\ \lambda_s\right] - \frac{1}{2}\left[\sigma^2_{\triangle\lambda_s}\right]$$

Note that for the unconditional version of this equation $\sigma_{r^{short}} \neq 0$. In particular, $\log E\left[R_{t+1}^{short}\right] = -\ln\beta - E\left[\triangle\ \lambda_{t+1}\right] - \frac{1}{2}\left[\sigma^2_{\triangle\lambda_{t+1}} + 2\rho_{\triangle\lambda_{t+1},r_{t+1}^{short}}\sigma_{\triangle\lambda_{t+1}}\sigma_{r_{t+1}^{short}}\right]$. The last term appears because although R_{t+1}^{short} is known at t and has, therefore a zero conditional variance, it is still a random variable whose unconditional variance is non-zero.

B.2 Nominal Short Rate

The nominal short rate is :

$$
\begin{aligned}
1 &= E_t\left[\beta\frac{\Lambda_{t+1}}{\Lambda_t}\frac{R_{t+1}^{short}}{\Pi_{t+1}}\right] \text{ with } R_{t+1}^{short} = \frac{1}{P_t^{short}} \\
0 &= \ln\beta + \ln\left(E_t\left[\exp\left(\triangle\ \lambda_{t+1} + r_{t+1}^{short} - \pi_{t+1}\right)\right]\right) \\
\text{with } \lambda &= \ln\Lambda \text{ and } r^{short} = \ln R^{short} \text{ and } \pi = \ln\Pi \\
0 &= \ln\beta + E_t\left[\triangle\ \lambda_{t+1} + r_{t+1}^{short} - \pi_{t+1}\right] + \\
&+\frac{1}{2}\left[\begin{array}{c} \sigma^2_{\triangle\lambda,t} + \sigma^2_{r^{short},t} + \sigma^2_{\pi,t} \\ +2\rho_{\triangle\lambda,r^{short},t}\sigma_{\triangle\lambda,t}\sigma_{r^{short},t} - 2\rho_{\triangle\lambda,\pi,t}\sigma_{\triangle\lambda,t}\sigma_{\pi,t} - 2\rho_{\pi,r^{short},t}\sigma_{\pi,t}\sigma_{r^{short},t} \end{array}\right] \\
\log E_t\left[R_{t+1}^{short}\right] &= -\ln\beta - E_t\left[\triangle\ \lambda_{t+1}\right] + E_t\left[\pi_{t+1}\right] - \frac{1}{2}\left[\sigma^2_{\triangle\lambda,t} + \sigma^2_{\pi,t}\right] \ + \rho_{\triangle\lambda,\pi,t}\sigma_{\triangle\lambda,t}\sigma_{\pi,t} \\
\text{because } \sigma_{r^{short},t} &= 0
\end{aligned}
$$

B.3 Equity Premium

The equity premium is defined as the difference between the nominal stock and the nominal risk free rate.

The stock return is:

$$
\begin{aligned}
1 &= E_t\left[\beta\frac{\Lambda_{t+1}}{\Lambda_t}\frac{R_{t+1}^{stock}}{\Pi_{t+1}}\right] \\
0 &= \ln\beta + E_t\left[\triangle\ \lambda_{t+1} + r_{t+1}^{stock} - \pi_{t+1}\right] + \\
&+\frac{1}{2}\left[\begin{array}{c} \sigma^2_{\triangle\lambda,t} + \sigma^2_{r^{stock},t} + \sigma^2_{\pi,t} \\ +2\rho_{\triangle\lambda,r^{stock},t}\sigma_{\triangle\lambda,t}\sigma_{r^{stock},t} - 2\rho_{\pi,r^{stock},t}\sigma_{\pi,t}\sigma_{r^{stock},t} - 2\rho_{\triangle\lambda,\pi,t}\sigma_{\triangle\lambda,t}\sigma_{\pi,t} \end{array}\right] \\
\log E_t\left[R_{t+1}^{stock}\right] &= -\ln\beta - E_t\left[\triangle\ \lambda_{t+1}\right] + E_t\left[\pi_{t+1}\right] - \frac{1}{2}\left[\sigma^2_{\triangle\lambda,t} + \sigma^2_{\pi,t}\right] - \rho_{\triangle\lambda,r^{stock},t}\sigma_{\triangle\lambda,t}\sigma_{r^{stock},t} \\
&+\rho_{\pi,r^{stock},t}\sigma_{\pi,t}\sigma_{r^{stock},t} + \rho_{\triangle\lambda,\pi,t}\sigma_{\triangle\lambda,t}\sigma_{\pi,t}
\end{aligned}
$$

with:

$$R_{t+1}^{stock} = \frac{P_{t+1}^{stock} + D_{t+1}}{P_t^{stock}}$$

So that the difference between the stock return and the short-term riskless bond, or the equity premium is:

$$r^{stock} - r^{short} = -\rho_{\triangle\lambda_s,r^{stock}}\sigma_{\triangle\lambda_s}\sigma_{r^{stock}} + \rho_{\pi,r^{stock}}\sigma_{\pi}\sigma_{r^{stock}}$$

B.4 Term Premium

The term premium is the difference between the nominal short term bond and the nominal long term bond.

The long-term bond return is:

$$
\begin{aligned}
1 &= E_t\left[\beta\frac{\Lambda_{t+1}}{\Lambda_t}\frac{R_{t+1}^{long}}{\Pi_{t+1}}\right] \\
0 &= \ln\beta + E_t\left[\triangle\ \lambda_{t+1} + r_{t+1}^{long} - \pi_{t+1}\right] + \\
&\quad +\frac{1}{2}\left[\begin{array}{c} \sigma_{\triangle\lambda,t}^2 + \sigma_{r^{long},t}^2 + \sigma_{\pi,t}^2 \\ +2\rho_{\triangle\lambda,r^{long},t}\sigma_{\triangle\lambda,t}\sigma_{r^{long},t} - 2\rho_{\pi,r^{long},t}\sigma_{\pi,t}\sigma_{r^{long},t} - 2\rho_{\triangle\lambda,\pi,t}\sigma_{\triangle\lambda,t}\sigma_{\pi,t}\end{array}\right] \\
\log E_t\left[R_{t+1}^{long}\right] &= -\ln\beta - E_t\left[\triangle\ \lambda_{t+1}\right] + E_t\left[\pi_{t+1}\right] - \frac{1}{2}\left[\sigma_{\triangle\lambda,t}^2 + \sigma_{\pi,t}^2\right] \\
&\quad -\rho_{\triangle\lambda,r^{long},t}\sigma_{\triangle\lambda,t}\sigma_{r^{long},t} + \rho_{\pi,r^{long},t}\sigma_{\pi,t}\sigma_{r^{long},t} + \rho_{\triangle\lambda,\pi,t}\sigma_{\triangle\lambda,t}\sigma_{\pi,t}
\end{aligned}
$$

with:

$$
\begin{aligned}
R_{t+1}^{long} &= \frac{P_{t+1}^{long} + C_{t+1}^{long}}{P_t^{long}} \\
&= \frac{P_{t+1}^{long} + \overline{C^{long}}}{P_t^{long}}
\end{aligned}
$$

So that the difference between the long-term bond return and the short-term riskless bond, or the term premium is:

$$
r^{long} - r^{short} = -\rho_{\triangle\lambda_s,r^{long}}\sigma_{\triangle\lambda_s}\sigma_{r^{long}} + \rho_{\pi,r^{long}}\sigma_{\pi}\sigma_{r^{long}}
$$

B.5 Sharpe Ratio

The equity Sharpe ratio is:

$$
\frac{\log E_t\left[R_{t+1}^{stock}\right] - r_{t+1}^{short}}{\sigma_{r,t}} = -\rho_{\triangle\lambda,r^{stock},t}\sigma_{\lambda,t} + \rho_{\pi,r^{stock},t}\sigma_{\pi,t}
$$

The bond Sharpe ratio is:

$$
\frac{\log E_t\left[R_{t+1}^{long}\right] - r_{t+1}^{short}}{\sigma_{r^{long},t}} = -\rho_{\triangle\lambda,r^{long},t}\sigma_{\lambda,t} + \rho_{\pi,r^{long},t}\sigma_{\pi,t}
$$

C Equity Premium Decomposition in Payout Risk, SDF Risk and Inflation Risk

Following Campbell (1994) and Jermann (1998) we propose the decomposition of the premium $\log\left(E_t R_{t,t+1}^{D_k}/R_{t,t+1}^1\right)$ on an asset that pays a dividend D_{t+k} (a strip) at time $t+k$ in excess of the risk free rate R_{t+1}^f as:

$$\log\left(E_t\left[\frac{R^{D_k}_{t,t+1}}{R^f_{t+1}\Pi_{t+1}}\right]\right) = -cov\left(\lambda_{t+1}, E_{t+1}d_{t+k}\right) \tag{14}$$
$$-cov(\lambda_{t+1}, E_{t+1}\lambda_{t+k} - \lambda_{t+1})$$
$$+cov_t\left(\lambda_{t+1}, E_{t+1}\left(p_{t+k} - p_{t+1}\right)\right)$$
$$+cov_t\left(p_{t+1}, E_{t+1}d_{t+k}\right)$$
$$+cov_t\left(p_{t+1}, E_{t+1}\lambda_{t+k}\right)$$
$$+cov_t\left(p_{t+1}, E_{t+1}p_{t+k}\right)$$

where $R^{D_k}_{t,t+1}$ is the return on the strip, R^f_{t+1} is the real risk free rate, Π_{t+1} is inflation and p_{t+1} is the log price level. The first term is the premium that arises from payout uncertainty, the second term arises from uncertainty in future marginal utility and the last four terms arise only when there is inflation in the model.

To obtain the above decomposition start from defining the one-period holding return for the strip as:

$$R^{D_{t+k}}_{t,t+1} = \frac{V_{t+1}\left[D_{t+k}\right]}{V_t\left[D_{t+k}\right]} \tag{15}$$

where $V_t\left[D_{t+k}\right] = \frac{\beta^k E_t[D_{t+k}\Lambda_{t+k}/P_{t+k}]}{\Lambda_t/P_t}$. Then,

$$\begin{aligned}
V_{t+1}\left[D_{t+k}\right] &= E_{t+1}\left[\frac{\beta^{k-1}\Lambda_{t+k}D_{t+k}P_{t+1}}{\Lambda_{t+1}P_{t+k}}\right] \\
&= \beta^{k-1}\exp\left(E_{t+1}\left[\lambda_{t+k} - \lambda_{t+1} + d_{t+k} - p_{t+k} + p_{t+1}\right]\right) \\
&\quad * \exp\left(\frac{1}{2}V_{t+1}\left[\lambda_{t+k} + d_{t+k} - p_{t+k}\right]\right) \\
E_t\left[V_{t+1}\left[D_{t+k}\right]\right] &= \\
&= \beta^{k-1}\exp\left(E_t\left[\lambda_{t+k} - \lambda_{t+1} + d_{t+k} - p_{t+k} + p_{t+1}\right]\right) \\
&\quad * \exp\left(\frac{1}{2}V_t\left\{E_{t+1}\left[\lambda_{t+k} - \lambda_{t+1} + d_{t+k} - p_{t+k} + p_{t+1}\right]\right\}\right) \\
&\quad * \exp\left(\frac{1}{2}V_{t+1}\left[\lambda_{t+k} + d_{t+k} - p_{t+k}\right]\right)
\end{aligned}$$

and

$$\begin{aligned}
V_t\left[D_{t+k}\right] &= E_t\left[\frac{\beta^k\Lambda_{t+k}D_{t+k}P_t}{\Lambda_t P_{t+k}}\right] \\
&= \beta^k\exp\left(E_t\left[\lambda_{t+k} - \lambda_t + d_{t+k} - p_{t+k} + p_t\right] + \frac{1}{2}V_t\left[\lambda_{t+k} + d_{t+k} - p_{t+k} + p_t\right]\right)
\end{aligned}$$

Given that the real return on a one-period bond $R_{t,t+1}\left[1_{t+1}\right]$ is the risk free rate R^f_t, we have:

$$
\begin{aligned}
R_t^f &\equiv R_{t,t+1}\left[1_{t+1}\right] \\
&= \frac{1}{E_t\left[\beta\frac{\Lambda_{t+1}}{\Lambda_t}\right]} \\
&= \frac{1}{\beta\exp\left(E_t\left[\lambda_{t+1}-\lambda_t\right]+\frac{1}{2}V_t\left[\lambda_{t+1}\right]\right)} \\
&= \beta^{-1}\exp\left(-E_t\left[\lambda_{t+1}-\lambda_t\right]-\frac{1}{2}V_t\left[\lambda_{t+1}\right]\right)
\end{aligned}
$$

By replacing in equation (15) and taking the conditional expectation, we have:

$$
\begin{aligned}
E_t R_{t,t+1}^{D_{t+k},N} &= \beta^{-1}\exp\left(E_t\left[\lambda_{t+k}-\lambda_{t+1}+d_{t+k}-p_{t+k}+p_{t+1}\right]\right) \\
&\quad *\exp\left(\frac{1}{2}V_t\left\{E_{t+1}\left[\lambda_{t+k}-\lambda_{t+1}+d_{t+k}-p_{t+k}+p_{t+1}\right]\right\}\right)*\exp\left(\frac{1}{2}V_{t+1}\left[\lambda_{t+k}+d_{t+k}-p_{t+k}\right]\right) \\
&\quad *\exp\left(-E_t\left[\lambda_{t+k}-\lambda_t+d_{t+k}-p_{t+k}+p_t\right]-\frac{1}{2}V_t\left[\lambda_{t+k}+d_{t+k}-p_{t+k}+p_t\right]\right) \\
&= \frac{1}{\beta}\exp\left(-E_t\left[\lambda_{t+1}-\lambda_t\right]-\frac{1}{2}V_t\left(\lambda_{t+1}\right)\right) \\
&\quad *\exp\left(E_t\left(p_{t+1}-p_t\right)+\frac{1}{2}V_t\left(p_{t+1}\right)\right) \\
&\quad *\exp\left(-cov_t\left(\lambda_{t+1},E_{t+1}d_{t+k}\right)-cov_t(\lambda_{t+1},E_{t+1}\lambda_{t+k}-\lambda_{t+1})+cov_t\left(\lambda_{t+1},E_{t+1}p_{t+k}-p_{t+1}\right)\right. \\
&\quad *\exp\left(cov_t\left(p_{t+1},E_{t+1}d_{t+k}\right)+cov_t\left(p_{t+1},E_{t+1}\lambda_{t+k}\right)+cov_t\left(p_{t+1},E_{t+1}p_{t+k}\right)\right)
\end{aligned}
$$

Taking logs, we recover equation (14).

The return on an asset that pays a stream of payouts D_{t+k} over k periods, $R_{t,t+1}\left[\{D_{t+k}\}_{k=1}^{\infty}\right]$, can be computed as the weighted sum of the return on k strips.

$$
\begin{aligned}
E_t\left\{R_{t,t+1}\left[\{D_{t+k}\}_{k=1}^{\infty}\right]\right\}/R_{t,t+1}^1 &= \\
&= \sum_{k=1}^{\infty} w_t\left[D_{t+k}\right]*\exp\left(RP\left(d_{t+k}\right)\right)
\end{aligned}
$$

where $w_t\left[D_{t+k}\right]=\frac{V_t[D_{t+k}]}{\sum_{k=1}^{\infty}V_t[D_{t+k}]}$. We compute the weights at the steady state:

$$
\begin{aligned}
\left[w_t\left[D_{t+k}\right]\right]_{SS} &= \left[\frac{\frac{\beta^k E_t[D_{t+k}\Lambda_{t+k}/P_{t+k}]}{\Lambda_t/P_t}}{\sum_{k=1}^{\infty}\frac{\beta^k E_t[D_{t+k}\Lambda_{t+k}/P_{t+k}]}{\Lambda_t/P_t}}\right]_{SS} \\
&= \frac{\beta^k}{\sum_{k=1}^{\infty}\beta^k} \\
&= \beta^{k-1}\left(1-\beta\right)
\end{aligned}
$$

D Bond yields

The price of zero-coupon bonds of maturity n is determined by the model's Euler equations:

$$P_t^{B,n} = E_t \left[P_{t+1}^{B,n-1} \beta \frac{\Lambda_{t+1}}{\Lambda_t \Pi_{t+1}} \right] \tag{16}$$

From a computational point of view this iteration can be problematic when evaluating longer maturity bonds. So instead of using (16) we follow Rudebusch and Swanson (2008) and approximate the price of the zero-coupon $n-$maturity bond by the price of a perpetuity whose coupon is 1 this period and then depreciates at a rate of δ_c. This rate of decay is chosen to set the perpetuity duration equal to the maturity of the zero-coupon bond.

The price of the perpetuity is:

$$P_t^{B,n} = 1 + \delta_c E_t \left[P_{t+1}^{B,n} \beta \frac{\Lambda_{t+1}}{\Lambda_t \Pi_{t+1}} \right] \tag{17}$$

The coupon is a geometrically declining function of δ_c and δ_c is such that the Macaulay duration of this bond , defined by:

$$Duration =_{n\rightarrow\infty} \frac{\sum_{t=1}^{n} \frac{t\delta_c^t}{(1+R^{nom}\_ss)^t} + \frac{nM}{(1+R^{nom}\_ss)^n}}{\sum_{t=1}^{n} \frac{\delta_c^t}{(1+R^{nom}\_ss)^t}} \tag{18}$$

is equal to 10 years. The denominator is the current bond price, and discounting uses the model's steady state risk free rate denoted here by r^f_ss. The perpetuity face value M is zero. For a steady state nominal rate of $R^N\_ss = 0.01/1.01$ per quarter, δ_c is set to 0.9947 to approximate a 40-period zero-coupon bond.

The continuously compounded yield to maturity y_{nt} equals:

$$y_{nt} \equiv \log \left(\frac{\delta_c P_t^{B,n}}{P_t^{B,n} - 1} \right) \tag{19}$$

and the realized (log) holding period return for the n-maturity bond is written as :

$$hpr_t^n \equiv \log(\frac{\delta_c P_t^{B,n} + (\exp(R_t^{nom}))}{P_{t-1}^{B,n} - 1})$$

to take into account the fact that the coupon payed today can be re-invested at the one period nominal rate R_t^{nom}. The spread and the (expected) excess holding period return are then defined as usual:

$$\begin{aligned} Eehrp_t^n &= E_t hpr_{t+1}^n - R_t^{nom} \\ spread_t^n &= y_{nt} - R_t^{nom} \end{aligned}$$

Term premia TP_t are defined as the difference the bond yield and its risk neutral counterpart,

$$TP_t = y_{nt} - \widetilde{y}_{nt}$$

where risk neutral prices $\widetilde{P}_t^n$ are defined by

$$\widetilde{P}_t^n = 1 + \delta_c E_t \left[\widetilde{P}_{t+1}^n R_t^{nom} \right]$$

and risk neutral bond yields $\widetilde{y}_{nt}$ are given by:

$$\widetilde{y}_{nt} \equiv \log \left(\frac{\delta_c \widetilde{P}_t^n}{\widetilde{P}_t^n - 1} \right)$$

To obtain the Campbell and Shiller regression for the perpetuity, start from (17) , use (19) to replace $(P_t^n - 1)$ and take logs:

$$\begin{aligned} P_t^{B,n} - 1 &\equiv \delta_c E_t \left[P_{t+1}^{B,n} \beta \frac{\Lambda_{t+1}}{\Lambda_t \Pi_{t+1}} \right] \\ \frac{P_t^{B,n}}{\exp(y_{nt})} &= E_t \left[P_{t+1}^{B,n} \beta \frac{\Lambda_{t+1}}{\Lambda_t \Pi_{t+1}} \right] \end{aligned}$$

and take logs of the 1st order approximation of the above equation :

$$\begin{aligned} y_{nt} &= E_t \log(P_{t+1}^{B,n}) - \log \left(P_t^{B,n} \right) + E_t \log(\beta \frac{\Lambda_{t+1}}{\Lambda_t \Pi_{t+1}}) \\ y_{nt} - R_t^{nom} &= E_t \log(P_{t+1}^{B,n}) - \log \left(P_t^{B,n} \right) \end{aligned}$$

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