ROADS TO EQUALITY
Wealth Distribution Dynamics with Public-Private Capital Complimentarity

by

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Abstract

This paper proposes a model of wealth distribution dynamics with a capital market imperfection and a production function where public capital is complementary to private capital. A unique invariant steady-state distribution is derived, with three social classes: subsistence workers, 'government dependent' middle-class entrepreneurs and 'private infrastructure owning' upper-class entrepreneurs. It is shown that there is a minimum level of public investment below which the middle class disappears, and that increases in non-targeted public investment over some range lead to unambiguously less inequality of opportunity, as well as to greater output. This provides an additional rationale for an active role for the government infrastructure, health and education provision, and implications for foreign aid.

Keywords: Wealth distribution; capital market imperfections; inequality; public investment.

JEL Nos.: D31, D63, E44, H54, O15.

1. Introduction:

Should the state be an active player in economic development, or should governments aim instead for a continued reduction in the share of their expenditure to GNP? In particular, how will changes in the 'size' of government affect the distributions of wealth and income? Can public investment projects which are not targeted specifically at the poor nevertheless contribute to alleviating inequality and poverty?

In the 1980s, the experience of structural adjustment led a large number of developing countries to reduce government intervention and seek a greater role for the private sector, even in the production of goods and services which had previously been the preserve of the state. This was in part influenced by advice from the World Bank and other international financial institutions, but also by a more general tendency: "More recently, the pendulum has swung the other way with a sizeable fraction of the herd of both politicians and economists charging in the direction of minimalist government, privatisation, and so on." (Stern, 1991a; p.250).

There were substantial reductions in government expenditures in many countries, and because personnel and debt financing costs proved remarkably resilient, the brunt of cuts was borne by infrastructure, health, education and welfare spending. While 24% of Brazil's total government expenditure was allotted to economic services[1] in 1980, only 9.3% was in 1992. In Mexico, the education share of expenditure fell from 18% to 13.9% over the same period, and health from 2.4% to 1.9%. Economic services collapsed from 31.2% to 13.4%. In Pakistan, that share fell from 37.2% to 11.6%, and health and education shares fell as well. Economic services as a share of total expenditure by the Filipino government more than halved from 56.9% to 26.8% (World Bank, 1994).

As policy-makers now ponder the wisdom of building upon these changes, brought about by structural adjustment experiences, with a development strategy more permanently based upon

[1] Among the expenditure categories used by the IMF Government Finance Statistics Yearbooks and reported in the World Bank's World Development Reports, "economic services" is the one which most closely approximates the general concept of 'infrastructure'.

privatization and low levels of public investment, questions have been raised as to the impacts of this on poverty and inequality (see e.g. Cornia, Jolly and Stewart, 1987, and Dreze and Sen, 1989). These concerns with negative impacts of government expenditure reduction on social welfare generally centre around Keynesian effects on the demand for labour, or on direct effects from changes in transfers to the poor. This paper suggests a third channel, namely the reduction in the free-of-charge provision by the government of inputs into production, such as infrastructure, education, or indeed health care. This reduction turns out to have potentially damaging effects on the long-run distribution of income, even when the inputs are made available equally to rich and poor, and there is no targeting to the worst-off.

To understand the nature of these "public inputs" into private production, it is best to take a broad view of the aggregate production process. The output of private firms depends not only on how much labour and private capital they employ, but also on the quality of the environment in which they operate. Many dimensions of this environment, such as the legal framework; the security services; the quality of public telecommunications; the transport network; the average nutritional and educational quality of the labour force; or the reliability of the power supply are either exclusively or partly supplied by the government. The idea that public capital (and investment) are important determinants of private output (and growth) is therefore both reasonable and familiar (see Stern, 1991b for a discussion and Barro, 1990 for an aggregated model).

While some of the output of the public sector consists of public goods, many others are rivalrous and excludable in their consumption, and can thus also be supplied privately (e.g. education, health services, power generation, telephone services). This paper will focus on the effects of changes in the public provision of these services, when ability to purchase the private substitutes differs along the distribution of wealth.

Whilst the examples of cuts in public investment mentioned above are from major countries, they clearly do not represent a random sample. A more detailed empirical investigation of relationships between changes in public investment and both growth and inequality is left to a companion paper (Ferreira, 1994). The present paper addresses theoretically the question of the impact of changes in public investment on the long-run distribution of wealth. It is concerned with the long-run effects of this massive redeployment of productive activity away from the public sector, in an environment of imperfect capital markets.

To do so it draws on the recent literature on wealth distribution dynamics in the presence of capital market imperfections. This was pioneered by Galor and Zeira (1993)[2] and Banerjee and Newman (1991). Other prominent contributions were made by Aghion and Bolton (1993), Banerjee and Newman (1993) and Piketty (1992); a survey of the incipient literature was carried out by Aghion and Bolton (1992).[3] These works suggest plausible specific causes for the persistence of wealth inequality, even in the absence of talent differences. All of these causes are capital market failures. They establish the existence of (unique or multiple) steady state distributions, and generally derive interventionist policy implications. Usually it is the case that "by redistributing wealth towards the poor or the middle-class borrowers the government can improve productive efficiency." (Aghion and Bolton, 1993, p.34). But this purely redistributive activity is a remarkably simplistic view of the role of the government in development. This paper suggests that more traditional activities, such as the provision of infrastructure, health and education, may also help to reduce long-run inequality, in addition to the well known microeconomic efficiency arguments for them, based on externalities and transaction costs.

The government portrayed in other papers in this literature is not involved in any productive activity. Its only policy option is to transfer wealth from rich to poor agents, and even though this is supposed to be a permanent policy, no incentive effects on rational recipients are considered. This picture of the government is naturally oversimplified, and in stark contrast to most current views on the role of the state in development, which encompass active participation in the production of certain goods and services in which, for some reason, it has a comparative advantage. These include at least the provision of some infrastructural services, education and health, besides pure public goods. The reasons why the government may have a cost-benefit advantage in the supply of these particular commodities has to do with the

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[2] A widely cited mimeo version dates to 1988.

[3] A much more detailed discussion of this literature and of how it bears on the present model is contained in an earlier draft of this paper, available from the author on request.

relative importance of market failures (eg. externalities) vis-a-vis government failures (eg. rent-seeking) in their markets. But they have been well studied elsewhere (see Stern, 1991a, and his references) and need not concern us here.

In this paper, the active role of the government arises from the existence of a public capital input into production (g), which is complementary to private capital (k). This g is efficiently produced by the government and distributed free of charge, in identical amounts, to every household-firm in the economy. The quantity each receives at any time t is g_g. Households can also purchase g privately through markets, albeit at a high fixed cost. The amount bought in this way is denoted g_p for that household, and the fixed cost is $\bar{g}_p$. Allowing for this private alternative to the supply of some public services reflects a very real recent tendency in many developing countries, particularly in Latin America. As The Economist has reported: "In Montevideo and Mexico City, businessmen fed up with inefficient public telephones have embraced cellular technology. In Buenos Aires and Caracas, private courier systems compete vigorously with the state-run postal system. In Cartagena and Lima many put their faith not in the national energy system but in private power generators. Across Latin America, consumers and businesses have been turning to private suppliers for services that had long been available only from the state." (The Economist, 1993, p.50).

This model describes the circumstances under which this sort of competition to the public sector arises, and who its customers are. It also shows why a replacement of state provisions by private ones may increase inequality and reduce output. It explains why "the result generally hits the poor hardest" (The Economist, 1993, p.50). Just as some agents are too poor to be given credit to invest in private capital, others turn out to be sufficiently wealthy to invest in private capital, but still unable to buy public capital.

The remainder of this introduction summarises the basic story. Section 2 presents the model; §3 describes the static equilibrium, §4 discusses the dynamics of the system and derives the steady-state distribution, and §5 performs some comparative static exercises on it. Section 6 concludes.

1.1. The Basic Story:

The model described in section 2 shows how reductions in the level of public investment (financed by foreign transfers) can lead to an increase in inequality, because the poor are more dependent on the government than the rich. To do so, I assume a large population of risk-neutral households, identical in every respect except their initial wealth. These households are also production units, and their size is normalised to one. No pooling of households is allowed. Each household maximises a utility function (of consumption and bequests), and may choose between a risky (entrepreneurial) production function and a deterministic subsistence one.

Expected returns on the entrepreneurial production function are high enough that everyone would prefer to invest in it if they could. But to do so they must purchase private capital, and there is a fixed start up cost, without which it is impossible to produce. Capital markets do exist, but are imperfect: banks require collateral, on which the maximum size of any loan depends. This means that the poorest households are unable to gather the minimum amount required to invest in the risky production function. They become subsistence farmers and artisans. Richer people, who can buy enough private capital to start up, find their probability of success depending on the amount of public capital (infrastructure, health care, education services) available to them.

Public capital is partly provided by the government, free of charge, in identical amounts to all household-firms. But it is also available from markets, at a price and subject to another fixed start up cost. The middle-class finds that, whilst it can afford to buy private capital, its collateral is not large enough to buy both the minimum outlays of private capital and of private infrastructure. They are hence constrained to operate with the amount of infrastructure made available by the government, whilst their richer competitors can top it up with private power generators, private health insurance, private schooling, etc. The richest households therefore choose an optimal combination of both types of capital, and are able to allocate their investments more efficiently than the credit constrained middle-classes. As a result, they face a higher ex-ante expected rate of return.

The model is set up so that the transition process of wealth across generations is a linear Markov process, which means that no matter what the initial distribution of wealth was like, it will converge to a unique, invariant distribution in the steady state. Furthermore, for a plausible set of parameter values, all of the three classes described above will exist in that distribution. Increases in government investment will reduce the inequality of opportunity between the richer and the poorer entrepreneurs in that distribution, by reducing the disadvantage to the poorer ones from being unable to buy private infrastructure. It will bring the mix of inputs with which they operate closer to the optimal mix, and thus closer to the one employed by their richer counterparts.

Because this is an open economy, with an integrated capital market, there is an exogenous world interest rate. The capital account is equilibrated by net lending to or net borrowing from abroad, as required. But this exogenous interest rate means that, for very low levels of government investment, the credit-constrained entrepreneurs will face expected returns from investing at home which are so low, that they will prefer to lend all of their wealth in the open market. Hence, there will be a threshold level of public investment below which the middle-class disappears, and domestic investment becomes the exclusive preserve of the very rich, who can buy enough private infrastructure to make it worthwhile operating in the country. Any increases in public investment from below that threshold have the additional effect of increasing the number of domestic entrepreneurs, by making it worthwhile for increasingly poorer people to invest at home.

2. A Model of Wealth Dynamics yielding Different Degrees of Dependence on Government Investment:

I assume a continuum of risk-neutral agents with wealth distributed in $[0, w_f]$ with total mass one[4]. At any time t, their distribution is given by $G_t(w)$, which gives the measure of the population with wealth less than w. All projects are conducted by identical households of normalised size 1. Agents live for one period and have one child each. They seek to maximise:

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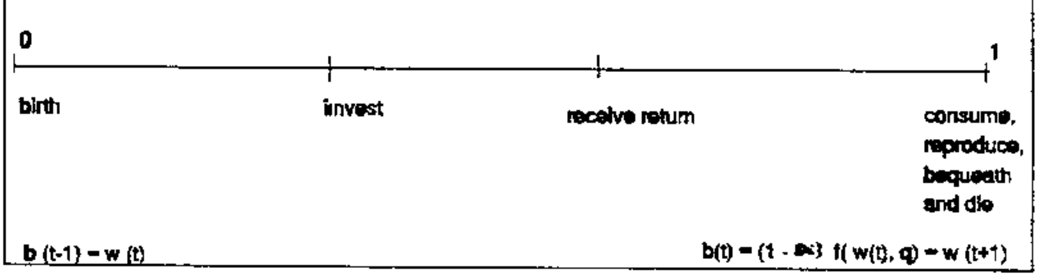
[4] w_f is very, very large.

$$U(c,b,e) = hc^{\alpha}b^{1-\alpha} - e \qquad (1)$$

where $0<\alpha<1$. c denotes the agent's total consumption, b denotes his bequest and $e \in (0,1)$ is his effort level in production. This formulation implies the "warm glow" bequest motive (see Andreoni, 1989), as well as that leisure is additively separable from consumption and bequests. There is one good, which can be consumed or invested, and if invested can be invested in private and/or public capital. The timing of an agent's life is represented in figure 1.

Figure 1



Since the large reductions in public investment which were observed in the 1980s were generally accompanied by comparable cuts in net transfers from abroad, I adopt a formulation in which all government expenditure is financed by aid transfers. This is not intended to be realistic, and the crucial issue of taxation is discussed in Ferreira (1994). But it is not merely a simplifying assumption either. In the context of structural adjustment policies, when the pressures forcing governments to slash investment were the twin deficits of the budget and the balance of payments - both of which were exacerbated by large cuts in foreign transfers - this assumption is quite appropriate. Long term aggregate net transfers[5] to all developing countries fell from U$46.5 billion in 1981 to U$11.5 billion in 1991, with the figures for each year between 1984 and 1988 actually negative. Transfers to the severely indebted middle

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[5] Long term aggregate net transfers are defined by the World Debt Tables of the World Bank as the sum of net resource flows on long term debt (excluding IMF) plus official grants (excluding technical assistance) and net direct foreign investment, minus interest payments on long term loans and foreign direct investment profits.

income countries fell from U$9.0 billion in 1981 to -U$11.0 billion in 1990, whereas severely indebted low income countries suffered a decline from U$6.6 billion in 1981 to -U$0.9 billion in 1990 (World Bank, 1991).

I assume that there is a transfer from abroad of size TR_t at time t. This is made to the government, which invests it in a socially efficient way in public capital, g. This allows us to capture the linkage between foreign capital flows and public investment, alluded to above. It is true to the spirit of expenditure reduction in the 1980s, when "debt crises and budget crunches ... led many governments to neglect their infrastructure." (The Economist, 1993, p.50).

Domestic production takes place according to the following production function:

$$\Omega(k_t, g_t, e_t) = 0 \qquad \text{if} \;\; e < 1 \tag{2}$$

$$\Omega(k_t, g_t, 1) = \Phi(k_t, g_t) \tag{3}$$

where k denotes the level of private capital used in production. This formulation requires an inelastic labour supply, as output is zero unless full effort is exerted. Furthermore, even if e = 1, there is a fixed start-up amount of k (= $\bar{k}$) which is required for production to take place. If both these conditions are satisfied, returns are given by the stochastic production function below:

$$\Phi(k_t, g_t) = 0 \qquad \text{with prob } 1 \qquad \text{if } k_t < \bar{k} \tag{4}$$

$$\begin{aligned} \Phi(k_t, g_t) &= \hat{r}k_t \qquad \text{with prob } q \mid k_t \geq \bar{k} \\ &= 0 \qquad \text{with prob } 1-q \mid k_t \geq \bar{k} \end{aligned} \tag{5}$$

$$\text{where} \quad q = f^{-1}\,(g/k) \qquad f' > 0 \tag{6}$$

Here, f(.) is defined over the domain [0,1], with f(.) = ∞ for any q $\notin$ [0,1]. For example:

$$g/k = q^{1/a} \qquad\qquad 0 < a < 1 \tag{7}$$

Then,

$$E\ [\Phi(k_t,g_t)\ |\ k_t \geq \bar{k}] = \hat{r}qk = \hat{r}k^{1-a}g^{a} \tag{8}$$

Now, let $g_t = g_{gt} + g_{pt}$, where the g subscript denotes the per capita stock of public capital provided by the government at time t, and the subscript p denotes the amount of public capital purchased privately by the agent in question at time t. As indicated above, we assume that:

$$g_{gt}\int_0^{\infty} dG(w) = TR_t \tag{9}$$

which is given exogenously. Three other assumptions complete the description of the production side of the economy. There is a fixed cost of purchasing public capital privately (A1), so that:

$$g_{pt} \in [\bar{g}_p, \infty) \qquad\qquad \bar{g}_p > 0 \tag{10}$$

and there is an exogenously given riskless world interest rate r, such that $1 < r < \hat{r}a^a(1-a)^{1-a}$ (A2)[6]. Finally, a 'subsistence technology' is also available to all agents, and is given by $s(e) = 0$ for $e < 1$, $s(1) = n$. (A3)[7]

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[6] This may be seen as a 'domestic viability condition' on $\hat{r}$, given the world interest rate and the domestic technological parameter a. The second inequality imposes a lower bound on $\hat{r}$, above which domestic investment is always profitable, provided k and g are combined in their optimal proportions. The optimal k/g ratio - (1-a)/a -equates the marginal revenue products (MRPs) of k and g. Substituting the ratio into the expression for either MRP, and requiring that it be positive, gives A2.

[7] n and $\hat{r}$ are such that:

$$\left(\frac{1}{1-\alpha} - r\right)\left(\bar{k} - \frac{\pi F}{r}\right) < n < \hat{r}\bar{k}^{1-a}\bar{g}_g^{\,a} - r\bar{k}$$

The first inequality rules out poverty traps, and the second ensures that the subsistence activity is only chosen by credit constrained people.

Four comments about this production structure may be appropriate at this stage. First, effort is an argument simply to acknowledge that there is a labour input, and to be explicit that there is no moral hazard problem associated with its supply, which is perfectly inelastic at 1. Both the main and the subsistence technologies only yield positive returns if $e = 1$. Note also that this specification rules out any labour pooling. Firms have a fixed employment level of 1, and may vary only their capital input.

Second, there are two sources of non-convexity in the production set. These arise from $\bar{k}$ and $\bar{g}_p$, which are minimum requirements for capital purchases. In particular, no production can take place with $k < \bar{k}$, and no private purchases of public capital can be smaller than $\bar{g}_p$. To rule out the trivial case, let $\bar{k} > \pi F/r$, where these are defined below.

Third, the complementarity between k and g comes from increases in g increasing the probability of success - and hence the expected value - of a project employing k. This suggests a world where observed returns to private investment are directly proportional to the amounts of private capital purchased, and not to the quality of roads or telephone systems, but where better roads and more reliable telephones increase the chances of the project being viable. Think of a farmer taking produce to the market. He owns a lorry (k), which he drives on a road (g). His returns depend on how many vegetables he can fit in the lorry, but the quality of the road determines the probability that he will make it to the market at all. Furthermore, no road ($g = 0$; $q = 0$) means no project: he would have to eat his produce by himself. Clearly, to build his own, private road, he would need a large fixed amount ($\bar{g}_p$).

Finally, notice that g is **not** a public good. In fact, its consumption must be rivalrous and excludable. g_g is simply the total transfer TR divided up equally amongst all the household-firms. The natural way to interpret g is as an entitlement, made freely available to residents by the state, to any good or service useful in production, which can also be substituted for private alternatives, albeit at a cost. g_g is the right to place a child in a state school, the right to use state hospitals, the right to be connected to public telephones or the power supply, the right to use a public road.

Turning now to the capital markets, I follow Banerjee and Newman (1993) in assuming that, although returns to investment projects are costlessly verifiable, repayment is not costlessly enforceable. Borrowers whose projects fail are forgiven, but successful ones - who must repay - face no cost of defaulting other than a fixed penalty F - unrelated to the size of their loans - which they are forced to pay if they are caught having defaulted. The probability that a defaulter is caught is π. Since all agents are risk-neutral, this means that the market for loans would only lend small amounts (up to πF/r), unless some institutional arrangement could be developed to increase agent willingness to repay. One obvious such institutional arrangement is a collateral requirement. Loan size is maximised by requiring that an agent's total wealth be left as collateral for any loan. I assume that this is the case in what follows and that lenders pay r on any collateral left with them, upon returning it to the borrower after repayment of the loan. All loans take place at this riskless rate r.

An agent that borrows an amount L, having left his wealth w as collateral will repay it iff:

$$V(L) - \pi F \leq V(L) - rL + rw \tag{11}$$

where V(L) is the value of the loan to the borrower. It follows directly that individual borrowers face a credit ceiling which is an affine function of their wealth, and which is given by:

$$L_c = w + \pi F/r \tag{12}$$

3. The Static Equilibrium:

From these simple utility and production functions, and using this design of the capital markets, we can now turn to the static equilibrium of the model. The goal of this section is to arrive at a description of the payoffs of all agents in any given period of time, depending on their initial wealth levels. To do so, I first identify the thresholds between social classes into which the continuum of wealth levels divides itself (Lemma 1) and the range of public investment levels of interest (Lemmas 2 and 3). I then determine the activities undertaken in each social class (Lemma 1 and propositions 1 and 2).

Lemma 1: $\exists$ w^*, w^{**}; $w^* < w^{**}$, such that a) agents with $w < w^*$ are subsistence lenders, and b) agents with $w^* < w < w^{**}$ are constrained to using public capital supplied by the government only.

<u>Proof:</u> a) The minimum investment necessary to become an entrepreneur is $\bar{k}$. Agents with $w < \bar{k}$ obviously need to borrow if they are to invest, but they can only borrow up to their credit ceilings. Hence, agents with wealth less than $w^* = \bar{k} - \pi F/r$ will not have access to the start up investment needed to become an entrepreneur. This is an incentive compatibility constraint. Their only alternative is then to dedicate their whole effort supply to the subsistence activity and lend all of their wealth at the riskless rate r.[8]

b) If an agent wishes to complement the stock of public capital made freely available to her by the government (g_g) by purchasing privately supplied public capital, then she must make a minimum outlay of $\bar{g}_p$. But because she must also have bought $\bar{k}$, without which her returns on g_p will be zero with certainty, it follows that agents with wealth less than $w^{**} = \bar{k} + \bar{g}_p - \pi F/r$ will not have access to sufficient funds to purchase private public capital. Since $\bar{g}_p > 0$, $w^{**} > w^*$. ■

The next two lemmas establish the existence of a range of values of government investment in which one observes a coexistence of projects using only g_g with those that use both g_g and g_p.

<u>Lemma 2:</u> $\exists\ \bar{g}_g$ such that for $g_g < \bar{g}_g$ no pure public capital (supplied by the government) projects are undertaken.

<u>Proof:</u> An agent's expected payoff from borrowing to purchase $k \geq \bar{k}$ and $g_p \geq \bar{g}_p$ and undertaking a project is:

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[8] I rule out the case in which they might prefer to forgo earnings from subsistence to enjoy leisure, by assuming that h is large enough so that:

$$hn\alpha^{\alpha}(1-\alpha)^{1-\alpha} > 1$$

$$\Pi = \hat{r}k_t^{1-a}(g_g+g_p)^a - r(k+g_p-w) \tag{13}$$

Hence,

$$\frac{\partial \Pi}{\partial k} = \hat{r}(1-a)k^{-a}(g_g+g_p)^a - r \tag{14}$$

which is decreasing in k and increasing in g. Hence, $\bar{g}_g$ is determined by:

$$\hat{r}(1-a)\bar{k}^{-a}g_g^{\,a} = r \tag{15}$$

At lower levels of government supplied public capital, expected returns to domestic private capital investment - in the absence of private purchases of public capital - are lower than riskless lending at the international rate (capital flight). Hence projects will only be undertaken by agents with wealth greater than $\tilde{w}$ = max (w**, w'). w' is defined by:

$$w' = \bar{k} + g'_p - \frac{\pi F}{r} \tag{16}$$

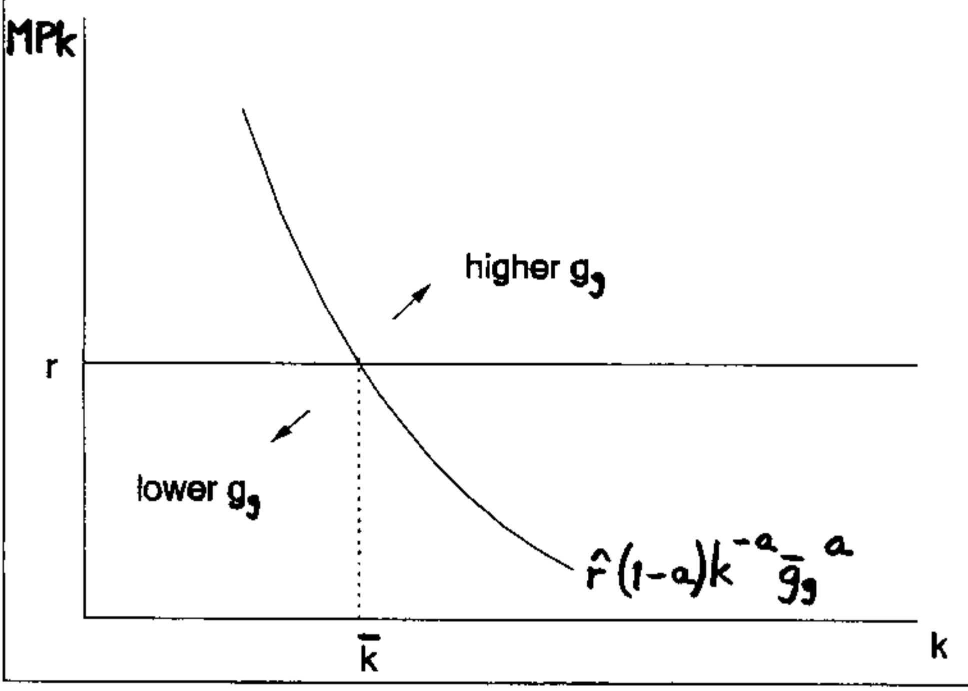
where g'_p (g_g) is given by:

$$\hat{r}(1-a)\bar{k}^{-a}(g_g+g_p)^a = r \tag{17}$$

■

It follows that if $g_g < \bar{g}_g$, all observed projects would necessarily use both g_g and g_p. In other words, the middle class disappears, and any entrepreneurs belong to the upper class. Figure 2 below illustrates the determination of $\bar{g}_g$: the expected marginal revenue product of k is a decreasing function of k, but increasing with g. Its curve thus shifts to the left as g falls. If it were to fall below the curve drawn in figure 2, any household constrained to investing only in k would prefer to lend its wealth at the going riskless rate r, rather than facing the lower prospective returns from investing $\bar{k}$ (or more) domestically.

Figure 2



Lemma 3: $\exists\ g_g^{\$}$ such that for $g_g > g_g^{\$}$, there is no demand for privately supplied public capital.

Proof: Consider $k_f = w_f + \pi F/r$. $g_g^{\$}$ is given by:

$$\frac{k_f}{g_g} = \frac{1-a}{a} \tag{18}$$

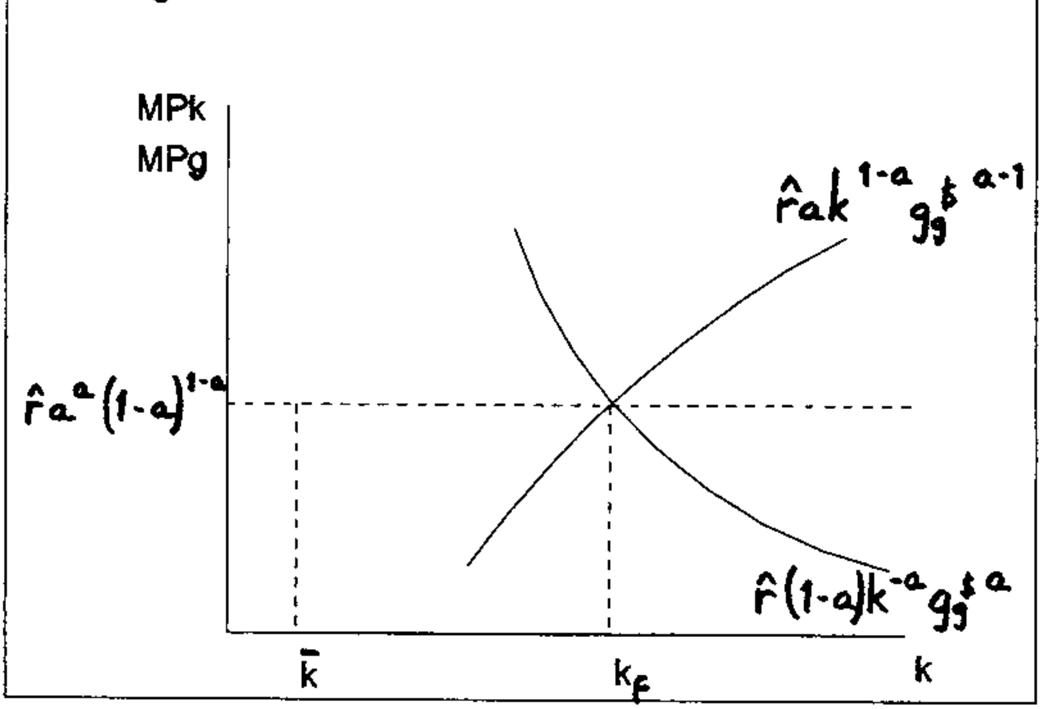
because for $k/g < (1-a)/a$,

$$\frac{\partial \Pi}{\partial k} = \hat{r}(1-a)k^{-a}g^{a} - r > \hat{r}k^{1-a}ag^{a-1} - r = \frac{\partial \Pi}{\partial g} \tag{19}$$

Hence, all private investment would be in private capital until $k = k_f$. But since the richest agent in society has $w = w_f$, then her credit constraint prevents her from borrowing any money to finance purchases of privately supplied public capital. All agents poorer than her would not even be able to reach the optimal private-public capital ratio, and would therefore invest only in k. When $g_g^{\$}$ is supplied by the government, there is therefore no demand for privately supplied public capital in this society, and all observed projects would use only g_g.■

Figure 3 below illustrates the determination of $g_g^{\$}$. The downward sloping curve denotes MPk, as in figure 2, and the upward sloping one denotes MPg. They always cross along the dotted line, which gives the value of both marginal products when the optimal input ratio is being used. If $g_g = g_g^{\$}$, MPk exceeds MPg for all values of k that can be afforded, even by the richest member of society, so there is never any demand for g_p.

Figure 3



The last two results can be summarized as follows: private and public capital are complementary inputs. Their respective marginal products rise with the quantity of the other input. For very low levels of g, therefore, the marginal product of private capital is so low that it becomes more profitable for capital to leave the country (lent at r) than to be invested domestically. Only the very rich ($w > \tilde{w}$) can afford to buy sufficient amounts of privately supplied public capital to make it worthwhile to invest at home. This is reminiscent of capital externalities of the Arrow (1962) type, and could lead to capital flows from poorer to richer countries, thus illustrating the expatriation of capital from poor countries with abysmal infrastructures, such as exist in Africa or Eastern Europe.

On the other hand, if g_g is very high, then the returns to investing in privately supplied public capital are lower than those to investing in private capital. The wealth level of the richest agent is not high enough to enable her to obtain a loan large enough to drive k/g_g to the optimal ratio. Therefore, no one has any incentive to invest in g_p.

One might claim that below $\bar{g}_g$, one is in a poor country where infrastructure, public health and education are so limited that minimum conditions for private investment are lacking, and one observes little incentive to save, capital flight and the existence of a small sector of large modern enterprises which build their own transport and communications network, as well as schools and health centres. In a world above $g_g^{\$}$, on the other hand, one might be in a rich country with an active and efficient government, which provides high quality infrastructure, a reliable health care system and such a good standard of public education that no one ever finds it in their interest to go to the market place for alternative provision of these services. Some European countries, notably in Scandinavia in the 1960s and 1970s, might have approximated this extreme. By and large, however, most countries in the world display a co-existence of government-provided infrastructure, health and education services with privately supplied substitutes. For this reason and because we are interested in the effects of public investment on equity and growth, we limit our attention for the moment to cases where $g_g \in (\bar{g}_g, g_g^{\$})$. This interval was established by Lemmas 2 and 3 as the range of public investment outlays in which projects relying exclusively on government-provided public capital will coexist with projects employing both g_g and g_p. It also implies that at $w = w^*$, there is demand for k. I also assume that $\bar{g}_p > \pi F/r$, thus ensuring that $w^{**} > \bar{k}$. This is only for simplicity.

We are now able to describe the social structure prevailing in this economy, according to the sort of economic activity undertaken at each wealth level.

Proposition 1: In general, agents with wealth $w \in [w^*, w^{**})$ are either borrowing or lending entrepreneurs. The only form of public capital they use is that supplied by the government.
Proof: Substituting the definition of w^* into equation (15) and considering only cases where $g_g > \bar{g}_g$, it follows that:

$$\hat{r}(1-a)(w* + \frac{\pi F}{r})^{-a} g_g^{\ a} \geq r \tag{20}$$

which states that the marginal product of capital is at least equal to the exogenous return on lending. Hence, the poorest individual in the interval will always be a borrower and an entrepreneur. As we move along the interval to higher wealth levels, there are three possible cases, depending on the exogenous level of g_g:

CASE 1)

$$\hat{r}(1-a)(w** + \frac{\pi F}{r})^{-a} g_g^{\ a} \geq r \tag{21}$$

In this case, g_g is high enough, so that it is profitable to invest in k for any $w \in (w^*, w^{**})$. All agents in that interval will therefore borrow the full amount to which they are constrained, and invest it.

CASE 2)

$$\hat{r}(1-a)(w** + \frac{\pi F}{r})^{-a} g_g^{\ a} < r < \hat{r}(1-a)(w**)^{-a} g_g^{\ a} \tag{22}$$

This is a marginal case: all agents in the interval invest their full wealth in k, although the richest portion may not borrow all they are able to. No one in the interval lends.

CASE 3)

$$\hat{r}(1-a)(w**)^{-a} g_g^{\ a} < r \tag{23}$$

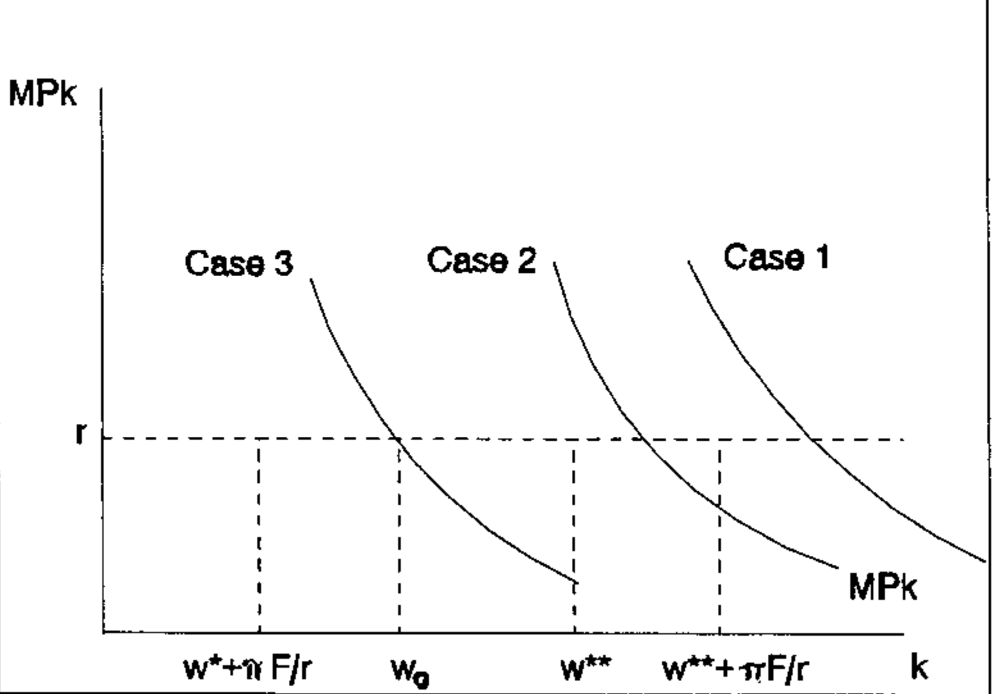
Then all agents with $w \in (w^*, w_0)$ borrow whatever they are allowed up to $(w_0 - w)$ to invest in k, while the richer people with $w \in (w_0, w^{**})$ invest w_0 in a project and lend $(w - w_0)$ at r. w_0 is given by:

$$\hat{r}(1-a) w_0^{\ -a} g_g^{\ a} = r \tag{24}$$

The fact that they use only g_g, and not g_p, follows from part (b) of the proof to Lemma 1. ■

Figure 4 below illustrates the three different cases. In case 1, public investment is large enough to make it profitable for everyone in the middle class interval to invest domestically. Case 2 is a borderline one, but still there are no lenders. In case 3, households richer than w_0 will choose to lend a fraction of their wealth in the international capital market, although they will invest up to w_0 domestically.

Figure 4



Proposition 2: Above w**, there are no lenders. All agents are entrepreneurs, borrowing up to their credit constraint and investing in k and g_p.

<u>Proof:</u> The expected returns to undertaking a project using technology (8) are a homogeneous of degree one function of both types of capital. It follows that there is an optimal k/g ratio. Equating the marginal products yields:

$$\left(\frac{k}{g}\right)_{opt} = \frac{1-a}{a} \qquad (25)$$

With wealth above w**, two cases can arise, depending again on the exogenous level of g_g. Define k' = w** + πF/r and let g_g' be defined by k'/g_g = (1-a)/a. That is, g_g' := ak'/(1-a).

Also, let $\lambda = k/(k+g_p)$ be the share of the loan used to purchase private capital k.

CASE 1) $g_g > g_g'$. Then at w**, the marginal revenue product of k exceeds that of g, so $\lambda = 1$ and $\exists\ w_s > w^{**}$ and $k_s = w_s + \pi F/r > k'$ such that $k_s/g_g = (1\text{-}a)/a$.
For $w > w_s$, $\lambda < 1$ and $d\lambda/dw < 0$, with $\lim_{w\to\infty}\lambda = 1\text{-}a$. Also, for $w > w_s$, the k/g ratio is always at its optimal value, given by (25).

CASE 2) $g_g \le g_g'$. Then at w**, $\lambda \le 1$, as a share of the loan is spent to purchase g_p, so as to bring k/g to its optimal value. As wealth increases, λ falls towards 1-a in the same way as in case 1, while k/g stays constant at (1-a)/a.
But at that ratio:

$$\frac{\partial\Pi}{\partial k} = \frac{\partial\Pi}{\partial g} = \hat{r}a^{a}(1-a)^{1-a} - r > 0 \tag{26}$$

where the inequality is assumed in (A2).
Π_k and Π_g are functions of the ratio (k/g) only, because of the homogeneity property. It is evident that for k/g greater than the optimal, Π_g will be greater than the expression above, whereas for k/g lower than the optimal, Π_k is greater than it. Hence, for any of the cases in proposition 2, agents wealthier than w** will borrow up to their credit constraint and invest fully in whatever input has the higher marginal product, until they equalise, and then will keep that optimal k/g ratio. ■

On the basis of lemma 1 and the two foregoing propositions, we can completely describe the payoffs of agents as a function of their initial wealth levels. Naturally, these will depend on which case of proposition 1 happens to hold (which essentially depends on the exogenous g_g). The flavour in all cases is similar, however, and I now limit consideration to case 1, for simplicity:

Agents with	receive	with probability
$w \in [0, w^*)$	$n + rw$	1

$w \in [w^*, w^{**})$	$\hat{r}\, w + (\hat{r} - r)\, \pi F/r$	q
	0	1-q
$w \in [w^{**}, \infty)$	$\hat{r}\lambda w + (\hat{r}\, \lambda - r)\, \pi F/r$	q
	0	1-q

These describe the three classes: subsistence workers, middle-class entrepreneurs who must rely on the government for their g, and the unconstrained upper class.

4. The Transitional Dynamics and the Steady State Distribution:

The utility formulation in (1) implies that a fixed fraction $1-\alpha$ of wealth at life-end is left as a bequest. Therefore we can identify the intergenerational dynamics for each lineage precisely, in the following manner. If $\theta \in \{0,1\}$ is defined as the random variable that indicates success for a project ($\theta = 1$ with probability q; $\theta = 0$ with probability 1-q)[9], and if it is clear that w_t indicates initial wealth at the beginning of period t, we can write:

$$w_{t+1} = (1 - \alpha) f(w_t, \theta_t) \tag{27}$$

where, of course,
$$f(w_t, \theta_t) = \begin{cases} n + r w_t & \text{if } w_t \in [0, w^*) \\ \theta_t[\hat{r} w_t + (\hat{r} - r)\, \pi F/r] & \text{if } w_t \in [w^*, w^{**}) \\ \theta_t[\lambda_t\, \hat{r} w_t + (\hat{r}\, \lambda_t - r)\, \pi F/r] & \text{if } w_t \in [w^{**}, \infty). \end{cases}$$

Different values for the parameters will generate different graphs for (27), but the parametric restriction in footnote 7 ensures that at least the two first classes exist. If parameter values are such that, in addition :

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9 Since $q_t = f^{-1}\,(g/k)_t$, θ_t is not i.i.d.. But - because $(g/k)_t$ is a function of the current value w_t alone, and not of w_{t-j}, $\forall\, j \neq 0$ - θ_t is independently distributed over time. The same is true of $\lambda_t = k_t/(k_t + g_{pt})$. This share of the loan used to buy k depends only on w_t and on time-invariant parameters π, F and r, but not on any previous value of w.
This implies that $\Pr(w_{t+1} \in A / w_t, w_{t-1}, ..., w_0) = \Pr(w_{t+1} \in A / w_t)$, which allows us to define the Markov process of the wealth variable in the unidimensional state space W, as in the proof of proposition 3 below.

$$w** < \bar{w} = \frac{1-\alpha}{1-\hat{r}\lambda(1-\alpha)}(\hat{r}\lambda - r)\frac{\pi F}{r} \tag{28}$$

also holds, then all three classes will exist in the steady state[10]. Figure 5 below illustrates a configuration of (27) compatible with case 1 of proposition 2, along with the behaviour of λ and k/g as wealth evolves. Figure 6 does the same for case 2.

$\bar{w}$ is obtained simply from equating w_{t+1} (θ_t=1) in (27) for $w_t \in [w^{**}, \infty)$, to w_t. It is, in other words, the wealth level which generates, for a successful project, a bequest identical to itself. Visual inspection of the transitional diagram suffices to see that $\bar{w}$ will constitute in some sense the upper bound of a limiting wealth distribution. This is because any individual receiving $w > \bar{w}$ as a bequest will necessarily bequeath to her child less than she received herself. Anyone in $[0, w^*)$ will bequeath more than she received, but in $[w^*, \bar{w}]$ there is a positive probability associated with either outcome. In that range, entrepreneurial success leads to upward social mobility, and failure leads to (a rather extreme form of) downward mobility.

Given the nature of the model outlined above, it turns out to be possible to be fairly specific about the long run behaviour of this stochastic dynamic system, as is shown by the following proposition.

Proposition 3: The stochastic process defined by equation (27) is a Markov process, with the property that the cross-section distribution $G_t(w)$ converges to a unique invariant limiting distribution G*, from any initial distribution $G_0(w)$.
Proof: see Appendix.

---

[10] For (28) to hold, it is clearly necessary - although not sufficient - that the consumption share of lifetime wealth, α, be large enough so that $\hat{r}\lambda(1 - \alpha) < 1$. I assume this to be the case. The intuition is that if savings are too large (α small), fuelling growth, there may be no upper bound on the limiting wealth distribution. On the other hand, if savings are too small (α too close to 1), (28) may not hold, indicating an 'impoverished' ergodic distribution.

Figure 5

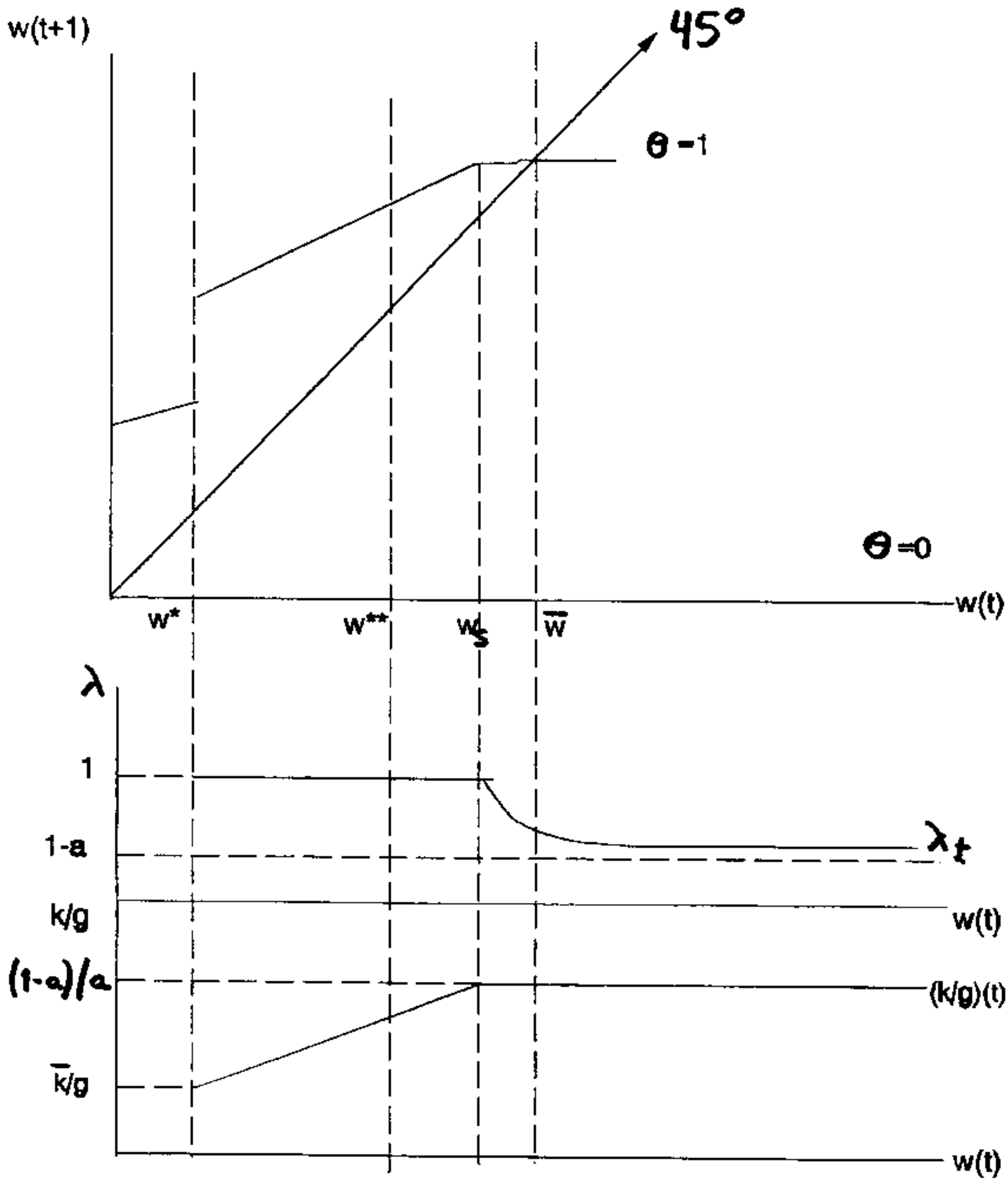
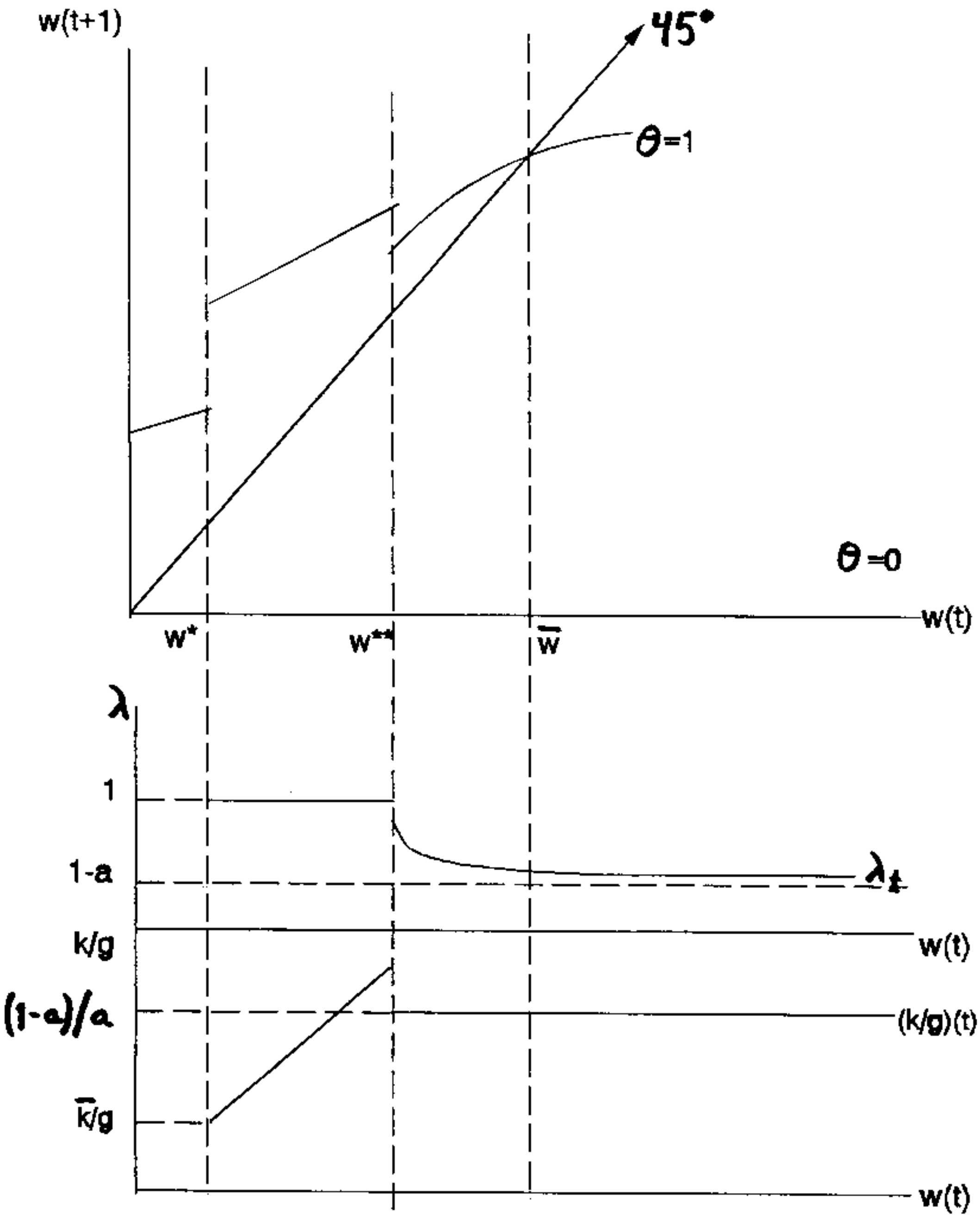


Figure 6



The intuition behind the proof is as follows. We first establish that the process in (27) is a linear Markov process in the set of all possible wealth values. This essentially requires that the expected value of w_{t+1} given w_t be the same as the expectation given the complete history of $\{w_t\}$. Then we show that there is a wealth interval to which any lineage with wealth outside that interval will eventually tend. This is the interval $[0, \tilde{w}]$. This is in turn used to show, drawing on an established Markov Convergence Theorem, that there exists a unique invariant probability measure which every lineage will eventually face. Finally, we establish the existence of a law of large numbers which allows us to reinterpret that probability measure as the steady state cross section distribution G*.

This limiting distribution G* is of interest because it is essentially the steady state - or equilibrium - distribution of the system. Proposition 3 is quite a strong result, in that it establishes the existence, uniqueness and stability of such a steady state distribution. It also follows from the analysis that G* has a compact support in $[0, \tilde{w}]$, and that w* must belong to this interval. If (28) holds, so will w**, so that the equilibrium wealth distribution in this economy contains three social classes. The poorest people are completely credit constrained. They can not borrow enough to acquire the minimum amount of private capital needed to start a project. They obtain their (deterministic) income from a subsistence activity and from lending their initial wealth at the ruling interest rate.

There is then a middle class which consists of entrepreneurs that use publicly provided infrastructure, government hospitals and schools, and so on. Their projects are risky, but their expected value is high (compared to the alternative subsistence activity). Above them there is a richer group, who also undertake entrepreneurial projects similar to those of the middle class. The difference is that the mix of public and private capital used is different. Whereas the middle classes are unable to borrow enough to buy privately supplied public capital, the rich are able to do so. Their input ratios are therefore unambiguously closer to (and indeed most often at) the optimal one. They are therefore able to produce more efficiently than their credit constrained fellow citizens.

5. Policy Impacts on the Steady State Distribution:

In the previous section, I established that the existence of capital market imperfections destroys the first best result that wealth inequalities tend to disappear in the long run. In this model, inequality persists in the long run invariant distribution, in the same way as in other models in this literature. Furthermore, I showed that if the production function uses two complementary inputs, one of which is partially (and freely) supplied by the government, and which I have identified with infrastructure, health or education services, and if there is a fixed cost for acquiring additional amounts of this input privately, then the investing class can be divided into two sub groups.

In this section I establish that, over a plausible range, increasing government expenditure in infrastructure not only increases the aggregate wealth level in steady state,[11] but also reduces inequality of opportunity between the middle and upper classes (Proposition 4). I also return to the case where $g_g < \hat{g}_g$, and show that in that case there is a second effect through which increases in government spending can reduce inequality, and have a positive impact on domestic output separate from that of the foreign transfer, which arises through a relaxation in the credit constraint (Proposition 5).

To do so, I wish to concentrate on ex-ante inequality - inequality in the distribution of expected end-of-period wealth at the beginning of the period - rather than on actual realized wealth levels. One reason for this is to abstract from the stochastic nature of the production function, which was made unrealistically stark for mathematical simplicity. Also, the expected payoff Π, defined in equation (13), and the expected rate of return Π/w, are eminently plausible concepts for discussing equality of opportunity.

Proposition 4: Consider case 2 of proposition 2. Then, for all $g_g \in [\bar{g}_g, g_g']$, any increase in g_g leads to expected income gains for all agents with wealth greater than w^*, but these gains are proportionately larger for those with wealth $w \in [w^*, w^{**})$ than for those with wealth w

[11] Which is obvious since all government expenditure in this model is financed by a transfer from abroad. The issue of taxation is addressed in a companion paper (Ferreira, 1994).

$\in$ [w**, $\bar{w}$]. That is to say, government spending helps reduce the disparity between the expected rates of return facing the rich - who have access to private infrastructure, education and health services - and poorer entrepreneurs, who are credit constrained.

Proof: Noting that we are still considering case 1 of proposition 1, we can use (13) to write the expected rate of return for agents with w $\in$ [w*, w**) as follows:

$$\frac{\Pi}{w} = \frac{\hat{r}}{w}\left(w + \frac{\pi F}{r}\right)^{1-a} g_g^{\,a} - \frac{\pi F}{w} \tag{29}$$

Hence:

$$\frac{\partial \Pi/w}{\partial g_g} = \frac{a\hat{r}}{w}\left[\frac{w + \frac{\pi F}{r}}{g_g}\right]^{1-a} \tag{30}$$

Analogously, the expected rate of return for agents with w $\in$ [w**, $\bar{w}$] can be written as:

$$\frac{\Pi}{w} = \frac{\hat{r}}{w}\left[\lambda(w + \frac{\pi F}{r})\right]^{1-a}\left[g_g + (1-\lambda)(w + \frac{\pi F}{r})\right]^{a} - \frac{\pi F}{w} \tag{31}$$

So:

$$\frac{\partial \Pi/w}{\partial g_g} = \frac{a\hat{r}}{w}\left[\frac{\lambda\ (w + \frac{\pi F}{r})}{g_g + (1-\lambda)(w + \frac{\pi F}{r})}\right]^{1-a} \tag{32}$$

Because $g_g \in [\bar{g}_g, g_g']$, $\lambda < 1$ for all households with w $\in$ [w**, $\bar{w}$]. This is case 2 of proposition 2. Since $0 < a, \lambda < 1$, the right hand side of (30) is unambiguously greater than the right hand side of (32) and

$$\left.\frac{\partial \Pi/w}{\partial g_g}\right|_{w\,\in\,[w*,w**)} > \left.\frac{\partial \Pi/w}{\partial g_g}\right|_{w\,\in\,[w**,\bar{w}]} \tag{33}$$

Hence, an increase in g_g increases the expected rate of return facing poorer entrepreneurs by more than the one facing richer entrepreneurs. ■

Now, (33) implies a reduction in inequality of opportunity iff:

$$\left.\frac{\Pi}{w}\right|_{w \in [w^*, w^{**})} < \left.\frac{\Pi}{w}\right|_{w \in [w^{**}, \bar{w}]} \tag{34}$$

But this must be true by self-selection, as agents facing Π/w given by (31) can choose to face (29) by setting control variable $\lambda = 1$. If they choose not to do so, as risk neutral agents, it must be because the right-hand side of (31) exceeds that of (29).

This result has therefore established that for a certain range, increasing public provision of infrastructure capital reduces inequality of opportunity, as measured by the ex-ante expected rate of return, between poorer entrepreneurs and their richer counterparts. Intuitively, this is because the constant returns to scale production function has an optimal input ratio, which maximises expected returns for a given input expenditure. Whereas the richer entrepreneurs are able to choose that ratio, those who are credit constrained - and therefore unable to purchase extra infrastructure privately - are not. The fact that they are poorer, that is, leads them to face a lower expected return, thus reducing their chances of upward social mobility. By increasing the stock of per capita public capital available, the government is not only increasing wealth, but also increasing equality of opportunity.[12]

Conversely, by reducing public investment (cutting g_g) the government will lead richer entrepreneurs to purchase greater quantities of the private substitute g_p, so as to stay at the optimal production ratio. Since this is impossible for the poor, the expenditure cut effectively worsens the impact of the moral hazard in repayment problem. It is because this problem affects only the poor entrepreneurs that it increases inequality of opportunity.

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[12] An appropriate disclaimer here is that the existence of subsistence lenders with deterministic incomes in this model prevents us from stating that increases in g_g will lead to increases in equality of opportunity **over the entire distribution.** They do not use any form of capital, and the increase in its stock makes both kinds of entrepreneurs richer relative to them. Bottom-sensitive inequality measures may therefore record an increase in inequality overall. This is why Proposition 4 is stated in terms of entrepreneurs only. The result is the more relevant the lower $\bar{k}$. Indeed, a special case of this model, with no fixed costs to the acquisition of private capital ($\bar{k} = 0$), would make Proposition 4 applicable to the whole distribution.

Whilst it is possible to make an unambiguous statement about the effect of extra government investment on equality of opportunity, it is much harder to derive analytical results in this model for equality of outcome (i.e. of realized wealth). This is essentially because we know nothing about the specific functional form of G*. The proof of proposition 3 establishes the existence of a unique λ^*, but says nothing of its shape. The only two categorical statements that can be made about G*, therefore, are: (1) that the support of the distribution increases with g_g, i.e: $\partial\bar{w}/\partial g_g > 0$. This is because, (a):

$$\frac{\partial \bar{w}}{\partial \lambda} = \frac{(1-\alpha)\hat{r}\frac{\pi F}{r}[1 - \hat{r}\lambda(1-\alpha)] + \hat{r}(1-\alpha)^2(\hat{r}\lambda - r)\frac{\pi F}{r}}{[1 - \hat{r}\lambda(1-\alpha)]^2} > 0 \tag{35}$$

(b):

$$\lambda = \frac{k}{k + g_p} , \qquad so \qquad \frac{\partial \lambda}{\partial g_p} < 0 \tag{36}$$

and (c): for $\lambda < 1$, $k / (g_g + g_p) = (1 - a)/ a$, so $\partial g_p/\partial g_g < 0$. Hence:

$$\frac{\partial \bar{w}}{\partial g_g} = \frac{\partial \bar{w}}{\partial \lambda}\frac{\partial \lambda}{\partial g_p}\frac{\partial g_p}{\partial g_g} > 0 \tag{37}$$

And (2), that the Generalized Lorenz Curve associated with the steady state distribution prior to an increase in g_g, G* (low g_g), **can not** dominate the Generalised Lorenz Curve associated with the distribution after such an increase, G* (high g_g). This is simply because the mean of the former is lower than that of the latter (see Cowell, 1995).

That is to say: ex-ante inequality (of opportunity) decreases unambiguously with public investment financed from transfers from abroad, over the interval $[\bar{g}_g, g_g']$; ex-post inequality (of realized wealth) behaves ambiguously, but we can be certain that the distribution with lower public investment per capita will not have second order stochastic dominance over that with higher g_g.

These results reflect the central manner in which increases in government spending can create more equality of opportunity in countries where there already is a reasonable amount of public capital (i.e, $g_g > \tilde{g}_g$). But as I stated in Lemma 2, for countries with an anaemic public sector ($g_g < \tilde{g}_g$), entrepreneurial activity is restricted only to the very rich, with everyone else consigned to subsistence work and lending at the exogenous rate. The impact of extra government spending in this scenario is more dramatic, and is the subject of the following proposition:

Proposition 5: If (i): a country has such a low level of g_g that $w' > w^{**}$,

(ii): a stronger version of assumption A3 holds, so that :

$$\hat{r}\bar{k}^{1-a}\bar{g}_g{}^a - r(\bar{k} + g_p{}') > n \tag{38}$$

then:

a) A unit increase in g_g will reduce the minimum wealth required for investment by a unit, thereby unambiguously increasing the number of agents involved in domestic entrepreneurial production;

b) This will lead to an increase in overall income additional to the increase in returns to those who were already investing .

Proof: a) Equation (17), which defines g_p', can be written explicitly as:

$$g_p{}' = \left[\frac{r}{\hat{r}(1-a)}\right]^{\frac{1}{a}}\bar{k} - g_g \tag{39}$$

So (16) can be expressed directly as a function of g_g:

$$w' = \left[1 + \left(\frac{r}{\hat{r}(1-a)}\right)^{\frac{1}{a}}\right]\bar{k} - g_g - \frac{\pi F}{r} \tag{40}$$

Clearly,

$$\partial w'/\partial g_g = -1 \tag{41}$$

which proves part (a).

b) Call w' before the increase in g_g: w'', and that after the increase: w'''. Then the gain in income from the increase is given by:

$$\Delta Y = \int_{w'''}^{w''} [E(\Pi(w)) - (n + rw)]\,dw \tag{42}$$

because the law of large numbers is applicable to the continuum of agents in [w''', w''][13]. This can be written out as:

$$\Delta Y = \int_{w'''}^{w''} [\tilde{r}k^{1-a}(g_g + g_p)^a - r(k + g_p) - n]\,dw \tag{43}$$

This expression is positive by (38), because, at w''', $g_p = g_p'$ and the first term within the brackets is equal to the first term in the left hand side of (38), by the definition of g_p'. As wealth increases from w''' to w'', the value of the bracketed expression only increases.■

Intuitively, proposition 5 captures the fact that when a country has so little infrastructure that it is not worthwhile even for those people who can afford to buy private capital to invest, and only those who can afford to buy that as well as a complement of privately supplied public capital are in business, then increases in government spending reduce the amount of g_p which is required to make investing at home as profitable as lending abroad. This in turn reduces the minimum wealth at which people find it worthwhile to invest (w'), thereby expanding opportunities to poorer people, and expanding domestic activity. These gains in output and income will occur until w' is driven down to w**. Any further increases in g_g will not lead to extra output until it reaches $\bar{g}_g$, because in that intermediate range, the agents which would like to invest are credit constrained and can not buy the necessary amounts of g_p.[14]

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[13] See step IV of the proof of proposition 3.

[14] Note that to be sure of preventing a collapse of the limiting invariant distribution to a single point (a poverty trap), the assumption in the first inequality of footnote 7 (A3), needs to be strengthened. Instead of $(\bar{k} - \pi F/r)$, write w'.

6- Conclusions:

This paper combines a capital market imperfection of the moral hazard in repayment variety with a production function where private capital is complementary to the output of public investment. This combination of plausible assumptions generates a unique, invariant steady state wealth distribution, to which the system converges from any initial distribution (Proposition 3). This distribution G* is non-degenerate - meaning that wealth inequality persists in the long-run - and a set of parameter values exists such that it will include three social classes. The poorest members of society are so tightly credit constrained that they are unable to afford the fixed start-up costs required to invest in any project. In this model, they dedicate their full effort supply to a subsistence activity, and lend whatever wealth they have in the open international capital markets (Lemma 1). Alternative stories might have been told, involving monitoring technologies à la Banerjee and Newman (1993), where they would have been employees instead of subsistence farmers. This may well be an interesting approach, but I have chosen to avoid the complications of an endogenous labour market in this paper.

The middle class is wealthy enough to be granted loans which enable them to buy the necessary amounts of private capital to invest domestically. But their credit ceilings prevent them from acquiring private substitutes for public infrastructure, health or education (Proposition 1). Only the richest class is able to complement the free public provision of these services with private alternatives, should it be profitable to do so. Unlike the middle class citizens, who might choose to lend some of their wealth at the international rate r, the rich always prefer to invest at home, given that they are able to allocate their investment optimally between the two inputs, and therefore face a higher expected return to their projects (Proposition 2).

To concentrate on the impact of changes in public investment which are brought about by changes in external financing, such as in the aftermath of the debt crisis of the 1980s, I assumed all government expenditure was financed by transfers from abroad. A lower bound in per capita levels of expenditure was derived, below which the middle class disappears. With minimal government expenditure on schools, hospitals, roads, telecommunications and the like, domestic production becomes unviable for all but the very rich, who can afford to build their own infrastructure (Lemma 2). These are highly polarised societies, with masses of poor subsistence farmers and artisans, and a small - highly privatised - modern sector. Increases in government investment in such an environment not only increase the returns to those who are already investing in the country, but also lead to an expansion in domestic entrepreneurial activity amongst poorer people, previously consigned to the subsistence sector, whose wealth was being lent in the global capital markets (proposition 5).

Even if public expenditure is above that lower bound, so that a middle class does exist, there is a range over which increases in public investment are shown to lead to an increase in (ex-ante) equality of opportunity among entrepreneurs. If the subsistence sector is relatively small, this can become equivalent to greater equality of opportunity for the whole society (Proposition 4). Although the stochastic nature of returns prevents us from being able to say much about the impact on (ex-post) equality of outcome, it is not possible for a distribution associated with a lower level of public investment to dominate (in the Generalized Lorenz Curve sense) one with higher public expenditure.

The general conclusions from this exercise are that the "pure redistribution" policy implications of the literature surveyed by Aghion and Bolton (1992) are too restrictive. In that literature, the long term consequences of transfers to the poor, in terms of effort supply in a rational expectations environment are not discussed. But those, as well as targeting and practical implementation difficulties, are the reasons why developing country governments find it so difficult to run generous rich-to-poor transfer schemes.[15] This paper shows that 'traditional' roles of the government, such as investing in sectors with high transaction costs or very large positive externalities, can also act to improve both equity and efficiency.

In particular, it is shown that productive public investment can alleviate inequality of opportunity even if expenditures are uniformly distributed, rather than targeted at the poor. This is relevant news for countries where the administrative or political difficulties associated

[15] I continue to ignore, as I have throughout the paper, considerations of political economy. This is despite the fact that they may very well provide the most realistic and important explanations for this absence. These results should be seen as a benchmark for the case of perfectly benign and competent governments.

with targeting are severe. Before this conclusion can be seized upon by advocates of universal benefits in general, however, the issue of optimal distribution of g_g should be addressed. What if the government is able to costlessly allocate g_g in different amounts to different households? What allocation pattern should it choose? There would appear to be scope for interesting further research into this issue, and into its interplay with different tax rates for different activities, using this dynamic distributional framework.

In any case, governments should be cautious as they embark in privatisation programmes, seek to reduce their budget deficits and search for "private sector involvement" everywhere. There are very good grounds for privatizing inefficient steel mills, and excellent reasons for avoiding running budget deficits over long periods. But if there are sectors where market failures outweigh state failures, so that the government has a comparative advantage in supply, and if there are large costs to acquiring the private substitutes, reductions to public investment in them can be detrimental to both long term efficiency and long term fairness and equity. Similar lessons apply to international agencies and bilateral donors involved in determining transfers from abroad to developing countries. Given that sudden reductions in these have often led to sharp cuts in public investment, it is necessary to take into account their pernicious impact upon growth and equality in the recipient countries.

APPENDIX

Proof of Proposition 3

Definitions:

Let $W \subseteq R^+$ be the set of all possible values for the wealth variable w, its state space.

Let Ω be the Borel algebra of W.

Let $\Phi(W, \Omega)$ be the set of all signed measures on (W, Ω); and let $\Lambda(W, \Omega)$ be the set of all probability measures in that measurable space.

Lemma A1: $\Phi(W, \Omega)$ is a vector space.

Proof: see chapter 11.3 in Stokey and Lucas (1989).

Let the total variation norm on this space be given by:

$$\|\lambda\| = \limsup \sum_{i=1}^{k} \lambda(A_i) \qquad \textbf{(A1)}$$

where the supremum is over all finite partitions of W into disjoint measurable sets.

Following Stokey and Lucas (1989), we say that a sequence of probability measures $\{\lambda_n\}$ converges in the total variation norm to the probability measure λ if $\lim_{n\to\infty} \|\lambda_n - \lambda\| = 0$.

Let $P(w, A) = \Pr[f(w_t, \theta_t) \in A]$, where $P: W \times \Omega \to [0,1]$ and $f(w, \theta)$ is defined by equation (27) above, be a candidate transition function.

Plan:

The proof is in four steps. Step I proves that P(w, A) is a transition function. Step II defines condition M and proves that the transition function satisfies it. Step III proves that the function thus satisfies the Strong Convergence Theorem for Markov Processes in infinite state spaces (Theorem 11.12 in Stokey and Lucas (1989)). Step IV applies a Law of Large Numbers to reinterpret the probability measure for a lineage as the expected long-run cross-section distribution at period t.

Step I:

P(w, A) is a transition function iff:

a) P(w, A) is a probability measure on (W, Ω), for each $w \in W$; and

b) P(w, A) is a Ω-measurable function for each $A \in \Omega$.

(a) is true because:

$P(w, \varnothing) = Pr\,[f(w, \theta) \in \varnothing] = 0$, by the definition of the probability operator.

$P(w, W) = Pr\,[f(w, \theta) \in W] = 1$, since W is the state space for all w.

$P(w, A) = Pr\,[f(w, \theta) \in A] \geq 0$ for all $A \in \Omega$, by the definition of the probability operator.

And for all disjoint $B \in \Omega$, $P(w, \cup^{\infty} B_i) = \sum_i^{\infty} P(w, B_i)$, because:

$Pr\,[f(w, \theta) \in \cup^{\infty} B_i] = \sum_i^{\infty} Pr\,[f(w, \theta) \in B_i]$, by the definition of the probability operator.

(b) is true because: $P(w, A) = Pr\,[f(w, \theta) \in A]$, which can be expressed as follows:

$$P(w,A) = \int_A f(w,\theta)\, dPr(\theta) = \int_A 1_A[\ \ ] f_A(w,\theta)\, dPr(\theta) \qquad \text{(A2)}$$

where 1_A is an indicator function, and f_A denotes the value of f when in A. But both of these functions are Ω-measurable for any $A \in \Omega$, because indicator functions are always measurable, and f_A is piecewise continuous, with a finite number of discontinuities (maximum of 2) in A. Piecewise continuous real-valued functions defined on bounded intervals of real numbers (such as A), are known to be measurable in the relevant Borel algebra (see Ash, 1972, p.34). That the product of two Ω-measurable functions is itself Ω-measurable follows from Theorem 1.5.6 in Ash (1972, p.39).

Hence P(w, A) is a transition function on (W, Ω), as this step set out to prove.

Step II:

Condition M is discussed in Onicescu (1969), and proved originally by Stokey and Lucas (1989), Chapter 11.4. It states that , in the complete metric space defined by Λ(W, Ω) and the total variation norm, there exist an integer $N \geq 1$, and a real number $\varepsilon > 0$, such that for any

$A \in \Omega$, either $P^N(w, A) > \varepsilon$, for all $w \in W$; or $P^N(w, A^c) > \varepsilon$, for all $w \in W$.[16]

In this case, let $A = [0, \bar{w}]$. Then,

a) all $w \geq 0$, as $W \subseteq R^+$.

b) if $w > \bar{w}$, $P(w, \{0\}) = 1-q$, and $\{0\} \subset A$. This establishes that $P^N(w, A) > \varepsilon$, for w such that $q(w) < 1-\varepsilon$ (with $N = 1$). But there may be w such that $q(w) > 1-\varepsilon$. In that case, $P(w_t, \{w: w < w_t\}) = 1$. Hence, $\exists\, M$, $1 < M < \infty$, such that $P^M(w_t, A) = 1 > \varepsilon$. This establishes for all $w > \bar{w}$, that there exist $N \in \{1, M\}$ and $\varepsilon > 0$ such that $P^N(w, A) > \varepsilon$.

c) if $w^* \leq w \leq \bar{w}$, $P(w, \{0\}) = 1-q$, and $\{0\} \subset A$.

For w such that $q(w) > 1-\varepsilon$, $P(w_t, \{w: w > w_t\}) = q$. Therefore, $\exists\, J \geq 1$ such that $P^J(w_t, A) > \varepsilon$. This is because either $w_{t+j} \in A$ or it belongs to $\{w: w > \bar{w}\}$, in which case item (b) above applies.

d) if $0 \leq w < w^*$, $P(w, A) = 1 > \varepsilon$.

Hence, for all $w \in W$, $\exists\, N \geq 1$, $\varepsilon > 0$, such that for $A = [0, \bar{w}]$, $P^N(w, A) > \varepsilon$.

P (w, A) satisfies condition M, as this step set out to prove.

Step III:

Theorem 11.12 in Stokey and Lucas (1989) establishes sufficient conditions for Strong Convergence of Markov Processes in infinite state spaces. Applied to this example, it states that: **if** (a) P is a transition function on (W, Ω); (b) T^* is the adjoint operator associated with P; and (c) P satisfies condition M for $N \geq 1$, $\varepsilon > 0$; **then** there exists a unique probability measure $\lambda^* \in \Lambda(W, \Omega)$ such that:

$$\|T^{*Nk}\lambda_0 - \lambda^*\| \leq (1-\epsilon)^k \|\lambda_0 - \lambda^*\| \qquad \textbf{(A3)}$$

for all $\lambda_0 \in \Lambda(W, \Omega)$, $k = 1, 2, \ldots$

Clearly, step I establishes (a) and step II establishes (c). For the definition of the adjoint operator in (b), see Stokey and Lucas (1989, Ch 8).

To see that (A2) indeed establishes that any λ_0 converges to λ^* in the total distribution norm,

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[16] $A^c = W \backslash A$, the complement of set A in the relevant state space.

note that it implies that $\lim_{k\to\infty} \| T^{*Nk} \lambda_0 - \lambda^* \| = 0$. This establishes the existence, uniqueness and stability of an invariant probability measure in the set of all probability measures in the measurable space of the state space W, to which any initial measure will converge in time through the transition function P, as this step set out to prove.

Step IV:

Because the problem was defined for a continuum of agents, λ^* can be reinterpreted as the limiting long run cross section distribution $G^*(w)$, by the law of large numbers, as this step sets out to show.

Let T be the earliest date at which every lineage in the population has converged to λ^*. At each $t > T$, there is then a continuum of agents indexed by i in [0, w upper bar], each drawing w_{it} from a distribution λ^*, such draws being independent. Let I = [0, w upper bar]. Using the Kolmogorov construction, let $\mathbf{H} = \mathbb{R}^I$ be the probability space in which all possible sequences of draws may be simultaneously represented, and Υ be its Borel algebra, derived as in Judd (1985). Finally, let $G_h(c) = l(\{i \backslash h(i) \leq c\})$, for an arbitrary $c \in \mathbb{R}$, be a sample distribution at a fixed time $t > T$. l simply denotes a Lebesgue measure.

For any Lebesgue measurable subset A of $\mathbb{R}$, and $A^t = \{h \in \mathbf{H} \backslash h(t) \in A\}$, $t \in I$, define the probability measure μ satisfying:

(i): $\mu(A^t) = \lambda^*(A)$, and

(ii): $\mu(A^{t1} \cap A^{t2} \cap \dots \cap A^{tn}) = \mu(A^{t1}) \times \mu(A^{t2}) \times \dots \times \mu(A^{tn})$; $t_i \in I$, $t_i \neq t_j$ for $i \neq j$, $n = 1,2, \dots$

Then the law of large numbers which allows us to state that the cross-section distribution of realized draws on the support $[0, \bar{w}]$ is λ^*, is given as follows:

$\mu(\{h \backslash G_h(.) = \lambda^*(.)\}) = 1$.

Theorem (1) in Judd (1985) proves that $N = \{h \in \mathbf{H} \backslash G_h$ fails to exist$\}$ is not μ-measurable, but for any $r \in [0,1]$, there exists an extension of μ, μ_r, such that $\mu_r(N) = r$. We choose $r = 0$, so N is a set of measure zero. Therefore G_h exists a.e. in $\mathbf{H}$. We call $\mu_0 = \mu_0$.

Theorem (2) in Judd (1985) proves that $L = \{h \backslash G_h$ exists and $G_h = \lambda^*\}$ is not μ_0-measurable, but there are extensions of μ_0 for which G_h exists almost surely and $G_h = \lambda^*$ holds with probability χ, **any** $\chi \in [0, 1]$. We choose $\mu^* = \mu_0(\chi)$ when $\chi = 1$.

Hence $G^* = G_h(w) = \lambda^*$ with probability 1. The law of large numbers can be stated, and it is true. This allows us to reinterpret the lineage probability measure λ^* as the long run cross-section distribution $G^*(w)$, as this step set out to show.

Therefore, $G_t(w)$ converges to a unique invariant distribution G^*, from any initial $G_0(w)$.

■

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