Indirect force control

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Outline

- Compliance control
  - Active compliance
- Impedance control
  - Active impedance
  - Inner motion control
  - Three-DOF impedance control
  - Six-DOF impedance control

Compliance control

- Active compliance

$$\boldsymbol{\gamma} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{T}(\boldsymbol{\varphi}_e) \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{K}_{Pp} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{K}_{Po} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{p}_{de} \\ \Delta\boldsymbol{\varphi}_{de} \end{bmatrix}$$

$$\Delta\boldsymbol{p}_{de} = \boldsymbol{K}_{Pp}^{-1}\boldsymbol{f}$$
$$\Delta\boldsymbol{\varphi}_{de} = \boldsymbol{K}_{Po}^{-1}\boldsymbol{T}^{\mathrm{T}}(\boldsymbol{\varphi}_e)\boldsymbol{\mu}$$

$$\boldsymbol{f} = \boldsymbol{K}_f(\boldsymbol{p}_e - \boldsymbol{p}_0)$$

… at steady state (position/force)

$$\boldsymbol{p}_{e,\infty} = \left(\boldsymbol{I} + \boldsymbol{K}_{Pp}^{-1}\boldsymbol{K}_f\right)^{-1}\left(\boldsymbol{p}_d + \boldsymbol{K}_{Pp}^{-1}\boldsymbol{K}_f\boldsymbol{p}_o\right)$$
$$\boldsymbol{f}_\infty = \left(\boldsymbol{I} + \boldsymbol{K}_f\boldsymbol{K}_{Pp}^{-1}\right)^{-1}\boldsymbol{K}_f(\boldsymbol{p}_d - \boldsymbol{p}_o)$$

Impedance control

- Active impedance

force/torque sensor

$$\boldsymbol{\tau} = \boldsymbol{B}(\boldsymbol{q})\boldsymbol{\alpha} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{J}^{\mathrm{T}}(\boldsymbol{q})\boldsymbol{h}$$
$$\boldsymbol{\alpha} = \boldsymbol{J}^{-1}(\boldsymbol{q})\left(\begin{bmatrix} \boldsymbol{a}_p \\ \boldsymbol{a}_o \end{bmatrix} - \dot{\boldsymbol{J}}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}\right)$$

$$\boldsymbol{a}_p = \ddot{\boldsymbol{p}}_d + \boldsymbol{K}_{Mp}^{-1}(\boldsymbol{K}_{Dp}\Delta\dot{\boldsymbol{p}}_{de} + \boldsymbol{K}_{Pp}\Delta\boldsymbol{p}_{de} - \boldsymbol{f})$$

$$\boldsymbol{a}_o = \boldsymbol{T}(\boldsymbol{\varphi}_e)(\ddot{\boldsymbol{\varphi}}_d + \boldsymbol{K}_{Mo}^{-1}(\boldsymbol{K}_{Do}\Delta\dot{\boldsymbol{\varphi}}_{de} + \boldsymbol{K}_{Po}\Delta\boldsymbol{\varphi}_{de} - \boldsymbol{T}^{\mathrm{T}}(\boldsymbol{\varphi}_e)\boldsymbol{\mu})) + \dot{\boldsymbol{T}}(\boldsymbol{\varphi}_e,\dot{\boldsymbol{\varphi}}_e)\dot{\boldsymbol{\varphi}}_e$$

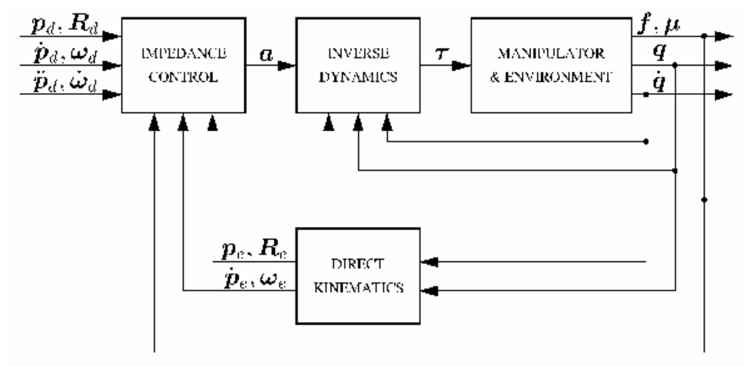
$$\Downarrow$$

$$\boldsymbol{K}_{Mp}\Delta\ddot{\boldsymbol{p}}_{de} + \boldsymbol{K}_{Dp}\Delta\dot{\boldsymbol{p}}_{de} + \boldsymbol{K}_{Pp}\Delta\boldsymbol{p}_{de} = \boldsymbol{f}$$
$$\boldsymbol{K}_{Mo}\Delta\ddot{\boldsymbol{\varphi}}_{de} + \boldsymbol{K}_{Do}\Delta\dot{\boldsymbol{\varphi}}_{de} + \boldsymbol{K}_{Po}\Delta\boldsymbol{\varphi}_{de} = \boldsymbol{T}^{\mathrm{T}}(\boldsymbol{\varphi}_e)\boldsymbol{\mu}$$

Impedance control (cont’d)

A.D. MCCXXIV

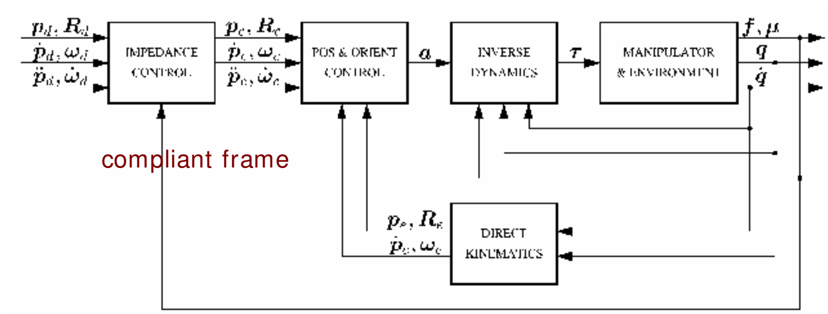
- Impedance control (w/ force/torque measurements)



Impedance control (cont’d)

A.D. MCCXXIV

- Inner motion control
  - Enhanced disturbance rejection



Impedance control (cont’d)

A.D. MCCXXIV

- Three-DOF impedance control
  - Translational impedance

$$\boldsymbol{M}_p \Delta \ddot{\boldsymbol{p}}_{dc} - \boldsymbol{D}_p \Delta \dot{\boldsymbol{p}}_{dc} - \boldsymbol{K}_p \Delta \boldsymbol{p}_{dc} - \boldsymbol{f}$$

$$\Delta \boldsymbol{p}_{dc} = \boldsymbol{p}_d - \boldsymbol{p}_c$$

  - Linear acceleration (inner motion loop)

$$\boldsymbol{a}_p = \ddot{\boldsymbol{p}}_c + \boldsymbol{K}_{Dp} \Delta \dot{\boldsymbol{p}}_{ce} + \boldsymbol{K}_{Pp} \Delta \boldsymbol{p}_{ce}$$

$$\Delta \boldsymbol{p}_{ce} = \boldsymbol{p}_c - \boldsymbol{p}_e$$

Impedance control (cont’d)

A.D. MCCXXIV

- Six-DOF impedance control
  - Rotational impedance (Euler angles)

$$\boldsymbol{M}_o \Delta \ddot{\boldsymbol{\varphi}}_{dc} + \boldsymbol{D}_o \Delta \dot{\boldsymbol{\varphi}}_{dc} + \boldsymbol{K}_o \Delta \boldsymbol{\varphi}_{dc} = \boldsymbol{T}^{\mathrm{T}}(\boldsymbol{\varphi}_c)\boldsymbol{\mu}$$

$$\Delta \boldsymbol{\varphi}_{dc} = \boldsymbol{\varphi}_d - \boldsymbol{\varphi}_c$$

  - Infinitesimal orientation displacement

$$\boldsymbol{\mu}_E = \boxed{\boldsymbol{T}^{-\mathrm{T}}(\boldsymbol{\varphi}_c)}\boldsymbol{K}_o \boxed{\boldsymbol{T}^{-1}(\boldsymbol{\varphi}_c)} \Delta \boldsymbol{\omega}_{dc} \mathrm{d}t$$

task geometric inconsistency

  - Angular acceleration (inner motion loop)

$$\boldsymbol{a}_o = \boldsymbol{T}(\boldsymbol{\varphi}_e)\left(\ddot{\boldsymbol{\varphi}}_c + \boldsymbol{K}_{Do} \Delta \dot{\boldsymbol{\varphi}}_{ce} + \boldsymbol{K}_{Po} \Delta \boldsymbol{\varphi}_{ce}\right) + \dot{\boldsymbol{T}}(\boldsymbol{\varphi}_e, \dot{\boldsymbol{\varphi}}_e)\dot{\boldsymbol{\varphi}}_e$$

$$\Delta \boldsymbol{\varphi}_{ce} = \boldsymbol{\varphi}_c - \boldsymbol{\varphi}_e$$

Impedance control (cont'd)

- Rotational impedance (alternative Euler angles)

$$\boldsymbol{M}_o\ddot{\boldsymbol{\varphi}}_{dc} + \boldsymbol{D}_o\dot{\boldsymbol{\varphi}}_{dc} + \boldsymbol{K}_o\boldsymbol{\varphi}_{dc} = \boldsymbol{T}^{\mathrm{T}}(\boldsymbol{\varphi}_{dc})\,{}^{c}\boldsymbol{\mu}$$

$${}^{c}\boldsymbol{R}_d = \boldsymbol{R}_c^{\mathrm{T}}\boldsymbol{R}_d \;\Longrightarrow\; \boldsymbol{\varphi}_{dc}$$

- Infinitesimal orientation displacement

$${}^{c}\boldsymbol{\mu}_E \simeq \boldsymbol{T}^{-\mathrm{T}}(\boldsymbol{0})\boldsymbol{K}_o\boldsymbol{T}^{-1}(\boldsymbol{0})\Delta{}^{c}\boldsymbol{\omega}_{dc}\mathrm{d}t$$

$$= \boldsymbol{K}_o\Delta{}^{c}\boldsymbol{\omega}_{dc}\mathrm{d}t$$

task geometric consistency

(XYZ Euler angles + diagonal stiffness)

- Angular acceleration (inner motion loop)

$$\boldsymbol{a}_o = \dot{\boldsymbol{\omega}}_d - \dot{\boldsymbol{T}}_e(\boldsymbol{\varphi}_{de},\dot{\boldsymbol{\varphi}}_{de})\dot{\boldsymbol{\varphi}}_{de} - \boldsymbol{T}_e(\boldsymbol{\varphi}_{de})\left(\ddot{\boldsymbol{\varphi}}_{dc} + \boldsymbol{K}_{Do}(\dot{\boldsymbol{\varphi}}_{dc} - \dot{\boldsymbol{\varphi}}_{de}) + \boldsymbol{K}_{Po}(\boldsymbol{\varphi}_{dc} - \boldsymbol{\varphi}_{de})\right)$$

Impedance control (cont'd)

- Rotational impedance (angle/axis)

$$\boldsymbol{M}_o\Delta{}^{c}\dot{\boldsymbol{\omega}}_{dc} + \boldsymbol{D}_o\Delta{}^{c}\boldsymbol{\omega}_{dc} + \boldsymbol{K}_o'\,{}^{c}\boldsymbol{o}_{dc} = {}^{c}\boldsymbol{\mu}$$

$${}^{c}\boldsymbol{o}_{dc} = f(\vartheta_{dc})\,{}^{c}\boldsymbol{r}_{dc} \qquad \boldsymbol{K}_o' = 2\psi\boldsymbol{\Omega}^{\mathrm{T}}({}^{c}\boldsymbol{r}_{dc},\vartheta_{dc})\boldsymbol{K}_o$$

$${}^{c}\dot{\boldsymbol{o}}_{dc} = \boldsymbol{\Omega}({}^{c}\boldsymbol{r}_{dc},\vartheta_{dc})\Delta{}^{c}\boldsymbol{\omega}_{dc}$$

- Infinitesimal orientation displacement

$${}^{c}\boldsymbol{\mu}_E \simeq 2\psi\left(f'(0)\right)^2\boldsymbol{K}_o\Delta{}^{c}\boldsymbol{\omega}_{dc}\mathrm{d}t \qquad \psi = 1/2(f'(0))^2$$

$$= \boldsymbol{K}_o\Delta{}^{c}\boldsymbol{\omega}_{dc}\mathrm{d}t$$

task geometric consistency

- Angular acceleration (inner motion loop)

$$\boldsymbol{a}_o = \dot{\boldsymbol{\omega}}_c + \boldsymbol{K}_{Do}\Delta\boldsymbol{\omega}_{ce} + \boldsymbol{K}_{Po}\boldsymbol{o}_{ce}'$$

$$\boldsymbol{o}_{ce}' = \frac{1}{2}\left(\boldsymbol{S}(\boldsymbol{n}_e)\boldsymbol{n}_c + \boldsymbol{S}(\boldsymbol{s}_e)\boldsymbol{s}_c + \boldsymbol{S}(\boldsymbol{a}_e)\boldsymbol{a}_c\right)$$

Impedance control (cont'd)

- Rotational impedance (quaternion)

$$M_o \Delta^c \dot{\omega}_{dc} + D_o \Delta^c \omega_{dc} + K_o'^{\,c}\epsilon_{dc} = {}^c\mu$$

$${}^cR_d = R_c^T R_d \implies {}^c\epsilon_{dc}$$

$$K_o' = 2E^T(\eta_{dc}, {}^c\epsilon_{dc})K_o$$

$$E(\eta, \epsilon) = \eta I - S(\epsilon)$$

- Infinitesimal orientation displacement

$${}^c\mu_E \simeq 2\psi \left(f'(0)\right)^2 K_o \Delta^c \omega_{dc} \mathrm{d}t$$

$$= \underline{K_o \Delta^c \omega_{dc} \mathrm{d}t}$$

task geometric consistency

- Angular acceleration (inner motion loop)

$$a_o = \dot{\omega}_c + K_{Do} \Delta\omega_{ce} + K_{Po} R_c {}^c\epsilon_{ce}$$