Robot Localization
and
Kalman Filters

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Outline

- Robot Localization
- Probabilistic Localization
- Kalman Filters
- Kalman Localization
- Kalman Localization with Landmarks

Robot Localization

- Localization a key problem
- Available location information
  - Relative Measurements
    - Driving: wheel encoders, accerelometers, gyroscopes
    - Frequent, but increasing error
  - Absolute Measurements
    - Sensing: GPS, vision, laser, landmarks
    - Less frequent, but bounded error

Probabilistic Localization

- Probabilistic approach
  - Consider whole space of locations
- Belief
  - $\mathrm{Bel}(x_k) = P(x_k \mid d_1, \ldots , d_k)$
  - Get belief as close to real distribution as possible
  - Prior Belief
    - $\mathrm{Bel}^{-}(x_k) = P(x_k \mid z_1, a_1, \ldots , z_{k-1}, a_{k-1})$
  - Posterior Belief
    - $\mathrm{Bel}^{+}(x_k) = P(x_k \mid z_1, a_1, \ldots , z_{k-1}, a_{k-1}, z_k)$

Probabilistic Localization

- Localization equations:
  - $\mathrm{Bel}^{-}(x_k) = P(x_k \mid z_1, a_1, \ldots, z_{k-1}, a_{k-1})$
    $= \int P(x_k \mid a_{k-1}, x_{k-1}) \cdot \mathrm{Bel}^{+}(x_{k-1})\, dx_{k-1}$
  - $\mathrm{Bel}^{+}(x_k) = P(x_k \mid z_1, a_1, \ldots, z_{k-1}, a_{k-1}, z_k)$
    $$= \frac{P(z_k \mid x_k)\, \mathrm{Bel}^{-}(x_k)}{P(z_k \mid z_1, a_1, \ldots, z_{k-1}, a_{k-1})}$$

Markov Assumption

- Implementation Issues:
  - Motion model: $P(x_k \mid a_{k-1}, x_{k-1})$
  - Measurement model: $P(z_k \mid x_k)$
  - Representation of belief

Kalman Filters

- Representation of belief
  - Gaussian function
  - Mean and (co)variance
  - Initial belief: $Bel(x_0) = N(x_0, P_0)$
- Motion model
  - $x_k = Ax_{k-1} + Ba_{k-1} + w_{k-1}$, where $w_k \sim N(0, Q_k)$
- Measurement model
  - $z_k = Hx_k + v_k$, where $v_k \sim N(0, R_k)$

Kalman Filters

- Representation of belief
  - Gaussian function
  - Mean and (co)variance
  - Initial belief: $\mathrm{Bel}(x_0) = N(x_0, P_0)$

- Motion model
  - $x_k = Ax_{k-1} + Ba_{k-1} + w_{k-1}$, where $w_k \sim N(0, Q_k)$
  - $P(x_k \mid a_{k-1}, x_{k-1}) = N(Ax_{k-1}, Q_k)$

- Measurement model
  - $z_k = Hx_k + v_k$, where $v_k \sim N(0, R_k)$
  - $P(z_k \mid x_k) = N(Hx_{k,} R_k)$

Kalman Filters

- Prior belief: $\text{Bel}^{-}(\, x_k\,) = N(\,\hat{x}_k^{-},\, P_k^{-}\,)$
  - Prior localization estimate

- Posterior belief: $\text{Bel}^{+}(\, x_k\,) = N(\,\hat{x}_k^{+},\, P_k^{+}\,)$

Kalman Filters

- Prior belief: $Bel^{-}(x_k) = N(\hat{x}_k^{-}, P_k^{-})$
  - Prior location estimate: $\hat{x}_k^{-}$
  - Prior uncertainty: P_k^{-}
- Posterior belief: $Bel^{+}(x_k) = N(\hat{x}_k^{+}, P_k^{+})$
  - Posterior location estimate: $\hat{x}_k^{+}$
  - Posterior uncertainty: P_k^{+}

Kalman Filters

- Prior belief: $\mathrm{Bel}^{-}(x_k) = N(\hat{x}_k^{-}, P_k^{-})$
  - $\hat{x}_k^{-} = A \cdot \hat{x}_{k-1}^{+} + B \cdot \hat{a}_{k-1}$
  - $P_k^{-} = A \cdot P_{k-1}^{+} \cdot A^T + B \cdot U_{k-1} \cdot B^T + Q_{k-1}$

Kalman Filters

- Prior belief: $Bel^{-}(x_k) = N(\hat{x}_k^{-}, P_k^{-})$
  - $\hat{x}_k^{-} = A \cdot \hat{x}_{k-1}^{+} + B \cdot \hat{a}_{k-1}$

    Prior location estimate

    Posterior location estimate

    Last relative measurement

  - $P_k^{-} = A \cdot P_{k-1}^{+} \cdot A^T + B \cdot U_{k-1} \cdot B^T + Q_{k-1}$

Kalman Filters

- Prior belief: $Bel^{-}(x_k) = N(\hat{x}_k^{-}, P_k^{-})$
  - $\hat{x}_k^{-} = A \cdot \hat{x}_{k-1}^{+} + B \cdot \hat{a}_{k-1}$

    Prior location estimate

    Posterior location estimate

    Last relative measurement

  - $P_k^{-} = A \cdot P_{k-1}^{+} \cdot A^T + B \cdot U_{k-1} \cdot B^T + Q_{k-1}$

    Prior uncertainty

    Posterior uncertainty

    Relative measurement uncertainty

    Motion uncertainty

Kalman Filters

- Posterior belief: $Bel^{+}(x_k) = N(\hat{x}_k^{+}, P_k^{+})$
  - $\hat{x}_k^{+} = \hat{x}_k^{-} + K_k \cdot (z_k - H \cdot \hat{x}_k^{-})$
  - $K_k = P_k^{-} \cdot H^T \cdot (H \cdot P_k^{-} \cdot H^T + R_k)^{-1}$
  - $P_k^{+} = (I - K_k \cdot H) \cdot P_k^{-}$

Kalman Filters

- Posterior belief: $\mathrm{Bel}^{+}(x_k) = N(\hat{x}_k^{+}, P_k^{+})$
  - $\hat{x}_k^{+} = \hat{x}_k^{-} + K_k \cdot \overbrace{(\, z_k - \underbrace{H \cdot \hat{x}_k^{-}}_{\text{Measurement prediction}}\,)}^{\text{Residual}}$

    Posterior state estimate

    Prior state estimate

    Kalman Gain

    True measurement

  - $K_k = P_k^{-} \cdot H^T \cdot (H \cdot P_k^{-} \cdot H^T + R_k)^{-1}$
  - $P_k^{+} = (I - K_k \cdot H) \cdot P_k^{-}$

Kalman Filters

- Posterior belief: $Bel^{+}(x_k) = N(\hat{x}_k^{+}, P_k^{+})$
  - $$\hat{x}_k^{+} = \hat{x}_k^{-} + K_k \cdot \overbrace{(z_k - \underbrace{H \cdot \hat{x}_k^{-}}_{\text{Measurement prediction}})}^{\text{Residual}}$$

    Posterior state estimate

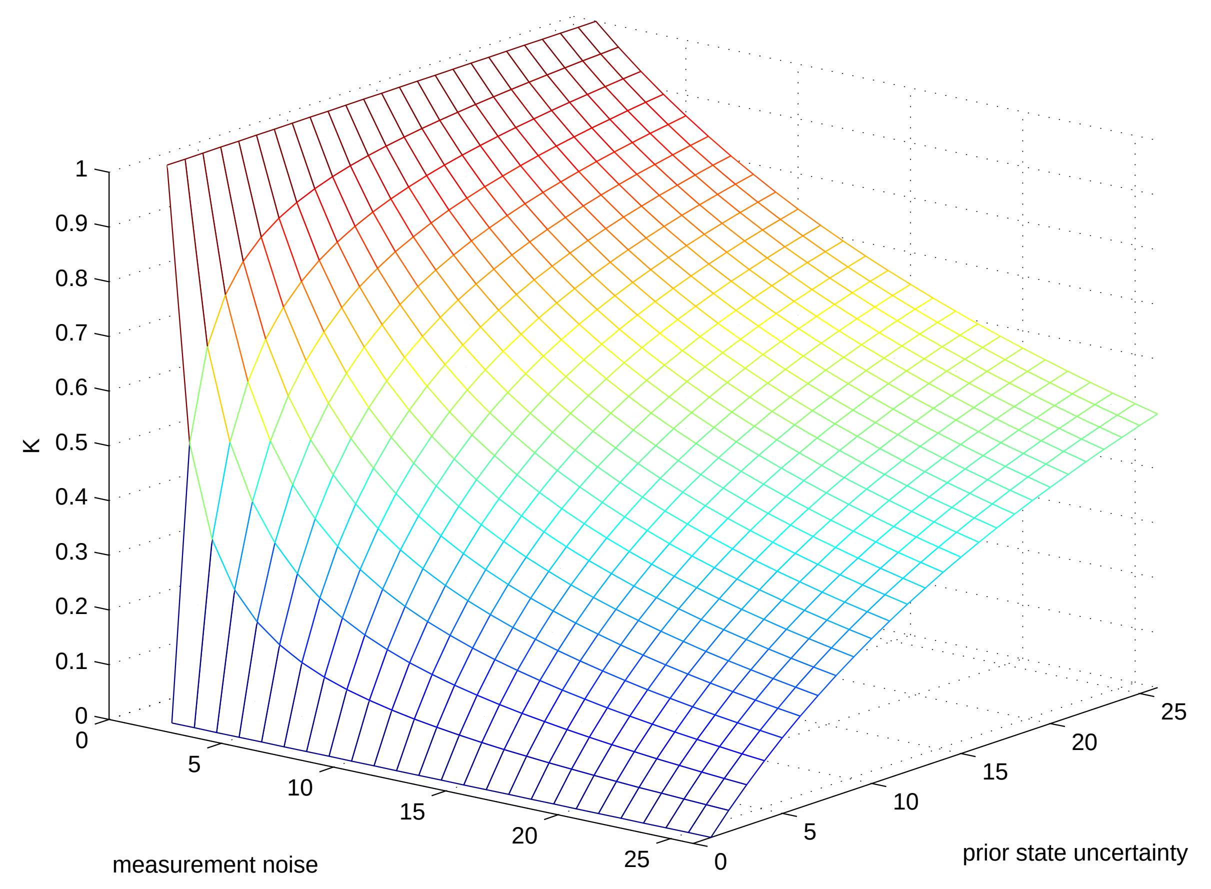
    Prior state estimate

    Kalman Gain

    True measurement

  - $$K_k = P_k^{-} \cdot H^T \cdot \underbrace{(H \cdot P_k^{-} \cdot H^T + R_k)}_{\text{Measurement residual uncertainty}}^{-1}$$
  - $$P_k^{+} = (I - K_k \cdot H) \cdot P_k^{-}$$

Kalman Gain


K
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
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0
5
10
15
20
25
measurement noise
0
5
10
15
20
25
prior state uncertainty

Kalman Filters

- Posterior belief: $\mathrm{Bel}^{+}(x_k) = N(\hat{x}_k^{+}, P_k^{+})$
  - $\hat{x}_k^{+} = \hat{x}_k^{-} + K_k \cdot \overbrace{(z_k - \underbrace{H \cdot \hat{x}_k^{-}}_{\text{Measurement prediction}})}^{\text{Residual}}$

    Posterior state estimate

    Prior state estimat

    Kalman Gain

    True measurement

  - $K_k = P_k^{-} \cdot H^{T} \cdot \underbrace{(H \cdot P_k^{-} \cdot H^{T} + R_k)}_{\text{Measurement residual uncertainty}}{}^{-1}$
  - $P_k^{+} = (I - K_k \cdot H) \cdot P_k^{-}$

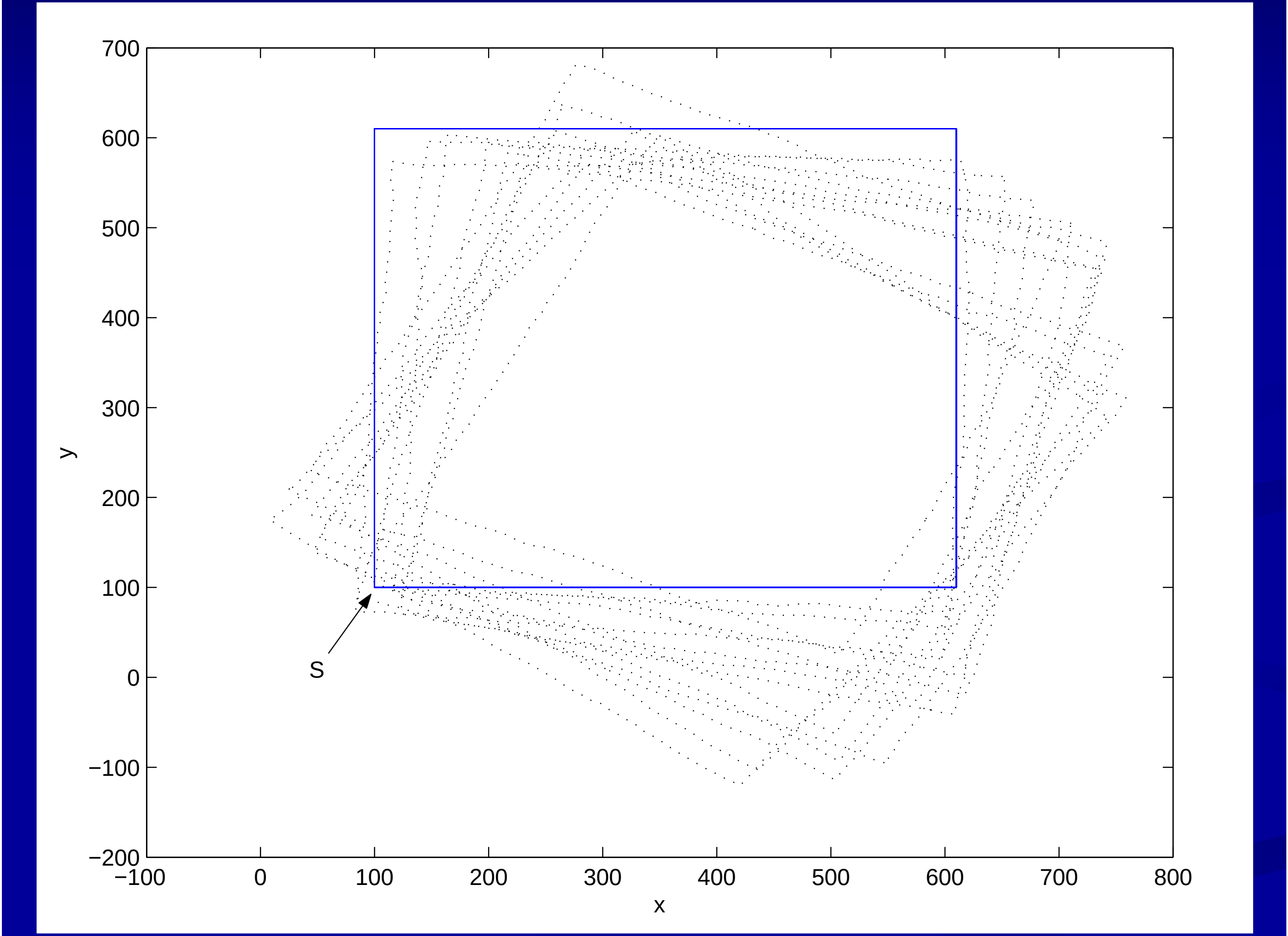
Extended Kalman Filter

- Nonlinear motion and measurement models
- Linearization around estimated trajectory
  - Partial derivatives of nonlinear model for A, B, H
  - Close to linear over uncertainty region
- Drawbacks
  - Evaluation at every time step
  - Linearization errors

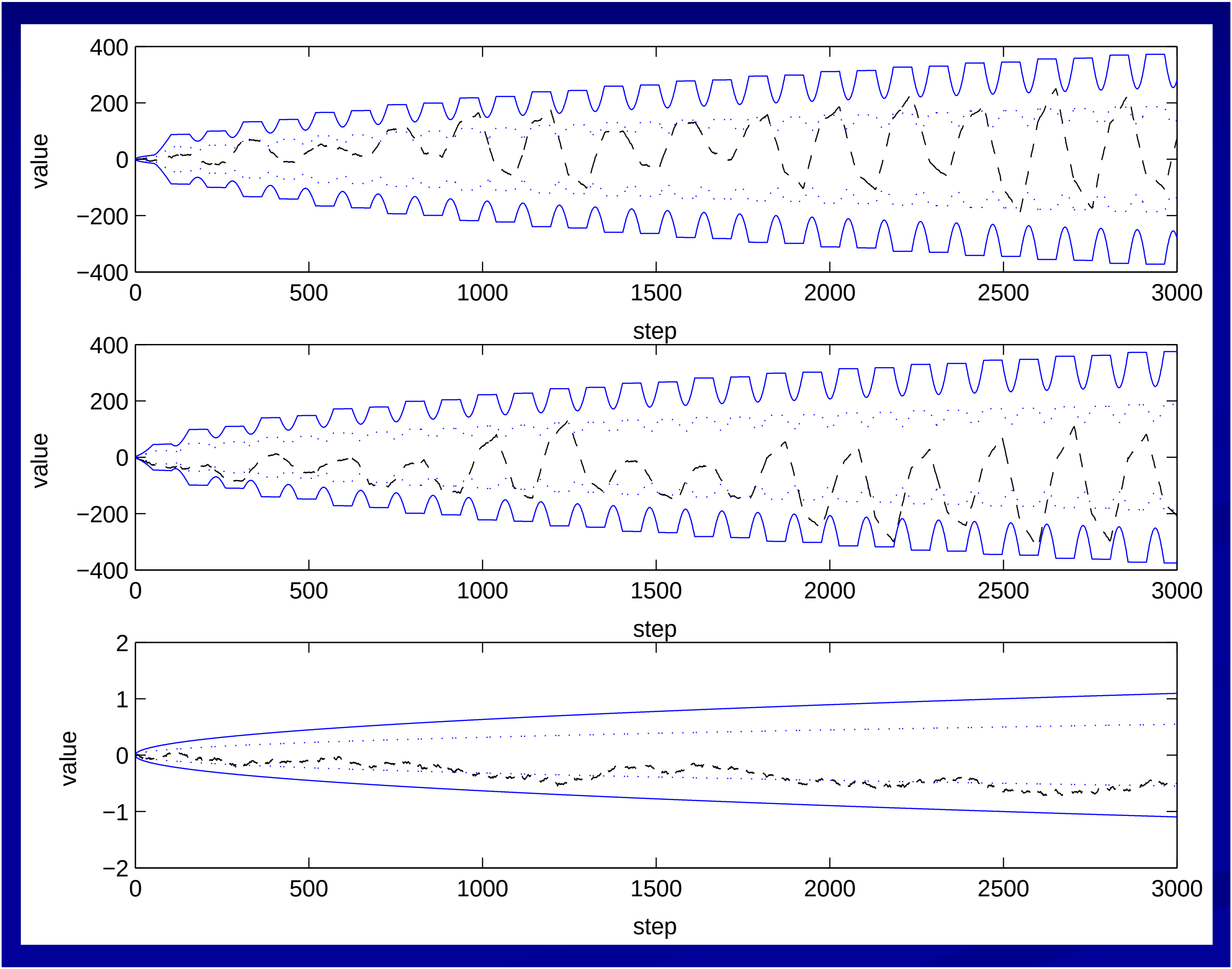
Kalman Localization

- Localization instances
  - Position Tracking
    - Initial belief with peak at true initial location
  - Global Localization
    - Initial uniform belief
  - Kidnapped Robot
    - Initial belief with peak far from true location

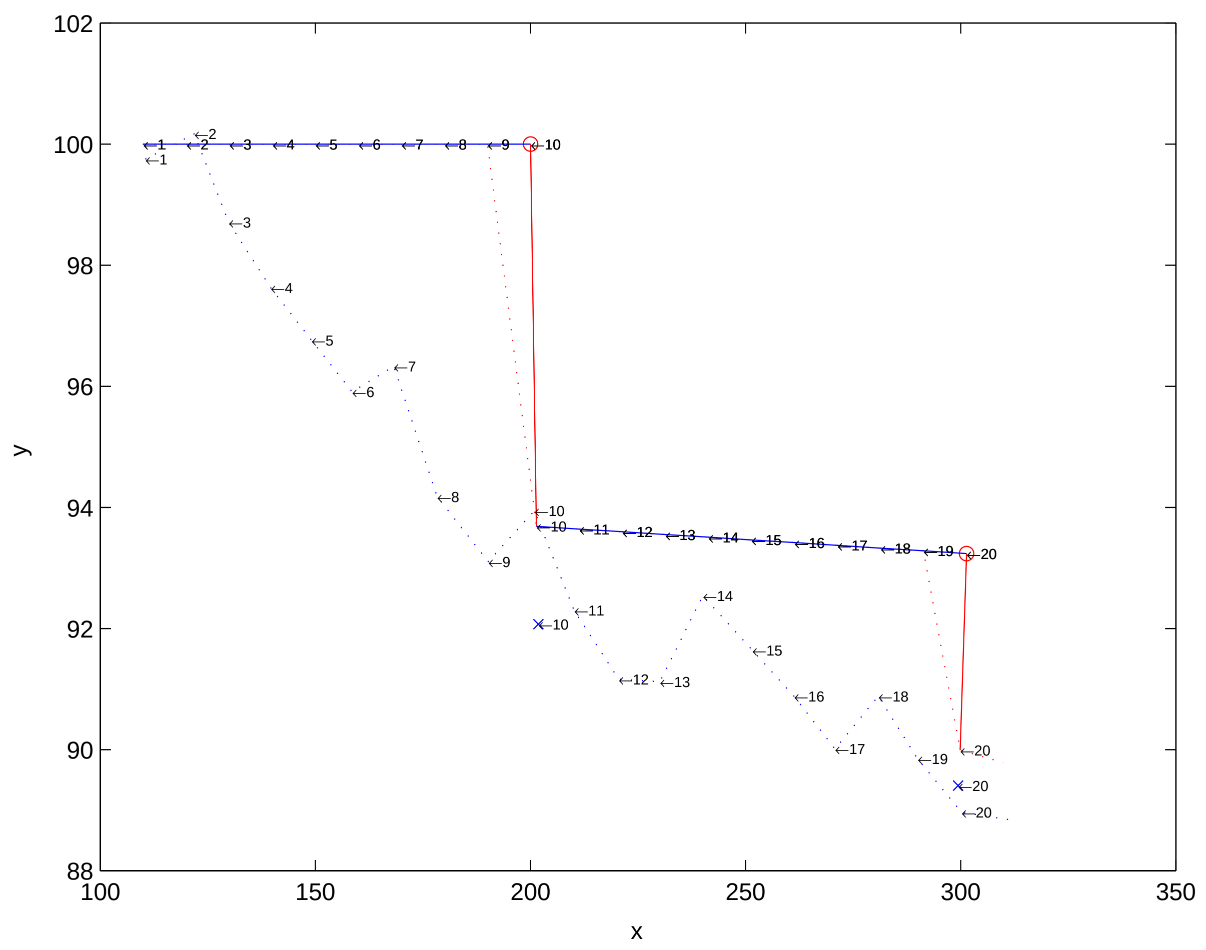
Position Tracking



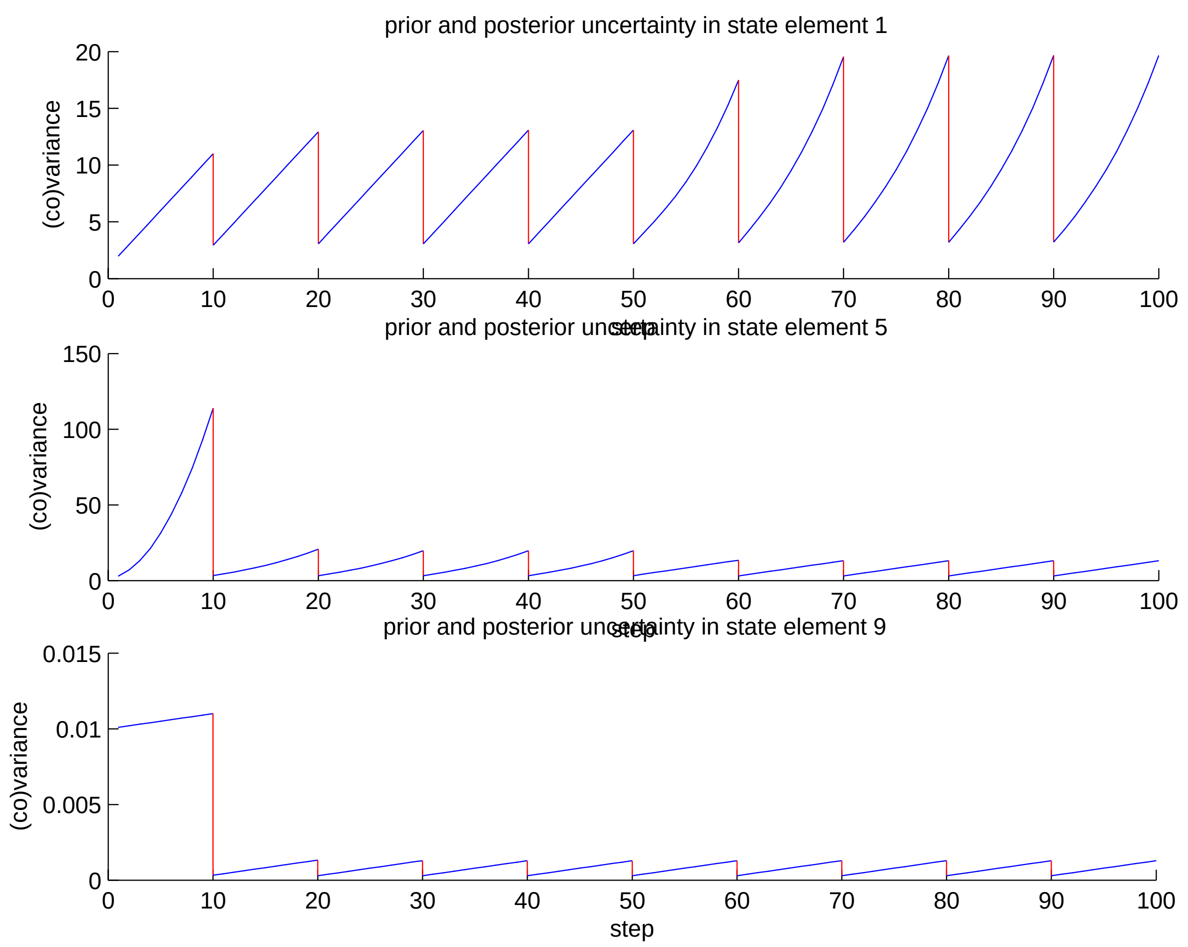
Position Tracking



Position Tracking



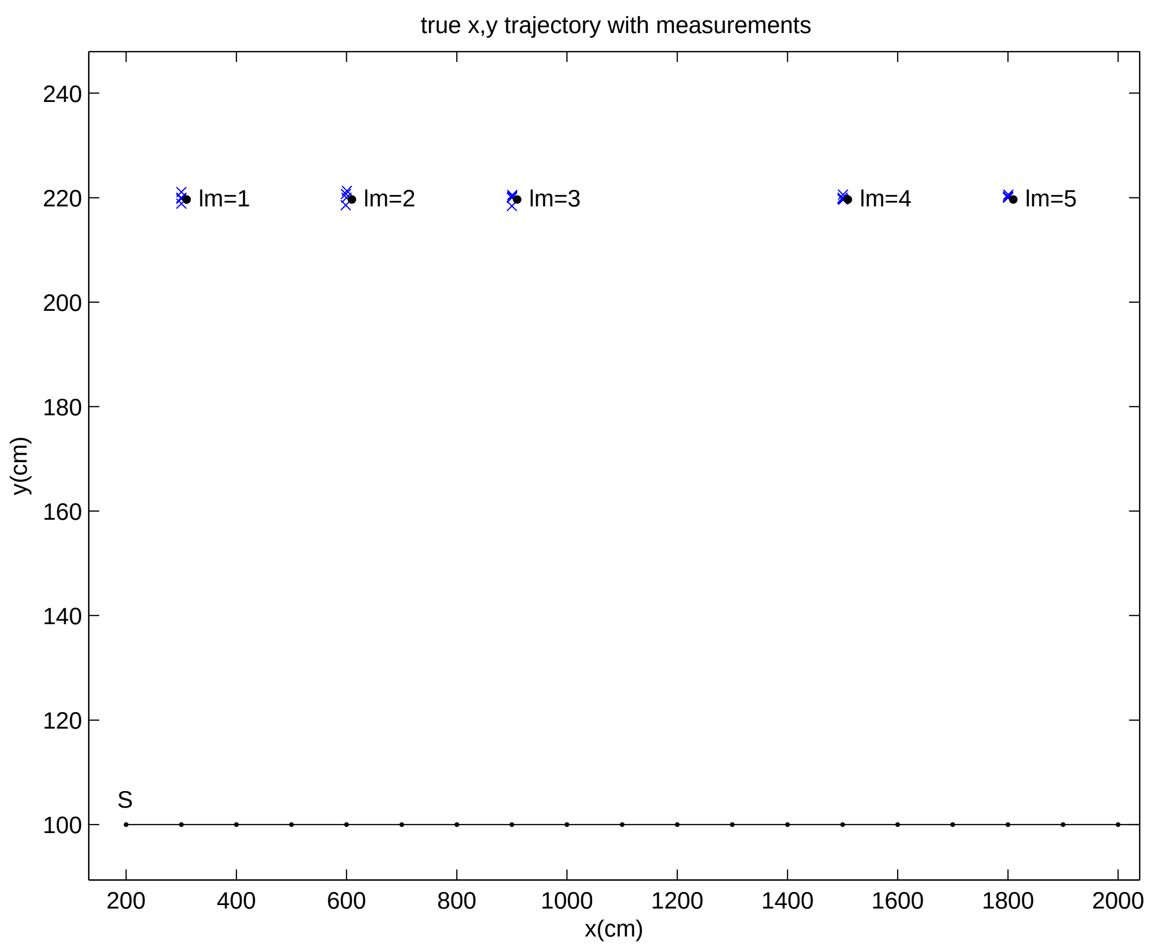
Infrequent Measurements



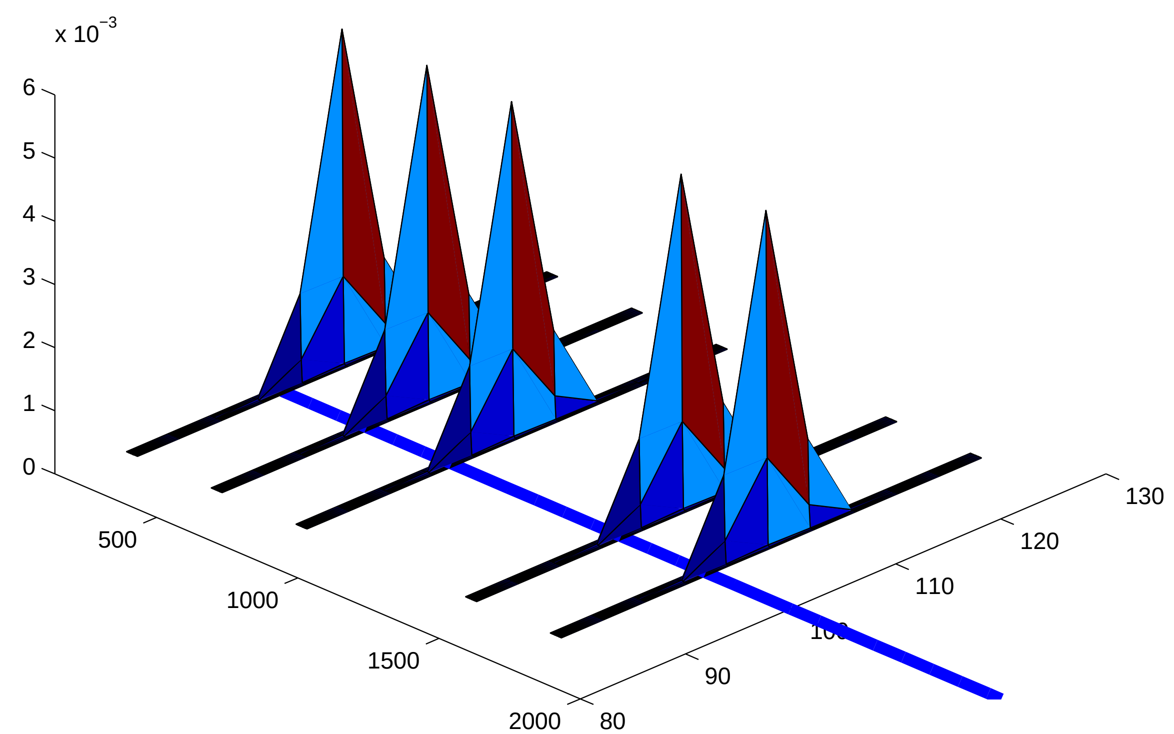
Kalman Localization with Landmarks

- Uniquely identifiable landmarks
    - 1:1 correspondence
- Type identifiable landmarks
    - 1:n correspondence
    - Kalman Filter framework extension
        - Multiple state beliefs
        - Probability for each belief

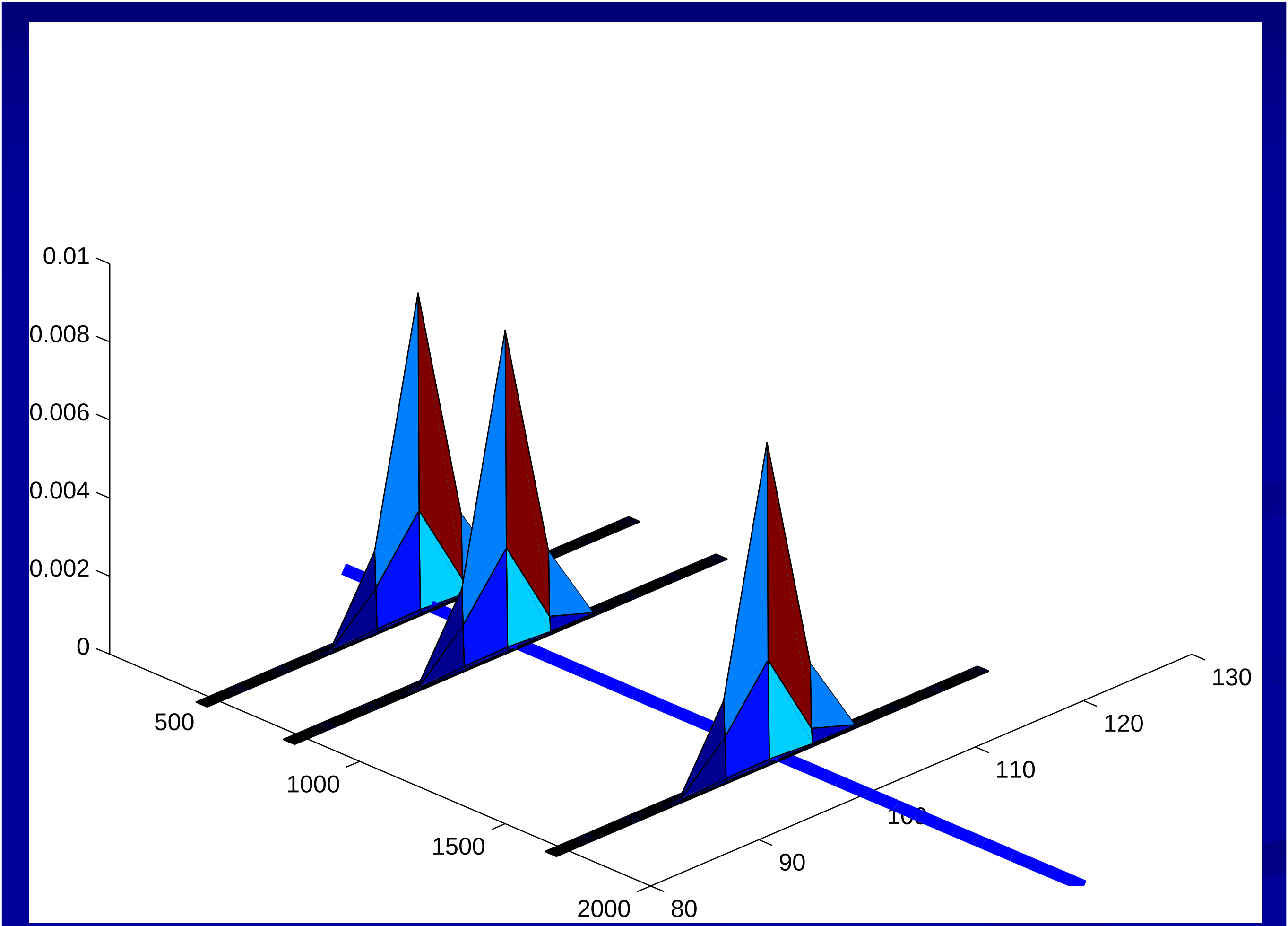
Type Identifiable Landmarks



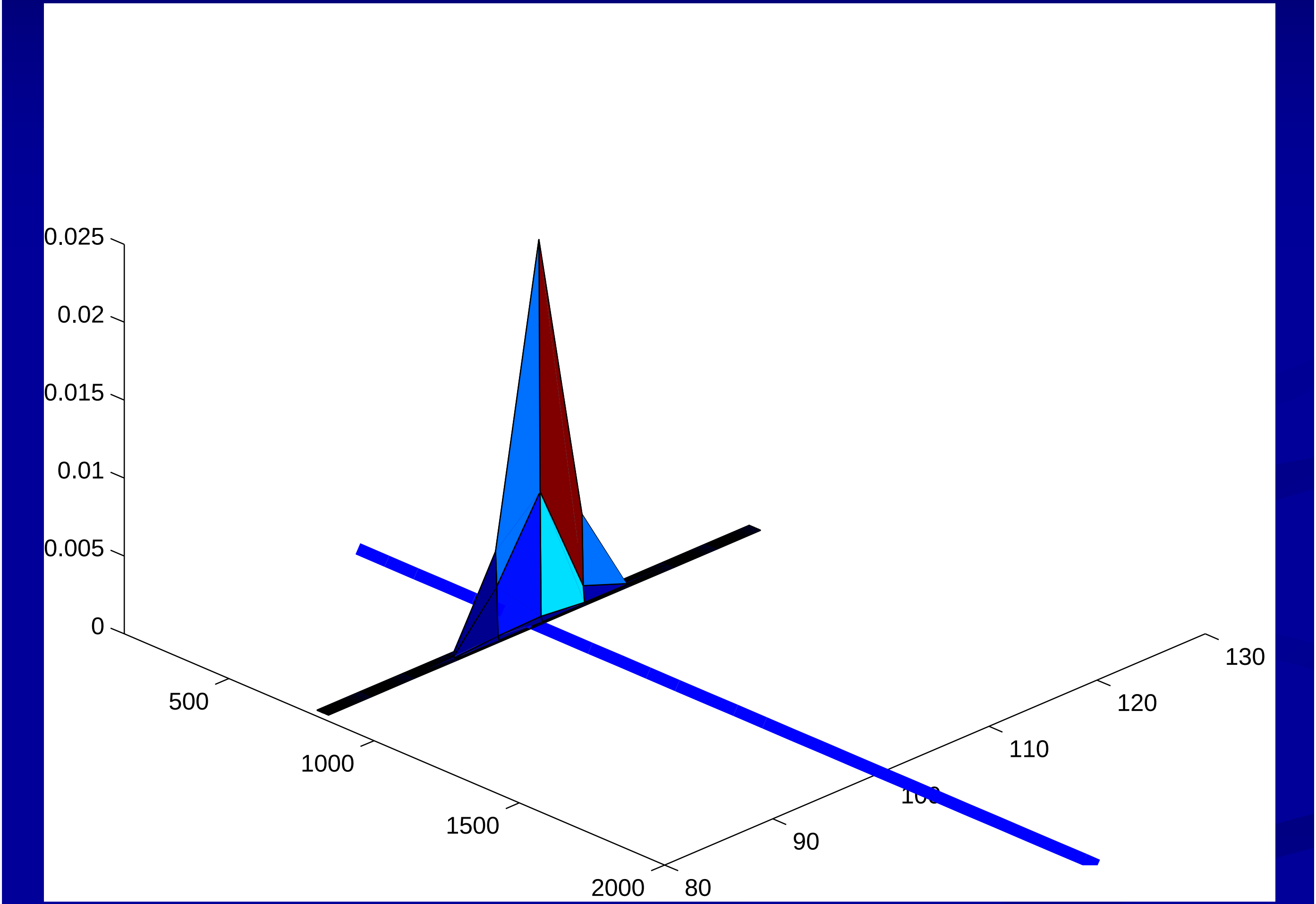
Type Identifiable Landmarks



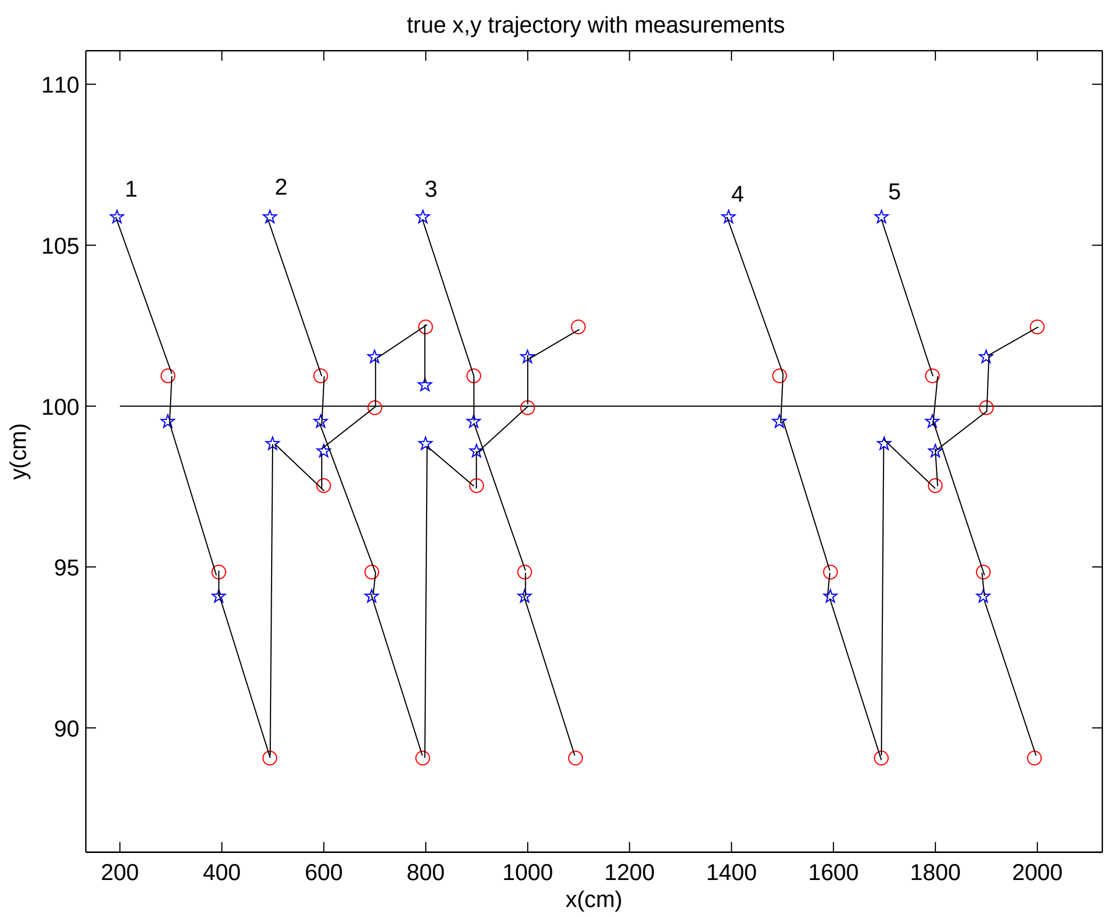
Type Identifiable Landmarks



Type Identifiable Landmarks



Type Identifiable Landmarks



Summary & Future

- Summary
    - Describing theory of localization and Kalman Filters
    - Illustrating applications of Kalman Filter to localization problems
    - Extension of Kalman Filter framework to multiple beliefs
- Future work
    - Practical application to robots
    - Possibilities of Kalman Filter extension
- Website: http://www.negenborn.net/kal_loc/

The end.