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Robot motion planning

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ABSTRACT

Manipulation and motion are the most common means that we use to act directly on the world. Vision and language serve us as inputs and we communicate with other human beings using speech, but we act on our surroundings by moving and manipulating. If AI is to deal with real life problems, autonomous intelligent systems will have to interact with the world in the way humans do. This tutorial is intended to present what robot motion planning is, what has been achieved through it so far, and what could be reasonably expected from it in the near future. The attention will be focused more on techniques for real-life applications than on theoretical formulations. Emphasis is on robot manipulators rather than on isolated moving objects, distinguising applications in 2D and 3D. The tutorial is organized on a performance basis instead of the usual methodological clasification. Three levels of performance are distinguished:(a) Geometric theoretical algorithms, (b) approaches that work at a computer simulation level and (c) techniques that have been implemented on actual robots, or which deal with problems for real-life robots. Some of the covered topics are: collision detection, fundamentals of the problem, geometric algorithms, basic motion planning techniques, moving obstacles, multiple robot coordination, non-holonomic motion, planning with uncertainty and future directions. Throughout the tutorial many examples and case studies will serve to illustrate successful applications as well as their underlying theoretical techniques.

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1.- Introduction

The collision-free planning of motion is a fundamental issue for Artificial Intelligence applications in Robotics [Brady et al., 1984]. Manipulation and motion are the most common means that we use to act directly on the world. Vision provides information about our environment and language and speech serve us to interact with other human beings, but we act on our surroundings by moving ourselves or by producing the motion of objects: by grasping, carrying, pushing, twisting, etc. Even the simplest task requires a complicated combination of movements. The AI community has been traditionally devoted to vision, language and different kinds of reasoning. However, a much deeper understanding of the reasoning involved in dealing with motion is needed. If AI is to be concerned with real life problems, intelligent systems will have to interact with the world in the way humans do: by moving and manipulating. If a robot is a machine with the ability of moving and/or manipulating, the problem we are considering can be called *robot motion planning* (RMP).

Robot Motion Planning is a common problem for both Robotics and Artificial Intelligence, since its final aim is the design of autonomous intelligent systems with the ability to plan how to perform a desired task including the computation of the sequence of movements involved in the task. The underlying issue of achieving higher level robot programming languages must also be considered. There exist many other questions which Robotics poses for AI, one of the most challenging is the connection of RMP to spatial reasoning.

Some classical material on Robotics and AI, including the problem of movement, can be found in [Brady et al., 1982], [Brady, 1984a], [Brady 1989], [Khatib et al. 1989], [Cox and Wilfong, 1990], the last one focusing mainly on mobile robots and RMP on the plane. For a good state of the art on Robotics [Albus, 1984] and [Korein and Ish-Shalom, 1987] can be mentioned, and [Hopcroft and Krafft, 1987] is an excellent summary of future directions in Robotics research from the point of view of Computer Science. The only published course in robot motion planning is [Latombe, 1991].

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2.- Spatial Representation and Motion Planning

The issue of representation has a crucial importance in Artificial Intelligence [Charniak and McDermott, 1985], [Davis, 1990]. Depending on how a representation is selected, the solution for a problem can be quite simple or very complex. In the context of spatial representations, several different models for 3D objects have been used mainly in the domain of computer graphics applications: Constructive Solid Geometry, Boundary Representation, spatial occupancy enumeration, cell decomposition, swept volumes, octrees or even prism-trees. These models are usually too graphics-oriented to be adequate as a paradigm for reasoning systems: they lack expressiveness and the elicitation of higher features of an object from these models is a very difficult task.

Generally, a trade-off between accuracy and simplicity is necessary, since a too detailed model may often be too complex, while a too simple model may be too inaccurate. A good representation in Artificial Intelligence must be one that is simple enough but, at the same time, contains all the necessary information to deal with the problem at hand, and one that can be refined as required. There are two possibilities to simplify the representation of an object: to decompose the object as a combination of simpler parts, on the one hand, or to compute an approximation of the shape of the object, on the other hand. According to Chazelle [1987], when we come to consider the situation of research regarding problems about approximation and decomposition of shapes in three dimensions, we find that relatively little is known. The mathematical theory of packing and covering [Rogers, 1964] deals with related problems; although it has a rather theoretical interest, its study is very useful as it provides us with a formal framework on which an object model may be built up.

Reviews of the various representations for 3D objects that have been used mainly in the domain of computer graphics applications can be found in [Reddy and Rubin, 1978], [Baer, Eastman and Henrion, 1979] and [Requicha, 1980]. Octrees are introduced in [Samet, 1984] and prism-trees, another interesting representation, is described in [Faugeras and Ponce, 1983]. A discussion of swept volume techniques can be found in [Wang and Wang, 1986]. An interesting representation paradigm based on generalized cylinders with good results for elongated objects is presented in [Agin and Binford, 1976]. The

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hierarchical nature of CSG has made possible some important results: Lee and Fu [1987] extract manufacturing features from it, Cameron [1989] uses it for intersection detection and Faverjon [1989] for collision avoidance.

Particular models have been developed for specific AI applications: namely [Brady, 1984b] and [Marr and Vaina, 1980] must be mentioned for their models to represent the shape of objects, as well as [Faugeras et al., 1984] in the context of artificial vision. A general survey of representations of space for commonsense reasoning can be found in [Davis, 1990]. In a previous work Davis [1988] deals with the problem of how to describe and predict the behavior of solid objects, reaching very interesting conclusions.

The simplicity of the sphere have motivated its use in many different fields. Many approaches to collision-free motion planning or other related problems in Robotics take advantage, in one way or another, of the easiness of working with spheres. Most of them, however, are based on a fixed, rather coarse approximation that cannot be improved in any way. Bajaj and Kim [1988] have studied a unique moving sphere, while Chen [1990] used a spherical model for the perceptual space. Early works in path-planning made a rather simple use of spheres by just enclosing every object inside one sphere [Pieper, 1968], [Widdoes, 1974]. More recent approaches utilize a certain set of spheres or circles to model the robot or the obstacles [Abramowski, 1988], [Singh, 1988], [Thakur, 1986], [de Pennington, Bloor and Balila, 1983], [Esterling and Van Rosendale, 1983], [Tornero, Hamlin and Kelley, 1991]. In the case of mobile robots, many approaches have been proposed that approximate the robot and the obstacles by enclosing them in a circle [Kambhampati and Davis, 1986], [Moravec, 1981], [Thorpe, 1984], [Ichikawa and Ozaki, 1985]. There exist also other publications dealing with special treatments to geometric planning problems in which the moving objects are circles [O'Dúnlaing and Yap, 1985], [Schwartz and Sharir, 1983b], [Yap, 1984], [Spirakis and Yap, 1984].

Badler, O'Rourke and Toltzis [1979] associated the concept of level with a spherical approximation for collision detection, but this model extends the representation to only two levels. More recently, Bonner and Kelley [1990] have developed a system that uses a sequence of rectangular sectors of spheres to efficiently find a path for a single object moving in straight lines in 3D space. In [del Pobil and Serna, 1991a, 1993a] a hierarchical object representation based

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on spheres is proposed, it is a twofold model for applications in Robotics and AI that approximates an object both by excess and default to any desired accuracy.

3.- Collision Detection

The issue of collision detection is a prior problem to RMP and it deals with the identification of a collision-free path, once it has been given. Obviously, we must be able to assure that a given path is safe before searching for a sequence of collision-free motions going from the start to the goal. The problem is first defined and structured and an overview of relevant approaches is presented.

3.1 Problem Definition

The problem can be stated in this way: given two sets of objects B and C and a description of their motions, decide whether two objects belonging to different sets will come into collision over a given time span.

To express this problem formally, we assume that a certain function $F(t)$ can be defined in such a way that for a particular time t_i, $F(t_i)A$ represents a transformation of the object A from its initial position at $t=t_0$, to its position at $t=t_i$. If an object is regarded as a set of points, then

$$F(t_i)A = \{ Q; \exists P \in A \mid Q = F(t_i)P \}.$$

If our time span is defined as $[t_0, t_f]$, then we can state that there will be no collision between B and C if:

$$\forall \ Sj_B \in B, \ Sk_C \in C \text{ and } \forall \ t \in [t_0, t_f], \ \ F(t)Sj_B \cap F(t)Sk_C = \varnothing,$$

Where $B = \{ Sj_B , j = 1, 2, ...\}$ and $C = \{ Sk_C , k= 1, 2, ...\}$.

3.2 Solution Approaches

The approaches to this problem can be classified into three groups [Cameron, 1985]:

a) *Multiple Intersection Test.* The problem is reduced to a sequence of intersection tests at certain moments $\{t_i\} \in [t_0, t_f]$ over the total time interval.

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b) *Swept Volume Intersection Test.* The volume SVA swept by an object A during its motion is given by:

$$SVA = \{ F(t)A \; ; t \in [t_0, t_f] \}.$$

Then, an absence of intersection between the volumes swept by two objects B and C (i.e., $SVB \cap SVC = \varnothing$) guarantees that the two objects did not come into collision over the analyzed time span.

c) *Four-Dimensional Intersection Test.* In this approach time is added as a new dimension and an intersection is tested in the resulting four-dimensional space.

Among these three alternatives the first one must be discarded for applications to motion planning. To plan a collision-free motion we must be able to generate safe trajectories. That amounts to guarantee that the infinite positions that belong to the trajectory are free of intersections. Obviously, this condition is not fulfilled in a multiple intersection test unless further considerations are made regarding minimum distances and expected maximum speeds [Cameron, 1985].

With respect to the four-dimensional intersection test approach, the mathematics involved may make it uneasy to deal with, although a very interesting contribution towards its understanding has been recently presented by Cameron [1990]. Moreover, the fact that in the case of spheres the technique becomes much simpler [Esterling and Van Rosendale, 1983] suggests a future application of the spherical model [del Pobil and Serna, 1993a] combined with the four-dimensional intersection test.

The main difficulty of the sweeping method is the computation of the actual swept volume in a general case, but this drawback is eliminated by using spherical models [de Pennington, Bloor and Balila, 1983] [del Pobil and Serna, 1992a].

Most approaches to collision detection reduce the problem to an instance of an intersection detection problem; [Shamos and Hoey, 1976], [Bentley and Ottmann, 1979] and [Chazelle and Edelsbrunner, 1988] use plane-sweep techniques for computing intersections on the plane. The question of detecting an intersection between two polyhedra has been studied by [Ahuja et al., 1985] and [Dobkin and Kirkpatric, 1985], while [Edelsbrunner, 1982] and [Hopcroft,

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Schwartz and Sharir, 1983b] deal with intersections among several objects, two sets of rectangular parallelepipeds in the first case, and a set of spheres in the second. For objects in motion, the multiple intersection test is usually applied in the domain of computer graphics: in [Uchiki, Ohashi and Tokoro, 1983] a typical system is described that is based on spatial occupancy enumeration; Hayward [1986] uses octrees for representing objects; Moore and Wilhelms [1988] present two approaches, the first one using multiple intersection test and boundary representations and the second applying implicit sweep volume for a cell decomposition of each surface into triangles.

Boyse [1979] proposed a method to detect collisions that was based on an implicit representation of swept volumes in order to avoid the complexity of explicitly computing swept volumes. Taking Boyse's work as starting point, an approach that characterizes the different types of collisions between vertices, edges and faces has been widely used (the most remarkable references are [Lozano-Pérez, 1987], [Canny, 87], [Donald, 1987] among many other). It is based on a boundary representation scheme and computes collisions by checking all possible intersections between the edges, vertices and faces of all objects. A rather different approach makes use of quaternions as a tool to represent the problem [Canny, 1986].

4.- The General Robot Motion Planning Problem

General motion planning can be generally stated as the problem of developing algorithms to automatically compute a continuous safe path for a given set of objects (possibly linked), in such a way that they move from an initial placement to a final placement avoiding collisions with obstacles.

4.1 Basic Concepts and Nomenclature

Two main issues can be distinguished when solving the motion planning problem. The first one is how to construct the space of Free Placements (*FP*) for the robot in its workspace. The second issue is how to search for a solution path inside the *FP* space. The first question is directly dependent on the selected collision detection scheme, since many candidate motions will have to be checked to determine if they are valid. In addition, this is highly influenced by the particular object model that is used by the off-line planner to represent the

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involved objects (obstacles and robot links): Constructive Solid Geometry (CSG), boundary representation, octrees, etc.

To solve the problem we also need the notion of *Configuration Space* (*CS*) for the robot and the obstacles [Lozano-Pérez, 1983]. We will denote as $B(q_1, q_2, ..., q_n)$ or simply $B(Q)$ a certain configuration of robot B; it will correspond to a point in an n-dimensional CS, so that each q_i represents an independent joint coordinate. Similarly, $B_i(Q)$ represents the location of link B_i for the given configuration, and $Bj_i(Q)$ the location for object Bj_i.

We define the *Space of Free Placements* (*FP*) as:

$$FP = \{ B(Q) \mid Q \in \Gamma \text{ and } B(Q) \cap C = \varnothing \},$$

where $\Gamma \subseteq \Re^n$ is the set of admissible configurations (as limited by the physical constraints of the joints) and C is the set of all obstacles. FP corresponds also to a subset of CS.

4.2 Statement of the General Motion Planning Problem

Now we can formally state our problem as follows: find a continuos path μ for robot B in an environment C from a start configuration Q^S to a goal configuration Q^G, in such a way that all the configurations on the path belong to FP, i.e., they are safe placements.

4.3 Crucial Features of the Problem and its Difficulty

It can be stated without exaggeration that motion planning is one of the most complex problems in the domain of Robotics [Hopcroft and Krafft, 1987]. Moreover, its complexity depends mainly on the answer to two essential questions, namely: *what is moving?* (a "flying object" or a manipulator) and *where is it moving?* (on a plane or in 3D space). Passing from two to three dimensions makes the problem inherently more difficult (see Fig. 1), while it becomes even much harder for robot manipulators as the number of degrees of freedom is increased (see [Brooks, 1983b] for a longer discussion of this point).

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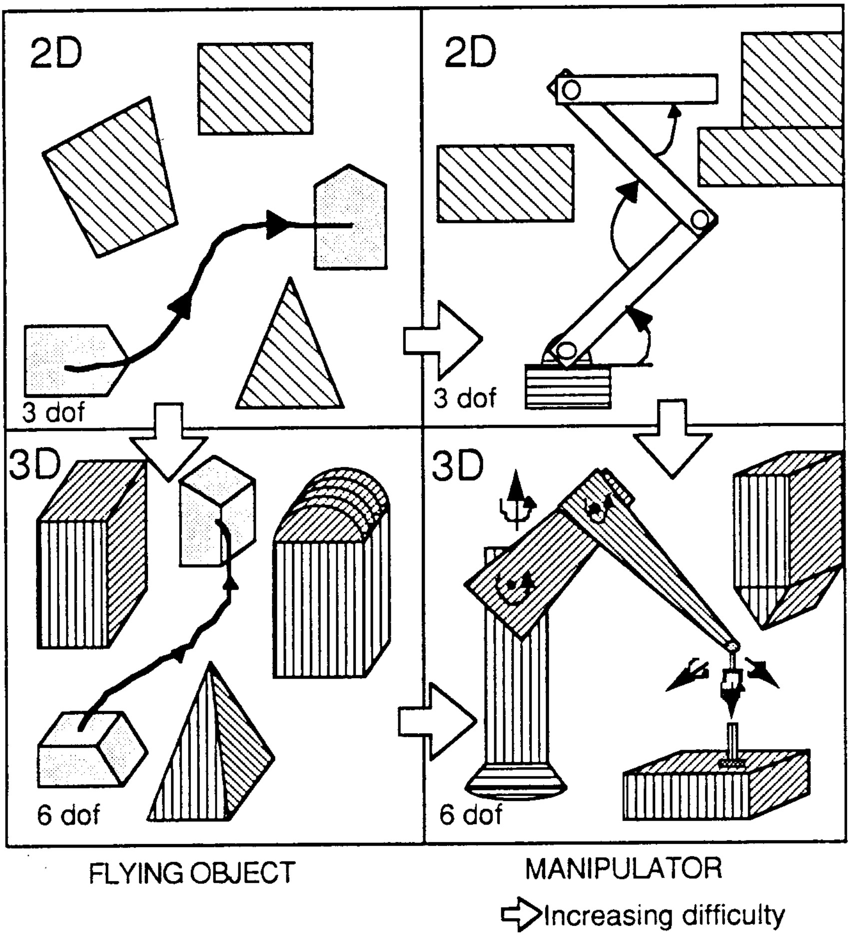

Figure 1 Difficulty of the motion planning problem

5.- Geometric Algorithms

Another research area within the field of motion planning is *geometric* or *algorithmic motion planning*. This area is briefly outlined due to its limited interest for applications. Its objective is to analyze the complexity of exact theoretical algorithms for motion planning problems. These approaches are

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extremely interesting from the point of view of algorithmics and computational geometry, but cannot usually be directly implemented. A review of these algorithms can be found in [Yap, 1987], [Schwartz and Sharir, 1988] and [Sharir, 1987]. A collection of papers on these topics has been published as [Schwartz, Hopcroft and Sharir, 1987]. Additional references are: [Reif, 1979], [Hopcroft, Joseph and Whitesides, 1984 and 1985], [Spirakis and Yap, 1984], [Schwartz and Sharir, 1983a], [Schwartz and Sharir, 1984], [Leven and Sharir, 1987a], [O'Dúnlaing, Sharir and Yap, 1983, 1986 and 1987], [O'Dúnlaing and Yap, 1985], [Kedem and Sharir, 1986], [Avnaim, Boissonat and Faverjon, 1988], [Tannenbaum and Yomdin, 1987], [Canny, 1988].

6.- Techniques at Computer Simulation Level

Approaches that work at a computer simulation level are considered more deeply, some of them will be the basis for robotic implementations to be discussed in the following section. First, different approaches for planning the motion of isolated objects (called *pianos*) are discussed both for two and three dimensions, these include freeway methods and cell decomposition methods. The gap between these approaches and its application to mobile robots is analyzed. Then, techniques for manipulator motion planning are considered: extension to previous freeway methods are introduced, and also configuration space approaches. In all cases the approximations that have been introduced are discussed as well as the various heuristic techniques. To end the section, two variants to the basic problem are presented: the case of obstacles that are not at rest, and the problem of how to coordinate the simultaneous movements of more than one robot that may collaborate in a given task.

6.1 Basic Motion Planning for *Pianos* in 2D and 3D

In the domain of motion planning, much work has been done for polygons moving on the plane: [Lozano-Pérez and Wesley, 1979], [Lozano-Pérez, 1983], [Brooks and Lozano-Pérez, 1985] and [Brooks 1983a] are seminal papers on this subject, in the second one the notion of Configuration Space is first applied to motion planning. Buckley [1989a] presents a rather different approach by using constrained optimization techniques. For polyhedra moving in 3D space Wong and Fu [985] consider a moving object with only three degrees of freedom, while Donald [1987] studies the motion of a polyhedron with six degrees of freedom. An algorithm called *roadmap* has been presented in [Canny,

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1988], it is very interesting from the point of view of complexity theory and it improves on a previous general algorithm by Schwartz and Sharir [1983]. Hierarchy has proved to be a very useful tool to solve the problem of our concern. Zhu and Latombe [1991] have successfully applied the notion of hierarchy to efficiently solve the problem of path planning in 2D.

6.2 Basic Motion Planning for Manipulators in 2D and 3D

In the case of manipulators, simplifications must be introduced for practical applications due to the inherent complexity of the problem: [Kantabutra and Kosaraju, 1986], [Chien, Zhang and Zhang, 1984] and [Chen and Vidyasagar, 1987] deal with robots represented as a set of linked rods moving on the plane. In [Ozaki, 1986] all intervening objects are constrained to be rectangular parallelepipeds. Gouzènes [1984] first points out the need for global planning and for an explicit construction of free space, the implemented model is a planar robot with 2 or 3 d.o.f. moving in an environment of rectangles.

Faverjon and Tournassoud [1988] have also applied the notion of hierarchy to motion planning, in this approach collision detection is based on an octree representation, as it is the case for other approaches that have been lately presented. Octrees, however, present important drawbacks when dealing with motion [Dupont, 1988], [Hayward, 1986], as the involved transformations (rotations and translations) are computationally very expensive, since the complete representation tree has to be computed anew for each placement of the robot. del Pobil and Serna [1992b, 1993b] have developed an approach that is based on their spherical representation for objects and can be applied to realistic 3D models of environments and 6 d.o.f. robots.

An interesting approach to manipulators in 3D is that of Brooks [1983b], who reduces the number of degrees of freedom of a PUMA arm to four and restricts the possible motions of the robot as well as the set of possible obstacles. Lozano-Pérez [1987] has proposed a technique called *slide projection* to build the configuration space for a manipulator: the range of possible values for each joint is discretized and the links are accordingly enlarged to assure that no collision occurs for the coordinate values that are not computed, the collisions are detected by checking all possible intersections between vertices and edges (or vertices, edges and faces in three dimensions).

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A quite different approach is that of the so-called *potential field* method [Khatib, 1986]. It must be regarded as a technique for collision avoidance rather than for motion planning: obstacles contribute in a negative way to the field while the goal has a positive contribution. Khosla and Volpe [1988] have suggested an alternative potential function. Canny and Lin [1990] have obtained good practical results by combining a simplified version of the roadmap algorithm [Canny, 1988] with techniques based on a potential field method.

6.3 Motion Planning with Moving Obstacles

Obstacles are allowed to move, generally following known paths with known velocities. As speed must play now an important role, most approaches take it as an additional coordinate: [Fujimura and Samet, 1989], [Kant and Zucker, 1990], [Maciejewski and Klein, 1985], [O'Dúnlaing, 1987], [Shih, Lee and Gruver, 1990].

6.4 Multiple Robot Coordination

Two or more independent robots move simultaneously and their motions must be coordinated to avoid collisions among them or to cooperate in a certain task. Some references that deal with these problems are: [Buckley, 1989b], [Erdmann and Lozano-Pérez, 1987], [Faverjon and Tournassoud, 1986], [Hopcroft, Schwartz and Sharir, 1984], [Lee and Lee, 1987], [Parsons and Canny, 1990], [Schwartz and Sharir, 1983b], [Yap, 1984], [Spirakis, 1984].

7.- Techniques for Real Robot Implementation

In this section, approaches that have been implemented on actual robots, or techniques aiming at applications for real-life robotic systems are presented. The limitations of a physical nature in mobile robots serve us to introduce the question of non-holonomic motion planning. Motion planning with uncertainties is another interesting problem that arises when dealing with reality. It may be due to discrepancies between the geometric models of the robot and the environment and the real world, or to a lack of accuracy in sensors or to tolerances in motor and control devices. Related to this kind of problem is fine manipulation and compliant motion.

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7.1 Mobile Robots and Non-Holonomic Motion Planning

In this case the robot movements are restricted by non-holonomic constraints: some of their equations are not algebraic, that is, they are expressed in terms of the derivatives of some coordinates. This situation arises frequently for mobile robots. References for this kind of problems are: [O'Dúnlaing, 1987], [Jacobs and Canny, 1990], [Tournassoud and Jehl, 1988].

7.2 Motion Planning with Uncertainty, Compliant Motion, Fine Manipulation.

Motion planning under uncertainty. If the environment is only partially known, or errors are introduced resulting in an uncertainty about the exact position of objects, then special techniques must be used for motion planning: [Erdmann, 1984], [Donald, 1988], [Brost, 1986], [Erdmann and Mason, 1986], [Latombe et al., 1991].

Fine motion and compliant motion. When objects are in contact [Hopcroft and Wilfong, 1986] or very close to a contact position, very precise movements are required that are usually beyond the controller capabilities [Mason, 1984], [Juan and Paul, 1986]. Particular approaches are then necessary [Lozano-Pérez, Mason and Taylor, 1984], one of the most important is compliant motion [Mason, 1982].

8.- Future Directions and Open Problems

To end the tutorial, it will be discussed what can be expected from robot motion planning as a result from future research, and what open questions remain still unsolved. The more recently emerging trends in RMP will also be presented.

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