# Robot Navigation with Model Predictive Equilibrium Point Control (MPEPC)

Jong Jin Park, Collin Johnson and Benjamin Kuipers
University of Michigan, USA

# Robot Navigation Faces Dynamic and Uncertain Environments





- Tight rectilinear spaces require high precision motion control
- Pedestrians and inaccurate robot model introduce dynamics and uncertainty
- Need to accommodate user preferences,
   e.g. aggressiveness and comfort

### Hierarchical Motion Planning Is Needed in Dynamic and Uncertain Environments

#### Global Planner

Approximate, longer term navigation plan in the environment

#### Local Planner

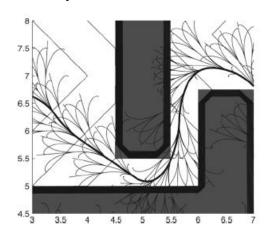
High fidelity local paths/trajectories in small scale space Generate-and-test search for trajectories

#### Control

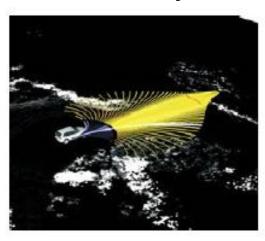
Low level controller for trajectory execution

# The Space of Trajectories is Continuous and Infinite

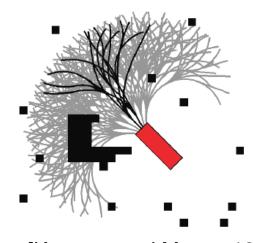
 Many current leading algorithms rely on a finite set of pre-determined candidate trajectories/paths.



[Ogren and Leonard 05]



[Hundelshausen et al. 08]



[Knepper and Mason 12]

- How to construct a good evaluation function is also an important question.
  - Determination of weights in multi-objective function, etc.

### Our MPEPC Approach: Objectives

- Efficient search for candidate trajectories
- Efficient evaluation of candidate trajectories, considering robot and pedestrian motion uncertainties
- Easy and straightforward implementation
- Accommodation of user preferences

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# Our MPEPC approach to Hierarchical Motion Planning and Control

#### Global Planner

Approximate navigation plan in the static environment Navigation Function (NF)

#### Local Trajectory Planner

High fidelity local trajectories in small scale space Dynamic replanning with receding-horizon MPC

#### Control

Low level controller for trajectory execution Pose-stabilizing feedback controller (EPC)

### Pose-stabilizing Feedback Control

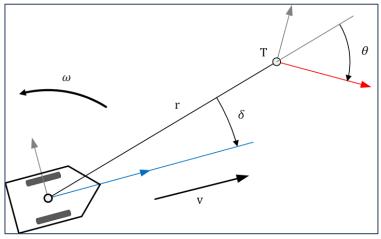
 We have developed a controller that allows the robot to reach an arbitrary target pose in a smooth curve.

[Park and Kuipers, ICRA-11]

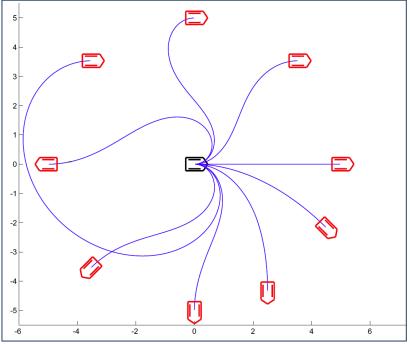
- While satisfying linear and angular velocity bounds, slowing down at high curvature points;
- Without singularity at the target.
- Target pose is exponentially stable.
- It allows us to compactly parameterize smooth and realizable robot trajectories in terms of the target pose and the gain value (4D).

### Pose-stabilizing Feedback Control

$$\omega = -\frac{v}{r} \left[ k_2 (\delta - \arctan(-k_1 \theta)) + \left(1 + \frac{k_1}{1 + (k_1 \theta)^2}\right) \sin \delta \right]$$



- $(r, \theta, \delta)$  describes the target T viewed from the vehicle in terms of the line of sight (LOS).
- At r=0, LOS is aligned with T.



[Park and Kuipers, ICRA-11]

### Pose-stabilizing Feedback Control

Curvature-dependent choice of linear velocity

$$v(\kappa) = v(r, \theta, \delta) = \frac{v_{\text{max}}}{1 + \beta |\kappa(r, \theta, \delta)|^{\lambda}}$$

- Guarantees bounded linear and angular velocities
- Slowdown rule near target pose

$$v = \min(\frac{v_{\text{max}}}{r_{\text{thresh}}}r, v(\kappa))$$

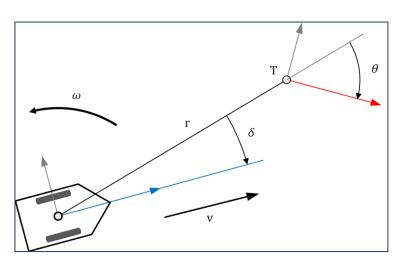
- Removes singularity at  $r \rightarrow 0$
- Target pose is exponentially stable
- $-v_{
  m max}$  can be viewed as a gain value

[Park and Kuipers, ICRA-11]

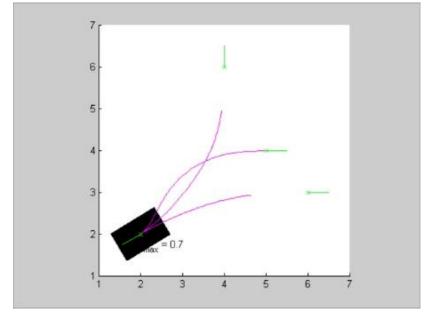
#### Combined Controller-Robot Model

- Closed-loop robot dynamic simulation with the controller target and gain,  $z_* = (r, \theta, \delta, v_{\text{max}})$ 
  - Non-holonomic, motor saturations, and P-controller for velocities (joystick)

 $-z_*$  parameterize the simulated responses of the robot system under the feedback controller.

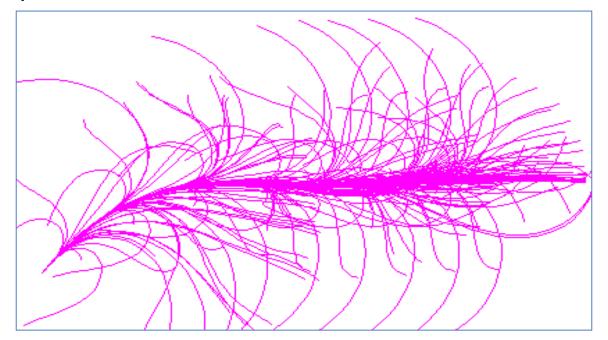


[Park and Kuipers, ICRA-11]



#### Defining Our Search Space: Controller-based Trajectory Parameterization

- Our 4D parameterization  $z_* = (r, \theta, \delta, v_{\text{max}})$  defines a continuous space of closed-loop trajectories.
  - It identifies a useful subspace of the infinite and continuous space of possible trajectories that are smooth and realizable by construction.
- Compact parameterization allows efficient search.



### Our MPEPC Approach: Objectives

- Efficient search for candidate trajectories
- Efficient evaluation of candidate trajectories, considering robot and pedestrian motion uncertainties
- Easy and straightforward implementation
- Accommodation of user preferences

### **Trajectory Evaluation**

• Trajectories parameterized by  $z_*$ :

$$q_{z_*}:[0,T]\to C$$

 Overall expected cost of a candidate trajectory, considering probability of collision

$$J(x, z_*, T) = E[\phi_{\text{progress}}] + E[\phi_{\text{collision}}] + E[\phi_{\text{action}}]$$
$$= E[\phi(q_{z_*})]$$

- Negative progress over the static plan (Navigation Function, NF)
- Penalty for probability of collision
- Quadratic action cost (on velocities)

# Incorporation of Motion Uncertainties Makes the Optimization Easier

- We construct probability weights as a function of robot and pedestrian motion uncertainties
  - We define simple approximations for:
    - Probability of collision and
    - Survivability of a trajectory segment.
  - Probability weights allow us to formulate the problem as unconstrained optimization over a smooth surface.

# Discrete Approximation to Probability of Collision and Survivability

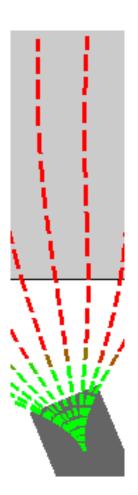
 For j-th sample along the trajectory, probability of collision to the i-th object in the map is approximated as:

$$p_c^i(d_i(j), \sigma_i) = exp(-d_i(j)^2/\sigma_i^2)$$

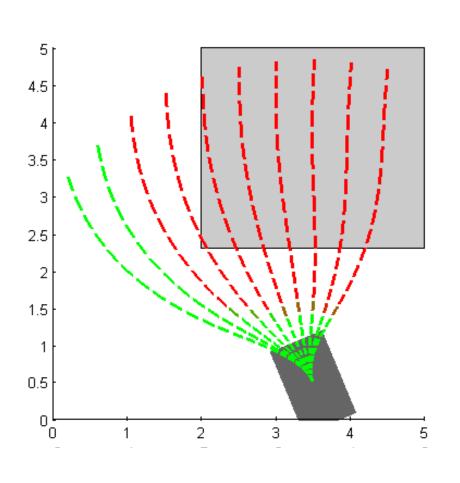
- $-d_i(j)$  is the minimum distance from any part of the robot body to any part of the *i*-th object in the map at time *j*.
- $\sigma_i$  are uncertainty parameters.
- Survivability of a trajectory segment is a probability that the trajectory segment will be collision free to any obstacles

$$p_s(j) \equiv \prod_{i=1}^{M} (1 - p_c^i(j))$$

 $-i \in [1...M], j \in [1...N]$ 



# Incorporating Probability Weights and Expected Values Creates a Smooth Optimization Surface



 Progress weighted by survivability

$$p_s(j) \cdot \Delta NF(j)$$

 Collision penalty weighted by probability of collision

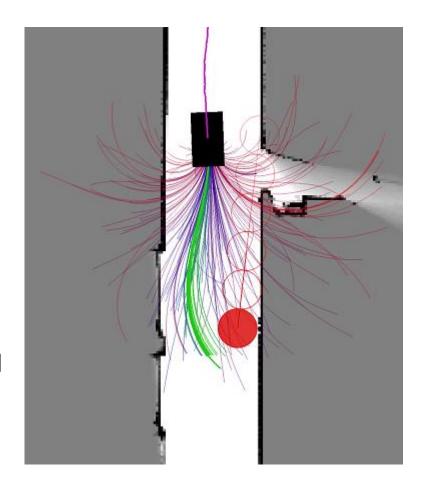
$$\sum_{i=1}^{M} p_c^i(j) \cdot \phi_{\text{collision}}^i(j)$$

 Additive action cost to modify robot behavior

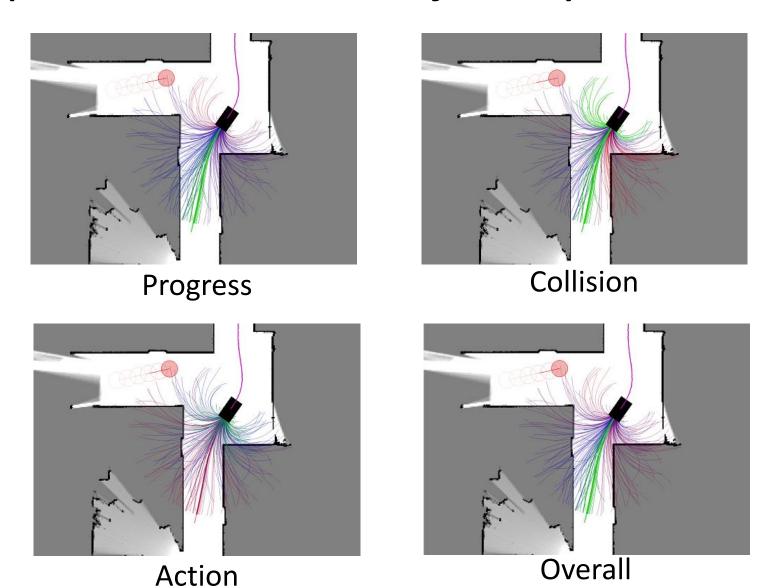
$$c_{\nu}v^{2}(j) + c_{\omega}\omega^{2}(j)$$

#### Expected Cost of a Trajectory Candidate

- The expected cost of a trajectory candidate is a probability-weighted time integral over [0, T]
- Probability weights create a smooth cost surface by setting physically meaningful soft boundaries around obstacles
- Weights on action cost can be tuned to match user preferences



#### **Expected Cost of a Trajectory Candidate**



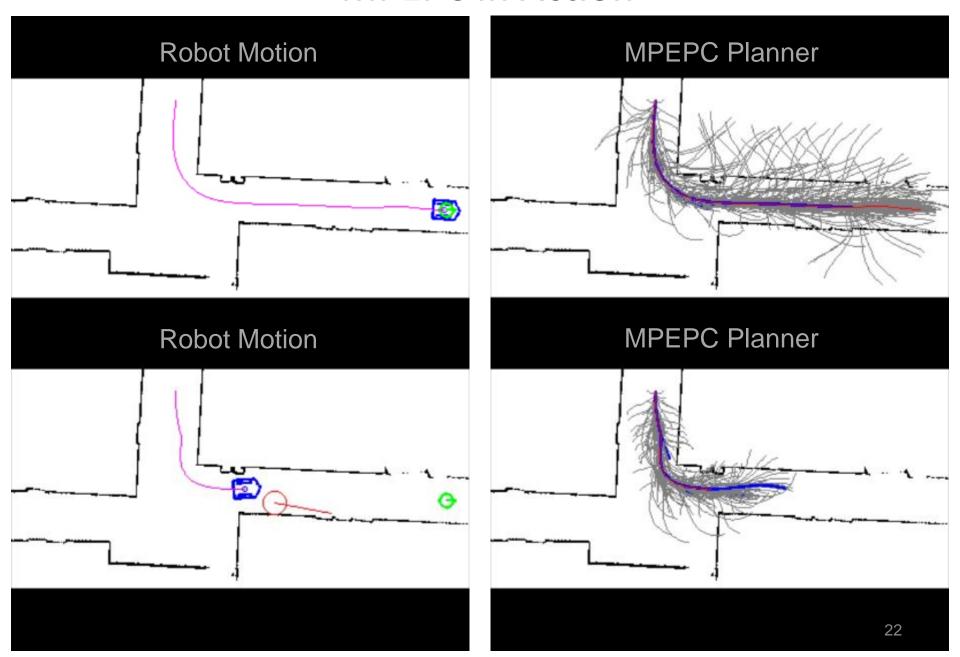
### Our MPEPC Approach: Objectives

- Efficient generation of motion hypothesis and fine motion control
- Efficient evaluation of candidate trajectories, considering robot and pedestrian motion uncertainties
- Implementation is easy and straightforward
- Action costs express user preferences

### Implementation is Straightforward

- Off-the-shelf optimization packages
  - Low-dimensional unconstrained optimization on continuous domain
  - No special post processing or optimization techniques
  - Real-time operation (C++)
- Two-phase optimization
  - Coarse pre-sampling of the search space to find a good initial condition.
  - Local gradient-based search from the best candidate from the pre-sampling phase.

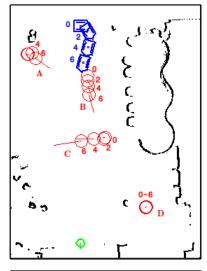
#### **MPEPC** in Action

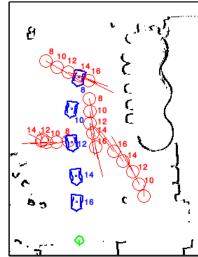


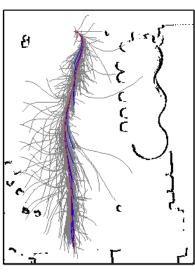
#### **Different People Have Different Preferences**

The proposed navigation algorithm handles multiple dynamic objects. We can shape robot behavior by changing weights in action cost.

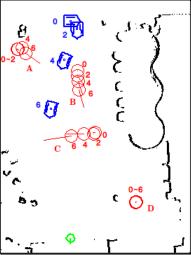
Moving slowly in a cluttered hall with multiple pedestrians (high weights on action cost)

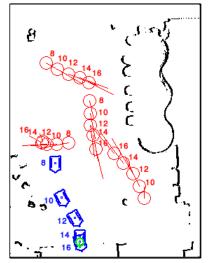


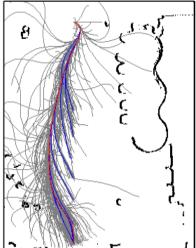




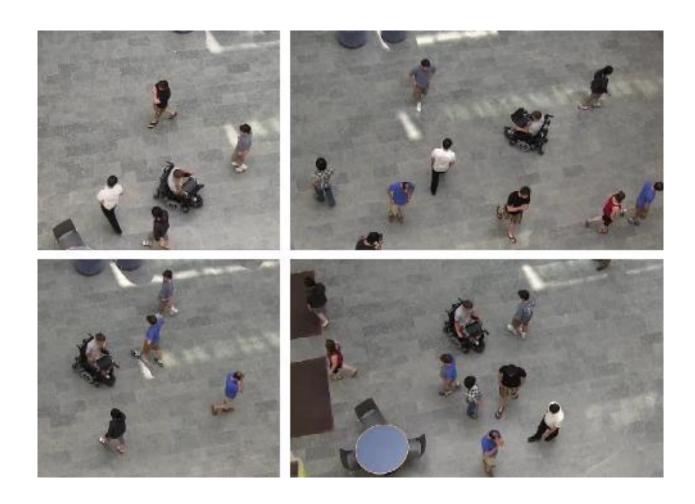
Moving
aggressively in a
cluttered hall with
multiple
pedestrians
(low weights on
action cost)







### Initial Tests on a Physical Platform



# Navigation is a Constant Decision-Making Process

- The navigation problem can be factored by decomposing the task in the hierarchical architecture.
- The search for the optimal trajectory can be made easier by integrating planning and control.
- Motion uncertainties need to be considered explicitly.
- What do they teach in driving school?

# Navigation is a Constant Decision-Making Process

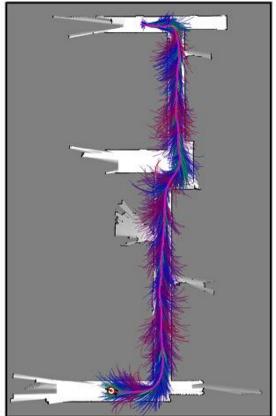
- The navigation problem can be factored by decomposing the task in the hierarchical architecture.
- The search for the optimal trajectory can be made easier by integrating planning and control.
- Motion uncertainties need to be considered explicitly.
- Identify, predict, decide and execute.
  - Minimize the probability that you might get in trouble, while progressing along the road.

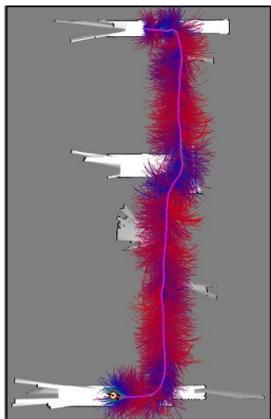
#### Conclusion

- We provide a compact representation of a space of smooth and realizable trajectories.
- We formulate local motion planning as an unconstrained optimization problem by computing expected values, using probability weights.
- The formulation allows straightforward low-dimensional optimization on a continuous domain.
- We have simple, easy to understand tunable parameters for qualitative robot behavior.

### Thank You







#### References

- [1] Jong Jin Park, Collin Johnson and Benjamin Kuipers, "Robot navigation with Model Predictive Equilibrium Point Control", *IROS-12*
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- [3] Knepper and Mason, "Path diversity is only part of a problem", ICRA-09
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- [6] Hundelshausen, Himmelsbach, Hecker, Mueller and Wuensche, "Driving with Tentacles: Integral structures for sensing and motion", *J. Field. Robot.*, 2008
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