Progress In Electromagnetics Research B, Vol. 23, 215–228, 2010

ROBUST ADAPTIVE BEAMFORMER USING INTERPOLATION TECHNIQUE FOR CONFORMAL ANTENNA ARRAY

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Abstract—A novel robust adaptive beamforming method for conformal array is proposed. By using interpolation technique, the cylindrical conformal array with directional antenna elements is transformed to a virtual uniform linear array with omni-directional elements. This method can compensate the amplitude and mutual coupling errors as well as desired signal point errors of the conformal array efficiently. It is a universal method and can be applied to other curved conformal arrays. After the transformation, most of the existing adaptive beamforming algorithms can be applied to conformal array directly. The efficiency of the proposed scheme is assessed through numerical simulations.

1. INTRODUCTION

Adaptive beamforming with conformal antenna arrays are of interest for future communication and defense applications [1, 2]. In ideal case, adaptive arrays can suppress the interference signals and noise while efficiently keeping the interest signals. However, in practice, there are all kinds of errors in the system, such as elements amplitude and phase errors, elements location errors, mutual coupling errors and

Received 15 June 2010, Accepted 14 July 2010, Scheduled 23 July 2010
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desired signal point errors, etc. Most traditional adaptive beamforming algorithms are sensitive to the environment. They often give poor performance when these errors are taken into consideration [3]. In past decades, many approaches have been proposed to improve the robustness. One of the efficient ways among them is to recast the sample covariance matrix, such as diagonal loading [4, 5]. How to choose the diagonal loading level based on the information of the uncertainty of the array steering vector is an open problem. Recently, many automatic diagonal loading approaches have been developed [6–10]. These methods have high performance and can be seen as robust adaptive beamformers.

Compared to uniform linear array (ULA), when conformal array with a small radius of curvature is used for adaptive signal processing or beamforming, some important problems must be considered. First, conformal array has "shadow effect" due to the metallic platforms. It means that an incident wave comes from a special angle, but not all of the antenna elements can receive this signal. Second, the radiation patterns of conformal antennas are always directional. Because each element has a different normal direction, the maximum radiation point is different. Hence, for any incident wave, different elements have different responses. Third, the mutual coupling between the elements cannot be ignored. It becomes more complex in the situation of conformal array due to the effects of the platform. It can only be analyzed by using some numerical methods, such as finite element method (FEM), finite differential time domain method (FDTD) and method of moment (MOM). These errors will seriously affect the performance of traditional beamformers if we do not compensate them. Unfortunately, most of the existing robust algorithms are based on the ideal case (i.e., ULA with omni-directional elements, without mutual coupling). Hence conventional algorithms often have a poor performance on conformal arrays and can hardly be applied directly.

In order to overcome the problems mentioned above, we propose a robust, interpolation based adaptive beamforming method to transform a cylindrical conformal array with directional antenna elements to a virtual ULA with omni-directional elements. The transformation can be seen as a procedure of array optimization. In the process, the mutual coupling effect is taken into consideration. After the transformation, the conformal array has the character of ideal ULA. Hence, all of the errors can be suppressed effectively. Moreover, most existing adaptive beamforming algorithms that can just be used to ULA can be applied to conformal array after the transformation. This method is simple and easy for implementation. It is a universal method and independent of the array configurations.

Simulation results show that the proposed method achieves a higher performance in conformal array than conventional methods. This paper is organized as follows: In Section 2, cylindrical conformal array beamforming based on interpolation method is proposed in detail. In Section 3, the performance of the proposed method and conventional methods are compared by some simulation examples. The new method is concluded in Section 4.

2. CYLINDRICAL CONFORMAL ARRAY BEAMFORMING BASED ON INTERPOLATION METHOD

It has been demonstrated that uniform circular arrays (UCA) have better performance, especially in azimuth-plane beamforming, than uniform rectangular arrays (URA). Most UCAs use dipole as the elements. Dipole antenna has an omni-directional radiation pattern at ϕ direction. But for conformal arrays, we cannot use dipole as the element because of the metallic platform. Microstrip antenna is a good choice for conformal array elements due to its low profile and unidirectional radiation characteristics.

2.1. Cylindrical Conformal Array

Consider a cylindrical conformal array with M identical microstrip antennas. As shown in Fig. 1, these antennas are mounted uniformly on the surface of the cylinder. The radius of the cylinder is r. At time t, $P\,(P < M)$ narrowband signals with azimuth angle $\phi_i\,(i = 1, 2, ..., P)$ impinge on the conformal array. The $M \times 1$ receiving data vector of

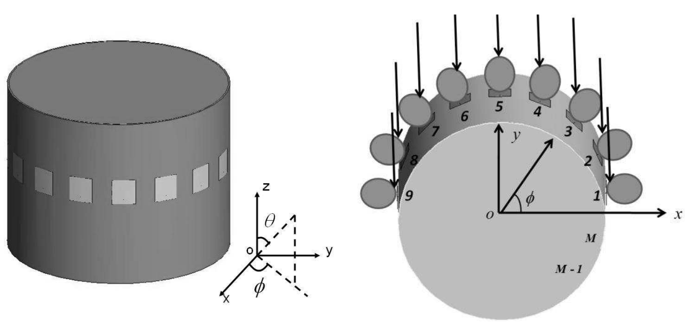

Figure 1. Cylindrical conformal array with directional antenna elements.

the array is given by

$$\mathbf{x}(t) = \mathbf{C}[\mathbf{F}(\phi) \cdot \mathbf{A}(\phi)\mathbf{S}(t)] + \mathbf{n}(t) \tag{1}$$

where $\mathbf{S}(t)$ (including the desired signal and the interferences) is the $P \times 1$ vector whose ith element denotes the ith signal. $\mathbf{A}$ is the $M \times P$ steering matrix, whose ith ($i = 1, 2, \ldots, P$) column is the steering vector of the ith signal.

$$\mathbf{A} = [\mathbf{a}(\phi_1), \mathbf{a}(\phi_2), \cdots, \mathbf{a}(\phi_P)] \tag{2}$$

$$a_m(\phi_i) = e^{jkr\cos(\phi_i - \beta_m)}, \ (m = 1, 2, \ldots, M) \tag{3}$$

where $\beta_m = 2\pi(m-1)/M$ and k is the wave number. The $M \times 1$ vector $\mathbf{n}(t)$ represents additive white noise. $\mathbf{F}$ is the $M \times P$ radiation pattern matrix whose m, ith elements denote the response of the mth ($m = 1, 2, \ldots, M$) antenna to the ith ($i = 1, 2, \ldots, P$) signal.

$$\mathbf{F} = [\mathbf{f}(\phi_1), \mathbf{f}(\phi_2), \ldots, \mathbf{f}(\phi_P)] \tag{4}$$

$$\mathbf{f}(\phi_i) = [f(\phi_i - \beta_1), f(\phi_i - \beta_2), \ldots, f(\phi_i - \beta_M)]^T \tag{5}$$

where $f(\phi_i - \beta_m)$ is the radiation pattern of the antenna element. $(\cdot)^T$ denotes the transpose. If the elements are point sources, $f(\phi_i - \beta_m) = 1$. From Fig. 1, we can see that for the conformal array, different elements have different responses to the signal. Assume that the direction of arrive (DOA) of the signal is $\phi = 90°$, then element 5 has the strongest response, while elements 1 and 9 can hardly receive this signal. Of course those elements behind the cylinder (element 10 to M) have zero response. $\mathbf{C}$ is the $M \times M$ circular symmetrical matrix [11–14] which represents the mutual coupling of the array

$$\mathbf{C} = Z_L(\mathbf{Z} + Z_L\mathbf{I})^{-1} \tag{6}$$

where Z_L is the load impedance (usually 50 ohms), and $\mathbf{Z}$ is the $M \times M$ mutual impedance matrix. $\mathbf{I}$ is the $M \times M$ unit matrix. Once $\mathbf{Z}$ is known, the mutual coupling $\mathbf{C}$ of the array can be determined by (6). The impedance matrix is difficult to determine analytically, but easy to get through S parameter measurement or numerical methods. Also, the radiation pattern matrix $\mathbf{F}$ can be determined in the same way.

The minimum variance distortionless response (MVDR) beamformer is to maintain the distortionless response to the desired signal while minimizing the output power

$$\min_{\mathbf{w}} \ \mathbf{w}^H\mathbf{R}\mathbf{w} \text{ subject to } \mathbf{w}^H\mathbf{a}(\phi_0) = 1 \tag{7}$$

where $\mathbf{w}$ is the $M \times 1$ beamformer weight vector, and $(\cdot)^H$ denotes the conjugate transpose. $\mathbf{R} = E\{\mathbf{x}(t)\mathbf{x}(t)^H\}$ is the array covariance matrix, and $E\{\cdot\}$ denotes the statistical expectation. $\mathbf{a}(\phi_0)$ is the steering vector of desired signal with the form of Equation (3), and ϕ_0 is the desired signal's DOA. The solution of this problem in the finite sample case is so called sample matrix inverse (SMI) beamformer and given by

$$\mathbf{w} = \mu^{-1}\hat{\mathbf{R}}^{-1}\mathbf{a}(\phi_0) \tag{8}$$

where $\mu = \mathbf{a}(\phi_0)^H\hat{\mathbf{R}}^{-1}\mathbf{a}(\phi_0)$ is a constant; $\hat{\mathbf{R}} = \sum_{j=1}^{K}\mathbf{x}(t)\mathbf{x}(t)^H/K$ is the sample estimate of $\mathbf{R}$; K is the number of snapshots, respectively. The output signal of the cylindrical conformal array can be written as

$$y(t) = \mathbf{w}^H\mathbf{x}(t) \tag{9}$$

It is clear to see from Equation (8) that the effects of mutual coupling and directional radiation pattern have been included in sample covariance matrix $\hat{\mathbf{R}}$ but not in the static steering vector $\mathbf{a}(\phi_0)$. If we use Equation (8) as the weight vector of conformal array without any compensation, big errors will be introduced. These errors will change not only the depth of the nulls but also their locations, which will result in a poor performance to the beamformer. These errors can be compensated by recasting Equation (8). Here we propose an alternative method by using the interpolation technique.

2.2. Interpolation Technique for Robust Adaptive Beamforming

Interpolation technique has been widely used for DOA estimation [15–18] but rare for adaptive beamforming [19, 20]. The main idea of interpolation method is dividing the field of view of the array into L sectors. The size of the sectors depends on the array geometry and desired accuracy. For example, there is a signal, whose DOA is in the sector $\mathbf{\Phi}$ ($\mathbf{\Phi} \in [\phi_1, \phi_2]$), where ϕ_1 and ϕ_2 are the left and right boundaries of this sector. Let $\Delta\phi$ as the interpolation step, then $\mathbf{\Phi}$ can be represented as

$$\mathbf{\Phi} = [\phi_1, \phi_1 + \Delta\phi, \phi_1 + 2\Delta\phi, \cdots, \phi_1 + n\Delta\phi, \phi_2] \tag{10}$$

the $\Delta\phi$ is determined by the desired accuracy. In this sector, the real array manifold is

$$\mathbf{CF}\cdot\mathbf{A} = \mathbf{C}[\mathbf{f}(\phi_1)\cdot\mathbf{a}(\phi_1), \mathbf{f}(\phi_1+\Delta\phi)\cdot\mathbf{a}(\phi_1+\Delta\phi), \cdots, \mathbf{f}(\phi_2)\cdot\mathbf{a}(\phi_2)] \tag{11}$$

In order to transform the real conformal array to a virtual ULA with omni-directional elements, we can construct a virtual ULA in the same sector $\mathbf{\Phi}$, whose steering matrix is

$$\bar{\mathbf{A}} = [\bar{\mathbf{a}}(\phi_1), \bar{\mathbf{a}}(\phi_1 + \Delta\phi), \cdots, \bar{\mathbf{a}}(\phi_1 + n\Delta\phi), \bar{\mathbf{a}}(\phi_2)] \tag{12}$$

where $\bar{\mathbf{a}}(\phi)$ is the steering vector of the virtual ULA, and its mth element is

$$\bar{a}_m(\phi) = e^{jk(m-1)d\cos\phi} \quad m = 1, 2, \ldots, M \tag{13}$$

Here d is the spacing of two adjacent elements. Next, we need to find a transformation matrix $\mathbf{B}$ to transform the real array into the virtual array, that is

$$\mathbf{B}^H[\mathbf{CF} \cdot \mathbf{A}(\phi)] = \bar{\mathbf{A}}(\phi), \ \phi \in \mathbf{\Phi} \tag{14}$$

The size of $\mathbf{B}$ is $M \times M$. It is impossible to find an ideal $\mathbf{B}$ to satisfy Equation (14). The accuracy of the interpolation can be examined by comparing the ratio of the Frobenius norms

$$\tau = \frac{||\bar{\mathbf{A}} - \mathbf{B}^H(\mathbf{CF} \cdot \mathbf{A})||}{||\mathbf{CF} \cdot \mathbf{A}||} \tag{15}$$

where $|| \cdot ||$ denotes the Frobenius norm. If τ is small enough then accept $\mathbf{B}$. If this ratio is not sufficiently small, we can reduce $\Delta\phi$ or change the configuration of the visual array then recalculate τ. The interpolation procedure is time consuming if the sector size and the number of interpolation angles n become very large. Fortunately, it is an off-line procedure. The matrix $\mathbf{B}$ just needs to be calculated only once for given array then stored in the system. More details about the interpolation can be found in [18] and references in it. The interpolation procedure is an optimization process. If τ is small enough, we can use the virtual ULA to replace the original conformal array. Note that the beamformer weight vector now is no longer the one in Equation (8), and it should be

$$\bar{\mathbf{w}} = \mu^{-1}\hat{\bar{\mathbf{R}}}^{-1}\bar{\mathbf{a}}(\phi_0) \tag{16}$$

where $\hat{\bar{\mathbf{R}}} = \sum_{j=1}^{K} \bar{\mathbf{x}}(t)\bar{\mathbf{x}}(t)^H/K$, $\bar{\mu}^{-1} = \bar{\mathbf{a}}^H(\phi_0)\hat{\bar{\mathbf{R}}}^{-1}\bar{\mathbf{a}}(\phi_0)$ and

$$\bar{\mathbf{x}}(t) = \mathbf{B}\mathbf{x}(t) \tag{17}$$

The receiving data of the conformal array $\mathbf{x}(t)$ are transformed to the data of the ULA $\bar{\mathbf{x}}(t)$ through Equation (17). Hence, the new covariance matrix $\hat{\bar{\mathbf{R}}}$ has the characteristics of ULA. Also, $\bar{\mathbf{a}}(\phi_0)$ in Equation (16) is the virtual ULA's steering vector. Therefore, the errors of the cylindrical conformal array can be compensated when we use the weight vector in Equation (16) for beamforming.

2.3. Implementation

The proposed method is summarized in Table 1. It is worth to note that to find the transformation matrix $\mathbf{B}$, Step 1.1 is necessary. Matrix

Table 1. Implemetation procedure.

Step 1: Initialization 1.1 Use numerical methods to calculate the matrix $\mathbf{F}$ and $\mathbf{C}$ of the conformal array in Equation (1); 1.2 Determine the steering vector $\bar{\mathbf{a}}(\phi)$ of the virtual array in Equation (13); 1.3 Set the interpolation parameters (the left and right boundary of sector $\mathbf{\Phi}$, interpolation step $\Delta\phi$, least interpolation error τ, etc.).
Step 2: Interpolation 2.1 Use Equations (11) and (12) to calculate the matrix $\mathbf{CF}\cdot\mathbf{A}$ and $\bar{\mathbf{A}}$; 2.2 Use Equation (14) to calculate the matrix $\mathbf{B}$ and Equation (15) to calculate the error τ; 2.3 If τ is small enough, accept $\mathbf{B}$. If not, go to step 1.2;
Step 3: Calculate the weight vector 3.1 Use Equation (17) to transform $\mathbf{x}(t)$ (the receiving date of the conformal array) to $\mathbf{x}(t)$ (the receiving data of the virtual ULA); 3.2 Use Equation (16) to calculate the weight vector $\bar{\mathbf{w}}$.

$\mathbf{C}$ and $\mathbf{F}$ are determined by the configuration of the array and the characteristics of antenna elements. They are easy to get by using numerical methods. Step 1 and Step 2 are off-line processing. They can be calculated and stored in the system beforehand. The advantage of this method is that it can do the transformation and calibration at the same time. Moreover, after the transformation, the array has the characteristics of ULA. Hence, most beamforming algorithms that can only be used on ULA (such as spatial smoothing methods for coherent signals [21]), can be applied directly to conformal array now.

3. SIMULATION RESULTS

In this section, several simulation results are given to evaluate the proposed approach. Consider a 16 elements cylindrical conformal array, and the array model is shown in Fig. 1, where $M = 16$. The elements of the array are linearly polarized microstrip antennas. The polarization of the antenna is along z axis. The height of the cylinder is 3λ, and the radius is 1.28λ. Here λ is the operating wavelength of the antenna. The radiation pattern of element 5 ($\phi = 90°$) is calculated by using the method of moment (MOM) and shown in Fig. 2. All the antenna elements have identical radiation pattern due to the symmetrical structure.

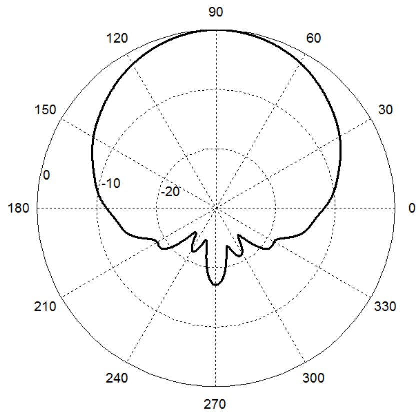

Figure 2. Normalized radiation pattern of element 5.

Without loss of generality, we chose elements 1–9 to do the transformation. The transformation sector is set to $\mathbf{\Phi} \in [60°, 120°]$, and the step $\Delta\phi$ is 0.1°. How to get the "best" virtual array is an optimization problem. The configuration of the virtual array can be arbitrary. It could be sparse, with different numbers of elements and element intervals. To simplify the problem, we use ULA as the virtual array. The virtual ULA consists of 9 isotropic point sources. The optimized location of these point sources are on the x axis, and the spacing is 0.25λ. The interpolation error τ in Equation (15) can be smaller than 0.003 under these conditions. The desired signal is located in the direction of $\phi = 90°$, and the SNR is 0 dB. An interference comes from $\phi = 65°$ with the interference in noise ratio (INR) of 20 dB. It should be noted that elements 8 and 9 cannot receive the interference due to the cylinder. Both the desired signal and the interference are narrow band and with the same polarization along z axis. The number of snapshots is fixed at 512. Traditional SMI algorithm and robust capon beamformer (RCB) [22] as well as their interpolated counterparts are used for testing the beamforming performance on this conformal array.

3.1. Conformal Array with Directional Element and Mutual Coupling

Figure 3 shows the normalized gain pattern of the four methods. The ideal case means that antenna elements are point sources, and mutual coupling is ignored. In the ideal case, both SMI and RCB algorithms have a -50 dB deep null at $\phi = 65°$. It can suppress the

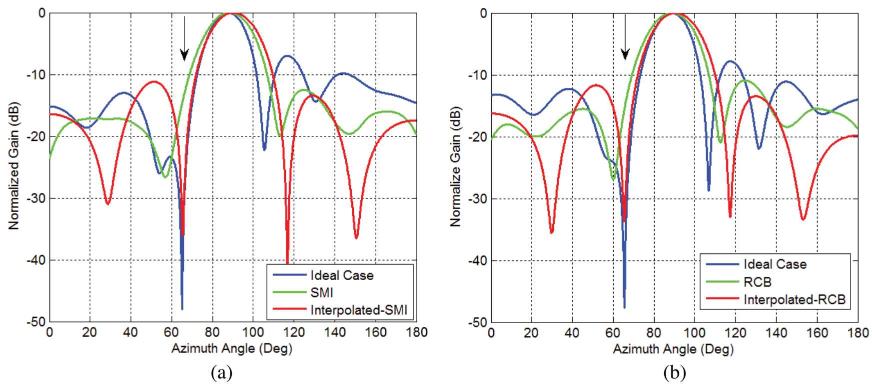

Figure 3. (Color online) Normalized gain pattern of different methods. (a) SMI and interpolated-SMI. (b) RCB and interpolated-RCB. The blue line is the idea case.

interference efficiently. However, once we use the microstrip antennas with radiation pattern shown in Fig. 2 instead of the point sources and take into account the mutual coupling, both of the algorithms are invalid. The null point disappears due to the big errors resulting from the conformal array.

On the contrary, the proposed method achieves a good performance in the array. Both the interpolated-SMI and interpolated-RCB algorithms have a null with −35 dB depth, and all the sidelobe levels are under −10 dB. It is needed to note that in the ideal case, the beam pattern has a relatively high sidelobe level. It is due to the omni-directional elements in UCA. Generally speaking, unlike ULA, the directional elements in UCA can be seen as amplitude weights. This can be used to lower the sidelobe level if designed properly. However, for adaptive beamforming, the ability of interference suppression is most important. Though the directional elements can lower the sidelobe level, it will change the depth and location of the null. Hence, calibration is necessary.

Figure 4 shows the comparison of the mean output signal-to-interference-plus-noise (SINR) versus input SNR of the four beamformers. The parameters are the same as aforementioned except the SNR of desired signal changes from 0 dB to 25 dB. 100 separate trials are performed. According to the results, we note that the proposed method achieves a higher performance than conventional methods. When SNR is low, the output SINR of interpolated-SMI and interpolated-RCB methods is 2 dB higher than SMI and RCB. With the SNR increasing, the advantage of the new method becomes

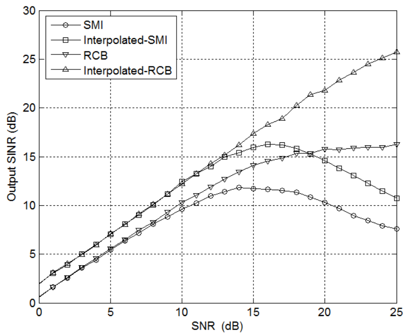

Figure 4. Comparison of beamformers mean output SINR versus input SNR.

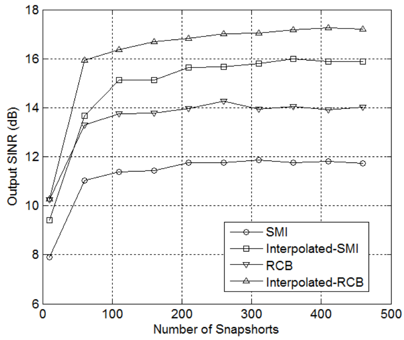

Figure 5. Mean SINR for varying number of snapshots.

more significantly. However, when SNR $>$ 16 dB, the output SINR of SMI based beamformes begins to decease. This is because for SMI based algorithms, at high values of SNR, the desired signal will be regarded as an interference to be suppressed. Since the RCB based algorithms are adaptive diagonal loading methods, the loading level can change automatically with changing SNR. Particularly, we can see that the interpolated-RCB method is insensitive to errors and seems to perform the best of all.

Figure 5 displays the mean output SINR versus the number of snapshots. In this experiment, the input SNR is fixed at 15 dB. Other parameters are the same as before. We find that the interpolated-RCB method outperforms other methods significantly. We can see from Fig. 4 and Fig. 5 that even without diagonal loading, the interpolated-SMI algorithm has a better performance than RCB algorithm if the value of SNR is not too high ($<$ 18 dB). The simulation results prove that the interpolated-SMI algorithm can provide a similar performance to diagonal loading based algorithms.

3.2. Conformal Array with Directional Element, Mutual Coupling and Desired Signal Point Error

Most of the adaptive beamformers are DOA based methods, which means that the DOA of desired signal should be estimated before the beamforming. In practice, due to the system mismatching and the noise, the DOA of desired signal cannot be estimated accurately. If the mainlobe of the beamformer points to an error direction, then the actual desired signal will be regarded as interference and suppressed by the beamformer [23]. This is the so called the problem of "desired signal point error".

Figure 6 shows the performance of output SINR versus desired signal point error. Both the errors of conformal array with directional element and mutual coupling are taken into consideration. The desired signal has a power of SNR = 15 dB, and other parameters are the same as above. It can be seen that the proposed interpolation based methods have a better SINR than conventional methods when the point error changes between $-3^\circ \sim 3^\circ$. Fig. 7 shows the beam patterns with a 2° desired signal point error. For the classical SMI algorithm, the mainlobe begins to split, and the sidelobe levels become very high. Though the RCB algorithm can maintain the mainlobe close to 90°,

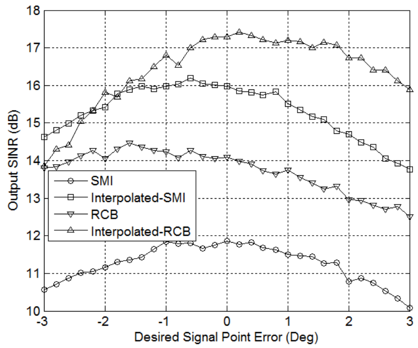

Figure 6. Output SINR versus desired signal point error.

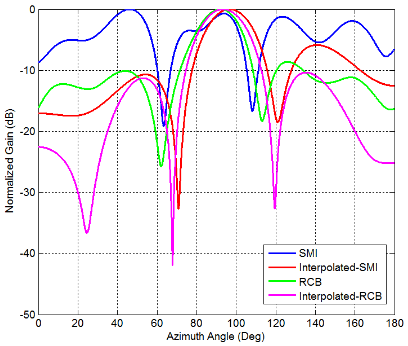

Figure 7. (Color online) Normalized gain patterns in the situation of 2° desired signal point error.

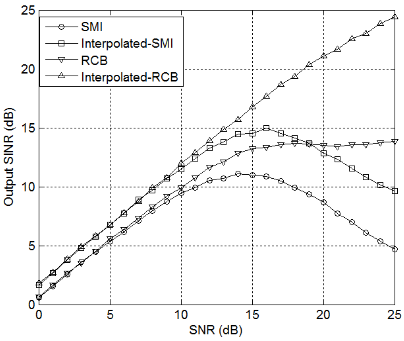

Figure 8. Output SINR versus SNR in the situation of 2° desired signal point error.

the location of the null changes. However, the interpolated methods achieve a relatively high performance. Particularly, it can be seen that the gain pattern of interpolated-RCB is very close to Fig. 3(b) (without desired signal point error). Hence, the interpolated-RCB has the strongest robustness. Fig. 8 shows the mean SINR versus input SNR with 2° desired signal point error. Again, the proposed method shows a better SINR performance.

4. CONCLUSION

In this paper, we propose a novel adaptive beamforming method based on the interpolation technique. This method gives a robust and high performance on conformal array compared with conventional methods. The errors resulting from different maximum radiating directions of the elements, mutual coupling and desired signal point can be calibrated by using the virtual array transformation. This method has the merits of simple, robust and high performance. It is more important that after the transformation, the conformal array has the characteristics of ideal ULA. Hence, most of the existing adaptive beamforming algorithms can be applied directly. This method is a universal method and easy to extend to other conformal arrays. A number of simulation examples clearly demonstrate that the proposed method is shown a significantly improved performance as compared with the conventional methods on conformal array beamforming.

ACKNOWLEDGMENT

This work is supported by NASF (No. 10876007) and GADF (No. 9140A01020109DZ0202).

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