Robust Adaptive Beamforming Based on Interference Covariance Matrix Reconstruction and Steering Vector Estimation

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***Abstract*—Adaptive beamformers are sensitive to model mismatch, especially when the desired signal is present in training snapshots or when the training is done using data samples. In contrast to previous works, this correspondence attempts to reconstruct the interference-plus-noise covariance matrix instead of searching for the optimal diagonal loading factor for the sample covariance matrix. The estimator is based on the Capon spectral estimator integrated over a region separated from the desired signal direction. This is shown to be more robust than using the sample covariance matrix. Subsequently, the mismatch in the steering vector of the desired signal is estimated by maximizing the beamformer output power under a constraint that prevents the corrected steering vector from getting close to the interference steering vectors. The proposed adaptive beamforming algorithm does not impose a norm constraint. Therefore, it can be used even in applications where gain perturbations affect the steering vector. Simulation results demonstrate that the performance of the proposed adaptive beamformer is almost always close to the optimal value across a wide range of signal to noise and signal to interference ratios.**

***Index Terms*—Covariance matrix reconstruction, robust adaptive beamforming, steering vector estimation.**

I. Introduction

Adaptive beamforming is a ubiquitous task in array signal processing and has been widely used in radar, sonar, radio astronomy, wireless communications, microphone array speech processing, medical imaging, and other areas (see, for example, [1], [2], and the references therein). However, the adaptive beamformer is also well-known to be sensitive to model mismatch, especially when the desired signal is present in the training data. Whenever a model mismatch exists, the conventional adaptive beamformer will suffer severe performance degradation. Therefore, robust adaptive beamforming has been an intensive research topic, and various robust adaptive beamforming techniques have been proposed in the past decades; see, e.g., [1] and [2].

In general, these robust techniques can be classified into two categories based on the fundamental Capon beamformer [3]. The first category covers techniques used solely to process the sample covariance matrix, because the exact interference-plus-noise covariance matrix is usually unavailable in practical applications. The most popular one in this category is the so-called diagonal loading technique [4], where a scaled identity matrix is added to the sample covariance matrix. However, choosing the optimal diagonal loading factor in different scenarios is a difficult task. Recently, the shrinkage estimate approach [5] can automatically compute the diagonal loading levels without specifying any user parameters. Unfortunately, this only produces an estimate of the theoretical received signal covariance matrix instead of the required interference-plus-noise covariance matrix. The second category of approaches simply processes the presumed steering vector of the signal, since the exact knowledge of the steering vector is not easy to obtain. In this category, the worst-case performance optimization-based adaptive beamforming technique [2], [6]–[8] makes explicit use of an uncertainty set of the signal steering vector. In practice, neither the mismatch vector nor its norm bound is known. A more recent approach in this category is to estimate the actual steering vector in an iterative way [9].

When the array manifold is fully known there is no difference between using the received signal covariance matrix also known sample matrix inversion (SMI) or minimum power distortionless response (MPDR) and using the interference-plus-noise covariance matrix known as minimum variance distortionless response (MVDR) [1]. However, when the array manifold is not completely known it is better to use the MVDR technique [1], [10]. In contrast, most of the robust techniques developed in recent years use generalizations of the MPDR technique. Recently, efforts to separate the effect of the interferers has been done by Khabbazibasmenj *et al.* [19] and by Mallipeddi *et al.* [20]. In both papers, the interference-plus-noise covariance matrix is replaced by a matrix of the form $\bar{\mathbf{C}} = \int_{\Theta} \mathbf{d}(\theta)\mathbf{d}(\theta)^H d\theta$. In [19], this is only used as part of a semidefinite optimization for correcting the desired signal steering vector. This leads to degraded performance at high signal to noise ratio. In [20], the integral is replaced by a discrete sum over interferences DOAs determined by minimizing the Capon spectral estimator. The fact that no power estimate of the interferes is included in the construction of the covariance matrix, will harm the method in high dynamic range applications, such as radio astronomical imaging of diffuse sources.

In this correspondence, we develop a new robust adaptive beamforming algorithm based on interference-plus-noise covariance matrix reconstruction and steering vector estimation. As mentioned above the MVDR beamformers are much more robust to array manifold errors than the MPDR beamformers [1], [10]. For this reason, we propose to reconstruct the interference-plus-noise covariance matrix using the spatial spectrum distribution instead of searching for the optimal diagonal loading factor for the sample covariance matrix. In addition, the presumed steering vector of the signal is subsequently corrected to maximize the beamformer output power under the constraint that the corrected steering vector does not converge to any interference. By combining them together, we obtain the interference-plus-noise covariance matrix reconstruction plus steering vector estimation-based adaptive beamformer. We also do not require a norm constraint on the steering vector. Hence, the proposed adaptive beamformer can be applied to many more scenarios than previous techniques. Numerical examples show that the performance of the proposed beamforming algorithm is almost always close to optimal performance both at low and high SNR outperforming previously proposed robust beamformers. By combining the interference-plus-noise covariance matrix reconstruction with improved estimate of the desired signal steering vector we overcome the problem of desired signal self-cancelation at high SNR while maintaining the good performance at low SNR. This leads to improved performance over current approaches over a wide range of signal-to-interference-plus-noise ratios.

II. The Signal Model

Assume that an array of M sensors receives signals from multiple narrowband sources. The array observation vector $\mathbf{x}(k) \in \mathcal{C}^M$ at time k can be modeled as

$$\mathbf{x}(k) = \mathbf{x}_s(k) + \mathbf{x}_i(k) + \mathbf{x}_n(k) \quad (1)$$

Manuscript received August 07, 2011; revised January 24, 2012; accepted March 14, 2012. Date of publication April 09, 2012; date of current version June 12, 2012. The associate editor coordinating the review of this manuscript and approving it for publication was Dr. Konstantinos Slavakis. This research was supported by the Israeli Science Foundation under Grant 1240/09.

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This paper has supplementary downloadable multimedia material available at http://ieeexplore.ieee.org provided by the authors. This includes “additional simulations.pdf,” which shows several other simulated experiments demonstrating the performance in other simulation scenarios.

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Digital Object Identifier 10.1109/TSP.2012.2194289

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where $\mathbf{x}_s(k)$, $\mathbf{x}_i(k)$, and $\mathbf{x}_n(k)$ are the statistically independent components of the desired signal, interference, and noise, respectively. The desired signal can be written as $\mathbf{x}_s(k) = \mathbf{a}s(k)$, where $s(k)$ is the signal waveform and $\mathbf{a} \in \mathcal{C}^M$ is the steering vector associated with the desired signal.

The adaptive beamformer output is given by

$$y(k) = \mathbf{w}^H \mathbf{x}(k) \tag{2}$$

where $\mathbf{w} = [w_1, \cdots, w_M]^T \in \mathcal{C}^M$ is the beamformer weight vector and $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and Hermitian transpose, respectively. The optimal beamformer weight vector $\mathbf{w}$ can be obtained by maximizing the output signal-to-interference-plus-noise ratio (SINR)

$$\mathrm{SINR} = \frac{\sigma_s^2 |\mathbf{w}^H \mathbf{a}|^2}{\mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w}} \tag{3}$$

where σ_s^2 is the signal power, $\mathbf{R}_{i+n} = E\{(\mathbf{x}_i(k) + \mathbf{x}_n(k))(\mathbf{x}_i(k) + \mathbf{x}_n(k))^H\} \in \mathcal{C}^{M \times M}$ is the interference-plus-noise covariance matrix, and $E\{\cdot\}$ is the statistical expectation. The problem of maximizing (3) is mathematically equivalent to the MVDR beamforming problem

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w} \quad \text{subject to} \quad \mathbf{w}^H \mathbf{a} = 1, \tag{4}$$

and the solution is the MVDR beamformer, also referred to as the Capon beamformer,

$$\mathbf{w}_{\mathrm{opt}} = \frac{\mathbf{R}_{i+n}^{-1} \mathbf{a}}{\mathbf{a}^H \mathbf{R}_{i+n}^{-1} \mathbf{a}}, \tag{5}$$

which is a function of two factors $\mathbf{R}_{i+n}$ and $\mathbf{a}$.

Since $\mathbf{R}_{i+n}$ is unavailable even in signal-free applications, it is usually replaced by the sample covariance matrix $\hat{\mathbf{R}} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{x}(k)\mathbf{x}^H(k)$ with K snapshots, and the corresponding adaptive beamformer $\mathbf{w}_{\mathrm{SMI}} = \frac{\hat{\mathbf{R}}^{-1}\mathbf{a}}{\mathbf{a}^H \hat{\mathbf{R}}^{-1}\mathbf{a}}$ is called the sample matrix inversion (SMI) beamformer. Whenever the desired signal is present in the training samples, the SMI beamformer is in essence the MPDR beamformer [1] instead of the MVDR beamformer (4). As K increases, $\hat{\mathbf{R}}$ converges to the theoretical covariance matrix $\mathbf{R} = \sigma_s^2 \mathbf{a}\mathbf{a}^H + \mathbf{R}_{i+n}$, and the corresponding SINR will approximate the optimal value as $K \to \infty$ under stationary and ergodic assumptions. However, when the sample size K is small, there is a large gap between $\hat{\mathbf{R}}$ and $\mathbf{R}$. This gap is known to dramatically affect the performance, especially when the signal is present in the training samples [4], [11]. Furthermore, using $\hat{\mathbf{R}}$ is much more sensitive to steering vector errors [1], [10].

III. The Proposed Algorithm

In this section, a new adaptive beamforming algorithm is proposed. The basic idea is to reconstruct the interference-plus-noise covariance matrix first, and then estimate the steering vector of the desired signal. Therefore, the section is divided into two parts. First, we discuss the problem of reconstructing the interference-plus-noise covariance matrix. Then, we discuss the problem of estimating the signal steering vector.

A. Interference-Plus-Noise Covariance Matrix Reconstruction

Previous works on robust adaptive beamforming focused on finding the optimal diagonal loading factor for the sample covariance matrix $\hat{\mathbf{R}}$, which inevitably resulted in performance degradation at high SNRs. This degradation is caused by the fact that the signal is always present in any kind of diagonal loading technique, and its effect becomes more and more pronounced with increases in SNR. This explains the performance degradation of adaptive beamformers at high SNRs. Therefore, we will reconstruct the interference-plus-noise covariance matrix $\mathbf{R}_{i+n}$ directly instead of searching for an optimal diagonal loading factor.

Recall that $\mathbf{R}_{i+n} = \sum_{l=1}^{L} \sigma_l^2 \mathbf{a}(\theta_l)\mathbf{a}^H(\theta_l) + \sigma_0^2 \mathbf{I}$, where L is the number of interferers, σ_l^2 is the power of the interference impinging from direction θ_l and $\mathbf{a}(\theta_l)$ is the corresponding steering vector, σ_0^2 is the noise power, and $\mathbf{I}$ is the identity matrix. Generally, the number of interferers, as well as their actual steering vectors and powers, are usually unknown. Moreover, the noise power is also unknown. Hence, in order to reconstruct the interference-plus-noise covariance matrix $\mathbf{R}_{i+n}$, we need to know the spatial spectrum distribution over all possible directions. In this correspondence, we use the Capon spatial spectrum estimator

$$\hat{P}(\theta) = \frac{1}{\mathbf{d}^H(\theta)\hat{\mathbf{R}}^{-1}\mathbf{d}(\theta)} \tag{6}$$

which is easy to obtain by substituting the Capon beamformer (5) back into the objective function of (4) with $\hat{\mathbf{R}}$. Aside from the Capon spatial spectrum estimator (6), there are other candidate spatial spectrum estimators [12]. Using the Capon spatial spectrum (6), the interference-plus-noise covariance matrix $\mathbf{R}_{i+n}$ can be reconstructed as

$$\begin{aligned}\tilde{\mathbf{R}}_{i+n} &= \int_{\bar{\Theta}} \hat{P}(\theta)\mathbf{d}(\theta)\mathbf{d}^H(\theta)d\theta \\ &= \int_{\bar{\Theta}} \frac{\mathbf{d}(\theta)\mathbf{d}^H(\theta)}{\mathbf{d}^H(\theta)\hat{\mathbf{R}}^{-1}\mathbf{d}(\theta)}d\theta \end{aligned} \tag{7}$$

where $\mathbf{d}(\theta)$ is the steering vector associated with a hypothetical direction θ based on the known array structure (note that for the specific direction of the desired signal θ_s $\bar{\mathbf{a}} = \mathbf{d}(\theta_s)$), and $\bar{\Theta}$ is the complement sector of Θ. That is to say, $\Theta \cup \bar{\Theta}$ covers the whole spatial domain, and $\Theta \cap \bar{\Theta}$ is empty. Here, Θ is an angular sector in which the desired signal is located. The desired signal DOA θ_s can be estimated, for example, using low-resolution direction finding methods, and the width of Θ is determined by the resolution of the array and the propagation environment. The main requirement is that the signal's direction is in Θ while the interferers are not. Hence, $\tilde{\mathbf{R}}_{i+n}$ collects all information on interference and noise in the out-of-sector $\bar{\Theta}$. Consequently, the effect of the desired signal is removed from the reconstructed covariance matrix, as long as the desired signal's direction is located inside Θ.

Before continuing, we would like to point out that, in the case of look direction mismatch, the estimate of the interference covariance matrix will be accurate, as long as the choice of Θ separates the interference from the desired signal. Similarly, in the case of local incoherent scattering effects, we expect the estimate to be very good. In the case of random steering vector errors, the effect of the random errors will be averaged over the various directions of the interference, therefore providing reasonable estimates of the interference-plus-noise covariance matrix. These observations provide at list an insight into the ad hoc estimate (7).

Using $\tilde{\mathbf{R}}_{i+n}$ (7) in lieu of $\hat{\mathbf{R}}$ in the SMI beamformer yields the interference-plus-noise covariance matrix reconstruction-based adaptive beamformer

$$\mathbf{w}_{\mathrm{Rec}} = \frac{\tilde{\mathbf{R}}_{i+n}^{-1}\bar{\mathbf{a}}}{\bar{\mathbf{a}}^H \tilde{\mathbf{R}}_{i+n}^{-1}\bar{\mathbf{a}}} \tag{8}$$

with the presumed steering vector $\bar{\mathbf{a}} = \mathbf{d}(\theta_s)$.

The computational complexity of the proposed algorithm is $\mathcal{O}(M^2 S)$, where S is the number of sampling points in $\bar{\Theta}$ during the

TABLE I
RECONSTRUCTION-PLUS-ESTIMATION-BASED ADAPTIVE BEAMFORMING ALGORITHM

step 1: Reconstruct the interference-plus-noise covariance matrix $\tilde{\mathbf{R}}_{i+n}$ (7) based on the Capon spatial spectrum $\hat{P}(\theta)$ (6).
step 2: Estimate the mismatch vector component $\mathbf{e}_\perp$ by solving the QCQP problem (11).
step 3: Calculate the adaptive beamformer weight (13) based on the corrected steering vector $\tilde{\mathbf{a}}$ from (12) with $\mathbf{e}_\perp$.

reconstruction of covariance matrix $\tilde{\mathbf{R}}_{i+n}$ (7). Typically, $S \gg M$. The computational complexity of the simplest SMI beamformer is $\mathcal{O}(M^3)$ mainly because of the matrix inversion operation [2]. Note, however, that if the spatial estimate in the whole region is desired, the SMI beamformer has complexity $\mathcal{O}(M^2S)$ as well, assuming $S \gg M$. Therefore, the simplified version of our beamformer, obtained by (8), has complexity slightly larger than the SMI but significantly more robust performance.

In practice, the presumed steering vector $\bar{\mathbf{a}}$ is usually different than the actual steering vector $\mathbf{a}$. Therefore, in the following subsection, we will estimate the actual steering vector of the signal using the reconstructed covariance matrix $\tilde{\mathbf{R}}_{i+n}$ (7). This provides additional beamforming gain at the expense of increased computational complexity.

B. Desired Signal Steering Vector Estimation

The actual steering vector is difficult to obtain in practical applications by simply using the nominal DOA of the signal because of the complex propagation environment. Hence, here we will correct the presumed steering vector to maximize the beamformer output power.

Substituting the Capon beamformer (5) with the reconstructed covariance matrix $\tilde{\mathbf{R}}_{i+n}$ (7) back into the objective function of (4), the beamformer output power is

$$\tilde{P}(\mathbf{a}) = \frac{1}{\mathbf{a}^H \tilde{\mathbf{R}}_{i+n}^{-1} \mathbf{a}} \tag{9}$$

which now is a function of the steering vector $\mathbf{a}$ not just the nominal DOA θ as in Section III-A. The steering vector $\mathbf{a}$ can be estimated by maximizing $\tilde{P}(\mathbf{a})$ or, equivalently, by minimizing the denominator of $\tilde{P}(\mathbf{a})$. In order to exclude the trivial solution $\mathbf{a} = \mathbf{0}$, the presumed steering vector $\bar{\mathbf{a}}$ should be utilized, which can be obtained from the nominal DOA of the signal with the known array structure. Specifically, the optimization problem of estimating $\mathbf{a}$ can be transformed to estimate the mismatch vector $\mathbf{e}$ as

$$\begin{aligned} \min_{\mathbf{e}} \quad & (\bar{\mathbf{a}}+\mathbf{e})^H \tilde{\mathbf{R}}_{i+n}^{-1} (\bar{\mathbf{a}}+\mathbf{e}) \\ \text{subject to} \quad & (\bar{\mathbf{a}}+\mathbf{e})^H \tilde{\mathbf{R}}_{i+n} (\bar{\mathbf{a}}+\mathbf{e}) \le \bar{\mathbf{a}}^H \tilde{\mathbf{R}}_{i+n} \bar{\mathbf{a}} \end{aligned} \tag{10}$$

where the inequality constraint is a single constraint expression of $|c(\theta)(\bar{\mathbf{a}}+\mathbf{e})^H \mathbf{d}(\theta)| \le |c(\theta)\bar{\mathbf{a}}^H \mathbf{d}(\theta)|$ for all $\theta \in \bar{\Theta}$ with a weight coefficient $c(\theta) = \sqrt{\hat{P}(\theta)}$, which can be used to prevent the corrected steering vector $\bar{\mathbf{a}}+\mathbf{e}$ from converging to any interference located in $\bar{\Theta}$. Here, the choice of $c(\theta)$ is based on the fact that the interference with higher power should be more suppressed. Furthermore, the resulting constraint matrix is just the reconstructed covariance matrix. If $c(\theta)$ is chosen to be independent of θ, e.g., $c(\theta) = 1, \forall\, \theta \in \bar{\Theta}$, then the inequality constraint becomes $(\bar{\mathbf{a}}+\mathbf{e}_\perp)^H \bar{\mathbf{C}} (\bar{\mathbf{a}}+\mathbf{e}_\perp) \le \bar{\mathbf{a}}^H \bar{\mathbf{C}} \bar{\mathbf{a}}$ with the constraint matrix $\bar{\mathbf{C}} = \int_{\bar{\Theta}} \mathbf{d}(\theta)\mathbf{d}^H(\theta) d\theta$ [9]. Basically, this constraint boils down to the requirement that the weighted average angle between the corrected steering vector and the vectors in the interference region will not increase relative to the average angle between the nominal steering vector and the same vectors.

As we pointed out in [15], in some works on robust adaptive beamforming, the steering vector is assumed to have a fixed norm $\sqrt{M}$, i.e., $\|\mathbf{a}\| = \sqrt{M}$. Although it does relax the requirement that the received signals from different sensors should have the same gain as the classical assumption in array signal processing, this norm constraint is still restrictive because in practical applications, e.g., in wireless communications, the gain perturbations of different sensors cannot be regarded as small as previously, and the norm constraint no longer holds. Hence, unlike [9], [13], this norm constraint will not be used in our formulation (10).

The mismatch vector $\mathbf{e}$ can be further decomposed into two components. One denoted by $\mathbf{e}_\perp$ is orthogonal to $\bar{\mathbf{a}}$, and the other denoted by $\mathbf{e}_{\|}$ is parallel to $\bar{\mathbf{a}}$. $\mathbf{e}_{\|}$ does not affect the beamforming quality because it is a scaled copy of $\bar{\mathbf{a}}$ and any scaling of the steering vector does not impact the SINR. Therefore, the optimization problem (10) can be further simplified to search for the orthogonal component $\mathbf{e}_\perp$ by solving the following problem:

$$\begin{aligned} \min_{\mathbf{e}_\perp} \quad & (\bar{\mathbf{a}}+\mathbf{e}_\perp)^H \tilde{\mathbf{R}}_{i+n}^{-1} (\bar{\mathbf{a}}+\mathbf{e}_\perp) \\ \text{subject to} \quad & \bar{\mathbf{a}}^H \mathbf{e}_\perp = 0, \\ & (\bar{\mathbf{a}}+\mathbf{e}_\perp)^H \tilde{\mathbf{R}}_{i+n} (\bar{\mathbf{a}}+\mathbf{e}_\perp) \le \bar{\mathbf{a}}^H \tilde{\mathbf{R}}_{i+n} \bar{\mathbf{a}} \end{aligned} \tag{11}$$

where the equality constraint is introduced to maintain the orthogonality between $\mathbf{e}_\perp$ and $\bar{\mathbf{a}}$. Because $\tilde{\mathbf{R}}_{i+n} \succ 0$ is a positive-definite matrix, the optimization problem (11) is a feasible quadratically constrained quadratic programming (QCQP) problem and can be easily solved using convex optimization software, such as CVX [14].

Except the two constraints in (11), we do not need any other constraint. Hence, the additional eigendecomposition of the covariance matrix in [9], [13], [15] is effectively avoided in our approach. Considering that the objective of the beamformer design is to maximize SINR (3), the presumed steering vector $\bar{\mathbf{a}}$ is corrected as

$$\tilde{\mathbf{a}} = \bar{\mathbf{a}} + \mathbf{e}_\perp \tag{12}$$

as soon as the problem (11) is solved. Here, we emphasize once again that the normalization operation $\frac{\|\tilde{\mathbf{a}}\|}{\sqrt{M}} = 1$ is unnecessary, because we do not know *a priori* the norm of the actual steering vector and more importantly, this normalization operation does not affect the beamformer output SINR at all. Hence, we can eliminate the iteration operation in [9] and [13].

Up to now, both the reconstructed interference-plus-noise covariance matrix $\tilde{\mathbf{R}}_{i+n}$ (7) and the corrected steering vector $\tilde{\mathbf{a}}$ (12) have been obtained. Substituting them back into the Capon beamformer (5) together, the adaptive beamformer based on covariance matrix reconstruction plus steering vector estimation can be computed as

$$\mathbf{w}_{\mathrm{Rec-Est}} = \frac{\tilde{\mathbf{R}}_{i+n}^{-1} \tilde{\mathbf{a}}}{\tilde{\mathbf{a}}^H \tilde{\mathbf{R}}_{i+n}^{-1} \tilde{\mathbf{a}}}. \tag{13}$$

The reconstruction-plus-estimation-based adaptive beamforming algorithm is summarized in Table I. The computational complexity in this case is dominated by the solution of the QCQP problem, which is $O(M^{3.5})$. This makes the two step algorithm equivalent to other robust beamforming algorithms.

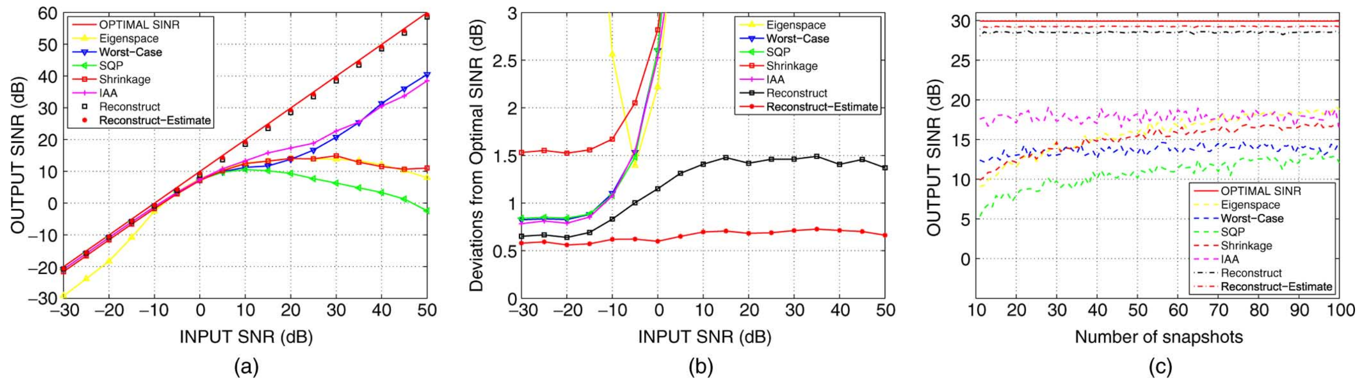

Fig. 1. First example. (a) Output SINR versus input SNR; (b) deviations from optimal SINR versus SNR; (c) output SINR versus number of snapshots.

IV. Simulation

In our simulations, a uniform linear array (ULA) with $M = 10$ omnidirectional sensors spaced a half wavelength apart is used. The additive noise is modeled as a complex circularly symmetric Gaussian zero-mean spatially and temporally white process. Two interfering sources are assumed to have DOAs -50° and -20°, respectively. The interference-to-noise ratio (INR) in each sensor is equal to 30 dB. The desired signal is assumed to be a plane-wave from the presumed direction $\theta_s = 5^\circ$. When comparing the performance of the adaptive beamforming algorithms in terms of the number of samples, the SNR in each sensor is set to be fixed at 20 dB. In the performance comparison of mean output SINR versus the input SNR, the number of snapshots is fixed to be $K = 30$. For each scenario, 200 Monte-Carlo runs are performed.

The proposed beamformers (8) and (13) are compared to the eigenspace-based beamformer [16], the worst-case-based beamformer [6], the sequential quadratic programming (SQP)-based beamformer [9], the shrinkage estimation-based beamformer [5], and the iterative adaptive approach (IAA) beamformer [17]. In the SQP-based beamformer and the proposed beamformers, the possible angular sector of the desired signal is set to be $\Theta = [0^\circ, 10^\circ]$ and the corresponding out-of-sector is $\bar{\Theta} = [-90^\circ, 0^\circ) \cup (10^\circ, 90^\circ]$. The value $\delta = 0.1$ and six dominant eigenvectors of the matrix $\mathbf{C} = \int_{\Theta} \mathbf{d}(\theta)\mathbf{d}^H(\theta)d\theta$ are used in the SQP-based beamformer, and the value $\epsilon = 0.3\ \mathrm{M}$ is used for the worst-case-based beamformer. The eigenspace-based beamformer is assumed to know the number of interference sources. The optimal SINR (3) is also shown in all figures, which is calculated from the exact interference-plus-noise covariance matrix and the actual desired signal steering vector. CVX software [14] was used to solve these convex optimization problems. It should be emphasized that in all simulations, the actual steering vector $\mathbf{a}$ was not normalized so that $\mathbf{a}^H\mathbf{a} = M$ was not always satisfied.

A. Example 1: Random Signal and Interference Look Direction Mismatch

In the first example, a scenario with random look direction mismatch is considered. The random DOA mismatch of both the desired signal and the interferers are uniformly distributed in $[-4^\circ, 4^\circ]$. That is to say, the DOA of the signal is uniformly distributed in $[1^\circ, 9^\circ]$, and the DOAs of two interferences are uniformly distributed in $[-54^\circ, -46^\circ]$ and $[-24^\circ, -16^\circ]$, respectively. Here, the random DOAs of the signal and the interferences change from run to run but remain fixed from snapshot to snapshot. Fig. 1(a) compares the output SINR of the aforementioned methods versus the SNR. Because the performance difference is not straightforward especially at low SNRs, their deviations from the optimal SINR are compared in Fig. 1(b). In Fig. 1(c), the output SINRs for the tested methods are illustrated against the number of snapshots K. It can be seen from these figures that the performances of the proposed beamformers are always close to the optimal SINR in a large range from -30 to 50 dB. Furthermore, the proposed algorithms enjoy much faster convergence rates than others. The signal power is 100 times the interference power in the case of $\mathrm{SNR} = 50$ dB, which can be used to illustrate the situation when the signal-to-interference ratio (SIR) approaches ∞.

B. Example 2: Signal Spatial Signature Mismatch due to Incoherent Local Scattering

A distributed or incoherent scattered source arises mainly from the multipath scattering effects caused by the presence of local scatterers, which is also commonplace in radar, sonar, radio astronomy and wireless communications applications. In this example, we assume incoherent local scattering of the signal. The signal is assumed to have a time-varying spatial signature that is different for each data snapshot and is modeled as

$$\mathbf{a}(k) = s_0(k)\bar{\mathbf{a}} + \sum_{p=1}^{4} s_p(k)\mathbf{d}(\theta_p) \tag{14}$$

where $s_0(k)$ and $s_p(k), p = 1, 2, 3, 4$ are independently and identically distributed (i.i.d.) zero-mean complex Gaussian random variables independently drawn from a random generator $\mathcal{N}(0, 1)$. The DOAs $\theta_p, p = 1, 2, 3, 4$ are independently normally distributed in $\mathcal{N}(\theta_s, 4^\circ)$ in each simulation run. It should be pointed out that the beamformers are implemented in a block adaptive manner, which means that θ_p changes from run to run while remains fixed from snapshot to snapshot. At the same time, the random variables $s_0(k)$ and $s_p(k)$ change not only from run to run but also from snapshot to snapshot. This corresponds to the case of incoherent local scattering [18]. In this scenario, the norm of the desired signal steering vector $\|\mathbf{a}\|$ keeps changing from snapshot to snapshot. Due to the fact that the signal covariance matrix $\mathbf{R}_s$ is no longer a rank-one matrix in this scenario, the output SINR should be rewritten as [11]

$$\mathrm{SINR} = \frac{\mathbf{w}^H\mathbf{R}_s\mathbf{w}}{\mathbf{w}^H\mathbf{R}_{i+n}\mathbf{w}} \tag{15}$$

instead of (3). The SINR (15) is maximized by [11]

$$\mathbf{w}_{\mathrm{opt}} = \mathcal{P}\left\{\mathbf{R}_{i+n}^{-1}\mathbf{R}_s\right\} \tag{16}$$

where $\mathcal{P}\{\cdot\}$ stands for the principal eigenvector of a matrix. It can be seen from Fig. 2 that although there are some performance degradations

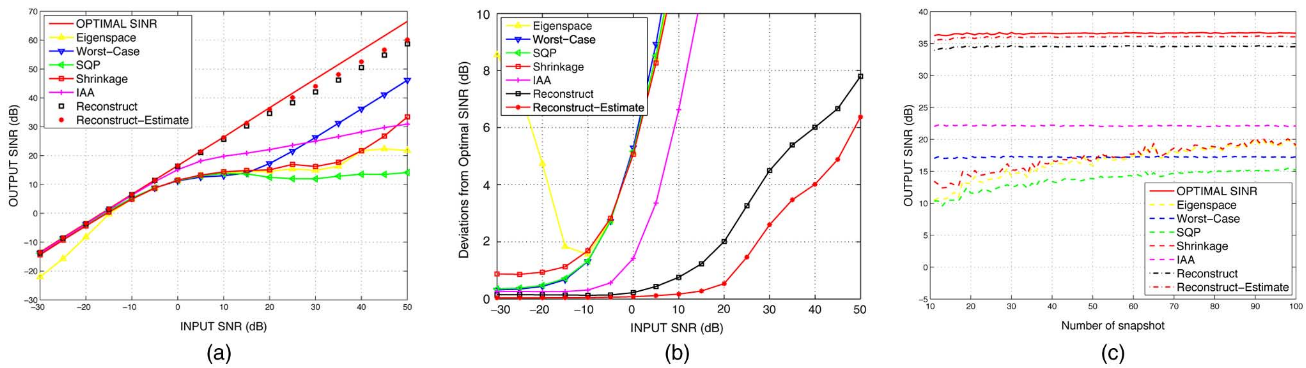

Fig. 2. Second example. (a) Output SINR versus input SNR; (b) deviations from optimal SINR versus SNR; (c) output SINR versus number of snapshots.

with the increase in SNR compared to the optimal value, the proposed beamforming algorithms still outperform the other methods tested.

The simulation results clearly demonstrate that the proposed beamformers outperform all other existing beamformers, and achieve performance that is consistently close to the optimal SINR for all values of SNR from -30 to 50 dB, which illustrates its high dynamic range. Specifically,

$$\mathrm{SINR}_{\mathrm{Rec-Est}} \simeq \|\mathbf{a}\|^2 \mathrm{SNR}. \tag{17}$$

In the first example, the actual steering vector satisfies the norm constraint and hence, the array output SINRs are approximately 10 dB higher than the SNR. By contrast, in the second example, the norm of the actual steering vector is magnified due to the effect of incoherent local scattering and the output SINRs of all tested beamformers are subsequently scaled up.

V. Conclusion

In this correspondence, we proposed an effective adaptive beamforming algorithm, which is robust not only to covariance matrix uncertainty but also to steering vector mismatch. With the knowledge of the presumed DOA of the desired signal, the interference-plus-noise covariance matrix can be reconstructed based on the spatial spectrum distribution, which provides a quasi signal-free environment. Based on the reconstructed covariance matrix, the presumed steering vector of the signal is corrected to maximize the array output power. In contrast to other algorithms, the proposed adaptive beamforming algorithm does not need the norm constraint of the steering vector, which thus widely extends its applicability. The simulation results demonstrate that the performance of proposed adaptive beamformer is almost always close to optimal in a very large range of SNR.

Acknowledgment

The authors would like to thank the associate editor Prof. Salvakis and the anonymous reviewers for their helpful comments and suggestions.

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