Robust Adaptive Beamforming via Estimating Steering Vector Based on Semidefinite Relaxation

Arash Khabbazibasmenj, Sergiy A. Vorobyov, and Aboulnasr Hassanien
Dept. of Electrical and Computer Engineering
University of Alberta, Edmonton, AB T6G 2V4 Canada
Email: khabbazi, vorobyov, hassanie@ece.ualberta.ca

***Abstract*—Most of the known robust adaptive beamforming techniques can be unified under one framework. This is to use minimum variance distortionless response principle for beamforming vector computation in tandem with sample covariance matrix estimation and steering vector estimation based on some information about steering vector prior. Motivated by such unified framework, we develop a new robust adaptive beamforming method based on finding a more accurate estimate of the actual steering vector than the available prior. The objective for finding such steering vector estimate is the maximization of the beamformer output power under the constraints that the estimate does not converge to an interference steering vector and does not change the norm of the prior. The resulting optimization problem is a non-convex quadratically constrained quadratic programming problem, which is NP hard in general, but can be efficiently and exactly solved in our specific case. Our simulation results demonstrate the superiority of the proposed method over other robust adaptive beamforming methods.**

I. Introduction

Robust adaptive beamforming is one of the classic array processing problems with ubiquitous applicability in wireless communications, radar, sonar, microphone array speech processing, radio astronomy, medical imaging, etc. Thus, various robust adaptive beamforming techniques gained a significant popularity due to their practical importance [1]. Among first robust adaptive beamforming techniques are the diagonal loading [2], [3] and the eigenspace-based beamformers [4]. More recent and more rigorous techniques are the worst-case-based adaptive beamforming [5]- [8], the probabilistically constrained robust adaptive beamforming [9], [10], doubly constrained robust Capon beamforming [11], and the method of [12], [13] based on steering vector estimation.

In general, most of the known robust adaptive beamforming techniques can be unified under one framework which can be summarized as follows. Use minimum variance distortionless response principle for beamforming vector computation in tandem with sample covariance matrix estimation and steering vector estimation based on some prior information about steering vector. For example, in the worst-case-based robust adaptive beamformer of [5], a presumed steering vector is assumed to be known, while the steering vector mismatch is modeled as an unknown norm bounded deterministic vector. It has been widely popularized that the method of [5] can be also obtained based on the covariance fitting approach [7] that involves a steering vector estimation based on the corresponding prior information. The probabilistically constrained robust adaptive beamformer of [9] uses different prior information, but it structurally reduces to the the worst-case-based approach and, thus, can also be understood in terms of the steering vector estimation-based framework. The doubly-constrained Capon robust adaptive beamformer of [11] explicitly estimates the steering vector using the same prior information as the worst-case-based approach, while the robust beamformer of [12] estimates the steering vector by maximizing the beamforming output power and restricting the estimate from convergence to any interference steering vector. The later is achieved in [12] by projecting a steering vector estimate to a certain subspace obtained from a matrix computed over an angular sector around the presumed steering vector.

Based on such unified framework to robust adaptive beamforming, we develop a new beamforming technique in which the steering vector is estimated by the beamformer output power maximization under the constraint on the norm of the steering vector estimate and the requirement that the estimate does not converge to an interference steering vector. To satisfy the latter requirement, we develop a new constraint which is different from the one in [12] and is convex quadratic. In general, our new robust adaptive beamforming technique differers from other techniques by the prior information about steering vector. The corresponding optimization problem is a non-convex (due to the steering vector normalization condition) quadratically constrained quadratic programming (QCQP) problem which satisfies the strong duality property (see [14] and [15] for similar problems). However, the analysis in [14] is based on extended version of the so-called S-lemma, while the analysis in [15] is based on the so-called rank reduction technique. Both approaches do not ideally fit the purposes of our study. Thus, we take a more common for array processing linear algebra-based approach while solving the corresponding optimization problem. It allows us to draw new links to the previously proposed robust adaptive beamforming techniques. Our simulation results demonstrate the superiority of the proposed method over other previously developed robust adaptive beamforming techniques.

II. System Model and Problem Formulation

The narrowband signal received by a linear antenna array with M omni-directional antenna elements at the time instant

This work is supported in parts by the Natural Science and Engineering Research Council (NSERC) of Canada and the Alberta Innovates – Technology Futures,, Alberta, Canada.

k is

$$\mathbf{x}(k) = \mathbf{s}(k) + \mathbf{i}(k) + \mathbf{n}(k) \tag{1}$$

where $\mathbf{s}(k)$, $\mathbf{i}(k)$, and $\mathbf{n}(k)$ are the $M \times 1$ vectors of the desired signal, interference, and noise, respectively. The desired signal, interference, and noise are assumed to be statistically independent to each other, and the desired signal can be written as $\mathbf{s}(k) = s(k)\mathbf{a}$, where $s(k)$ is the desired signal waveform and $\mathbf{a}$ is the desired signal steering vector.

The beamformer output at the time instant k is

$$y(k) = \mathbf{w}^H \mathbf{x}(k) \tag{2}$$

where $\mathbf{w}$ is the $M \times 1$ complex weight (beamforming) vector of the antenna array and $(\cdot)^H$ stands for the Hermitian transpose.

Using the available data sample covariance matrix

$$\hat{\mathbf{R}} = \frac{1}{K} \sum_{i=1}^{K} \mathbf{x}(i)\mathbf{x}^H(i) \tag{3}$$

instead of the unavailable interference-plus-noise covariance matrix $\mathbf{R}_{i+n} \triangleq E\{(\mathbf{i}(k) + \mathbf{n}(k))(\mathbf{i}(k) + \mathbf{n}(k))^H\}$, the beamforming problem can be formulated as the problem of finding such vector $\mathbf{w}$ that maximizes the beamformer output signal-to-noise-plus-interference ratio (SINR)

$$SINR = \frac{\sigma_s^2 |\mathbf{w}^H \mathbf{a}|^2}{\mathbf{w}^H \hat{\mathbf{R}} \mathbf{w}}. \tag{4}$$

Here K is the number of training data samples which also include the desired signal component, $E\{\cdot\}$ stands for the statistical expectation, and σ_s^2 is the desired signal power. The solution to this problem is given by the well-known minimum variance (MV) sample matrix inversion (SMI) beamforming, that is,

$$\mathbf{w}_{\mathrm{MV-SMI}} = \alpha \hat{\mathbf{R}}^{-1} \mathbf{a} \tag{5}$$

where $\alpha = 1/\mathbf{a}^H \hat{\mathbf{R}}^{-1} \mathbf{a}$.

Robust adaptive beamforming techniques address the situations when the steering vector $\mathbf{a}$ is not known precisely as well as when the sample estimate of the data covariance matrix (3) is inaccurate (for example, because of small sample size). It is usually assumed that some prior information about steering vector is available such as the presumed steering vector $\mathbf{p}$ as well as some information about the steering vector mismatch $\boldsymbol{\delta}$. The actual steering vector differs from the presumed one and can be expressed as $\mathbf{a} = \mathbf{p} + \boldsymbol{\delta}$ where $\boldsymbol{\delta}$ is unknown.

Most of the known robust adaptive beamforming methods can be represented in the form of (5) with an estimate of the steering vector $\hat{\mathbf{a}}$. The estimate $\hat{\mathbf{a}}$ is found using the prior given by the presumed steering vector $\mathbf{p}$. For example, in [12], [13], the estimate $\hat{\mathbf{a}}$ is found so that the beamformer output power is maximized while the convergence of $\hat{\mathbf{a}}$ to any interference steering vector is prohibited. Specifically, (5) can be written as a function of unknown $\boldsymbol{\delta}$, that is, $\mathbf{w}(\boldsymbol{\delta}) = \alpha \hat{\mathbf{R}}^{-1}(\mathbf{p} + \boldsymbol{\delta})$. Using the latter expression, the beamformer output power can be also expressed as

$$P(\boldsymbol{\delta}) = \frac{1}{(\mathbf{p} + \boldsymbol{\delta})^H \hat{\mathbf{R}}^{-1} (\mathbf{p} + \boldsymbol{\delta})}. \tag{6}$$

Thus, such estimate of $\boldsymbol{\delta}$ or, equivalently, $\mathbf{a}$ that maximizes (6) is the best estimate of the actual steering vector $\mathbf{a}$ under the constraints that the norm of $\hat{\mathbf{a}}$ equals $\sqrt{M}$ and $\hat{\mathbf{a}}$ does not converge to any interference steering vectors. The latter is guaranteed in [12], [13] by requiring that

$$\mathbf{P}^{\perp}(\mathbf{p} + \hat{\boldsymbol{\delta}}) = \mathbf{P}^{\perp}\hat{\mathbf{a}} = 0 \tag{7}$$

where $\mathbf{P}^{\perp} = \mathbf{I} - \mathbf{U}\mathbf{U}^H$, $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_L]$, $\mathbf{u}_l$, $l = 1, \ldots, L$ are the L dominant eigenvectors of the matrix

$$\mathbf{C} = \int_{\Theta} \mathbf{d}(\theta)\mathbf{d}^H(\theta)\, d\theta \tag{8}$$

with $\mathbf{d}(\theta)$ being the steering vector associated with direction θ and having the structure defined by the antenna geometry, Θ being the angular sector in which the desired signal is located, $\hat{\boldsymbol{\delta}}$ and $\hat{\boldsymbol{a}}$ standing for the estimate of the steering vector mismatch and the estimate of the actual steering vector, respectively, and $\mathbf{I}$ being the identity matrix.

III. New Robust Beamforming Problem Formulation

The output power (6) is maximized if the denominator of (6) is minimized. While maximizing (6) one needs to guarantee that the norm of the estimate $\hat{\mathbf{a}}$ equals $\sqrt{M}$, i.e., $\|\hat{\mathbf{a}}\| = \sqrt{M}$ and $\hat{\mathbf{a}}$ does not converge to any interference steering vector. We suggest to achieve the latter purpose by means of the following new constraint. Let us assume that the desired source is located in the angular sector $\Theta = [\theta_{\min}, \theta_{\max}]$ which can be obtained using low resolution direction finding methods and it is distinguishable from general locations of all interfering signals. Let us define the $M \times M$ matrix $\tilde{\mathbf{C}}$ as

$$\tilde{\mathbf{C}} = \int_{\tilde{\Theta}} \mathbf{d}(\theta)\mathbf{d}^H(\theta)\, d\theta \tag{9}$$

where $\tilde{\Theta}$ denotes the complement of the sector Θ. Then the new constraint can be expressed as

$$\hat{\mathbf{a}}^H \tilde{\mathbf{C}} \hat{\mathbf{a}} \leq \Delta_0 \tag{10}$$

where Δ_0 is the maximum value(s) of $\mathbf{d}^H(\theta)\tilde{\mathbf{C}}\mathbf{d}(\theta)$ within the sector Θ. Thus, as soon as the sector Θ is known, the value of Δ_0 is unique.

The constraint (10) forces the estimate $\hat{\mathbf{a}}$ not to converge to any interference steering vector with the directions within the angular sector $\tilde{\Theta}$. Indeed, let us consider as an example a uniform linear array (ULA) of 10 omni-directional antenna elements spaced half wavelength apart from each other. Let the range of the desired signal angular locations be $\Theta = [0°, 10°]$. Fig. 1 shows the values of the quadratic term $\mathbf{d}^H(\theta)\tilde{\mathbf{C}}\mathbf{d}(\theta)$ for different angles. The rectangular bar in the figure marks the directions within the angular sector Θ. It can be observed from this figure that the term $\mathbf{d}^H(\theta)\tilde{\mathbf{C}}\mathbf{d}(\theta)$ takes the smallest values within the angular sector Θ where the desired signal is located, and increases outside of this sector. Therefore, if Δ_0 equals to the maximum value of the term $\mathbf{d}^H(\theta)\tilde{\mathbf{C}}\mathbf{d}(\theta)$ within Θ, the constraint (10) guarantees that the estimate $\hat{\mathbf{a}}$ does not converge to any interference steering vectors. The constraint (10) is an alternative to the constraint (7). This can be further explained later.

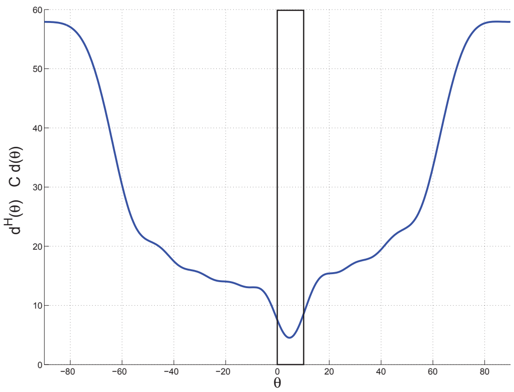

Fig. 1. The term $\mathbf{d}^H(\theta)\tilde{\mathbf{C}}\mathbf{d}(\theta)$ in the constraint (10) versus different angles.

The problem of finding the estimate $\hat{\mathbf{a}}$ can be then formulated as the following optimization problem

$$\begin{aligned} \min_{\hat{\mathbf{a}}} \quad & \hat{\mathbf{a}}^H\hat{\mathbf{R}}^{-1}\hat{\mathbf{a}} && (11)\\ \text{subject to} \quad & \|\hat{\mathbf{a}}\| = \sqrt{M} && (12)\\ & \hat{\mathbf{a}}^H\tilde{\mathbf{C}}\hat{\mathbf{a}} \leq \Delta_0. && (13) \end{aligned}$$

where the prior $\mathbf{p}$ is used only for selecting the sector Θ. Because of the non-convex equality constraint (12) the QCQP problems of type (11)–(13) are non-convex and an NP-hard in general. However, an exact and simple solution for the problem (11)–(13) can be found.

IV. Solution via Semidefinite Programming Relaxation

First, it can be verified that the problem (11)–(13) is feasible if and only if Δ_0/M is greater than or equal to the smallest eigenvalue of the matrix $\tilde{\mathbf{C}}$. Indeed, if the smallest eigenvalue of $\tilde{\mathbf{C}}$ is larger than Δ_0/M, then the constraint (13) can not be satisfied for any estimate $\hat{\mathbf{a}}$. However, Δ_0 selected so (see above) that the aforementioned feasibility condition is always satisfied and, thus the problem (11)–(13) is feasible.

Second, if the problem (11)–(13) is feasible, the equalities $\hat{\mathbf{a}}^H\hat{\mathbf{R}}^{-1}\hat{\mathbf{a}} = Tr(\hat{\mathbf{R}}^{-1}\hat{\mathbf{a}}\hat{\mathbf{a}}^H)$ and $\hat{\mathbf{a}}^H\tilde{\mathbf{C}}\hat{\mathbf{a}} = Tr(\tilde{\mathbf{C}}\hat{\mathbf{a}}\hat{\mathbf{a}}^H)$, where $Tr(\cdot)$ denotes the trace of a matrix, can be used to rewrite it as the following optimization problem

$$\begin{aligned} \min_{\hat{\mathbf{a}}} \quad & Tr(\hat{\mathbf{R}}^{-1}\hat{\mathbf{a}}\hat{\mathbf{a}}^H) && (14)\\ \text{subject to} \quad & Tr(\hat{\mathbf{a}}\hat{\mathbf{a}}^H) = M && (15)\\ & Tr(\tilde{\mathbf{C}}\hat{\mathbf{a}}\hat{\mathbf{a}}^H) \leq \Delta_0. && (16) \end{aligned}$$

Introducing the new variable $\mathbf{A} = \hat{\mathbf{a}}\hat{\mathbf{a}}^H$, the problem (14)–(16) can be further rewritten as

$$\begin{aligned} \min_{\mathbf{A}} \quad & Tr(\hat{\mathbf{R}}^{-1}\mathbf{A}) && (17)\\ \text{subject to} \quad & Tr(\mathbf{A}) = M && (18)\\ & Tr(\tilde{\mathbf{C}}\mathbf{A}) \leq \Delta_0 && (19)\\ & rank(\mathbf{A}) = 1 && (20) \end{aligned}$$

where $rank(\cdot)$ stands for the rank of a matrix.

The only non-convex constraint in (17)–(20) is the rank-one constraint (20). Using the SDP relaxation technique, the relaxed problem can be obtained by dropping the non-convex rank-one constraint (20) and replacing it by the semi-definiteness constraint $\mathbf{A} \succeq \mathbf{0}$. Thus, the problem (17)–(20) is replaced by the following relaxed convex problem

$$\begin{aligned} \min_{\mathbf{A}} \quad & Tr(\hat{\mathbf{R}}^{-1}\mathbf{A}) && (21)\\ \text{subject to} \quad & Tr(\mathbf{A}) = M && (22)\\ & Tr(\tilde{\mathbf{C}}\mathbf{A}) \leq \Delta_0 && (23)\\ & \mathbf{A} \succeq \mathbf{0}. && (24) \end{aligned}$$

In general, only approximate solution of the original problem can be found from the solution of the relaxed problem. However, it has been shown in [14] and [15] that the strong duality between the primal and dual problems holds for the considered type of optimization problems and, thus, the exact solution can be found. It is worth noting that the robust adaptive beamforming problem considered in [14] is just a replica of the beamforming problem developed in [11] that differs from the one in [11] only by adopting an ellipsoidal uncertainty region instead of spherical uncertainty region. Thus, the worst-case approach is taken in both aforementioned works and the choice of the ellipsoid in [14] as well as any physical meaning of the problem are not given. Moreover, to establish strong duality, [14] considered an extended version of S-lemma, while [15] uses the rank reduction technique.

In this paper, different from [14] and [15], we aim at straightforwardly studying the structure of the primal problem (11)–(13) that gives us the possibility to obtain and explain the solution in more traditional for array processing linear algebra terms. It also proves a necessary intuition for understanding the differences of proposed robust adaptive beamforming method over the other methods. Toward this end the following results are of importance.

First, we consider the result that connects the feasibility of the relaxed problem (21)–(24) to the feasibility of the original problem (11)-(13). This result states that problem (21)–(24) is feasible if and only if the problem (11)–(13) is feasible. Due to space limitations, the complete proof is not given here, but will be available in [16].

Second, we observe that if the relaxed problem (21)–(24) has a rank-one solution, then the principal eigenvector of the solution of (21)–(24) is exactly the solution to the original problem (11)–(13). However, if the relaxed problem (21)–(24) has a solution $\mathbf{A}^* = \mathbf{Y}\mathbf{Y}^H$ of a rank higher than one, i.e., $\mathbf{Y}$ is an $N \times r$ matrix with $r > 1$, the exact solution of the original problem (11)–(13) can be extracted from the solution of the relaxed problem (21)–(24) by means of algebraic operations given in the following main constructive result.

Result: If $\mathbf{A}^* = \mathbf{Y}\mathbf{Y}^H$ is the rank r optimal minimizer of the relaxed problem (21)–(24), the estimate $\hat{\mathbf{a}}$ can be found as $\hat{\mathbf{a}} = \mathbf{Y}\mathbf{v}$ where $\mathbf{v}$ is an $r \times 1$ vector such that $\|\mathbf{Y}\mathbf{v}\| = \sqrt{M}$ and $\mathbf{v}^H\mathbf{Y}^H\tilde{\mathbf{C}}\mathbf{Y}\mathbf{v} = Tr(\mathbf{Y}^H\tilde{\mathbf{C}}\mathbf{Y})$. Note that one possible choice for the vector $\mathbf{v}$ is proportional to the sum of the eigenvectors of the following $r \times r$ matrix

$$\mathbf{D} = \frac{1}{M}\mathbf{Y}^H\mathbf{Y} - \frac{\mathbf{Y}^H\tilde{\mathbf{C}}\mathbf{Y}}{Tr(\mathbf{Y}^H\tilde{\mathbf{C}}\mathbf{Y})}. \tag{25}$$

The proof of this result is lengthy and is given in [16].

The last result aims at showing when the solution of the relaxed problem (21)–(24) has rank one. It is worth noting that any phase rotation of $\hat{\mathbf{a}}$ does not change the SINR at the output of the corresponding robust adaptive beamformer. Therefore, we say that the solution $\hat{\mathbf{a}}$ is unique regardless of any phase rotation if the value of the output SINR (output power) is the same for any $\hat{\mathbf{a}}' = \hat{\mathbf{a}}e^{j\phi}$. Then our last result stands that under the condition that the solution of the original problem (11)–(13) is unique in the aforementioned sense, the solution of the relaxed problem (21)–(24) always has rank one. The proof of this result is also given in [16].

In summary, under the condition of the last result, the solution of the relaxed problem (21)–(24) is rank-one and the solution of the original problem (11)–(13) can be found as a dominant eigenvector of the optimal solution of the relaxed problem (21)–(24). However, when the uniqueness condition of the last result is not satisfied for the problem (11)–(13), we resort to the constructive second result, which shows how to find the rank-one solution of (11)–(13) algebraically. As compared to the problem in [12], in which the constraint (7) enforces the estimated steering vector to be a linear combination of L dominant eigenvectors $\mathbf{U} = [\mathbf{u}_1, \ldots, \mathbf{u}_L]$, the estimated steering vector by the new method is not restricted by such linear combination constraint due to the new quadratic constraint (10) and as a result has more degrees of freedom. It is thus expected that the new robust adaptive beamforming method will outperform the one of [12] that can indeed be seen from the following simulations.

V. Simulation Results

A ULA of 10 omni-directional antenna elements with the inter-element spacing of half wavelength is considered. Additive noise in antenna elements is modeled as spatially and temporally independent complex Gaussian noise with zero mean and unit variance. Two interfering sources are impinging on the antenna array from the directions 30° and 50°, while the presumed direction towards the desired signal is 3°. The interference-to-noise ratio (INR) equals 30 dB and the desired signal is always present in the training data. For obtaining each point in the examples, 100 independent runs are used.

The proposed robust adaptive beamforming method is compared with three other methods in terms of the output SINR. These robust adaptive beamformers are (i) the worst-case-based robust adaptive beamformer of [5], (ii) the robust adaptive beamformer of [12], and (iii) the eigenspace-based beamformer of [4]. For the proposed robust adaptive beamformer and the beamformer of [12], the angular sector of interest Θ is assumed to be $\Theta = [\theta_p - 5°, \theta_p + 5°]$ where θ_p is the presumed direction of arrival of the desired signal. The value $\delta = 0.1$ and 6 dominant eigenvectors of the matrix $\mathbf{C}$ are used in the robust adaptive beamformer of [12] and the value $\varepsilon = 0.3M$ is used for the worst-case-based beamformer as it

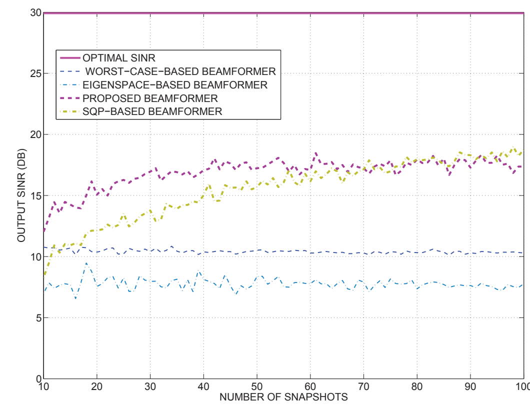

Fig. 2. Output SINR versus training sample size K for fixed $\mathrm{SNR} = 20$ dB and $\mathrm{INR} = 30$ dB. The case of exactly known signal steering vector.

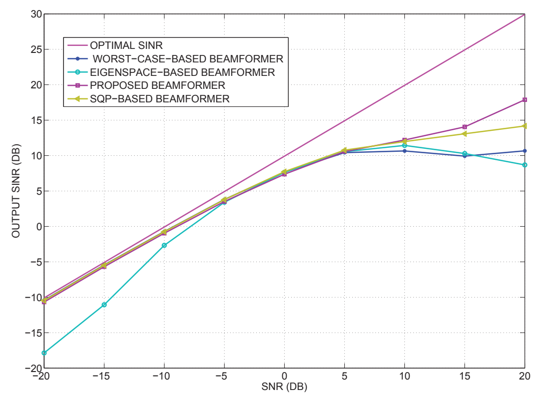

Fig. 3. Output SINR versus SNR for training data size of $K = 30$ and $\mathrm{INR} = 30$ dB. The case of exactly known signal steering vector.

has been recommended in [5]. The dimension of the signal-plus-interference subspace is assumed to be always estimated correctly for the eigenspace-based beamformer and equals 3.

First, we consider the case when the actual desired signal steering vector is known exactly. Even in this case, the presence of the signal of interest in the training data can substantially reduce the convergence rates of adaptive beamforming algorithms as compared to the signal-free training data case. In Fig. 2, the mean output SINRs for the aforementioned methods are illustrated versus the number of training snapshots for the fixed single-sensor $\mathrm{SNR} = 20$ dB. Fig. 3 displays the mean output SINR of the same methods versus the SNR for fixed training data size of $K = 30$. It can be seen from these figures that the proposed beamforming technique outperforms the other techniques even in the case of exactly known signal steering vector. It is especially true for small sample size.

Second, we consider the situation when the signal steering vector is distorted by wave propagation effects in an inhomogeneous medium. Independent-increment phase distortions

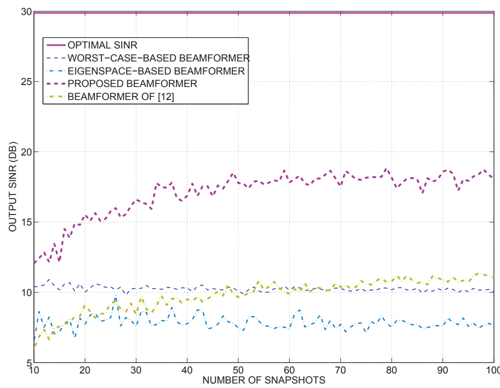

Fig. 4. Output SINR versus training sample size K for fixed SNR $= 20$ dB and INR $= 30$ dB. The case of signal steering vector mismatch due to wavefront distortion.

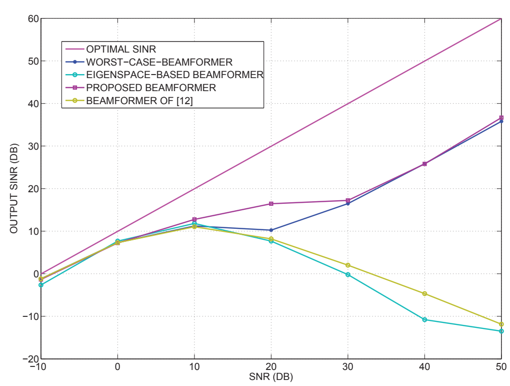

Fig. 5. Output SINR versus SNR for training data size of $K = 30$ and INR $= 30$ dB. The case of signal steering vector mismatch due to wavefront distortion.

are accumulated by the components of the presumed steering vector. It is assumed that the phase increments remain fixed in each simulation run and are independently chosen from a Guassian random generator with zero mean and variance 0.04. The performance of the methods tested is shown versus the number of training snapshots for fixed single-sensor SNR$= 20$ dB in Fig. 4 and versus the SNR for fixed training data size $K = 30$ in Fig. 5. It can be seen from these figures that the proposed beamforming technique outperforms all other beamforming techniques. Interestingly, it outperforms the eigenspace-based beamformer even at high SNR. This performance improvement compared to the eigenspace-based beamformer can be attributed to the fact that the knowledge of sector which includes the desired signal steering vector is used in the proposed beamforming technique. Fig. 5 also illustrates the case when SNR$\gg$INR where INR stands for interference-to-noise ratio. This case aims to illustrate the situation when SIR$\rightarrow \infty$. As it can be expected, the proposed and the worst-case-based methods perform almost equivalently.

VI. Conclusion

A new approach to robust adaptive beamforming in the presence of signal steering vector errors has been developed. According to this approach, the actual steering vector is first estimated using its presumed value, and then this estimate is used to find the optimal beamformer weight vector. It has been shown that the corresponding optimization problem for estimating the actual steering vector can be solved using the SDP relaxation technique and the exact solution for the signal steering vector can be found efficiently. As compared to the eigespace-based method, the proposed technique does not suffer from the subspace swap phenomenon since it does not use eigenvalue decomposition of the sample covariance matrix. Moreover, our simulation results demonstrate the superior performance for the proposed method over the existing state of the art robust adaptive beamforming methods.

References

[1] A. B. Gershman, "Robust adaptive beamforming in sensor arrays," *Int. J. Electron. Commun.*, vol. 53, pp. 305-314, Dec. 1999.

[2] H. Cox, R. M. Zeskind, and M. H. Owen, "Robust adaptive beamforming," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-35, pp. 1365-1376, Oct. 1987.

[3] Y. I. Abramovich, "Controlled method for adaptive optimization of filters using the criterion of maximum SNR," *Radio Eng. Electron. Phys.*, vol. 26, pp. 87-95, Mar. 1981.

[4] L. Chang and C. C. Yeh, "Performance of DMI and eigenspace-based beamformers," *IEEE Trans. Antennas Propagat.*, vol. 40, pp. 1336-1347, Nov. 1992.

[5] S. A. Vorobyov, A. B. Gershman, Z.-Q. Luo,"Robust adaptive beamforming using worst-case performance optimization: A solution to the signal mismatch problem," *IEEE Trans. Signal Processing*, vol. 51, pp. 313–324, Feb. 2003.

[6] S. A. Vorobyov, A. B. Gershman, Z.-Q. Luo, and N. Ma, "Adaptive beamforming with joint robustness against mismatched signal steering vector and interference nonstationarity," *IEEE Signal Processing Lett.*, vol. 11, pp. 108-111, Feb. 2004.

[7] J. Li, P. Stoica, and Z. Wang, "On robust Capon beamforming and diagonal loading," *IEEE Trans. Signal Processing*, vol. 51, pp. 1702-1715, July 2003.

[8] R. G. Lorenz and S. P. Boyd, "Robust minimum variance beamforming," *IEEE Trans. Signal Process.*, vol. 53, pp. 1684-1696, May 2005.

[9] S. A. Vorobyov, H. Chen, and A. B. Gershman, "On the relationship between robust minimum variance beamformers with probabilistic and worst-case distrortionless response constraints," *IEEE Trans. Signal Processing*, vol. 56, pp. 5719–5724, Nov. 2008.

[10] S. A. Vorobyov, A. B. Gershman, and Y. Rong, "On the relationship between the worst-case optimization-based and probability-constrained approaches to robust adaptive beamforming," in *Proc. IEEE ICASSP*, Honolulu, HI, Apr. 2007, pp. 977-980.

[11] J. Li, P. Stoica, and Z. Wang, "Doubly constrained robust Capon beamformer," *IEEE Trans. Signal Processing*, vol. 52, pp. 2407–2423, Sept. 2004.

[12] A. Hassanien, S. A. Vorobyov, and K. M. Wong, "Robust adaptive beamforming using sequential programming: An iterative solution to the mismatch problem," *IEEE Signal Processing Lett.*, vol. 15, pp. 733–736, 2008.

[13] A. Hassanien, S. A. Vorobyov, and K. M. Wong, "Robust adaptive beamforming using sequential quadratic programming," in *Proc. IEEE ICASSP*, Las Vegas, NV, Apr. 2008, pp. 2345–2348.

[14] A. Beck and Y. C. Eldar, "Doubly constrained robust Capon beamformer with ellipsoidal uncertainty sets," *IEEE Trans. Signal Processing*, vol. 55, pp. 753–758, Feb. 2007.

[15] Y. Huang and D. P. Palomar, "Rank-constrained separable semidefinite programming with applications to optimal beamforming," *IEEE Trans. Signal Processing*, vol. 58, pp. 664–678, Feb. 2010.

[16] A. Khabbazibasmenj, S. A. Vorobyov, and A. Hassanien, "Unified framework to robust adaptive beamforming and a new method," submitted to *IEEE Trans. Signal Processing*.