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ROBUST ADAPTIVE NEURAL CONTROL FOR A CLASS OF TIME-VARYING DELAY SYSTEMS WITH BACKLASH-LIKE HYSTERESIS INPUT

Xiuyu Zhang, Zhi Li, Chun-Yi Su, Xinkai Chen, Jianguo Wang, and Linlin Xia

ABSTRACT

This paper proposes a robust adaptive dynamic surface control (DSC) scheme for a class of time-varying delay systems with backlash-like hysteresis input. The main features of the proposed DSC method are that 1) by using a transformation function, the prescribed transient performance of the tracking error can be guaranteed; 2) by estimating the norm of the unknown weighted vector of the neural network, the computational burden can be greatly reduced; 3) by using the DSC method, the explosion of complexity problem is eliminated. It is proved that the proposed scheme guarantees all the closed-loop signals being uniformly ultimately bounded. The simulation results show the validity of the proposed control scheme.

Key Words: Dynamic surface control, unknown time-varying delay, prescribed tracking error performance, backlash-like hysteresis.

I. INTRODUCTION

Recently, the control schemes in dealing with the hysteresis have become the hotpot due to more and more applications of smart material-based actuators (*i.e.*, piezoceramics and shaped memory alloys, etc.) in precision control systems [1,2]. Control of nonlinear systems with hysteresis as an input is quite a challenging task, because hysteresis possesses non-differentiable, multi-valued and non-memoryless characters that severely limit system performance by exhibiting undesirable properties such as inaccuracy, oscillation and instability [3]. The research on dealing with the hysteresis in control systems can be classified into two categories: one is to construct an inverse operator to cancel the hysteresis effect [4,5]. Another is to develop robust adaptive schemes without constructing the hysteresis inverse. The typical examples include [6,7], where for a class of nonlinear plants preceded by the hysteresis described by the backlash-like model, the developed robust adaptive control schemes guarantee the tracking errors to certain precision. Following the same line, the works of [2,8–10] are the latest results to cope with the hysteresis in order to improve the tracking performance by using the robust adaptive backstepping or dynamic surface control (DSC) schemes.

Other than hysteresis, time-delay phenomena are also commonly found in, for example, chemical processes, biological systems, economic systems, and hydraulic/pneumatic systems, and the existence of time delays in a control system is also a source of instability and may degrade the control performance [11–13]. Owing to the great challenge both in academic research and in industrial applications, control of nonlinear time-delay systems have received a lot of attention [14,15]. Lyapunov–Krasovskii functionals [16] and Lyapunov–Razumikhin functionals [17] are the two main tools for the controller designs of the nonlinear time-delay systems. References [18,19] dealt with the tracking problem for a class of strict-feedback nonlinear time-delay systems with parameters uncertainties by using the Lyapunov–Krasovskii functionals and backstepping method. In [20,21], the adaptive neural

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Xiuyu Zhang (corresponding author, e-mail:zhangxiuyu80@163.com), Jianguo Wang, and Linlin Xia are with the School of Automation Engineering, Northeast Dianli University, Jilin, 132012, China.

Zhi Li is with the Department of Electrical Engineering, Eindhoven University of Technology, Eindhoven 5600 MB, the Netherlands.

Chun-Yi Su is with the Department of Mechanical and Industrial Engineering, Concordia University, Montreal, Canada, H3G 1M8.

Xinkai Chen is with the Department of Electronic and Information Systems, Shibaura Institute of Technology, Minuma-ku, Saitama-shi, Saitama 337-8570, Japan.

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stabilizing control was proposed with the help of a novel Lyapunov–Krasovskii functional. For other schemes dealing with time delays, readers may refer to [8,22–24], where the Lyapunov-Krasovskii functionals also play a key role. In particular, in [12] and [23], the DSC method is also applied to the time delay systems to solve the problem of explosion of complexity. However, it seems that these schemes cannot make the tracking error converge to an arbitrarily small residual set.

Although some works on control of the nonlinear systems with the hysteresis or time delay have been conducted, the results dealing with both time delays and hysteresis inputs are rare, with the exception of [8,25] due to the complexity of such systems. In [8], Ren *et al.* developed an adaptive backstepping control scheme to mitigate the effects of both hysteresis and time delays. In [25], an adaptive variable structure control scheme was proposed for turning metal cutting system which includes the backlash-like hysteresis and time delays. However, the tracking performance could be further improved.

In this paper, inspired by [8] and [25], an adaptive neural DSC scheme is proposed for a class of unknown nonlinear time-varying delays systems preceded by unknown backlash-like hysteresis with the following features:

- Compared with [26,27], the proposed adaptive DSC scheme is able to guarantee the prescribed transient performance of the tracking error for the unknown nonlinear time-varying delay systems preceded by unknown hysteresis.
- The limitation on time-delay functions are relaxed compared with [8,22], where some knowledge of time-delay functions needs to be known for the purpose of eliminating the possibly undesirable "bursting" phenomenon [13] and obtaining a desired tracking error.
- By estimating the vector norm of unknown parameters at each controller design step, the computational burden is greatly reduced.
- To our best knowledge, this is the first attempt to fuse adaptive DSC technique with nonlinear systems having both backlash-like hysteresis model and unknown time-varying delays.

The rest of this paper is organized as follows. In Section II, the class of controlled nonlinear time-varying delay systems preceded by the backlash-like hysteresis is introduced and the control objective is formulated. In Section III, the design procedure of the adaptive DSC is presented. Section IV gives the stability analysis for the proposed scheme. Finally, a simulation example is given to demonstrate the effectiveness of the proposed design method.

II. PROBLEM STATEMENT AND PRELIMINARIES

2.1 Plant to be controlled

We consider the following nonlinear time-varying delay plant preceded by unknown hysteresis:

$$\begin{aligned} \dot{x}_i &= g_i(\bar{x}_i)x_{i+1} + f_i(\bar{x}_i) + h_i(\bar{x}_{i\tau}) + d_i(t), i = 1, \cdots, n-1, \\ \dot{x}_n &= g_n(\bar{x}_n)w(v) + f_n(\bar{x}_n) + h_n(\bar{x}_{n\tau}) + d_n(t), \\ y &= x_1, \end{aligned} \tag{1}$$

where $x_1, x_2, \cdots, x_n$ are state variables and $\bar{x}_i := [x_1, x_2, \cdots, x_i]^T \in R^i$, $i = 1, \cdots, n$; $\bar{x}_{i\tau} := [x_1(t-\tau_1(t)), x_2(t-\tau_2(t)), \cdots, x_i(t-\tau_i(t))]^T \in R^i$ are time-varying delay state variables with $\tau_i(t)$ being unknown time-varying delays; $f_i(\cdot)$ are unknown smooth functions; $h_i(\cdot)$ are unknown time delay functions; d_i denote the unknown perturbed terms; $g_i(\bar{x}_i) \in R$ are unknown smooth functions; $y \in \mathbb{R}$ is the output of the controlled plant; $w \in \mathbb{R}$ is the unknown backlash-like hysteresis and can be expressed as

$$w(t) = P(v(t)) \tag{2}$$

with v as the input signal to be designed. The hysteresis operator P will be discussed in detail below.

We emphasize that system (1) is used to describe many practical nonlinear systems preceded by unknown hysteresis such as metal cutting mechanical systems [25] and some systems with smart material-based actuators [9,10,29].

For the controlled plant, we make the following assumptions.

A1. The unknown time-delay functions $h_i(\bar{x}_i(t))$, $i = 1, \cdots, n$, satisfy the following inequalities:

$$\left|h_i(\bar{x}_i(t))\right| \le \sum_{j=1}^{i} \varphi_{i,j}(x_j(t)), \tag{3}$$

where $\varphi_{i,j}(\cdot)$ are unknown continuous functions.

A2. The disturbances $d_i(t)$, $i = 1, \cdots, n$, satisfy

$$\left|d_i(t)\right| \le \bar{d}_i, \tag{4}$$

where $\bar{d}_i$ are unknown positive constants.

A3. The desired trajectory y_r is smooth and available with $y_r(0)$ at designer's disposal; $[y_r, \dot{y}_r, \ddot{y}_r]^T$ belongs to a known compact set for all $t \geq 0$.

A4. The unknown time-varying state delays $\tau_i(t)$, $i = 1, \cdots, n$, satisfy the following inequalities

$$\dot{\tau}_i(t) \leq \bar{\tau}_{\max} < 1. \tag{5}$$

Remark 1. Assumption A1 is the same as that in [10,30], which implies that the smooth functions $g_i(\cdot)$, $i = 1, \cdots, n$, are strictly either positive or negative and it is the controllable condition of system (1). Assumption A1 relaxes the assumption in [8,22], in which it was assumed that $\varphi_{i,j}(\cdot)$ are known. Assumptions A2–A4 are common in the dynamic surface control method.

2.2 Backlash-like hysteresis model

In this paper, the backlash-like hysteresis nonlinearity is described by the following differential equation [5]:

$$\frac{dw}{dt} = \alpha \left| \frac{dv}{dt} \right| (\lambda v - w) + \psi \frac{dv}{dt}, \tag{6}$$

where α, $\lambda(> 0)$ and ψ are unknown constant parameters with $\lambda > \psi$. The solution of (6) is

$$w = \lambda v + d(v) \tag{7}$$

with

$$\begin{aligned} d(v) = & (w_0 - \lambda v_0) \exp[-\alpha(v - v_0)\mathrm{sgn}(\dot{v})] \\ & + \exp(-\alpha v \mathrm{sgn}(\dot{v})) \int_{v_0}^{v} (\psi - \lambda) \exp[\alpha \xi \mathrm{sgn}(\dot{v})] d\xi, \end{aligned} \tag{8}$$

where $v_0 = v(t_0)$ and $w_0 = w(v_0)$. It can be proved that $d(v)$ is bounded for any $v \in \mathbb{R}$; furthermore,

$$\lim_{v \to -\infty} d(v) = \lim_{v \to -\infty} [w(v; v_0, w_0) - \lambda v] = (\lambda - \psi)/\alpha, \tag{9}$$

$$\lim_{v \to +\infty} d(v) = \lim_{v \to +\infty} [w(v; v_0, w_0) - \lambda v] = -(\lambda - \psi)/\alpha. \tag{10}$$

That is, α determines the rate at which w switches between $-(\lambda - \psi)/\alpha$ and $(\lambda - \psi)/\alpha$: The larger the parameter α is, the faster the transition frequency in w is going to be [5]. Fig. 1 illustrates the class of backlash-like hysteresis described by (6).

Now, taking (7) into consideration, (1) can be rewritten as

$$\begin{aligned} \dot{x}_i &= g_i(\bar{x}_i)x_{i+1} + f_i(\bar{x}_i) + h_i(\bar{x}_{i\tau}) + d_i(t), i = 1, \cdots, n-1, \\ \dot{x}_n &= \beta v + g_n(\bar{x}_n)d(v) + f_n(\bar{x}_n) + h_n(\bar{x}_{n\tau}) + d_n(t), \\ y &= x_1, \end{aligned} \tag{11}$$

where

$$\beta = g_n(\cdot)\lambda, \beta > 0, \tag{12}$$

and $d(v)$ is a bounded hysteresis term satisfying

$$|d(v)| \leq D, \tag{13}$$

with D being a positive unknown constant.

A5. The signs of $g_i(\bar{x}_i)$, $i = 1, \cdots, n$, are known. Without loss of generality, it is assumed that $g_i(\bar{x}_i) > 0$ and there exist two constants $g_{\min}$ and $g_{\max}$ satisfying $0 < g_{\min} \leq |g_i| \leq g_{\max}$. Also, there exists the constants $\beta_{\min}$, $\beta_{\max}$, and $D_{\lambda\max}$, such that $\beta_{\min} \leq \beta(\cdot) \leq \beta_{\max}$ and $D/\lambda \leq D_{\lambda\max}$.

Remark 2. Due to λ being an unknown positive constant, Assumption A5 is reasonable. Also, we emphasize that $g_{\min}$, $g_{\max}$, $\beta_{\min}$, $\beta_{\max}$ and $D_{\lambda\max}$ are not required in implementation of the proposed control design. They are used for analysis only.

2.3 Radial basis function neural network (RBFNN) approximation

In this paper, the RBFNN is employed to approximate a continuous function on a given compact set.

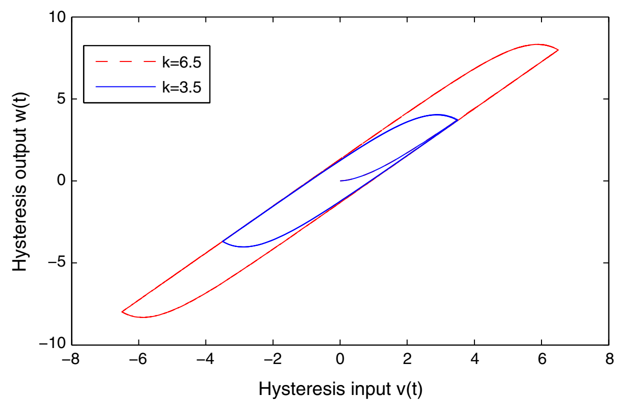

Fig. 1. Hysteresis curves given by (7), where the parameters $\alpha = 1, \lambda = 1.432, \psi = 0.105$, and the input $v(t) = k \sin(2.3t)$ with $k = 3.5$ and $k = 6.5$, respectively.

Mathematically, an RBFNN can be expressed by [31,32]:

$$Y = \vartheta^{\mathrm{T}} \rho(\xi), \tag{14}$$

where $Y \in \mathbb{R}$ is the RBFNN output, $\xi \in \mathbb{R}^n$ is the RBFNN input, $\vartheta \in \mathbb{R}^N$ is an N-dimensional vector of synaptic weights and $\rho(\xi)$ is an N-dimensional vector of regressor terms constructed by $\rho_k(\xi)$ named basis functions, $k = 1, \dots, N$. Generally, the so-called radial basis function is used as a basis function with the following form:

$$\rho_k(\xi) = \exp\left(-\frac{||\xi - \zeta_k||^2}{\sigma_k^2}\right), \sigma_k > 0, k = 1, \dots, N, \tag{15}$$

where $\zeta_k \in \mathbb{R}^n$ are constant vectors called the center of the basis function, and σ_k are real positive numbers called the width of the basis function.

Now, let F_i, $i = 1, \cdots, n$, denote the unknown functions to be estimated by the RBFNN and Ω_{ξ_i} denote the given compact sets in the ith step of Section III. Then by using the approximated properties of the neural network in [28,31], it follows that

$$F_i = \theta_{\delta_i}^{*T} \psi_i(\xi_i) + \delta_i(\xi_i),\ \forall \xi_i \in \Omega_{\xi_i},\ i = 1, \cdots, n, \tag{16}$$

where $\theta_{\delta_i}^{*T}$ are optimal weight vectors; $\psi_i(\xi_i)$ and ξ_i denote, respectively, the vector valued functions and the RBFNN input with proper dimensions that will be given in Section III; $\delta_i(\xi_i)$ are the network approximation errors satisfying $|\delta_i(\xi_i)| \leq \delta_i^*$ with δ_i^* being an unknown constant.

The objective of this paper is to design a control law v in (11) based on the DSC technique, such that the prescribed performance of the tracking error can be obtained and all the closed-loop signals are uniformly ultimately bounded [34]. Here, the prescribed performance was proposed for the first time in [26] and it can provide a systematic procedure to accurately compute the required bounds, thus making tracking error converge to a redefined arbitrarily small residual set, with convergence rate no less than a prescribed value, exhibiting a maximum overshoot less than a sufficiently small pre-assigned constant.

III. ADAPTIVE DSC DESIGN

In this section, we will apply the adaptive DSC technique to a class of nonlinear time-varying delay systems preceded by unknown hysteresis described by the backlash-like model (1). First of all, to guarantee a prescribed tracking performance, both the performance and the error transformation functions are introduced.

3.1 Performance and the error transformation functions

Let the tracking error be

$$e := x_1 - y_r, \tag{17}$$

where y_r is the desired trajectory. By [26], a performance function $\varpi(t) : \mathbb{R}_+ \to \mathbb{R}_+ - \{0\}$ is defined as a smooth and decreasing positive function such that for all $t \geq 0$,

$$\begin{cases} -\sigma\varpi(t) < e(t) < \varpi(t), \text{if } e(0) > 0, \\ -\varpi(t) < e(t) < \sigma\varpi(t), \text{if } e(0) < 0, \end{cases} \tag{18}$$

where $0 < \sigma \leq 1$ and $\lim_{t\to\infty} \varpi(t) = \varpi_\infty > 0$ with ϖ_∞ the maximum allowable value of steady state tracking error. Fig. 3 in Section V graphically shows the performance function (18).

To transform (18) into an equivalent unconstrained one, the error transformation function is defined as

$$\Phi(S_1) = \varpi(t)/e(t), \tag{19}$$

where S_1 is the transformed error and $\Phi(S_1)$ is a smooth, strictly increasing and thus the invertible function $\Phi(S_1)$ possesses the following properties:

$$\begin{aligned} &-\sigma < \Phi(S_1) < 1, \text{ if } e(0) > 0, \\ &-1 < \Phi(S_1) < \sigma, \text{ if } e(0) < 0, \end{aligned} \tag{20}$$

$$\begin{cases} \lim_{S_1\to-\infty} \Phi(S_1) = -\sigma,\ \lim_{S_1\to+\infty} \Phi(S_1) = 1, \text{ if } e(0) > 0, \\ \lim_{S_1\to-\infty} \Phi(S_1) = -1,\ \lim_{S_1\to+\infty} \Phi(S_1) = \sigma, \text{ if } e(0) < 0. \end{cases} \tag{21}$$

From (21), if S_1 is bounded, (20) holds, which, together with $\varpi(t) > 0$ and (19), implies that $-\sigma\varpi(t) < \varpi(t)\Phi(S_1) = e(t) < \varpi(t)$ (if $e(0) > 0$) or $-\varpi(t) < \varpi(t)\Phi(S_1) = e(t) < \sigma\varpi(t)$ (if $e(0) < 0$). That is, (18) holds. Hence, to achieve the prescribed tracking performance, one only needs to show is $S_1 \in_\infty$. Then, from (19), we have,

$$S_1 = \Phi^{-1}\left(\frac{e(t)}{\varpi(t)}\right). \tag{22}$$

Noting that in case of $e(0) = 0$, one can incorporate it into $e(0) > 0$ or $e(0) < 0$ without any effect on the system analysis; however, σ can not be chosen as zero due to $S_1(0)$ being infinite.

3.2 Adaptive RBFNN DSC law design

The whole DSC design procedure [33] contains n steps, and the actual control law will be deduced at the last controller design step.

Step 1. Let S_1 given by (22) be the first surface error. Taking (11) into consideration, it follows that

$$\dot{S}_1 = \Psi\left[-\frac{\dot{\varpi}}{\varpi}e + g_1 x_2 + f_1 + h_1(x_{1\tau}) + d_1 - \dot{y}_r\right], \tag{23}$$

where

$$\Psi := \frac{1}{\varpi}\frac{\partial \Phi^{-1}}{\partial(e/\varpi)}, \tag{24}$$

which, owing to the definitions of $\Phi(\cdot)$ and ϖ, satisfies $\Psi > 0$. Therefore the following quadratic function is considered,

$$V_1 = \frac{1}{2}\left(S_1^2 + \frac{g_m}{\gamma_{\vartheta_1}}\tilde{\vartheta}_1^2\right), \tag{25}$$

where γ_{ϑ_1} is the positive design parameter; $g_m = \min\{g_{\min}, \beta_{\min}\}$, therein, $g_{\min}$ and $\beta_{\min}$ is defined in Assumption A5; $\tilde{\vartheta}_1 := \hat{\vartheta}_1 - \vartheta_1^*$ with $\hat{\vartheta}_1$ being the estimation of $\vartheta_1^* = \frac{1}{g_m}\left\|\theta^*_{\delta_1}\right\|^2$. Then, the time derivative of V_1 yields

$$\dot{V}_1 = S_1\dot{S}_1 + \frac{g_m}{\gamma_{\vartheta_1}}\tilde{\vartheta}_1\dot{\hat{\vartheta}}_1. \tag{26}$$

Also, by using Assumptions A1 and A2, the following inequalities hold,

$$\Psi S_1 h_1(x_1(t-\tau_1)) \le \frac{1}{2}\Psi^2 S_1^2 + \frac{1}{2}\varphi_{1,1}^2(x_{1\tau}), \tag{27}$$

$$\Psi S_1 d_1 \le \frac{1}{2}\Psi^2 S_1^2 + \frac{1}{2}\bar{d}_1^2. \tag{28}$$

Therefore, from (23)–(28), we have*

$$\begin{aligned}\dot{V}_1 \le{}& S_1\Psi\left[-\frac{\dot{\varpi}}{\varpi}e + g_1 x_2 + f_1 - \dot{y}_r + S_1\Psi\right]\\ &+ \frac{1}{2}\bar{d}_1^2 + \frac{1}{2}Q_1(x_{1\tau}) + \frac{g_m}{\gamma_{\vartheta_1}}\tilde{\vartheta}_1\dot{\hat{\vartheta}}_1,\end{aligned} \tag{29}$$

where $Q_1(x_{1\tau})$ is defined below with $k = 1$:

$$Q_k(\bar{x}_{k\tau}) := \sum_{l=1}^{k}\varphi_{k,l}^2(x_l(t-\tau_l)), k = 1, \cdots, n. \tag{30}$$

By adding and subtracting $\frac{1}{(1-\bar{\tau}_{\max})}\tanh^2(\frac{S_1}{\varepsilon_1})Q_1(x_1)$ at the right hand side of (29) with ε_1 a positive constant yields

$$\begin{aligned}\dot{V}_1 \le{}& S_1\Big[g_1\Psi x_2 - \frac{\dot{\varpi}}{\varpi}\Psi e + \Psi f_1 - \Psi\dot{y}_r + S_1\Psi^2\\ &+ \frac{1}{S_1(1-\bar{\tau}_{\max})}\tanh^2\left(\frac{S_1}{\varepsilon_1}\right)Q_1(x_1) + \frac{3}{2}g_1^2\Psi^2 S_1\Big]\\ &- \frac{1}{(1-\bar{\tau}_{\max})}\tanh^2\left(\frac{S_1}{\varepsilon_1}\right)Q_1(x_1) + \frac{1}{2}\bar{d}_1^2 + \frac{1}{2}Q_1(x_{1\tau})\\ &- \frac{3}{2}g_1^2\Psi^2 S_1^2 + \frac{g_m}{\gamma_{\vartheta_1}}\tilde{\vartheta}_1\dot{\hat{\vartheta}}_1,\end{aligned} \tag{31}$$

where $Q_1(x_1)$ is defined below with $k = 1$:

$$Q_k(\bar{x}_k) := \sum_{l=1}^{k}\varphi_{k,l}^2(x_l), k = 1, \cdots, n. \tag{32}$$

Now, noting (16), the RBFNN is used to approximate the unknown terms† in (31) on a given compact set Ω_{ξ_1}, *i.e.*,

$$\begin{aligned}\theta^{*T}_{\delta_1}\psi_1(\xi_1) + \delta_1(\xi_1) = \frac{1}{g_{1\Psi}}\Big[&-\frac{\dot{\varpi}}{\varpi}\Psi e + \Psi f_1 - \Psi\dot{y}_r + S_1\Psi^2\\ &+ \frac{1}{S_1(1-\bar{\tau}_{\max})}\tanh^2\left(\frac{S_1}{\varepsilon_1}\right)Q_1(x_1) + \frac{3}{2}g_1^2\Psi^2 S_1\Big],\end{aligned} \tag{33}$$

where

$$\xi_1 := (x_1, \varpi, \dot{\varpi}, \Psi, S_1) \in \Omega_{\xi_1} \subset \mathbb{R}^5.$$

Thus, using the following inequality

$$\begin{aligned}g_1\Psi S_1\left(\theta^{*T}_{\delta_1}\psi_1 + \delta_1(\xi_1)\right) \le{}& \frac{g_m\Psi^2\alpha_1^2 S_1^2\vartheta_1^*\psi_1^T\psi_1}{2}\\ &+ \frac{g_{\max}^2}{2\alpha_1^2} + \frac{1}{2}g_1^2\Psi^2 S_1^2 + \frac{1}{2}\delta_1^{*2},\end{aligned} \tag{34}$$

where α_1 is a positive design parameter and $g_{\max}$ is defined in Assumption A5, we have

*The derivation of $Q_1(x_1)$, $Q_1(x_{1\tau})$ below in this step and $Q_k(\bar{x}_k)$, $Q_k(\bar{x}_{k\tau})$, $k = 1, \ldots, n$, in the next steps is just for the purpose of making the expression more succinct.

†Note that, $\lim_{S\to 0}\frac{1}{S}\tanh^2(S/\varepsilon) = 0$, where ε is a positive constant.

$$\begin{aligned}\dot{V}_1 \le{}& S_1\left[g_1\Psi x_2 + \frac{g_m\Psi^2\alpha_1^2 S_1\vartheta_1^*\psi_1^T\psi_1}{2}\right]\\ &-\frac{k}{(k-\bar{\tau}_{\max})}\tanh^2\left(\frac{S_1}{\varepsilon_1}\right)Q_1(x_1)+\frac{1}{2}Q_1(x_{1\tau})\\ &-g_1^2\Psi^2S_1^2+\frac{g_m}{\gamma_{\vartheta_1}}\tilde{\vartheta}_1\dot{\hat{\vartheta}}_1+\frac{1}{2}\bar{d}_1^2+\frac{g_{\max}^2}{2\alpha_1^2}+\frac{1}{2}\delta_1^{*2},\end{aligned} \tag{35}$$

which suggests us to choose the virtual control signal x_{2d} as

$$x_{2d} = \left[-k_1S_1 - \frac{\Psi^2\alpha_1^2S_1\hat{\vartheta}_1\psi_1^T\psi_1}{2}\right]/\Psi \tag{36}$$

with $\hat{\vartheta}_1$ being the estimation of ϑ_1^*, and updated by

$$\dot{\hat{\vartheta}}_1 = \gamma_{\vartheta_1}\left[\frac{\alpha_1^2\Psi^2S_1^2\psi_1^T\psi_1}{2} - \sigma_{\vartheta_1}\hat{\vartheta}_1\right], \tag{37}$$

where σ_{ϑ_1} are positive design parameters. Let x_{2d} pass through a first-order filter to obtain a new state variable z_2 with time constant ς_2:

$$\varsigma_2\dot{z}_2 + z_2 = x_{2d}, z_2(0) = x_{2d}(0). \tag{38}$$

Step i ($2 \le i \le n-1$). Define the ith surface error

$$S_i = x_i - z_i, \tag{39}$$

whose time derivative by considering (11) is

$$\dot{S}_i = \dot{x}_i - \dot{z}_i = g_ix_{i+1} + f_i + h_i(x_{i\tau}) + d_i - \dot{z}_i. \tag{40}$$

To stabilize (40), consider the following quadratic function

$$V_i = \frac{1}{2}\left(S_i^2 + \frac{g_m}{\gamma_{\vartheta_i}}\tilde{\vartheta}_i^2\right), \tag{41}$$

where γ_{ϑ_i} are positive design parameters; $\tilde{\vartheta}_i := \hat{\vartheta}_i - \vartheta_i^*$ with $\hat{\vartheta}_i$ as the estimation of $\vartheta_i^* = \frac{1}{g_m}\left\|\theta_{\delta_i}^*\right\|^2$. The time derivative of V_i yields

$$\dot{V}_i = S_i\dot{S}_i + \frac{g_m}{\gamma_{\vartheta_i}}\tilde{\vartheta}_i\dot{\hat{\vartheta}}_i. \tag{42}$$

Also, by using Assumptions A1 and A2, the following inequality holds:

$$\begin{aligned}S_ih_i(x_i(t-\tau_i)) &\le \frac{i}{2}S_i^2 + \frac{1}{2}\sum_{l=1}^{i}\varphi_{i,l}^2(x_l(t-\tau_l)),\\ S_id_i &\le \frac{1}{2}S_i^2 + \frac{1}{2}\bar{d}_i^2.\end{aligned} \tag{43}$$

Therefore, from (40)–(43), we have

$$\begin{aligned}\dot{V}_i \le{}& S_i\left[g_ix_{i+1} + f_i - \dot{z}_i + \frac{i+1}{2}S_i\right]\\ &+\frac{1}{2}Q_i(\bar{x}_{i\tau}) + \frac{g_m}{\gamma_{\vartheta_i}}\tilde{\vartheta}_i\dot{\hat{\vartheta}}_i + \frac{1}{2}\bar{d}_i^2,\end{aligned} \tag{44}$$

where $Q_i(\bar{x}_{i\tau})$ is defined by (30). By adding and subtracting $\frac{k}{(k-\bar{\tau}_{\max})}\tanh^2(\frac{S_i}{\varepsilon_i})Q_i(\bar{x}_i)$ at the right hand side of (44) with ε_i a positive constant and $Q_i(\bar{x}_i)$ defined by (32), we obtain

$$\begin{aligned}\dot{V}_i \le{}& S_i\Bigg[g_ix_{i+1} + f_i - \dot{z}_i + \frac{i+1}{2}S_i\\ &+\frac{1}{S_i(1-\bar{\tau}_{\max})}\tanh^2\left(\frac{S_i}{\varepsilon_i}\right)Q_i(\bar{x}_i) + \frac{3}{2}g_i^2S_i\Bigg]\\ &-\frac{1}{(1-\bar{\tau}_{\max})}\tanh^2\left(\frac{S_i}{\varepsilon_i}\right)Q_i(\bar{x}_i) + \frac{1}{2}Q_i(\bar{x}_{i\tau})\\ &+\frac{g_m}{\gamma_{\vartheta_i}}\tilde{\vartheta}_i\dot{\hat{\vartheta}}_i - \frac{3}{2}g_i^2S_i^2 + \frac{1}{2}\bar{d}_i^2.\end{aligned} \tag{45}$$

Similar to Step 1, the RBFNN is used to approximate the following unknown terms on a given compact set Ω_{ξ_i}:

$$\begin{aligned}\theta_{\delta_i}^{*T}\psi_i(\xi_i) + \delta_i(\xi_i) ={}& \frac{1}{g_i}\Bigg[f_i - \dot{z}_i + \frac{i+1}{2}S_i\\ &+\frac{1}{S_i(1-\bar{\tau}_{\max})}\tanh^2\left(\frac{S_i}{\varepsilon_i}\right)Q_i(\bar{x}_i) + \frac{3}{2}g_i^2S_i\Bigg],\end{aligned} \tag{46}$$

where

$$\xi_i := (\bar{x}_i, S_i) \in \Omega_{\xi_i} \subset \mathbb{R}^{i+1}.$$

Then, similar to (34), by using the following inequality

$$\begin{aligned}g_iS_i\left(\theta_{\delta_i}^{*T}\psi_i + \delta_i(\xi_i)\right) \le{}& \frac{g_m\alpha_i^2S_i^2\vartheta_i^*\psi_i^T\psi_i}{2}\\ &+\frac{g_{\max}^2}{2\alpha_i^2} + \frac{1}{2}g_i^2S_i^2 + \frac{1}{2}\delta_i^{*2},\end{aligned} \tag{47}$$

where α_i are positive design parameters, we have

$$\dot{V}_i \leq S_i\left[g_i x_{i+1} + \frac{g_m \alpha_i^2 S_i \vartheta_i^* \psi_i^T \psi_i}{2}\right] - \frac{1}{(1-\bar{\tau}_{\max})}\tanh^2\left(\frac{S_i}{\epsilon_i}\right) Q_i(\bar{x}_i) + \frac{1}{2}Q_i(\bar{x}_{i\tau}) - g_i^2 S_i^2 + \frac{g_m}{\gamma_{\vartheta_i}}\tilde{\vartheta}_i\dot{\hat{\vartheta}}_i + \frac{1}{2}\bar{d}_i^2 + \frac{g_{\max}^2}{2\alpha_i^2} + \frac{1}{2}\delta_i^{*2}, \tag{48}$$

which suggests us to choose the virtual control signal as

$$x_{i+1d} = -k_i S_i - \frac{\alpha_i^2 \hat{\vartheta}_i S_i \psi_i^T \psi_i}{2} \tag{49}$$

with $\hat{\vartheta}_i$ being the estimation of ϑ_i^* and updated by

$$\dot{\hat{\vartheta}}_i = \gamma_{\vartheta_i}\left[\frac{\alpha_i^2 S_i^2 \psi_i^T \psi_i}{2} - \sigma_{\vartheta_i}\hat{\vartheta}_i\right], \tag{50}$$

where σ_{ϑ_i} are positive design parameters. Let x_{i+1d} pass through a first-order filter to obtain a new state variable z_{i+1} with time constant ς_{i+1}:

$$\varsigma_{i+1}\dot{z}_{i+1} + z_{i+1} = x_{i+1d},\ z_{i+1}(0) = x_{i+1d}(0). \tag{51}$$

Step *n*. Define the nth surface error

$$S_n = x_n - z_n, \tag{52}$$

whose time derivative by considering (11) is

$$\dot{S}_n = \dot{x}_n - \dot{z}_n = \beta v + g_n(\cdot)d_1(v) + f_n + h_n(\bar{x}_{n\tau}) + d_n - \dot{z}_n. \tag{53}$$

Consider the following quadratic function

$$V_n = \frac{1}{2}\left(S_n^2 + \frac{\beta_{\max}}{\gamma_{D_\lambda}}\tilde{D}_\lambda^2 + \frac{g_m}{\gamma_{\vartheta_n}}\tilde{\vartheta}_n^2\right), \tag{54}$$

where $\beta_{\max}$ is defined in Assumption A5, γ_D and γ_{ϑ_n} are positive design parameters, $\tilde{D}_\lambda = \hat{D}_\lambda - D_{\lambda\max}$, $\tilde{\vartheta}_n = \hat{\vartheta}_n - \vartheta_n^*$ with $D_{\lambda\max}$ being defined in Assumption A5, $\hat{D}_\lambda$, $\hat{\vartheta}_n$ being as the estimation of the $D_\lambda = D/\lambda$, and $\vartheta_n^* = \frac{1}{g_m}\left\|\theta_{\delta_n}^*\right\|^2$, respectively. Then the time derivative of V_n yields

$$\dot{V}_n = S_n\dot{S}_n + \frac{\beta_{\max}}{\gamma_{D_\lambda}}\dot{\hat{D}}_\lambda\tilde{D}_\lambda + \frac{g_m}{\gamma_{\vartheta_n}}\dot{\hat{\vartheta}}_n\tilde{\vartheta}_n. \tag{55}$$

By using Assumption A1, the following inequality holds

$$S_n h_n(x_n(t-\tau_n)) \leq \frac{n}{2}S_n^2 + \frac{1}{2}\sum_{l=1}^{n}\varphi_{i,l}^2(x_l(t-\tau_l)). \qquad S_n d_n \leq \frac{1}{2}S_n^2 + \frac{1}{2}\bar{d}_n^2. \tag{56}$$

Therefore, from (53)–(56), we have

$$\dot{V}_n \leq S_n\left[\beta v + f_n - \dot{z}_n + \frac{n}{2}S_n\right] + \frac{1}{2}\bar{d}_n^2 + \beta\left|S_n\right|D_\lambda + \frac{\beta_{\max}}{\gamma_{D_\lambda}}\dot{\hat{D}}_\lambda\tilde{D}_\lambda + \frac{1}{2}Q_n(\bar{x}_{n\tau}) + \frac{g_m}{\gamma_{\vartheta_n}}\dot{\hat{\vartheta}}_n\tilde{\vartheta}_n, \tag{57}$$

where $Q_n(\bar{x}_{n\tau})$ is defined by (30). By adding and subtracting $\frac{1}{(1-\bar{\tau}_{\max})}\tanh^2(\frac{S_n}{\epsilon_n})Q_n(\bar{x}_n)$ at the right hand side of (57) with ϵ_n being a positive constant and $Q_n(\bar{x}_n)$ defined by (32), we obtain

$$\dot{V}_n \leq S_n\left[\beta v + f_n - \dot{z}_n + \frac{n+1}{2}S_n + \frac{1}{S_n(1-\bar{\tau}_{\max})}\tanh^2\left(\frac{S_n}{\epsilon_n}\right)Q_n(\bar{x}_n) + \frac{1}{2}\beta^2 S_n\right] + \frac{1}{2}\bar{d}_n^2 + \beta\left|S_n\right|D_\lambda - \frac{1}{(1-\bar{\tau}_{\max})}\tanh^2\left(\frac{S_n}{\epsilon_n}\right)Q_n(\bar{x}_n) + \frac{1}{2}Q_n(\bar{x}_{n\tau}) - \frac{1}{2}\beta^2 S_n^2 + \frac{\beta_{\max}}{\gamma_{D_\lambda}}\dot{\hat{D}}_\lambda\tilde{D}_\lambda + \frac{g_m}{\gamma_{\vartheta_n}}\dot{\hat{\vartheta}}_n\tilde{\vartheta}_n. \tag{58}$$

Then, the RBFNN is used to approximate the following unknown terms in (58) on a given compact set Ω_{ξ_n}:

$$\theta_{\delta_n}^{*T}\psi_n(\xi_n) + \delta_n(\xi_n) = \frac{1}{\beta(\cdot)}\left[f_n - \dot{z}_n + \frac{n+1}{2}S_n + \frac{1}{S_n(1-\bar{\tau}_{\max})}\tanh^2\left(\frac{S_n}{\epsilon_n}\right)Q_n(\bar{x}_n) + \frac{1}{2}\beta^2 S_n\right], \tag{59}$$

where

$$\xi_n := (\bar{x}_n, S_n) \in \Omega_{\xi_n} \subset \mathbb{R}^{n+1}.$$

Similar to (34), using the following inequality

$$\beta S_n\left(\theta_{\delta_n}^{*T}\psi_n + \delta_n(\xi_n)\right) \leq \frac{g_m\alpha_n^2 S_n^2 \vartheta_n^* \psi_n^T \psi_n}{2} + \frac{\beta_{\max}^2}{2\alpha_n^2} + \frac{1}{2}\beta_n^2 S_n^2 + \frac{1}{2}\delta_n^{*2}, \tag{60}$$

where α_n is a positive design parameter, it follows that

$$\dot{V}_n \leq S_n \left[\beta v + \frac{g_m \alpha_n^2 S_n \vartheta_n^* \psi_n^T \psi_n}{2} \right] + \frac{1}{2}\bar{d}_n^2 - \frac{1}{(1-\bar{\tau}_{\max})} \tanh^2\left(\frac{S_n}{\varepsilon_n}\right) Q_n(\bar{x}_n) + \beta \left|S_n\right| D_\lambda + \frac{1}{2} Q_n(\bar{x}_{n\tau}) + \frac{g_m}{\gamma_{\vartheta_n}} \dot{\hat{\vartheta}}_n \tilde{\vartheta}_n + \frac{\beta_{\max}}{\gamma_{D_\lambda}} \dot{\hat{D}}_\lambda \tilde{D}_\lambda + \frac{\beta_{\max}^2}{2\alpha_n^2} + \frac{1}{2}\delta_n^{*2}. \tag{61}$$

Based on (61), the control law v is designed as

$$v = -k_n S_n - \frac{\alpha_n^2 S_n \hat{\vartheta}_n \psi_n^T \psi_n}{2} - \tanh\left(\frac{S_n}{\varepsilon}\right) \hat{D}_\lambda, \tag{62}$$

where ε is a positive constant. $\hat{\vartheta}_n$ and $\hat{D}_\lambda$ are updated by

$$\dot{\hat{\vartheta}}_n = \gamma_{\vartheta_n} \left[\frac{\alpha_n^2 S_n^2 \psi_n^T \psi_n}{2} - \sigma_{\vartheta_n} \hat{\vartheta}_n \right], \quad \dot{\hat{D}}_\lambda = \begin{cases} \gamma_{D_\lambda} \left(\left|S_n\right| - \sigma_{D_\lambda} \hat{D}_\lambda \right), \text{ if } 0 < \hat{D}_\lambda \leq D_{\lambda\max}, \\ -\gamma_{D_\lambda} \sigma_{D_\lambda} \hat{D}_\lambda, \text{ if } \hat{D}_\lambda > D_{\lambda\max} \end{cases} \tag{63}$$

with σ_{ϑ_n}, σ_{D_λ} being positive design parameters.

Remark 3. In [8,22], the bounding functions $\varphi_{i,j}(\cdot)$ in (3) should be known to avoid "bursting" phenomena [13]. In this paper, $\varphi_{i,j}(\cdot)$, $i = 1, \cdots, n$, are unknown continuous functions to avoid such an undesirable behavior owing to the prescribed tracking performance technique.

Remark 4. In the above design procedures, inspired by [30], the vector norms $\vartheta_i^* = \left\|\theta_{\delta_i}^*\right\|^2$, $i = 1, ..., n$ (see (37), (50), and (63)) are estimated at each controller design step, instead of the estimation of the complet neural networks weight vectors $\theta_{\delta_i}^*$, $i = 1, ..., n$. Therefore, the computational burden is reduced.

Remark 5. To avoid the "Chattering" phenomenon of the controller, instead of the $sgn(\cdot)$ function to compensate the hysteresis, the continuous function $\tanh(\cdot)$ is used to compensate the hysteresis in the control law in the form of $-\tanh(\frac{S_n}{\varepsilon})\hat{D}_\lambda$.

IV. STABILITY ANALYSIS

In this section, the analysis of stability for the proposed DSC scheme will be presented. To begin with, we define

$$y_2 = z_2 - x_{2d} = z_2 + \left[k_1 S_1 + \frac{\Psi^2 \alpha_1^2 S_1 \hat{\vartheta}_1 \psi_1^T \psi_1}{2} \right] e/\Psi,$$

$$y_{i+1} = z_{i+1} - x_{i+1d} = z_{i+1} + k_i S_i + \frac{\alpha_i^2 \hat{\vartheta}_i S_i \psi_i^T \psi_i}{2}, \quad i = 2, \cdots, n-1, \tag{64}$$

where x_{2d} and x_{i+1d} are given by (36) and (49), respectively. From (38) and (51), it follows that

$$\dot{z}_i = \frac{(x_{id} - z_i)}{\tau_i} = -\frac{y_i}{\tau_i}, \; i = 2, \cdots, n. \tag{65}$$

Therefore, the time derivation of (64) can be written as

$$\begin{aligned} \dot{y}_2 &= -\frac{y_2}{\tau_2} + \frac{k_1(\dot{S}_1\Psi - S_1)}{\Psi^2} + \frac{\alpha_1^2}{2}\Big[\dot{\Psi}\hat{\vartheta}_1 S_1 \psi_1^T \psi_1 + \Psi \dot{\hat{\vartheta}}_1 S_1 \psi_1^T \psi_1 \\ &\quad + \Psi \hat{\vartheta}_1 \dot{S}_1 \psi_1^T \psi_1 + 2\Psi \hat{\vartheta}_1 S_1 \psi_1^T \left(\frac{\partial \psi_1}{\partial x_1} \dot{x}_1 + \frac{\partial \psi_1}{\partial S_1} \dot{S}_1 \right) \Big] \\ &= -\frac{y_2}{\tau_2} + B_2\left(S_1, S_2, y_2, \hat{\theta}_{g_1}, \varpi, \dot{\varpi}, \ddot{\varpi}, \hat{\vartheta}_1, y_r, \dot{y}_r, \ddot{y}_r\right), \\ \dot{y}_{i+1} &= -\frac{y_{i+1}}{\tau_{i+1}} + k_i \dot{S}_i + \frac{\alpha_i^2 \dot{\hat{\vartheta}} S_i \psi_i^T \psi_i}{2} + \frac{\alpha_i^2 \hat{\vartheta} \dot{S}_i \psi_i^T \psi_i}{2} \\ &\quad + \alpha_i^2 \hat{\vartheta}_i S_i \psi_i^T \times \sum_{j=1}^{i} \left(\frac{\partial \psi_i}{\partial x_j} \dot{x}_j + \frac{\partial \psi_i}{\partial S_i} \dot{S}_i \right) \\ &= -\frac{y_{i+1}}{\tau_{i+1}} + B_{i+1}\left(S_1, \cdots, S_{i+1}, y_2, \cdots, y_{i+1}, \hat{\theta}_{g_1}, \cdots, \hat{\theta}_{g_i}, \varpi, \dot{\varpi}, \ddot{\varpi}, \hat{\vartheta}_1, \cdots, \hat{\vartheta}_i, y_r, \dot{y}_r, \ddot{y}_r\right), \end{aligned} \tag{66}$$

where B_{i+1}, $i = 1, \cdots, n-1$, are continuous functions.

Define a Lyapunov function candidate as

$$V = \sum_{i=1}^{n} V_i + \frac{1}{2}\sum_{i=1}^{n} V_{Q_i} + \frac{1}{2}\sum_{i=1}^{n-1} y_{i+1}^2, \tag{67}$$

where V_i, $i = 1, \cdots, n$, are given by (25), (41), and (54), respectively; V_{Q_i} are the so called Lyapunov-Krasovskii functionals:

$$V_{Q_i} = \frac{1}{(1-\bar{\tau}_{\max})} \int_{t-\tau_i}^{t} Q_i(\bar{x}_i(\eta)) d\eta, \; i = 1, \cdots, n. \tag{68}$$

We are now ready to establish the main theorem of this paper.

Theorem 1. Consider the closed-loop system consisting of the time-delay plant in (11) with backlash-like hysteresis input described by (7), the control law (62) and the updated law (63), subject to Assumptions A.1-A.5. For a positive number $p > 0$, if the initial value of V in (67) satisfies

$$V(0) \le p, \tag{69}$$

then, all the signals in the closed-loop system are semi-globally uniformly ultimately bounded and the prescribed tracking performance of the tracking error can be guaranteed by properly choosing the design parameters $k_i, \tau_i, \gamma_{\vartheta_i}, \gamma_{D_\lambda}, \sigma_{\vartheta_i}, \sigma_{D_\lambda}, i = 1, \cdots, n$.

Proof. The derivative of the Lyapunov function candidate (67) by considering (68) is

$$\dot{V} = \sum_{i=1}^{n} \dot{V}_i + \sum_{i=1}^{n} \frac{1}{2(1-\bar{\tau}_{\max})} \left[Q_i(\bar{x}_i) - Q_i(\bar{x}_{i\tau})(1-\dot{\tau}_i(t)) \right] + \sum_{i=1}^{n-1} y_{i+1}\dot{y}_{i+1}. \tag{70}$$

Using (39), (49), and (64), we have

$$x_{i+1} = S_{i+1} + y_{i+1} + x_{i+1d}, \ i = 1, \cdots, n-1. \tag{71}$$

Substituting (71), (36) into (35) yields

$$\begin{aligned} \dot{V}_1 \le{}& S_1 \left[-k_1 g_1 S_1 - \frac{g_m \Psi^2 \alpha_1^2 S_1 \tilde{\vartheta}_1 \psi_1^T \psi_1}{2} + g_1 \Psi S_2 + g_1 \Psi y_2 \right] \\ &- \frac{1}{(1-\bar{\tau}_{\max})} \tanh^2\left(\frac{S_1}{\varepsilon_1}\right) Q_1(x_1) \\ &+ \frac{1}{2} Q_1(x_{1\tau}) - g_1^2 \Psi^2 S_1^2 + \frac{g_m}{\gamma_{\vartheta_1}} \tilde{\vartheta}_1 \dot{\hat{\vartheta}}_1 + \frac{1}{2}\bar{d}_1^2 + \frac{g_{\max}^2}{2\alpha_1^2} + \frac{1}{2}\delta_1^{*2}. \end{aligned} \tag{72}$$

Similarly, substituting (71) and (49) into (48) yields

$$\begin{aligned} \dot{V}_i \le{}& S_i \left[-k_i g_i S_i - \frac{g_m \alpha_i^2 S_i \tilde{\vartheta}_i \psi_i^T \psi_i}{2} + g_i S_{i+1} + g_i y_{i+1} \right] \\ &- \frac{1}{(1-\bar{\tau}_{\max})} \tanh^2\left(\frac{S_i}{\varepsilon_i}\right) Q_i(\bar{x}_i) \\ &+ \frac{1}{2} Q_i(\bar{x}_{i\tau}) - g_i^2 S_i^2 + \frac{g_m}{\gamma_{\vartheta_i}} \tilde{\vartheta}_i \dot{\hat{\vartheta}}_i + \frac{1}{2}\bar{d}_i^2 + \frac{g_{\max}^2}{2\alpha_i^2} + \frac{1}{2}\delta_i^{*2}. \end{aligned} \tag{73}$$

Also, substituting (62) into (61) and using the inequality $|x| < x\tanh(\frac{x}{\varepsilon}) + 0.2785\varepsilon$ with $\varepsilon > 0$ yields

$$\begin{aligned} \dot{V}_n \le{}& S_n \left[-k_n \beta S_n - \frac{g_m \alpha_n^2 S_n \vartheta_n^* \psi_n^T \psi_n}{2} \right] \\ &- \frac{1}{(1-\bar{\tau}_{\max})} \tanh^2\left(\frac{S_n}{\varepsilon_n}\right) Q_n(\bar{x}_n) - \beta \left|S_n\right| \tilde{D}_\lambda \\ &+ \frac{1}{2} Q_n(\bar{x}_{n\tau}) + \frac{g_m}{\gamma_{\vartheta_n}} \dot{\hat{\vartheta}}_n \tilde{\vartheta}_n + \frac{\beta_{\max}}{\gamma_{D_\lambda}} \dot{\hat{D}}_\lambda \tilde{D}_\lambda + \frac{1}{2}\bar{d}_n^2 \\ &+ \frac{\beta_{\max}^2}{2\alpha_n^2} + 0.2785\varepsilon\beta_{\max} D_{\lambda\max} + \frac{1}{2}\delta_n^{*2}. \end{aligned} \tag{74}$$

For mitigating the hysteresis, considering the updated law (63) and Assumption A5, we have

$$\begin{aligned} &- \beta \left|S_n\right| \tilde{D}_\lambda + \frac{\beta_{\max}}{\gamma_{D_\lambda}} \dot{\hat{D}}_\lambda \tilde{D}_\lambda \\ &\le -\beta_{\max} \sigma_{D_\lambda} \tilde{D}_\lambda \hat{D}_\lambda. \end{aligned} \tag{75}$$

Substituting (75) into (74), it follows that

$$\begin{aligned} \dot{V}_n \le{}& S_n \left[-k_n \beta S_n - \frac{g_m \alpha_n^2 S_n \vartheta_n^* \psi_n^T \psi_n}{2} \right] \\ &- \frac{1}{(1-\bar{\tau}_{\max})} \tanh^2\left(\frac{S_n}{\varepsilon_n}\right) Q_n(\bar{x}_n) + \frac{1}{2} Q_n(\bar{x}_{n\tau}) \\ &- \beta_{\max} \sigma_{D_\lambda} \tilde{D}_\lambda \hat{D}_\lambda + \frac{1}{\gamma_{\vartheta_n}} \dot{\hat{\vartheta}}_n \tilde{\vartheta}_n + \frac{\beta_{\max}^2}{2\alpha_n^2} \\ &+ 0.2785\varepsilon\beta_{\max} D_{\lambda\max} + \frac{1}{2}\delta_n^{*2}. \end{aligned} \tag{76}$$

By using Assumption A4, the inequality

$$-\frac{1}{2(1-\bar{\tau}_{\max})} Q_i(\bar{x}_{i\tau})(1-\dot{\tau}_i(t)) \le -\frac{1}{2} Q_i(\bar{x}_{i\tau}), \tag{77}$$

holds. In view of (70) and (77), substituting the update laws (37), (50), and (63) into (72), (73), and (76), respectively, by using the following inequalities

$$
\begin{aligned}
&g_1\Psi S_1 S_2 \leq \frac{1}{2}g_1^2\Psi^2 S_1^2 + \frac{1}{2}S_2^2,\\
&g_1\Psi S_1 y_2 \leq \frac{1}{2}g_1^2\Psi^2 S_1^2 + \frac{1}{2}y_2^2,\\
&g_i S_i S_{i+1} \leq \frac{1}{2}g_i^2 S_i^2 + \frac{1}{2}S_{i+1}^2, i = 1,\cdots,n-1,\\
&g_i S_i y_{i+1} \leq \frac{1}{2}g_i^2 S_i^2 + \frac{1}{2}y_{i+1}^2, i = 1,\cdots,n-1,
\end{aligned} \tag{78}
$$

we have

$$
\begin{aligned}
\dot V \leq& -k_1 g_m S_1^2 - \sum_{i=2}^{n-1}\left(k_i g_m - \frac{1}{2}\right)S_i^2 + \frac{1}{2}\sum_{i=1}^{n-1} y_{i+1}^2\\
&+ \sum_{i=1}^{n}\sigma_{\vartheta_i}\tilde\vartheta_i\hat\vartheta_i - \beta_{\max}\sigma_{D_\lambda}\tilde D_\lambda \hat D_\lambda\\
&+ \sum_{i=1}^{n-1}\left(-\frac{y_{i+1}^2}{\varsigma_{i+1}} + |y_{i+1}B_{i+1}|\right) + \frac{1}{2}\left(\sum_{i=1}^{n}\delta_i^{*2} + \bar d_i^2\right)\\
&+ \sum_{i=1}^{n-1}\frac{g_{\max}^2}{2\alpha_i^2} + \frac{\beta_{\max}^2}{2\alpha_n^2} + \frac{1}{2}\sum_{i=1}^{n}\delta_i^{*2}\\
&+ \frac{1}{2(1-\bar\tau_{\max})}\sum_{i=1}^{n}\left[1 - 2\tanh^2\left(\frac{S_i}{\varepsilon_i}\right)\right]Q_i(\bar x_i)\\
&+ \sum_{i=1}^{n-1} y_{i+1}\dot y_{i+1} + 0.2785\varepsilon\beta_{\max}D_{\lambda\max}.
\end{aligned} \tag{79}
$$

To deal with the terms $|y_{i+1}B_{i+1}|$ in the above inequality, by using Assumption A3, we note that the set

$$
\Pi_1 := \left\{(y_r, \dot y_r, \ddot y_r) : y_r^2 + \dot y_r^2 + \ddot y_r^2 \leq B_0\right\} \tag{80}
$$

is compact in $\mathbb{R}^3$ for $B_0 > 0$. Moreover, for any given $p > 0$, the set

$$
\Pi_2 := \left\{\sum_{j=1}^{i}\left(S_j^2 + V_{Q_j}\right) + \sum_{j=2}^{i} y_j^2 \leq 2p\right\} \tag{81}
$$

is compact in $\mathbb{R}^{3n-1+\sum_{j=1}^n N_j}$. Note that $\Pi_1\times\Pi_2$ is also compact in $\mathbb{R}^{3n+2+\sum_{j=1}^n N_j}$. Therefore, $|B_{i+1}|$, for $i = 1,\cdots,n-1$, have maximums, say, M_{i+1} on $\Pi_1\times\Pi_2$. Using Young's inequality, one has

$$
|y_{i+1}B_{i+1}| \leq \frac{y_{i+1}^2 M_{i+1}^2}{2\mu} + \frac{\mu}{2},\ i = 1,\cdots,n-1. \tag{82}
$$

On the other hand, the following inequalities hold:

$$
\begin{aligned}
&-\sigma_{\vartheta_i}\tilde\vartheta_i\hat\vartheta_i \leq -\frac{\sigma_{\vartheta_i}}{2}\tilde\vartheta_i^2 + \frac{\sigma_{\vartheta_i}}{2}\vartheta_i^{*2}, i = 1,\cdots,n,\\
&-\beta_{\max}\sigma_{D_\lambda}\tilde D_\lambda\hat D_\lambda \leq -\frac{\beta_{\max}\sigma_{D_\lambda}}{2}\tilde D_\lambda^2 + \frac{\beta_{\max}\sigma_{D_\lambda}}{2}D_\lambda^2.
\end{aligned} \tag{83}
$$

Let

$$
\frac{1}{\varsigma_{i+1}} = \frac{1}{2} + \frac{M_{i+1}^2}{2\mu} + \frac{\varrho_i}{2}, i = 1,\cdots,n-1, \tag{84}
$$

where ϱ_i are positive design parameters,

$$
\begin{aligned}
C_{11} &= \min\left\{2g_m k_1, \gamma_{\vartheta_1}\sigma_{\vartheta_1}, \varrho_1\right\},\\
C_{i1} &= \min\left\{2\left(g_m k_i - \frac{1}{2}\right), \gamma_{\vartheta_i}\sigma_{\vartheta_i}, \varrho_i\right\}, i = 2,\cdots,n-1,\\
C_{n1} &= \min\left\{2\left(g_m k_n - \frac{1}{2}\right), \gamma_{\vartheta_n}\sigma_{\vartheta_n}, \gamma_{D_\lambda}\sigma_{D_\lambda}\right\},\\
C_2 &= \frac{1}{2}\left(\sum_{i=1}^{n}\delta_i^{*2} + \bar d_i^2\right) + \sum_{i=1}^{n-1}\frac{g_{\max}^2}{2\alpha_i^2}\\
&\quad + \frac{\beta_{\max}^2}{2\alpha_n^2} + 0.2785\varepsilon\beta_{\max}D_{\lambda\max}.
\end{aligned} \tag{85}
$$

Now, substituting the inequalities (82)–(85) into (79), we obtain

$$
\begin{aligned}
\dot V \leq& -C_1\left(2V - \sum_{i=1}^{n}V_{Q_i}\right) + C_2\\
&+ \frac{1}{2(1-\bar\tau_{\max})}\sum_{i=1}^{n}\left[1 - 2\tanh^2\left(\frac{S_i}{\varepsilon_i}\right)\right]Q_i(\bar x_i),
\end{aligned} \tag{86}
$$

where $C_1 = \min\left\{C_{11},\cdots,C_{n1}\right\}$. Note that the term $-C_1(2V - \sum_{i=1}^n V_{Q_i})$ is negative definite and C_2 is a positive constant in the above inequality (86). To deal the terms $\frac{1}{2(1-\bar\tau_{\max})}\sum_{i=1}^n[1 - 2\tanh^2(\frac{S_i}{\varepsilon_i})]Q_i(\bar x_i)$, the following three cases need to be considered.

Case 1. $S_i \in \Omega_{S_i}$[‡], $\forall\, i = 1,\cdots,n$. Since S_i are bounded, from the update laws (37), (50), (63), it is clear that $\hat\vartheta_i$ and $\hat D_\lambda$ are bounded, which implies V is bounded from (67).

Case 2. $S_i \notin \Omega_{S_i}$, $\forall\, i = 1,\cdots,n$. By [13], it can obtain that $[1 - 2\tanh^2(\frac{S_i}{\varepsilon_i})] \leq 0$, which together with the fact that

‡By [28], the compact set Ω_{S_i} is defined as $\Omega_{S_i} := \{S_i\,||S_i| < 0.8814\varepsilon_i\}$ for $i = 1,\cdots,n$. Note that for any $S_i \notin \Omega_{S_i}$, the inequality $(1 - 2\tanh^2(\frac{S_i}{\varepsilon_i})) \leq 0$ holds with $\varepsilon_i > 0$.

$Q_i(\bar{x}_i) = \sum_{l=1}^{i} \varphi_{i,l}^2(x_l) \geq 0$, $i = 1, \cdots, n$, leading to

$$\dot{V} \leq -C_1 \left(2V - \sum_{i=1}^{n} V_{Q_i} \right) + C_2. \tag{87}$$

The above inequality (87) implies that $\dot{V} \leq 0$ on $V = p$, when $C_1 > C_2/(2p - \sum_{i=1}^{n} V_{Q_i})$. Hence, $V \leq p$ is an invariant set, *i.e.*, if condition (69) is satisfied ($V(0) \leq p$), then, $V(t) \leq p$, for all $t \geq 0$.

Case 3. $S_p \notin \Omega_{S_p}$, $S_q \in \Omega_{S_q}$, $\forall\, p, q = 1, \cdots, n$, and $p \neq q$, $p + q = n$. In this case, similar to [24], it can be proved that if condition (69) is satisfied ($V(0) \leq p$), we still have $V(t) \leq p$, for all $t \geq 0$ by considering both Case 1 and Case 2.

Therefore, V and all the closed-loop signals are semi-globally uniformly ultimately bounded. Then, from (18) and (19), the prescribed tracking performance of the tracking error is achieved. This completes the proof.

Remark 6. In [23,24], the authors obtained the following inequality:

$$\dot{V} \leq -\gamma \left(2V - \sum_{i=1}^{n} V_{Q_i} \right) + C, \tag{88}$$

from which it is easy to verify that $V(t)$ is eventually bounded by

$$\frac{C}{2\gamma} + \gamma \sum_{i=1}^{n} \left(\int_0^{\infty} e^{-2\gamma(t-\tau)} \int_{\tau-\tau_i}^{\tau} Q(\bar{x}_i(\eta)) d\eta d\tau \right). \tag{89}$$

Therefore, from the definition of V in [23,24], one has the following compact set:

$$D = \Bigg\{ \bar{s}_n, y_2, \cdots, y_n, \tilde{\psi}_1, \cdots, \tilde{\psi}_n, \tilde{W}_1, \cdots, \tilde{W}_n \,\Bigg|\, \sum_{i=1}^{n} \left(s_i^2 + \tilde{\psi}_i^2 + \|\tilde{W}_i\|^2 \right) + \sum_{i=1}^{n-1} y_{i+1}^2 < \frac{C}{\gamma} + 2\gamma \times \sum_{i=1}^{n} \left(\int_0^{\infty} e^{-2\gamma(t-\tau)} \int_{\tau-\tau_i}^{\tau} Q(\bar{x}_i(\eta)) d\eta d\tau \right) \Bigg\}, \tag{90}$$

which cannot be kept arbitrarily small by increasing γ.

V. SIMULATION RESULTS

In this section, the following second-order time-varying delay nonlinear system with unknown backlash-like hysteresis is given to illustrate the validity of the proposed scheme:

$$\begin{aligned} \dot{x}_1 &= g_1(\bar{x}_1)x_2 + f_1(\bar{x}_1) + h_1(\bar{x}_{1\tau}) + d_1(t), \\ \dot{x}_2 &= g_2(\bar{x}_2)w(v) + f_2(\bar{x}_2) + h_2(\bar{x}_{2\tau}) + d_2(t), \\ y &= x_1, \end{aligned} \tag{91}$$

where w is the output of the hysteresis, $g_1(\bar{x}_1)$, $g_2(\bar{x}_2)$ are unknown smooth functions, and d_1, d_2 are disturbances. In the simulation, we choose $g_1(\bar{x}_1) = 1 + 0.05\cos(x_1)$, $g_2(\bar{x}_2) = 1 + 0.1\cos(x_2)$, $f_1(x_1) = x_1^2 + \sin(x_1)$, $f_2(\bar{x}_2) = x_1 x_2^2$, $h_1(x_1) = \sin(x_1)$, $h_2(\bar{x}_2) = x_1 x_2$, $d_1 = 0.1\sin(t)$ and $d_2 = 0.1\cos(t)$. The hysteresis is described by (7) with $\lambda = 1.432$, $\psi = 0.105$,and $\alpha = 1$; The time-varying delays $\tau_1(t) = \tau_2(t) = 1 - 0.5\cos(t)$. The control objective is to make the state $x_1 (= y)$ follow $y_r = \sin(t)$. According to Section III, the design procedure is as follows.

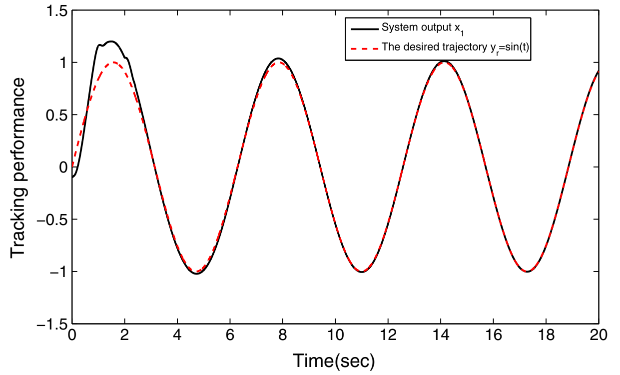

Fig. 2. y and y_r (solid and dashed lines, respectively).

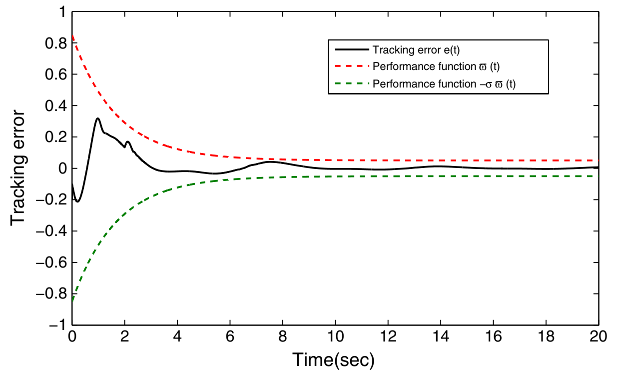

Fig. 3. Tracking error and prespecified transient performance.

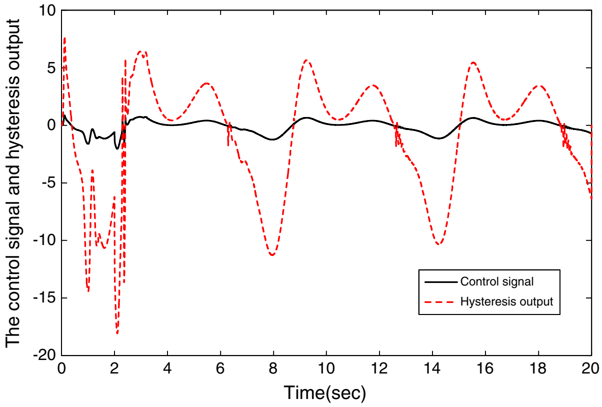

Fig. 4. Control signal and hysteresis output.

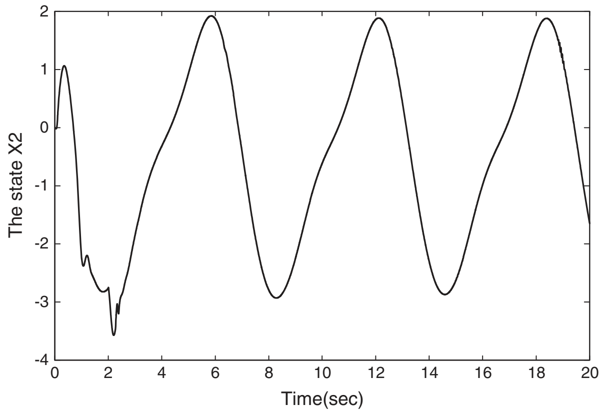

Fig. 5. The state $x_2(t)$.

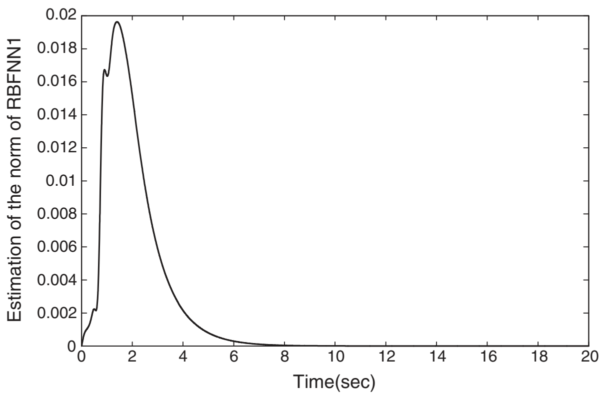

Fig. 6. Estimation of RBFNN $\hat{\vartheta}_1$.

Step 1. The first surface error is

$$S_1 = \Phi^{-1}\left(\frac{e(t)}{\varpi(t)}\right) = \tan\left(\frac{\pi}{2}\frac{e(t)}{\varpi(t)}\right), \tag{92}$$

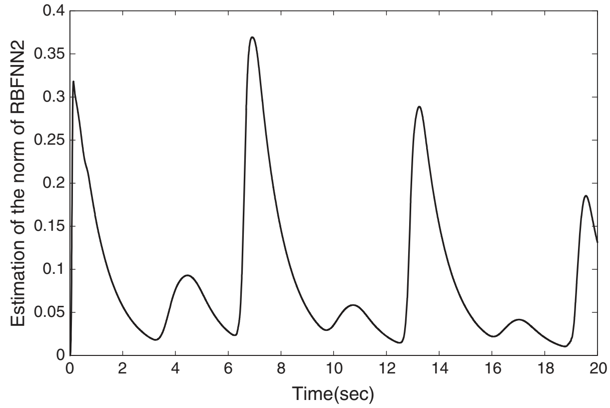

Fig. 7. Estimation of RBFNN.

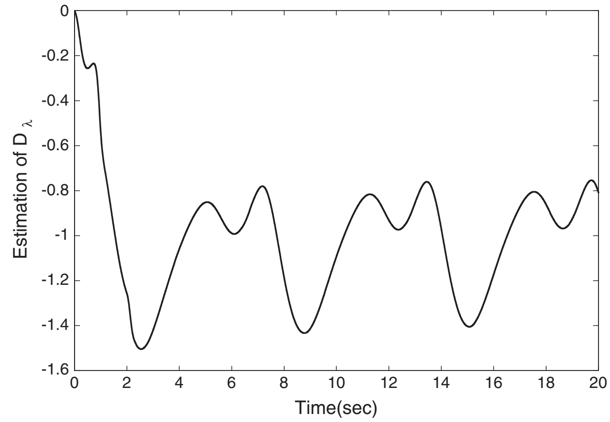

Fig. 8. Estimation of parameter D_λ.

where $\Phi(S_1) = \frac{2}{\pi}\arctan(S_1)$, $e = x_1 - y_r$ and $\varpi(t) = 0.8 * e^{(-0.6*t)} + 0.05$. By (36), the virtual control signal

$$x_{2d} = \left[-k_1 S_1 - \frac{\Psi^2 \alpha_1^2 S_1 \hat{\vartheta}_1 \psi_1^T \psi_1}{2}\right] / \Psi, \tag{93}$$

where $\hat{\vartheta}_1$ and $\hat{\theta}_{g_1}$ are updated by (37).

Step 2. The second surface error is

$$S_2 = x_2 - z_2. \tag{94}$$

Since the controlled plant is a second-order system, from (62), the control law is

$$v = -k_n S_2 - \frac{\alpha_2^2 S_2 \hat{\vartheta}_2 \psi_2^T \psi_2}{2} - tanh\left(\frac{S_2}{\varepsilon}\right)\hat{D}_\lambda, \tag{95}$$

where $\hat{\vartheta}_2$, $\hat{D}_\lambda$ are updated by (63).

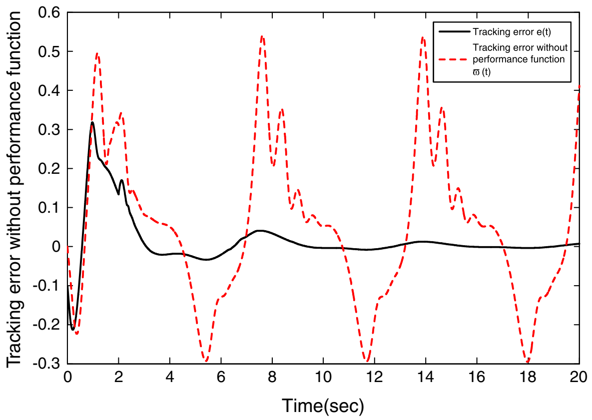

Fig. 9. Tracking error with and without performance function $\varpi(t)$.

In this simulation, the inputs of the RBFNN are chosen as $\xi_1 = (x_1, \Psi, S_1) \in \mathbb{R}^3$ and $\xi_2 = (x_1, x_2, S_2) \in \mathbb{R}^3$. For NNs $\psi(\xi_1)$, we choose 81 nodes with the centers of the basis functions ζ_k, $k = 1, \cdots, 41$, being evenly spaced in $[-2, +2]\times[-2, +2]\times[-2, +2]$ and width $\sigma_k = 1$, $k = 1, \cdots, 41$. For NNs $\psi(\xi_2)$, we choose 161 nodes with the centers of the basis functions ζ_k, $k = 1, \cdots, 81$, being evenly spaced in $[-2, +2] \times [-2, +2] \times [-2, +2]$ and width $\sigma_k = 1$, $k = 1, \cdots, 81$. The initial values of update laws are selected as $\hat{\vartheta}_1(0) = \hat{\vartheta}_2(0) = \hat{D}_\lambda = 0$,. In addition, the design parameters are chosen as $k_1 = 4$, $k_2 = 0.9368$, $\alpha_1 = \alpha_2 = 1$, $\gamma_{\vartheta_1} = \gamma_{\vartheta_2} = 2$, $\gamma_{D_\lambda} = 3$, $\varepsilon = 0.1$. The small gains are selected as $\sigma_{\vartheta_1} = \sigma_{\vartheta_2} = \sigma_{D_\lambda} = 0.00001$ and the time constant of the low pass filter is chosen as $\varsigma_2 = 0.01$. The initial states are chosen as $x_1(0) = -0.1$ and $x_2(0) = 0.2$.

The simulation results are shown in Figs 2–9. From Fig. 2, it can be seen that the system output $y = x_1$ satisfies the prescribed performance. Fig. 3 illustrates that the tracking error $e(t)$ is kept between $-\sigma\varpi(t)$ and $\varpi(t)$ for all $t \geq 0$, which shows the validity of our proposed dynamic surface control method. Fig. 4 is the control signal and the hysteresis output corresponding to the control signal which shows the effectiveness of the hysteresis nonlinearities. Fig. 5 shows the curve of the state x_2 in (94). Figs 4 and 7 illustrate the estimations of the norms of the neural network weighted vectors $\vartheta_1^* = \frac{1}{g_m}\left\|\theta_{\delta_1}^*\right\|^2$ and $\vartheta_2^* = \frac{1}{g_m}\left\|\theta_{\delta_2}^*\right\|^2$, respectively. Fig. 8 shows the estimation of $D_\lambda = D/\lambda$. Note that though ϑ_1^*, ϑ_2^* and D_λ, may not converge to the true value, we still achieve the objective of this paper. Fig.9 displays the tracking error with and without the performance function $\varpi(t)$ which shows the effectiveness of the prescribed performance.

VI. CONCLUSION

In this paper, a robust adaptive dynamic surface control has been proposed for a class of nonlinear time-varying delay systems preceded by unknown backlash-like hysteresis. We have shown that by using the proposed control scheme, the prescribed tracking error performance can be achieved; by estimating the norm of unknown weighted vector of the neural network, the computational burden can be greatly reduced. Simulation results are presented to demonstrate the validity of the proposed scheme.

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Xiuyu Zhang received the B.S. and M.S. degree from Northeast Dianli University, Jilin, China, in 2003 and 2006, respectively, and the Ph.D. degree from Beijing University of Aeronautics and Astronautics (BUAA), Beijing, China, in 2012. He is currently Associate Professor with the School of Automation Engineering, Northeast Dianli University, Jilin, China. His research interests include robust and adaptive control for nonlinear systems with smart material based actuators.

Zhi Li received the M.Sc. degree in control engineering from Northeastern University, Shenyang, China, in 2009 and the Ph.D. degree in mechanical engineering from Concordia University, Montreal, Canada, in 2015. He was a visiting researcher at Concordia University, Montreal, Canada, from September 2010 to August 2011. He is currently Postdoctoral Research Fellow with the Department of Electrical Engineering, Eindhoven University of Technology, Eindhoven, Netherlands. His research interests include modeling and control of the smart material-based actuators, hysteresis modeling and compensation, modeling based battery management, battery management systems. Dr. Li was the recipient of the Concordia University Graduate Fellowship and Concordia Full Tuition Recruitment Award in 2011 and the T. J. Tarns Best Paper Award on Control Applications at the 11th World Congress on Intelligent Control and Automation, 2014.

Chun-Yi Su received his Ph.D. degrees in control engineering from South China University of Technology in 1990. After a seven-year stint at the University of Victoria, he joined the Concordia University in 1998, where he is currently Professor of Mechanical and Industrial Engineering and holds the Concordia Research Chair in Control. He has also held several short-time visiting positions including a Chang Jiang Chair Professorship by Chinas Ministry of Education and JSPS Invitation Fellowship from Japan. Dr. Sus research covers control theory and its applications to various mechanical systems, with a recent focus on control of systems involving hysteresis nonlinearities. He is the author or co-author of over 300 publications, which have appeared in journals, as book chapters and in conference proceedings. Dr. Su has served as Associate Editor of IEEE Transactions on Automatic Control and IEEE Transactions on Control Systems Technology. He is on the Editorial Board of 18 journals, including IFACs Mechatronics. Dr. Su has also severed for many conferences as an organizing committee member, including General Co-Chair of the 2012 IEEE International Conference on Mechatronics and Automation and Program Chair of the 2007 IEEE Conference on Control Applications.

Xinkai Chen received the B.S. and M.S. degrees in mathematics from Hebei University, Baoding, China, in 1986 and 1989, respectively, the M.S. degree in electrical engineering from Mie University, Tsu, Japan, in 1995, and the Ph.D. degree in engineering from Nagoya University, Nagoya, Japan, in 1999. He is currently Professor with the Department of Electronic and Information Systems, Shibaura Institute of Technology, Tokyo, Japan. His research interests include adaptive control, robust nonlinear control, motion control, and machine vision.

Jianguo Wang received the B.S. and M.S. degree from Tianjin University, Tianjin, China, in 1985 and 1988, respectively, and the Ph.D. degree from North China Electric Power University, Beijing, China, in 2001. He is currently Professor with the School of Automation Engineering, Northeast Dianli University, Jilin, China. His current research interests include robust adaptive control, fault diagnosis of the electrical device in power plant.

Linlin Xia was born in 1980. She received B.S. degree in Automation from Harbin Engineering University, Harbin, China, in 2003, and M.S degree in Navigation, Guidance and Control from Harbin Engineering University, Harbin, China, in 2006, and Ph.D. degree in Navigation, Guidance and Control from Harbin Engineering University, Harbin, China, in 2008. Now she is Associated professor at School of Automation Engineering, Northeastern Dianli University, Jilin, China. Her research interests include Artificial intelligence, intelligent control and information processing.