Robust Consensus Problem of Heterogeneous Uncertain Second-Order Multi-Agent Systems Based on Sliding Mode Control

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This paper investigates the robust consensus problem for heterogeneous second-order multi-agent systems with uncertain parameters. Based on the sliding mode control method, novel robust consensus protocols are designed for the linear multi-agent systems with uncertain parameters and a class of uncertain nonlinear multi-agent systems. Finally, numerical simulations are given to verify the effectiveness of the proposed protocols.

Keywords: sliding mode, control, robust consensus, multi-agent systems, uncertain parameters

1 INTRODUCTION

A multi-agent system (MAS) is composed of many subsystems connected by a network, where each subsystem is called an agent. All the agents transmit state information via the interconnecting network and affect each other according to a control protocol. The research topics on multi-agent system theory include mathematical models, simulations, collective behavior analysis, protocol design and so on. Collective behavior analysis of multi-agent systems is an important issue of multi-agent system theory, which has a wide background on physics and engineering such as the self-organization of a colony, coordinated control of robots, synchronization of satellites, formation control of unmanned aerial vehicles (UAVs).

In recent years, many mathematical models of MASs have been widely studied and lots of research contributions have been published Ren and Atkins (2007); Ren and Beard (2005); Ren (2007); Liu and Guo (2008, 2009), including Vicsek model Liu and Guo (2008, 2009); Vicsek et al. (1995), Kuramoto model Kuramoto (1975), integrator multi-agent systems Olfati-Saber et al. (2007); Olfati-Saber and Murray (2004), general high-order linear multi-agent systems Ma and Zhang (2010) and nonlinear multi-agent systems Ren and Chen (2015). A protocol is an interaction rule that specifies the information exchange between an agent and its neighbours on the network. Consensus problem is just to design a protocol such that the states of agents reach consensus, which has been studied from several different perspectives in the existing literatures Olfati-Saber and Murray (2004)-Hou et al. (2009). In Olfati-Saber and Murray (2004); Tang et al. (2012); Xu et al. (2013), consensus protocols for the first order multi-agent systems are developed. While in Islam et al. (2018); Su et al. (2011); Yu et al. (2009), consensus protocols for the second order multi-agent systems are proposed.

For an actual system, it is usually difficult to get an exact mathematical model since uncertain parameters and external disturbances often exist. Thus it is significant to investigate this kind of multi-agent systems with agents having different dynamics and being affected by uncertainties and disturbances. Sliding mode control is an important technique in robust control field, whose main advantage lies in the robustness with respect to uncertainties and disturbances Utkin (1978, 1993);
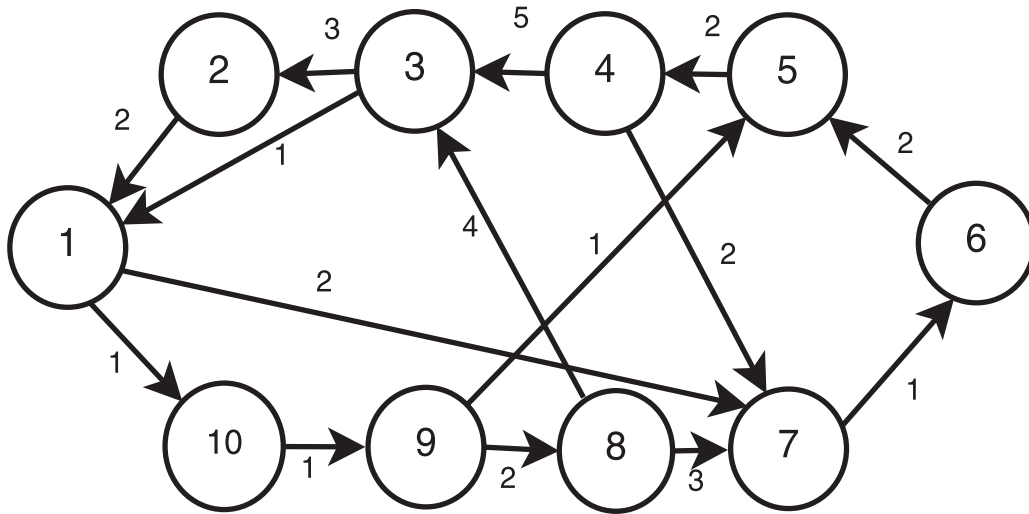
FIGURE 1 | The digraph of the multi-agent system.

Utkin et al. (2009). At present, sliding mode technique has been used in multi-agent systems to control some collective behaviors. For solving finite-time consensus problem, sliding mode control method has been adopted in many cases Khoo et al. (2009, 2008); Cao et al. (2010); Yu and Long (2015). The well-known high-order sliding mode control is effective in diminishing the chattering phenomenon Levant (1993); Moreno and Osorio (2008) and has been used to solve some formation control problems Jia et al. (2007); Galzi and Shtessel (2006). In Zhang et al. (2017) and Islam et al. (2018), sliding mode control method has been used to solve some consensus problem of multi-agent systems with uncertainties. Some recent results on robust consensus can be seen in Xue et al. (2021) and Yu et al. (2021). In Xue et al. (2021), structured uncertainties are considered, and in Yu et al. (2021) adaptive sliding mode control is designed. In our recent paper Zhao and Zhu (2020), we have investigated a class of linear uncertain multi-agent systems. However, in Zhao and Zhu (2020), one has to solve linear matrix inequalities, and the nonlinear uncertain multi-agent systems have not been considered.

In this paper, we investigates the robust consensus problem for the heterogeneous uncertain second-order multi-agent systems based on sliding mode control. For the linear uncertain multi-agent systems, the uncertain parameters appear in both the state coefficients and the control coefficients. A similar result is obtained for a class of nonlinear multi-agent systems with uncertain parameters. Simulations show the effectiveness of the proposed protocols in realizing robust consensus.

2 ROBUST CONSENSUS OF THE LINEAR HETEROGENEOUS SECOND-ORDER MULTI-AGENT SYSTEMS

We consider the networked multi-agent system with the fixed topology $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$, which is a weighted digraph of order m

with the set of nodes $\mathcal{V} = \{1, 2, \ldots, m\}$, set of directed edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a nonnegative adjacency matrix $\mathcal{A} = (\alpha_{ij})_{m \times m}$. A directed edge of $\mathcal{G}$ is denoted by $j \rightarrow i$, which means node j can transmit its state information to node i. The matrix $\mathcal{A}$ satisfies

$$\alpha_{ij} \begin{cases} > 0, & j \rightarrow i \in \mathcal{E}, \\ = 0, & \text{otherwise.} \end{cases} \tag{1}$$

Suppose that each node of the digraph is a dynamic agent with dynamics.

$$\dot{x}_i = v_i, \tag{2}$$
$$\dot{v}_i = a_{1i}x_i + a_{2i}v_i + w_i + b_i u_i, \tag{3}$$

where $(x_i, v_i)^{\mathrm{T}} \in \mathbf{R}^2$ is the state vector, $u_i(t) \in \mathbf{R}$ the external control, a_{1i}, a_{2i} and b_i the general uncertain parameters, w_i the external disturbance. For uncertain multi-agent systems, it is usually to assume that all the agents have the same uncertainty Chen et al. (2010). Obviously, it is more reasonable to admit different uncertainties for different agents. This is just why we call the system as heterogeneous uncertain multi-agent system.

The *robust consensus problem* is just to design a distributed feedback controller u_i for each $i = 1, 2, \ldots, m$ such that the closed-loop system satisfies

$$\lim_{t \rightarrow \infty} \left(x_j(t) - x_i(t) \right) = 0, \ \lim_{t \rightarrow \infty} \left(v_j(t) - v_i(t) \right) = 0, \ (i, j = 1, \ldots, m) \tag{4}$$

for some kinds of uncertain parameters a_{1i}, a_{2i}, b_i and external disturbances.

Assumption 1. All of the uncertain parameters and the disturbances are uniformly bounded, i.e., there exist known positive constants η and δ such that

$$|a_{1i}| \leq \eta, \ |a_{2i}| \leq \eta, \ |w_i| \leq \eta, \ |b_i - b_0| \leq \delta, \tag{5}$$
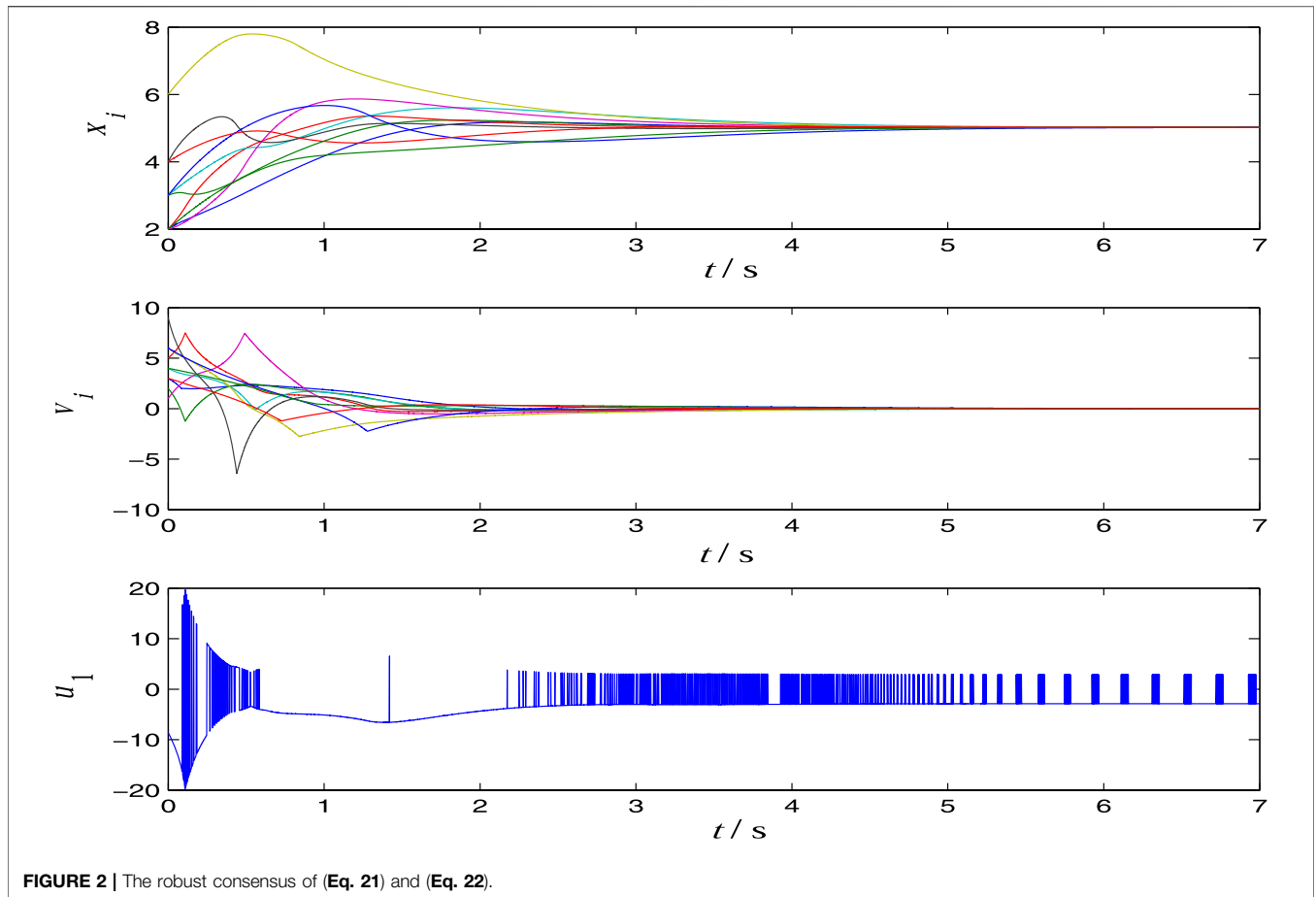
where b_0 is a known constant and $\delta < |b_0|$.

FIGURE 2 | The robust consensus of (**Eq. 21**) and (**Eq. 22**).

Theorem 1. Consider the heterogeneous uncertain multi-agent system described by **Eq. 2** and **Eq. 3**. If digraph $\mathcal{G}$ has a spanning tree and Assumption 1 holds, then the robust consensus problem is solved by the distributed sliding mode controller

$$u_i = -\frac{1}{b_0}(L_i + \eta)\text{sgn}(s_i), \qquad (6)$$

where

$$L_i = \frac{\eta|x_i| + \eta|v_i| + \eta}{1-r} + \frac{k}{1-r}\sum_{j=1}^{m}\alpha_{ij}|v_j - v_i|, \qquad (7)$$

$$s_i = v_i - k\sum_{j=1}^{m}\alpha_{ij}(x_j - x_i), \qquad (8)$$

$$0 < r = \frac{\delta}{b_0} < 1 \text{ and } k > 0$$

Proof. First, we prove that the trajectory of the closed-loop system will stay in the sliding surfaces $s_i = 0$ ($i = 1, 2, \ldots, m$) after a finite time. Let $V_i = \frac{s_i^2}{2}$. Then

$$\dot{V}_i = s_i\dot{s}_i$$

$$= s_i\left(a_{1i}x_i + a_{2i}v_i + w_i + b_iu_i - k\sum_{j=1}^{m}\alpha_{ij}(v_j - v_i)\right)$$

$$= s_i\left(a_{1i}x_i + a_{2i}v_i + w_i - k\sum_{j=1}^{m}\alpha_{ij}(v_j - v_i) + b_0\left(1 + \frac{b_i - b_0}{b_0}\right)u_i\right)$$

$$= s_i\left(a_{1i}x_i + a_{2i}v_i + w_i - k\sum_{j=1}^{m}\alpha_{ij}(v_j - v_i) + \left(1 + \frac{b_i - b_0}{b_0}\right)u_i^*\right).$$

(9)

By **Eq. 7**, it is easy to see

$$\left|\frac{a_{1i}x_i + a_{2i}v_i + w_i - k\sum_{j=1}^{m}\alpha_{ij}(v_j - v_i)}{1 + \frac{b_i - b_0}{b_0}}\right| \qquad (10)$$

$$\leq \frac{\eta|x_i| + \eta|v_i| + \eta}{1-r} + \frac{|k|}{1-r}\sum_{j=1}^{m}\alpha_{ij}|v_j - v_i|$$
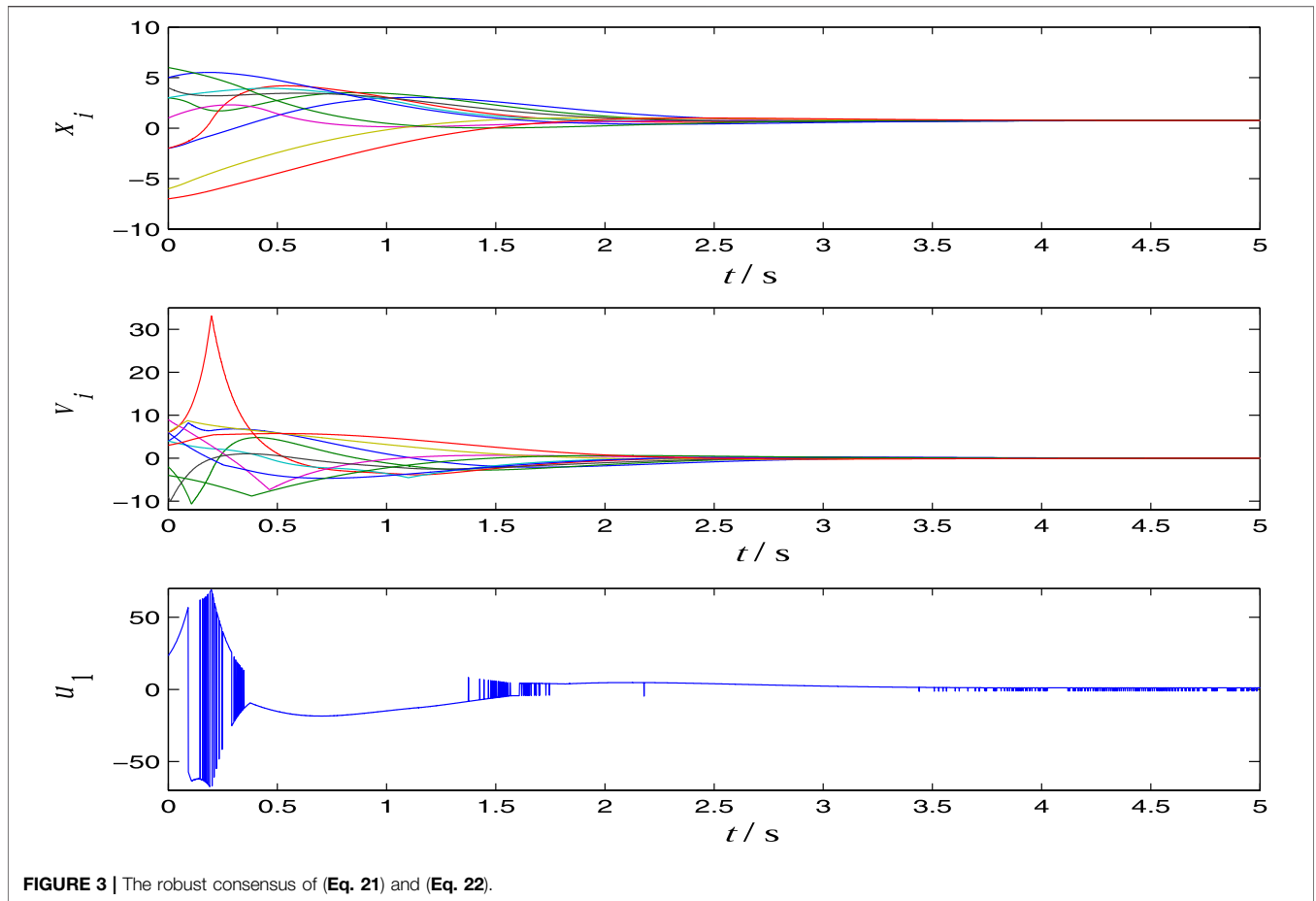
$$= L_i.$$

By **Eq. 9**, we have

FIGURE 3 | The robust consensus of (**Eq. 21**) and (**Eq. 22**).

$$
\begin{aligned}
\dot{V}_i &\leq \left(1 + \frac{b_i - b_0}{b_0}\right)|s_i|L_i + \left(1 + \frac{b_i - b_0}{b_0}\right)s_i u_i^\star \\
&= \left(1 + \frac{b_i - b_0}{b_0}\right)|s_i|L_i - \left(1 + \frac{b_i - b_0}{b_0}\right)s_i(L_i + \eta)\mathrm{sgn}(s_i) \\
&= -\left(1 + \frac{b_i - b_0}{b_0}\right)\eta|s_i| \\
&\leq -(1 - r)\eta|s_i| \\
&= -\sqrt{2}(1 - r)\eta V_i^{1/2} < 0.
\end{aligned}
\tag{11}
$$

From **Eq. 11**, one obtains that $s_i(t) = 0$ as $t \geq \max\limits_{1 \leq i \leq m} \frac{\sqrt{2V_i(0)}}{(1-r)\eta}$.

Second, we consider the dynamics limited on the sliding mode surfaces. While the trajectory stays in the sliding mode surfaces $s_i = 0$, we have

$$
v_i - k\sum_{j=1}^{m}\alpha_{ij}(x_j - x_i) = 0.
\tag{12}
$$

From **Eq. 12** and **Eq. 2**, it follows that the dynamics limited on the sliding mode surfaces is described by

$$
\dot{x}_i = -k\sum_{j=1}^{m}\alpha_{ij}(x_j - x_i), \quad i = 1, 2, \ldots, m.
\tag{13}
$$

In Olfati-Saber and Murray (2004), the consensus of system (13) has been proved as follows: there exists a constant C, which is dependent on the initial states $(x_1(0), x_2(0), \ldots, x_m(0))^{\mathrm{T}}$, such that $x_i(t) \to C$ as $t \to +\infty$ for every $1 \leq i \leq m$. Considering each sliding mode function s_i described by **Eq. 8**, we conclude by $s_i = 0$ that $v_i(t) \to 0$ as $t \to +\infty$. Therefore, the consensus described by **Eq. 4** is realized.

Remark 1. The proposed conditions are somewhat conservative since the designed controller depends on the bounds of the uncertain parameters. If the bounds are unknown, one can consider adopt adaptive control method. In many practical systems, there are errors in the measurement of parameters, but generally speaking the measurement accuracy of the instrument is known. Therefore, in this case the variation range of parameters in Assumption 1 can be accurately determined. Moreover, the boundary of disturbance signals needs to be obtained through experience and experiments.

FIGURE 4 | The robust consensus of (**Eq. 21**) and (**Eq. 22**).
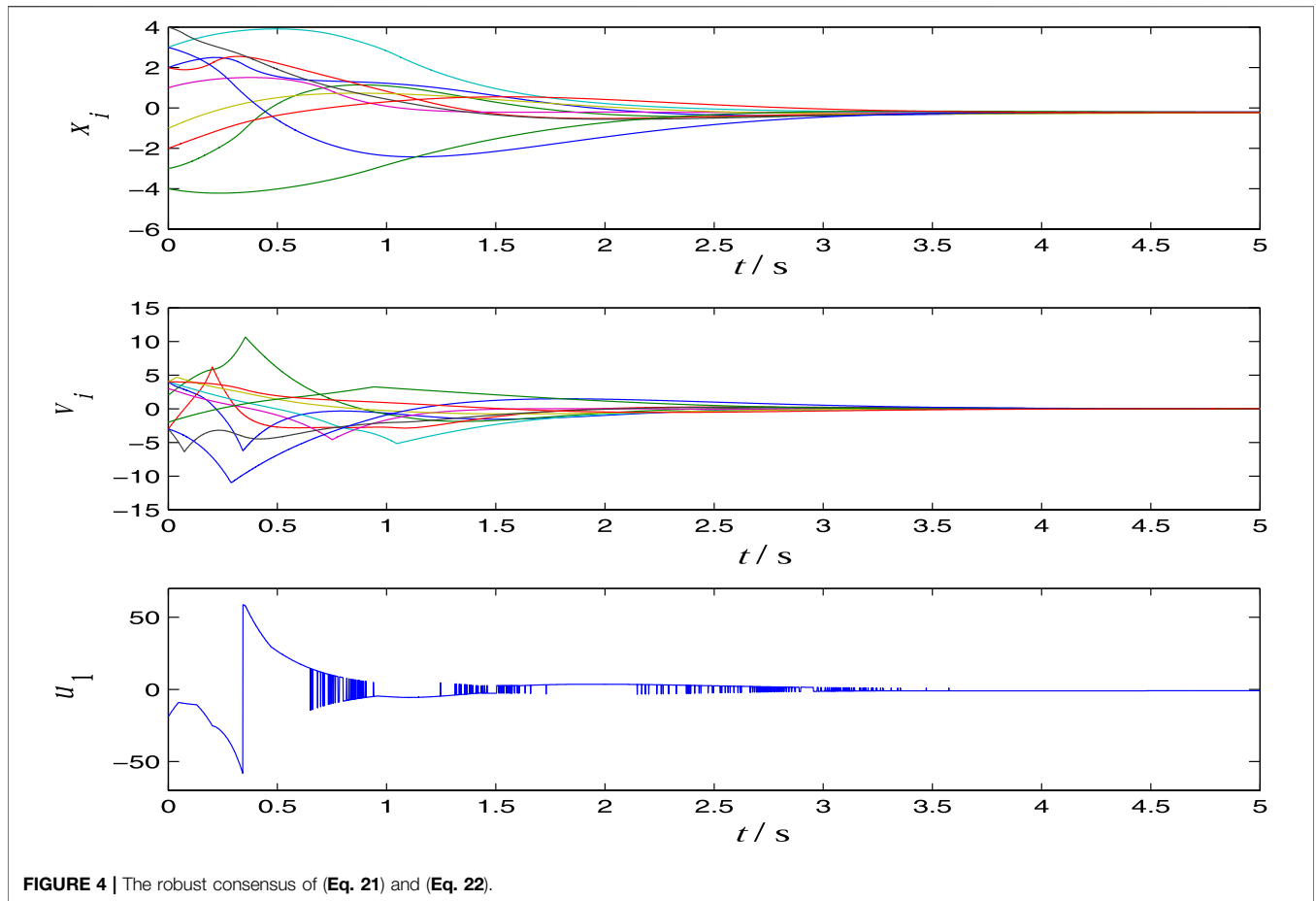
3 ROBUST CONSENSUS OF A CLASS OF UNCERTAIN NONLINEAR MULTI-AGENT SYSTEMS

In this section, we consider a kind of uncertain nonlinear multi-agent systems. Suppose that each node of the digraph has a nonlinear dynamics as follows:

$$\dot{x}_i = v_i, \tag{14}$$

$$\dot{v}_i = a_i(x_i, v_i) + b_i(x_i, v_i)u_i, \tag{15}$$

where $(x_i(t), v_i(t))^{\mathrm{T}} \in \mathbf{R}^2$ is the state vector of the ith agent, $u_i(t) \in \mathbf{R}$ is the control, $a_i(x_i, v_i)$ and $b_i(x_i, v_i)$ are uncertain nonlinear functions for every $i = 1, 2, \ldots, m$.

Assumption 2. For the uncertain nonlinear functions $a_i(x_i, v_i)$ and $b_i(x_i, v_i)$, there exists a known function $L(x_i, v_i)$ satisfying

$$\left| \frac{a_i(x_i, v_i)}{b_i(x_i, v_i)} \right| \leq L(x_i, v_i), \tag{16}$$

where $b_i(x_i, v_i) \geq b_0$ for known constant $b_0 > 0$.

Theorem 2. For the uncertain nonlinear multi-agent system described by **Eq. 14** and **Eq. 15**, If digraph $\mathcal{G}$ has a spanning tree

and Assumption 2 holds, then the robust consensus problem of the system can be solved by the distributed sliding mode controller

$$u_i = -\beta_i \mathrm{sgn}(s_i), \tag{17}$$

where

$$\beta_i = L(x_i, v_i) + \frac{k}{b_0} \sum_{j=1}^{m} \alpha_{ij} |v_j - v_i| + \beta_0 \tag{18}$$

with a known constant $\beta_0 > 0$ and

$$s_i = v_i - k \sum_{j=1}^{m} \alpha_{ij}(x_j - x_i) \tag{19}$$

with any constant $k > 0$.

Proof. Let $V_i = \frac{s_i^2}{2}$. Then

$$
\begin{aligned}
\dot{V}_i &= s_i \dot{s}_i \\
&= s_i \left(\dot{v}_i - k \sum_{j=1}^{m} \alpha_{ij}(\dot{x}_j - \dot{x}_i) \right) \\
&= s_i \left(a_i(x_i, v_i) + b_i(x_i, v_i)u_i - k \sum_{j=1}^{m} \alpha_{ij}(v_j - v_i) \right)
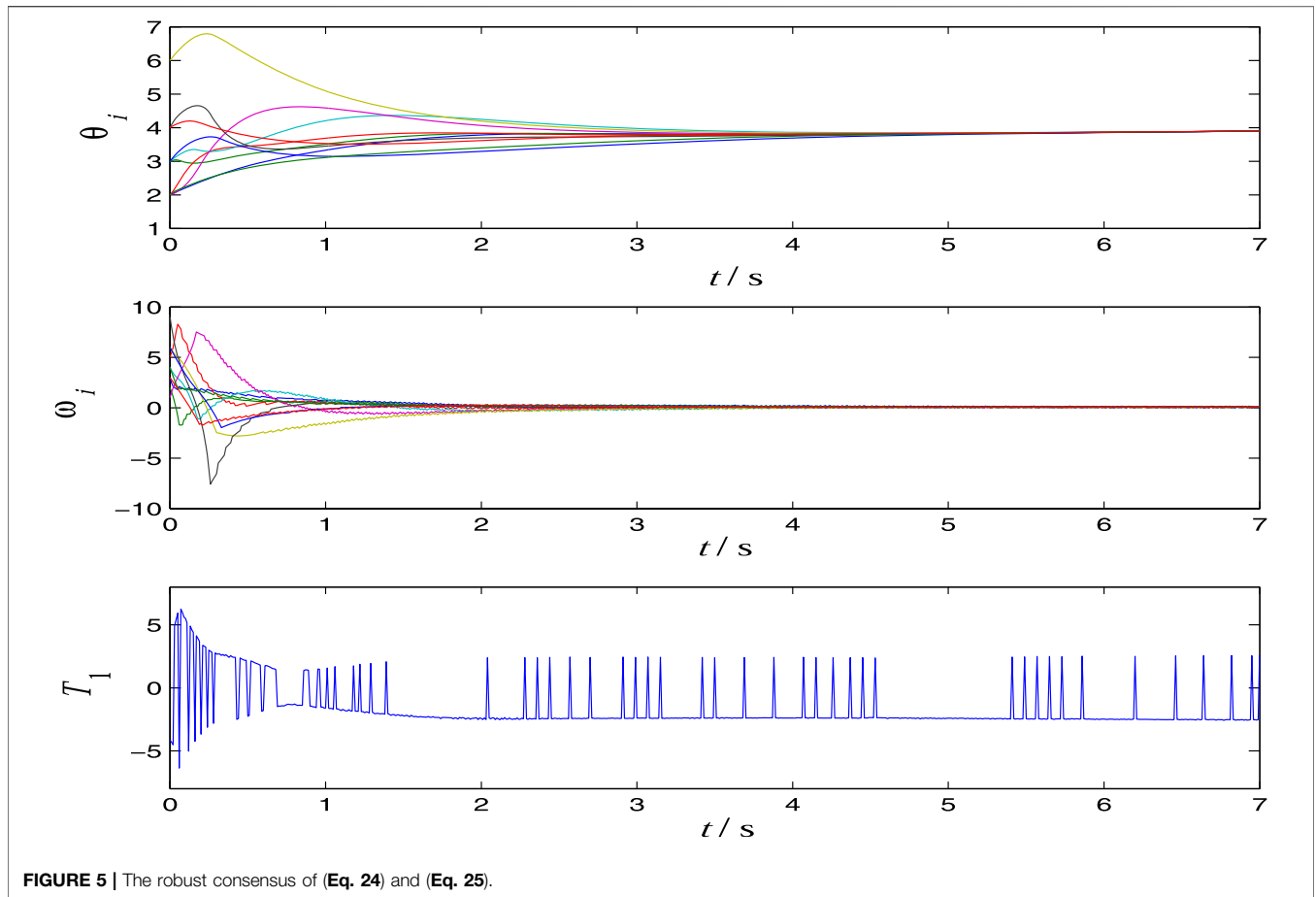\end{aligned}
$$

FIGURE 5 | The robust consensus of (**Eq. 24**) and (**Eq. 25**).

$$= s_i \left(a_i (x_i, v_i) - k \sum_{j=1}^{m} \alpha_{ij} (v_j - v_i) \right) + s_i b_i (x_i, v_i) u_i$$

$$= s_i b_i (x_i, v_i) \frac{a_i (x_i, v_i) - k \sum_{j=1}^{m} \alpha_{ij} (v_j - v_i)}{b_i (x_i, v_i)} + s_i b_i (x_i, v_i) u_i.$$

Applying Assumption 2, (17) and (18), we have

$$\dot{V}_i \leq |s_i| b_i (x_i, v_i) \left(L (x_i, v_i) + \frac{k}{g_0} \sum_{j=1}^{m} \alpha_{ij} |v_j - v_i| \right) + s_i b_i (x_i, v_i) u_i$$

$$= b_i (x_i, v_i) |s_i| \left(L (x_i, v_i) + \frac{k}{g_0} \sum_{j=1}^{m} \alpha_{ij} |v_j - v_i| \right) - b_i (x_i, v_i) s_i \mathrm{sgn} (s_i) \beta$$

$$= b_i (x_i, v_i) |s_i| \left(L (x_i, v_i) + \frac{k}{g_0} \sum_{j=1}^{m} \alpha_{ij} |v_j - v_i| - \beta \right)$$

$$= -b_i (x_i, v_i) \beta_0 |s_i|$$

$$\leq -b_0 \beta_0 |s_i|$$

$$= -\sqrt{2} b_0 \beta_0 V_i^{1/2}. \tag{20}$$

From **Eq. 20**, it follows that $s_i(t) = 0$ as $t \geq \max_{1 \leq i \leq m} \frac{\sqrt{2V_i(0)}}{b_0 \beta_0}$.

The rest of the proof is as same as that of the proof of Theorem 1.

Remark 2. In practice, the upper bound functions satisfying Assumption 2 are also needed to be designed via experience and experiments. The bound of nonlinearity is used to design the controller, which can lead to high-gain feedback. This is a disadvantage of our paper.

4 SIMULATIONS

Example 1. Consider the multi-agent systems with the topology $\mathcal{G}$ shown in **Figure 1** and the dynamics as follows

$$\dot{x}_i = v_i, \tag{21}$$

$$\dot{v}_i = a_{1i} x_i + a_{2i} v_i + w_i + b_i u_i, \tag{22}$$

where $i = 1, 2, \ldots, 10$. Let $x = [x_1, x_2, \ldots, x_{10}]$ and $v = [v_1, v_2, \ldots, v_{10}]$ be the position vector and the velocity vector of the multi-agent system. Let $a_1 = [a_{11}, a_{12}, \ldots, a_{1,10}]$, $a_2 = [a_{21}, a_{22}, \ldots, a_{2,10}]$ and $b = [b_1, b_2, \ldots, b_{10}]$ be the uncertain parameters satisfying $|a_{ij}| \leq 0.3$, $|b_i - 1| \leq 0.3$. Let $w(t) = [w_1(t), w_2(t), \ldots, w_{10}]$ be composed of the disturbance signals satisfying $|w_i(t)| \leq 0.3$. By **Figure 1**, the adjacency matrix of $\mathcal{G}$ is
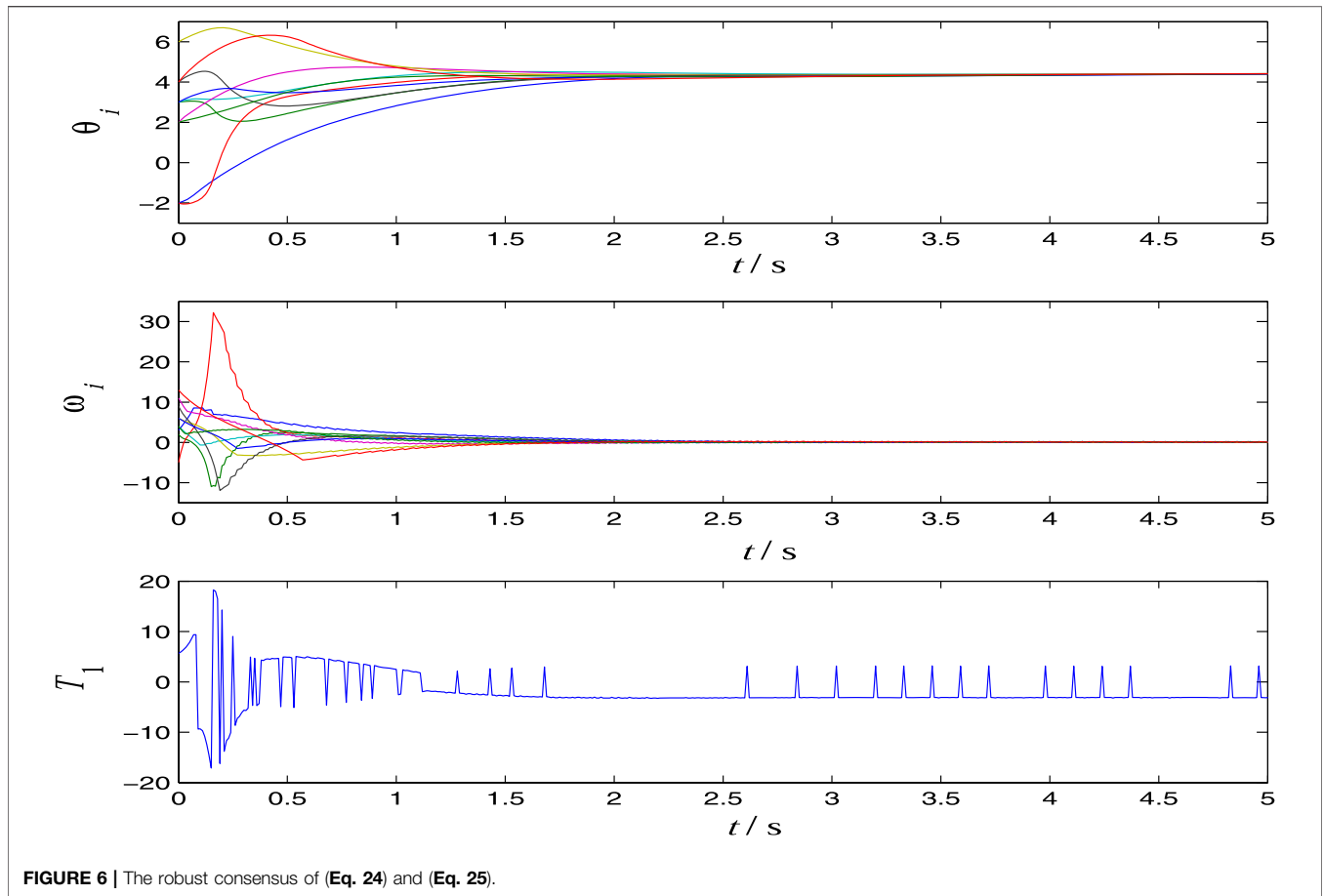
FIGURE 6 | The robust consensus of (**Eq. 24**) and (**Eq. 25**).

$$\mathcal{A} = \begin{bmatrix} 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 2 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{23}$$

The controller is designed as (6)–(8) with $\eta = \delta = 0.3$, $k = 1$, $b_0 = 1$ and $r = \delta/b_0 = 0.3$. In order to show the robustness of the designed controller, we choose three groups of parameters and disturbance signals in the simulations. In **Figure 2**, the time response curves and the first control component are displayed with

$$x(0) = [2, 3, 2, 3, 2, 6, 4, 3, 2, 4], \quad v(0) = [3, 2, 5, 4, 1, 6, 9, 6, 4, 3],$$
$$a_1 = [0.1, 0.2, -0.3, 0.05, 0.2, -0.1, 0.05, -0.3, 0.1, 0],$$
$$a_2 = [0.2, -0.1, 0.05, 0.2, -0.3, 0.05, -0.2, 0.1, 0.1, -0.1],$$
$$b = [1.1, 1.2, 0.9, 0.8, 1.1, 1.2, 1.2, 0.9, 0.8, 1.1],$$
$$w = [0.2, -0.1, 0.05, 0.2, -0.1, 0.05, -0.2, 0.1, 0.1, -0.1].$$

In the simulations shown in **Figure 3** and **Figure 4**, we set

$$x(0) = [-2, 3, -2, 3, 1, -6, 4, 5, 6, -7], \quad v(0) = [4, -2, 6, 4, 9, 6, -9, 6, -4, 3],$$
$$a_1 = [-0.1, 0.3, 0.3, -0.05, 0.2, 0.1, 0.15, -0.2, 0.3, -0.2],$$
$$a_2 = [0.3, 0.1, 0.15, -0.2, -0.3, 0.3, -0.2, 0.1, -0.1, 0.1],$$
$$b = [1.2, 1.3, 0.9, 0.7, 1.1, 1.1, 0.7, 1.2, 0.7, 1.2],$$
$$w(t) = [-0.2 \sin(t), 0.1 \cos(t), 0.15 \sin(2t), 0.2, 0.1, -0.25, -0.2, 0.2, 0.1, -0.2 \sin(t)].$$

and

$$x(0) = [2, -3, 2, 3, 1, -1, 4, 3, -4, -2], \quad v(0) = [4, 2, -3, 4, 3, 4, -3, -3, -2, 4],$$
$$a_1 = [-0.2, 0.1, 0.25, -0.15, 0.1, 0.3, -0.15, -0.3, -0.3, 0.12],$$
$$a_2 = [0.2, 0.15, -0.15, -0.25, 0.13, 0.23, 0.2, 0.3, -0.2, 0.15],$$
$$b = [1.3, 1.1, 0.8, 1.3, 0.8, 1.2, 0.8, 1.3, 0.7, 0.9],$$
$$w(t) = [0.3 \sin(t), -0.1, -0.2, 0.3 \cos(t), 0.2, -0.2, -0.3, -0.2, 0.3 \sin(2t), 0.3],$$

respectively. From **Figure 2**, **Figure 3** and **Figure 4**, we see that the controller is robust for the parameters and disturbances. From the time response curves of $u_1(t)$, we see that the typical characteristic chattering phenomenon exists in the sliding mode control.

Example 2. Consider a practical multi-agent system: a dynamical network composed of pendulums:

$$\dot{\theta}_i = \omega_i, \tag{24}$$

$$\dot{\omega}_i = -\frac{g_0}{l_i} \sin(\theta_i) - \frac{k_0}{m_i} \omega_i + \frac{1}{m l_i^2} T_i, \tag{25}$$

where θ_i, ω_i, T_i, m_i, and l_i are angular displacement, angular velocity, external torque, mass, and length of the ith pendulum, respectively. The constants k_0 and g_0 are coefficient of friction and acceleration due to the gravity, respectively. The known parameter information include $0.05 < m_i < 0.2$, $0.9 < l_i < 1.1$, $0.01 < k_0 < 0.05$, and $9.8 < g_0 < 9.9$. By Theorem 2, we need to determine $L(\theta_i, \omega_i)$ satisfying Assumption 2. It is easy to check that

$$\left| \frac{-\frac{g_0}{l_i}\sin(\theta_i) - \frac{k_0}{m_i}\omega_i}{\frac{1}{ml_i^2}} \right| \leq m_i l_i g_0 |\sin(\theta_i)| + k_0 l^2 |\omega_i| \leq 2.178|\sin(\theta_i)|$$
$$+ 0.0605|\omega_i|.$$

$$(26)$$

By **Eq. 26**, we let $L_i(\theta_i, \omega_i) = 2.178|\sin(\theta_i)| + 0.0605|\omega_i|$. Since $\frac{1}{m_i l_i^2} \geq 4.1322$, one can let $b_0 = 4$. So the controller can be determined by **Eq. 17**-**Eq. 19**. The simulations are performed by using $k = \beta_0 = 1$ and the digraph shown in **Figure 1**. **Figure 5** and **Figure 6** show the robust consensus of the closed-loop system with two group of parameter.

5 CONCLUSION

In this paper, we have investigated the robust consensus problem for the heterogeneous uncertain second order multi-agent systems based on sliding mode control. Novel robust consensus protocols have been proposed based on sliding mode control to realize the consensus. Numerical simulations have shown the effectiveness of the proposed protocols. The main disadvantage lies in the chattering phenomenon of the designed controller. How to reduce the chattering phenomenon will be discussed in our future research work.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/**Supplementary Material**, further inquiries can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

SUPPLEMENTARY MATERIAL

The Supplementary Material for this article can be found online at: https://www.frontiersin.org/articles/10.3389/fcteg.2021.744027/full#supplementary-material

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