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ROBUST CONTROL FOR A CLASS OF UNCERTAIN STATE-DELAYED SINGULARLY PERTURBED SYSTEMS

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Abstract: This paper considers the problem of robust control for a class of uncertain state-delayed singularly perturbed systems with norm-bounded nonlinear uncertainties. The system under consideration involves state time delay and norm-bounded nonlinear uncertainties in the slow state variable. It is shown that the state feedback gain matrices can be determined to guarantee the stability of the closed-loop system for all $\varepsilon \in (0, \infty)$ and independently of the time delay. Based on this key result and some standard Riccati inequality approaches for robust control of singularly perturbed systems, a constructive design procedure is developed. We present an illustrative example to demonstrate the applicability of the proposed design approach.

Keywords: Robust stability; Disturbance attenuation; singularly perturbed systems; Time delay

1. INTRODUCTION

Singularly perturbed systems often occur naturally because of the presence of small parasitic parameters multiplying the time derivatives of some of the system states. Singularly perturbed control systems have been intensively studied for the past three decades; see, (for example, Kokotovic, *et al*., 1986). A popular approach adopted to handle these systems is based on the so-called reduced technique (OMalley, 1974). The composite design based on separate designs for slow and fast subsystems has been systematically reviewed by Saksena, *et al*. (1984). Recently, the robust stabilization of singularly perturbed systems based on a new modeling approach has been investigated by Karimi and Yazdanpanah (2000).

The stability problem (ε-bound problem) in singularly perturbed systems differs from conventional linear systems, which can be designed as: characterizing an upper bound ε_0 of the positive perturbing scalar ε such that the stability of a reduced-order system would guarantee the stability of the original full-order system for all $\varepsilon \in (0, \varepsilon_0)$ (Chen and Lin, 1990). It is known, by the lemma of Klimushchev and Krasovskii (OMalley, 1974; Kokotovic, *et al*., 1986), that if the reduced-order system is an asymptotically stable, then this upper bound

ε_0 always exists. Researchers have tried various ways to find either the stability bound ε_0 or a less conservative lower bound for ε_0, see (Chen and Lin, 1990; Kokotovic, *et al.*, 1986; Tsai, *et al.*, 1991). Also, Shao and Rowland (1994) considered a linear time-invariant singularly perturbed system with single time delay in the slow states. Then, the research on time-scale modeling was extended to include singularly perturbed systems with multiple time delays in both slow and fast states (Pan, *et al.*, 1996). Recently, the problem of robust stabilization and disturbance attenuation for a class of uncertain singularly perturbed systems with norm-bounded nonlinear uncertainties has been considered by Karimi and Yazdanpanah (2001b). Also, the robust stability analysis and stability bound improvement of perturbed parameter (ε) in the singularly perturbed systems by using linear fractional transformations and structured singular values approach (μ) has been investigated by Karimi and Yazdanpanah (2001a).

Continuing the work of Karimi and Yazdanpanah (2001b), this paper presents new results on control synthesis for robust stabilization and robust disturbance attenuation for linear state-delayed singularly perturbed systems with norm-bounded nonlinear uncertainties. The class of plants considered in this paper consists of systems in state-space form with linear nominal parts and norm-bounded nonlinear uncertainties only in the slow state variable. Robust stabilization and disturbance attenuation of such systems is investigated using the Hamiltonian approach. The state feedback gain matrices can be constructed from the positive definite solutions to a certain Riccati inequality. Another advantage to this approach is that we can preserve the characteristic of the composite controller, i.e., the whole-dimensional process can be separated into two subsystems (Chiou, *et al.*, 1999; Cheng, *et al.*, 1992). Moreover, the presented stabilization design insures the stability for all $\varepsilon \in (0,\infty)$ and independently of the time delay.

2. PROBLEM FORMULATION

Consider a linear time-invariant state-delayed singularly perturbed system with norm-bounded nonlinear uncertainties in the form:

$$\begin{bmatrix} \dot{x}_1(t) \\ \varepsilon\,\dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} x_1(t-h) + \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} w(t) + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u(t) + \begin{bmatrix} \Delta_1(x_1(t)) \\ \Delta_2(x_1(t)) \end{bmatrix} \tag{1}$$

$$x_1(t) = \varphi(t) \quad t \in [-h, 0] \tag{2}$$

$$z(t) = c_1\, x_1(t) + c_2\, x_2(t) + D\,u(t) \tag{3}$$

where $x_1 \in R^{n1}$, $x_2 \in R^{n2}$, $n(=n_1+n_2)$ is the order of the whole system, $u \in R^m$, $w \in R^k$, $z \in R^l$ are control vector, disturbance and controlled output, respectively, $\Delta_i(x_1)$ $(i=1,2)$ are nonlinear terms of the uncertainty space. The certain matrices $a_{11} \in R^{n1\times n1}$, $a_{12} \in R^{n1\times n2}$, $a_{21} \in R^{n2\times n1}$, $a_{22} \in R^{n2\times n2}$, $b_1 \in R^{n1\times m}$, $b_2 \in R^{n2\times m}$, $d_1 \in R^{n1\times k}$, $d_2 \in R^{n2\times k}$, $r_1 \in R^{n1\times n1}$ and $r_2 \in R^{n2\times n1}$ are constant and $\varepsilon \geq 0$ is scalar and real. For a vector ν, v^T is its transpose, and $\|v\|$ is its Euclidean norm and L^2 is the Lebesque space of square integrable functions.
We shall make the following assumption for system (1).

Assumption 1. There exist the known real constant matrixes G_1, G_2 such that the known nonlinear uncertainties $\Delta_i(x_1(t))$ $(i=1,2)$ satisfy the following bounded condition,

$$\|\Delta_i(x_1(t))\| \leq \|G_i\, x_1(t)\| \quad \forall x_1(t) \in R^{n1} \tag{4}$$

Denote the corresponding uncertainty set by

$$\Xi_i(x_1) = \{\Delta_i(x_1(t)) : \|\Delta_i(x_1(t))\| \leq \|G_i\, x_1(t)\|\} \quad (i=1,2) \tag{5}$$

Definition 1.
1) A state feedback $u = -k_1\, x_1 - k_2\, x_2$, $k_1 \in R^{m\times n1}$, $k_2 \in R^{m\times n2}$ is said to achieve robust global asymptotic stability if for $w=0$ and any $\Delta_i(x_1) \in \Xi_i(x_1)$ $(i=1,2)$ the closed-loop system

$$\begin{bmatrix} \dot{x}_1 \\ \varepsilon\,\dot{x}_2 \end{bmatrix} = \begin{bmatrix} a_{11}-b_1 k_1 & a_{12}-b_1 k_2 \\ a_{21}-b_2 k_1 & a_{22}-b_2 k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} x_1(t-h) + \begin{bmatrix} \Delta_1(x_1) \\ \Delta_2(x_1) \end{bmatrix} \tag{6}$$

is globally asymptotically stable in the Lyapunov sense for all $\varepsilon \in (0,\infty)$ and independently of the time delay (h).

2) A state feedback $u=-k_1 x_1 - k_2 x_2$ is said to achieve robust disturbance attenuation if under zero initial condition there exists $0 \le \gamma < \infty$ for which the performance bound is such that:

$$\|z(t)\| < \gamma \, \|w(t)\| \quad \forall w \in L^2, \Delta_i(x_1) \in \Xi_i(x_1) \quad for\ i=1,2 \tag{7}$$

The main objective of the paper is to design $k_1 \in R^{m\times n_1}$, $k_2 \in R^{m\times n_2}$ such that the state feedback $u=-k_1 x_1 - k_2 x_2$ achieves simultaneously robust global asymptotic stability and robust disturbance attenuation for all $\varepsilon \in (0,\infty)$ and independently of the time delay (h). The main approach employed here is the standard HJI method. Hence, we define a quadratic energy function in the form:

$$E(x_1,x_2) = x_1^T P_1 x_1 + \varepsilon\, x_2^T P_2 x_2 + \int_{t-h}^{t} x_1^{\,T}(\sigma) Q\, x_1(\sigma)\, d\sigma \tag{8}$$

where $P_1>0, P_2>0$ and $Q>0$ are to be determined. Define the Hamiltonian function

$$H[u,w,\Delta_1(x_1),\Delta_2(x_1)] = z^T z - \gamma^2 w^T w + \frac{dE}{dt} \tag{9}$$

where derivative of $E(t)$ is evaluated along the trajectory of the closed-loop system. It is well known that a sufficient condition for achieving robust disturbance attenuation is that the inequality

$$H[u,w,\Delta_1(x_1),\Delta_2(x_1)] < 0, \quad \forall w \in L^2, \Delta_i(x_1) \in \Xi_i(x_1),\ i=1,2 \tag{10}$$

results in an $E(x)$ which is strictly radially unbounded (Wang and Zhan, 1996), $E(x)$ may be regulated as a Lyapunov function for the closed-loop systems, and hence, robust stability is guaranteed for all $\varepsilon \in (0,\infty)$ and independently of the time delay (h).

In this paper we will establish conditions under which

$$Inf_u\ Sup_{\Delta_i \in \Xi_i}\ Sup_{w\in L^2} H[u,w,\Delta_1,\Delta_2] < 0 \quad for\ i=1,2 \quad such\,that\ \Delta_i := \Delta_i(x_1),\ \Xi_i := \Xi_i(x_1). \tag{11}$$

3. MAIN RESULTS

Before deriving the main results, some preliminary lemmas are reviewed.

Lemma 1(Zhou and Khargonekar, 1988)**.** For any matrices X and Y with appropriate dimensions and for any constant $\eta > 0$, we have:

$$X^T Y + Y^T X \le \eta X^T X + \frac{1}{\eta} Y^T Y. \tag{12}$$

Lemma 2. For an arbitrary positive scalar $\varepsilon_i > 0$ and a positive definite $P_i > 0$, we have:

$$\Delta_i^T(x_i(t))\, P_i\, x_i(t) + x_i^T(t)\, P_i\, \Delta_i(x_i(t)) \le x_i^T(t)\, (\varepsilon_i P_i^2 + \frac{1}{\varepsilon_i} G_i^T G_i)\, x_i(t) \tag{13}$$

Proof of Lemma 2. By using assumption 1 and lemma1, we can conclude (13).

One of the key technical contributions of this paper is utilization of Lemma 2, which establishes a representation of the nonlinear uncertainty set by the certain terms. This observation leads to the following Theorem, which is the main result of this paper. The approach employed here is the standard method of Riccati inequalities, which have been used, extensively in linear control for state-space systems (Wang and Zhan, 1996).

Theorem. Let the matrix $D^T D$ be nonsingular. If there exist positive scalars $\varepsilon_1, \varepsilon_2$ and positive definite solutions $P_1>0, P_2>0$ and $Q>0$ to the Matrix inequality

$$\begin{bmatrix} \overline{R}_{11} + P_1 r_1 Q^{-1} r_1^T P_1 & \overline{R}_{12} + P_1 r_1 Q^{-1} r_2^T P_2 \\ \overline{R}_{12}^T + P_2 r_2 Q^{-1} r_1^T P_1 & \overline{R}_{22} + P_2 r_2 Q^{-1} r_2^T P_2 \end{bmatrix} < 0 \tag{14}$$

such that

$$\overline{R}_{11} = P_1 a_{11} + a_{11}^T P_1 + Q + c_1^T c_1 + \gamma^{-2} P_1 d_1 d_1^T P_1 + \varepsilon_1 P_1^2 + \frac{1}{\varepsilon_1} G_1^T G - (P_1 b_1 + c_1^T D)\,(D^T D)^{-1} (P_1 b_1 + c_1^T D)^T$$

$$\overline{R}_{22} = P_2 a_{22} + a_{22}^T P_2 + c_2^T c_2 + \gamma^{-2} P_2 d_2 d_2^T P_2 + \varepsilon_2 P_2^2 + \frac{1}{\varepsilon_2} G_2^T G_2 - (P_2 b_2 + c_2^T D)\,(D^T D)^{-1} (P_2 b_2 + c_2^T D)^T$$

$$\overline{R}_{12} = P_1 a_{12} + a_{21}^T P_2 + c_1^T c_2 + \gamma^{-2} P_1 d_1 d_2^T P_2 - (P_1 b_1 + c_1^T D)(D^T D)^{-1} (P_2 b_2 + c_2^T D)^T.$$

then, the control law

$$u(t) = -(D^T D)^{-1}\, ((b_1^T P_1 + D^T c_1) x_1 + (b_2^T P_2 + D^T c_2) x_2) \tag{15}$$

achieves robust global asymptotic stability and robust disturbance attenuation in the sense of (6) and (7), respectively and independently of the time delay (h).

Proof. We will prove the Theorem by showing that the control law (15) will guarantee the inequality of (10).

Noting to the expression (8) and according to (9), we have:

$$\begin{aligned} H(u,w,\Delta_1,\Delta_2) &= x_1^T(a_{11}^T P_1 + P_1 a_{11} + c_1^T c_1 + Q)x_1 \\ &+ x_2^T(a_{22}^T P_2 + P_2 a_{22} + c_2^T c_2)x_2 \\ &+ x_1^T(P_1 a_{12} + a_{21}^T P_2 + c_1^T c_2)x_2 \\ &+ x_2^T(a_{12}^T P_1 + P_2 a_{21} + c_2^T c_1)x_1 \\ &+ u^T(b_1^T P_1 + D^T c_1)x_1 \\ &+ u^T(b_2^T P_2 + D^T c_2)x_2 \\ &+ x_1^T(c_1^T D + P_1 b_1)u \\ &+ x_2^T(c_2^T D + P_2 b_2)u + u^T D^T D u \\ &- \gamma^2 w^T w + w^T(d_1^T P_1 x_1 + d_2^T P_2 x_2) \\ &+ (x_1^T P_1 d_1 + x_2^T P_2 d_2)w - x_h^T Q x_h \\ &+ x_h^T(r_2^T P_2 x_2 + r_1^T P_1 x_1) \\ &+ (r_2^T P_2 x_2 + r_1^T P_1 x_1)^T x_h + \Delta_1^T P_1 x_1 \\ &+ x_1^T P_1 \Delta_1 + \Delta_2^T P_2 x_2 + x_2^T P_2 \Delta_2 \end{aligned} \tag{16}$$

such that $x_h \triangleq x_1(t-h)$.

It is easy to show that the worst case disturbance occurs when

$$w^* = \gamma^{-2}(d_1^T P_1 x_1 + d_2^T P_2 x_2). \tag{17}$$

It follows that

$$\begin{aligned} H_1(u,\Delta_1,\Delta_2) &= \underset{w\in L^2}{Sup}\, H(u,w,\Delta_1,\Delta_2) \\ &= x_1^T R_{11} x_1 + x_2^T R_{22} x_2 + x_1^T R_{12} x_2 + x_2^T R_{12}^T x_1 \\ &+ u^T G_1^T(x_1,x_2) + G_1(x_1,x_2)u + u^T D^T D u \\ &- x_h^T Q x_h + x_h^T G_2(x_1,x_2) + G_2^T(x_1,x_2)x_h \\ &+ x_1^T P_1 \Delta_1 + \Delta_1^T P_1 x_1 + x_2^T P_2 \Delta_2 + \Delta_2^T P_2 x_2 \end{aligned} \tag{18}$$

where

$$R_{11} = a_{11}^T P_1 + P_1 a_{11} + c_1^T c_1 + \gamma^{-2} P_1 d_1 d_1^T P_1 + Q$$

$$R_{22} = a_{22}^T P_2 + P_2 a_{22} + c_2^T c_2 + \gamma^{-2} P_2 d_2 d_2^T P_2$$

$$R_{12} = a_{21}^T P_2 + P_1 a_{12} + c_1^T c_2 + \gamma^{-2} P_1 d_1 d_2^T P_2$$

$$G_1(x_1,x_2) = x_1^T(P_1 b_1 + c_1^T D) + x_2^T(P_2 b_2 + c_2^T D)$$

$$G_2(x_1,x_2) = r_1^T P_1 x_1 + r_2^T P_2 x_2.$$

According to Lemma 2, we have

$$\begin{aligned} \underset{\Delta_i\in\Xi_i}{Sup}\, H_1(u,\Delta_1,\Delta_2) &\le x_1^T(R_{11} + \varepsilon_1 P_1^2 + \frac{1}{\varepsilon_1} G_1^T G_1)x_1 \\ &+ x_2^T(R_{22} + \varepsilon_2 P_2^2 + \frac{1}{\varepsilon_2} G_2^T G_2)x_2 \\ &+ x_1^T R_{12} x_2 + x_2^T R_{12}^T x_1 + u^T G_1^T(x_1,x_2) \\ &+ G_1(x_1,x_2)u + u^T D^T D u - x_h^T Q x_h \\ &+ x_h^T G_2(x_1,x_2) + G_2^T(x_1,x_2)x_h \end{aligned} \tag{19}$$

The optimal control law, which minimizes the right-hand side of (19), is given by

$$u(t) = -(D^T D)^{-1}((b_1^T P_1 + D^T c_1)x_1 + (b_2^T P_2 + D^T c_2)x_2). \tag{20}$$

As a result, we have:

$$\underset{u}{Inf}\ \underset{\Delta_i\in\Xi_i}{Sup}\, H_1(u,\Delta_1,\Delta_2) \le F(x_1,x_2,x_h) \tag{21}$$

where

$$\begin{aligned} F(x_1,x_2,x_h) &\triangleq \begin{bmatrix} x_1 \\ x_2 \\ x_h \end{bmatrix}^T M \begin{bmatrix} x_1 \\ x_2 \\ x_h \end{bmatrix} \\ &= x_1^T \overline{R}_{11} x_1 + x_2^T \overline{R}_{22} x_2 + x_1^T \overline{R}_{12} x_2 + x_2^T \overline{R}_{12}^T x_1 \\ &- x_h^T Q x_h + x_h^T G_2(x_1,x_2) + G_2^T(x_1,x_2)x_h \end{aligned} \tag{22}$$

and

$$\begin{aligned} \overline{R}_{11} &= P_1 a_{11} + a_{11}^T P_1 + Q + c_1^T c_1 + \gamma^{-2} P_1 d_1 d_1^T P_1 + \varepsilon_1 P_1^2 \\ &+ \frac{1}{\varepsilon_1} G_1^T G - (P_1 b_1 + c_1^T D)(D^T D)^{-1}(P_1 b_1 + c_1^T D)^T \end{aligned}$$

$$\begin{aligned} \overline{R}_{22} &= P_2 a_{22} + a_{22}^T P_2 + c_2^T c_2 + \gamma^{-2} P_2 d_2 d_2^T P_2 \\ &+ \varepsilon_2 P_2^2 + \frac{1}{\varepsilon_2} G_2^T G_2 - (P_2 b_2 + c_2^T D)(D^T D)^{-1}(P_2 b_2 + c_2^T D)^T \end{aligned}$$

$$\begin{aligned} \overline{R}_{12} &= P_1 a_{12} + a_{21}^T P_2 + c_1^T c_2 + \gamma^{-2} P_1 d_1 d_2^T P_2 \\ &- (P_1 b_1 + c_1^T D)(D^T D)^{-1}(P_2 b_2 + c_2^T D)^T \end{aligned}$$

$$M = \begin{bmatrix} \overline{R}_{11} & \overline{R}_{12} & P_1 r_1 \\ \overline{R}_{12}^T & \overline{R}_{22} & P_2 r_2 \\ r_1^T P_1 & r_2^T P_2 & -Q \end{bmatrix}. \tag{23}$$

Consequently, if there exist positive definite solutions $P_1 > 0, P_2 > 0$ and $Q > 0$ to the Matrix inequality

$$M < 0$$

then we have

$$\begin{aligned} &H[u,w,\Delta_1(x_1(t)),\Delta_2(x_1(t))] < 0, \\ &\quad \forall\ w \in L^2,\ \Delta_i(x_1(t)) \in \Xi_i(x_1(t)),\ i = 1,2 \end{aligned} \tag{24}$$

Furthermore, by noting that the matrix Q is positive definite then the matrix M is negative definite if the following inequality holds:

$$\begin{bmatrix} \overline{R}_{11} & \overline{R}_{12} \\ \overline{R}_{12}^T & \overline{R}_{22} \end{bmatrix} + \begin{bmatrix} P_1 r_1 \\ P_2 r_2 \end{bmatrix} Q^{-1} \begin{bmatrix} r_1^T P_1 & r_2^T P_2 \end{bmatrix} < 0$$

or

$$\begin{bmatrix} \overline{R}_{11} + P_1 r_1 Q^{-1} r_1^T P_1 & \overline{R}_{12} + P_1 r_1 Q^{-1} r_2^T P_2 \\ \overline{R}_{12}^T + P_2 r_2 Q^{-1} r_1^T P_1 & \overline{R}_{22} + P_2 r_2 Q^{-1} r_2^T P_2 \end{bmatrix} < 0 \quad (25)$$

Hence, (25) completes the proof.

4. EXAMPLE

Consider a fourth-order singularly perturbed system with time delay in the slow state variable:

$$\begin{bmatrix} \dot{x}_{s1}(t) \\ \dot{x}_{s2}(t) \\ \varepsilon \dot{x}_{f1}(t) \\ \varepsilon \dot{x}_{f2}(t) \end{bmatrix} = \begin{bmatrix} -9 & 0 & 0 & 0.1 \\ 0.1 & -8 & 0.05 & 0.1 \\ 0 & 0 & -15 & 0 \\ 0.01 & 0.003 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_{s1}(t) \\ x_{s2}(t) \\ x_{f1}(t) \\ x_{f2}(t) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} x_1(t-h) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0.5 \\ 0.5 & 0 \end{bmatrix} u(t) + \begin{bmatrix} 0.1 \\ 0.2 \\ 0.2 \\ 0.5 \end{bmatrix} w(t) + \begin{bmatrix} \Delta_1(x_1(t)) \\ \Delta_2(x_1(t)) \end{bmatrix} \quad (26)$$

$$x_1(t) = \begin{bmatrix} 0.5 & -0.5 \end{bmatrix}^T \quad \forall t \in [-h, 0]$$

$$z(t) = \begin{bmatrix} 0.4 & 0.15 \\ 0.1 & 0.6 \end{bmatrix} \begin{bmatrix} x_{s1}(t) \\ x_{s2}(t) \end{bmatrix} + \begin{bmatrix} 0.2 & -0.1 \\ -0.2 & 0.9 \end{bmatrix} \begin{bmatrix} x_{f1}(t) \\ x_{f2}(t) \end{bmatrix} + \begin{bmatrix} 0.25 & 0.5 \\ 0.5 & 0.1 \end{bmatrix} u(t)$$

where $x_1 = [x_{s1} \quad x_{s2}]^T$, $x_2 = [x_{f1} \quad x_{f2}]^T$ and the uncertainty terms $\Delta_i(x_1)$ $(i = 1, 2)$, are assumed to be norm-bounded such that the matrixes G_1, G_2 have been considered as follows:

$$G_1 = G_2 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.3 \end{bmatrix}.$$

Consider also $\gamma = 0.1$ as the performance bound, $\varepsilon = 0.1$ as the perturbed parameter and $h = 2$ second as the time delay parameter. From (14), we can choose the positive definite solutions $P_1 > 0$, $P_2 > 0$ and $Q > 0$ as follows:

$$P_1 = \begin{bmatrix} 2.1137 & -0.7639 \\ -0.7639 & 0.6814 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 2.0760 & -0.1406 \\ -0.1406 & 0.1390 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1.5 & 0 \\ -1500 & 0.015 \end{bmatrix}.$$

Also, positive numbers of $\varepsilon_1, \varepsilon_2$ are obtained as follows:

$$\varepsilon_1 = 1.8, \quad \varepsilon_2 = 1$$

The required state feedback control law is given by

$$u = -k_1 x_1 - k_2 x_2$$

$$k_1 = \begin{bmatrix} -13.5405 & 5.0117 \\ 11.2441 & -6.5131 \end{bmatrix}, k_2 = \begin{bmatrix} -6.1792 & -1.9220 \\ -0.1409 & 1.4103 \end{bmatrix}$$

Robust stability and disturbance attenuation of the slow and fast dynamics in the presence of disturbance (Gussian noise) have been depicted in Figures 1 and 2. Therefore, we conclude that system (26) can be stabilized by the control law (15) for all $\varepsilon \in (0, \infty)$ and independently of the time delay (h), which has been depicted in Figure 3 and the correctness of the attenuation level of the disturbance on the controlled output has been depicted in Figure 4.

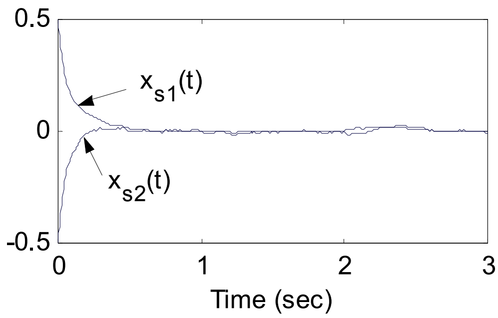

Fig. 1. Robust stability and disturbance attenuation of slow dynamics

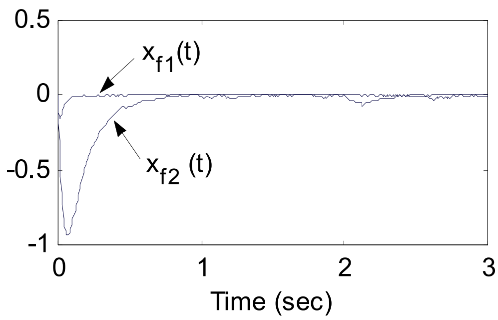

Fig. 2. Robust stability and disturbance attenuation of fast dynamics

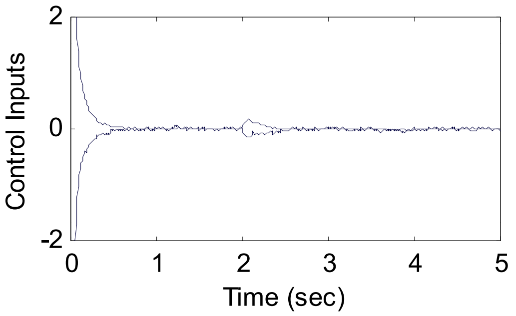

Fig. 3. Control law by means of state feedback

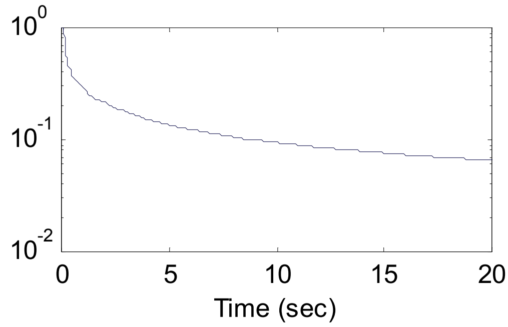

Fig. 4. Attenuation level of the disturbance on the controlled output

5. CONCLUSION

For a class of continuous uncertain state-delayed singularly perturbed system, this paper has presented a robust control design methodology to achieve the robust stabilization and disturbance attenuation for all $\varepsilon \in (0, \infty)$ and independently of time delay. Then, this paper has three major contributions: One is that the type of norm-bounded nonlinear uncertainties considered in this class of systems coincides with the certain terms by utilization of Lemma 2. The other is that the state feedback gain matrices can be determined independently from one certain Riccati inequality, and the last is that the closed-loop system is stable for all $\varepsilon \in (0, \infty)$ and independently of time delay. In this paper, the results are presented on the two-time-scale case, and the extension of results to multiple-time-scale is a topic currently under study.

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