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Robust Control for Seeker Scan Loop using Sliding Modes

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Abstract: This paper presents robust control of seeker system used in missile target scanning and tracking. The mathematical model for seeker system is considered. A sliding mode controller (SMC) is designed to compensate uncertain coupling effect and external disturbances during scan phase. The model of the plant is based on first-principle method. The dynamic equations of motion are formulated with the assumption that the seeker system is a rigid body with no mass imbalance. These equations of motion are considered in cascade with DC motor equations which provides the actuating torque to the gimbal axes of the seeker system. The performance of the controller is validated in simulations.

Keywords: Sliding Mode Control, seeker system.

1. INTRODUCTION

Seeker plays very important role in a guided missile system. In general, a missile can be defined as an aerospace vehicle with varying guidance capabilities. These vehicles are fabricated for air-to-air, surface-to-air, or surface-to-surface roles.
Specifically, a guided missile is typically divided into four subsystems: (1) the airframe, (2) guidance, (3) motor (or propulsion), and (4) warhead.

Guidance is the means by which a missile is steered to a target. The main part of guidance system is a seeker.

Control of a Seeker system is a challenging problem. This is due to coupled nonlinear dynamics of seeker system. Although classical and PID controllers are easy to design, their performance is not robust on the face of uncertainties. Several control methods have been reported in literature see for example [1]- [8] and the references therein. The Robust controllers such as H_∞ have been reported [1]-[4]. However major drawbacks associated with it are tuning of the weighing martices is time consuming and the controller order becomes large.

A Feedback linearization method have been discussed in [5]. An extended Kalman filter (EKF) has been formulated for the estimation of line-of-sight(LOS) rate from measurements of relative angular displacement between seeker gimbals. Feedback linearizing control is an approach to the design of nonlinear controllers which has attracted a great deal of research interest in recent years. However, actual applications resulting in the implementation of such control algorithms have been few. Usually, feedback linearizing control does not guarantee robustness in the presence of model uncertainty and disturbances. A suboptimal method(θ-D) has been proposed in [6] to design an integrated guidance and control system for missiles. However the issue is robustness.

In [7], Adaptive controller has been used for stabilizing the seeker. However the adaptive controller has a limitation over fast varying disturbance. SMC is known to be robust [9] [10] [11]. Disturbance rejection capability, simplicity and order reduction are the accomplishments of this theory.

1.1 Motivation

Precise and accurate performance of the seeker is important for the successful operation of the missile. The main operation includes the control of two gimbals driven by DC servo motors. Each of the gimbals exhibits disturbance to the other due to cross coupling. SMC has the capability to reject disturbance and yield robust performance.

A sliding mode controller has been used in [8]. The plant has been considered to have decoupled yaw and pitch dynamics. A Lyapunov approach is used to design control.

Herein the torque due to cross coupling is computed analytically and its effect is compensated by updating the control. The effect of further variation in disturbance torque due to parametric variation and external disturbance is taken care of by SMC. Reaching law is used to synthesize SMC.

1.2 Structure of Paper

The brief outline of the paper is as follows:
In the next Section mathematical model is described. In Section 3 controller design is presented. Section 4 presents simulation results. Section 5 concludes the work.

2. MATHEMATICAL DESCRIPTION

Most of the seekers of the gimballing variety have two gimbal axes, namely, yaw (azimuth) and pitch (elevation). The gimbal axes are rotated through desired angles by a DC motor which provides the actuating torque. The basic structure is shown in Fig. 1,

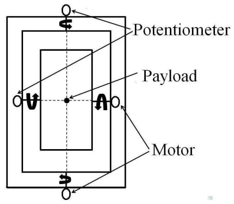


Fig. 1 Basic schematic of Seeker

To describe dynamic behavior of the complete system, mathematical model of actuator(DC motor) and 2 axis gimbal system is considered.

2.1 Modeling of DC Motor

The DC motor which provides the actuating torque can be schematically represented as shown in Fig. 2,

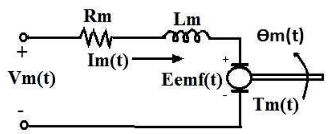


Fig. 2 Schematic of DC servo motor

The parameters are
V_m = DC input voltage in volts, I_m = DC current in Amps, R_m and L_m = Equivalent resistance (Ω) and inductance (henry) of motor, E_{emf} = Equivalent back emf in volts, T_m = Equivalent motor torque in Nm, T_l = Equivalent torque at load in Nm, θ_m = Motor angular displacement in degrees, θ_l = Load angular displacement in degrees, J_m = Motor Inertia in kgm^2, K_t = Motor torque constant in Nm, K_m = Back emf constant, K_g = Total gear ratio, η_m = Motor efficiency, η_g = Gearbox efficiency.
Using Kirchhoff's voltage law,

$$V_m - R_m I_m - L_m \frac{dI_m}{dt} - E_{emf} = 0. \tag{1}$$

Since, $L_m << R_m$, the motor inductance can be discarded, therefore,

$$I_m = \frac{V_m - E_{emf}}{R_m}. \tag{2}$$

Substituting $\theta_m = K_g \theta_l$ and $T_m = \eta_m K_t I_m$,

$$\frac{T_m}{\eta_m K_t} = \frac{V_m}{R_m} - \frac{K_m K_g \dot{\theta}_l}{Rm}. \tag{3}$$

Applying Newton's 2nd law of motion to the motor shaft,

$$J_m \ddot{\theta}_m = T_m - \frac{T_l}{\eta_g K_g}. \tag{4}$$

Substituting (3) in (4),

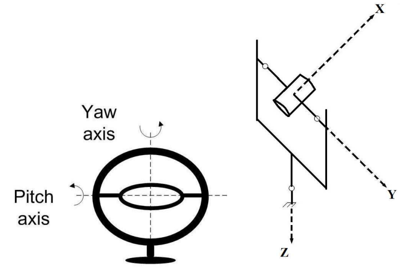
$$T_l = \frac{\eta_g \eta_m K_t K_g V_m}{R_m} - \frac{\eta_g \eta_m K_t K_m {K_g}^2 \dot{\theta}_l}{Rm} - \eta_g J_m K_g^2 \ddot{\theta}_l. \tag{5}$$

This is output torque provided by the motor.
Substituting the values of parameters and constants in (5), DC motor dynamics becomes

$$T_l = 0.128 V_m - 0.068 \dot{\theta}_l - 1.014 \times 10^{-2} \ddot{\theta}_l. \tag{6}$$

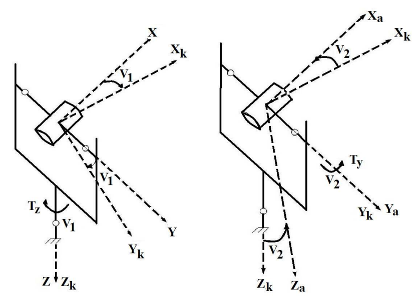
2.2 Modeling of 2 Axis Gimbal System

Yaw and pitch motion of the payload is obtained by using two axis gimbal. The two axes; yaw pitch gimbal system is shown in Fig. 3(a) and the fixed body frame is depicted in Fig. 3(b).



(a)2 axis gimbal system (b)Body fixed frame B

Fig. 3 Schematic of Gimbal System



(a)Yaw frame (b)Pitch frame

Fig. 4 Rotation to Yaw frame by angle v_1 and then to Pitch frame by angle v_2

Three reference frames have been introduced. Fuselage body-fixed frame B with coordinate axes x, y and z. Frame K fixed to the yaw gimbal with coordinate axes x_k , y_k and z_k. Frame A fixed to the pitch gimbal with coordinate axes x_a , y_a and z_a.
The yaw gimbal rotation is about the z axis. x_k , y_k and z_k are the yaw gimbal frame axes and the angle of rotation is v_1 degrees. Thus system rotates from body fixed frame B to yaw frame K. Refer Fig. 4(a). The pitch gimbal rotation is about the y axis. x_a , y_a and z_a are the pitch gimbal frame axes and the angle of rotation is v_2 degrees. Thus system rotates from yaw frame K to pitch frame A. Refer Fig. 4(b). The roll, pitch and yaw components of angular velocity of frames B, K and A are as follows, $\begin{bmatrix} \mathrm{p} & \mathrm{q} & \mathrm{r} \end{bmatrix}^T$; $\begin{bmatrix} \mathrm{p_k} & \mathrm{q_k} & \mathrm{r_k} \end{bmatrix}^T$; $\begin{bmatrix} \mathrm{p}_a & \mathrm{q}_a & \mathrm{r}_a \end{bmatrix}^T$. Here p, q and r represent the roll, pitch and yaw components respectively.
Quaternion method is used to obtain the instantaneous values p, q and r during the missile flight. Quaternion describes angular orientation with four parameters. Three define the axis of rotation and the fourth specifies the amount of rotation. Quaternion algebra [12] formulated by Sir W R Hamilton is defined as a hypercomplex number $[Q] = q_0 + q_1 i + q_2 j + q_3 k = (q_0, q_1, q_2, q_3)$
where q_0, q_1, q_2, q_3 are real numbers. Quaternion computation has the advantage that it does not require intensive trigonometric evaluations as with other methods. Also

it can be written in terms of direction cosines and Euler angles. Quaternion representation has associated linear differential equations that can be integrated easily, given the initial conditions and the forcing function.

2.2.1 Pitch Channel Dynamics

The modeling of pitch or inner gimbal is done initially in this Section. The inertia matrix of pitch gimbal is considered for roll, pitch and yaw rotations as,

$$J_A = \begin{bmatrix} J_{ax} & D_{xy} & D_{xz} \\ D_{xy} & J_{ay} & D_{yz} \\ D_{xz} & D_{yz} & J_{az} \end{bmatrix}. \tag{7}$$

The moments of inertia are denoted by J and products of inertia by D. The angular momentum $\overline{H}$ of the pitch gimbal system can be given as,

$$\bar{H} = \begin{bmatrix} H_x & H_y & H_z \end{bmatrix}^T = J_A \begin{bmatrix} p_a & q_a & r_a \end{bmatrix}^T. \tag{8}$$

The net torque on a rotating body is given by Euler's equations of motion. These equations relate the net torque on a rigid rotating body to the moment of inertia tensor and angular momentum.

$$\bar{T} = \frac{d\bar{H}}{dt} + \bar{\omega} \times \bar{H}. \tag{9}$$

Where $\overline{\omega} = \begin{bmatrix} p_a & q_a & r_a \end{bmatrix}^T$. Using (7) and (8), (9) can be solved mathematically to give the total external torque T_y as,

$$\begin{aligned} T_y &= J_{ay}\dot{q}_a + (J_{ax} - J_{az})p_a r_a + D_{xz}({r_a}^2 - {p_a}^2) \\ &+ \mathrm{D}_{xy}(\dot{p}_a + q_a r_a) + \mathrm{D}_{yz}(\dot{r}_a - p_a q_a). \end{aligned} \tag{10}$$

T_y is the actuating torque provided by the DC motor with the assumption that there will not any transmission loss,

$$T_y = T_l. \tag{11}$$

Eqn (10) can be rewritten as, $T_y + T_D = J_{ay}\dot{q}_a$, where

$$\begin{aligned} T_D &= (J_{az} - J_{ax})p_a r_a + D_{xz}(p_a^2 - r_a^2) \\ &- \mathrm{D}_{yz}(\dot{r}_a - p_a q_a) - D_{xy}(\dot{p}_a + q_a r_a). \end{aligned}$$

T_D includes cross coupling effect and nonlinearities.
If d_p is a smooth matched bounded lumped external disturbance in the pitch channel,the dynamics becomes,

$$T_y + T_D + d_p = J_{ay}\dot{q}_a. \tag{12}$$

Since, $\dot{q_a} = \dot{q_k} + \ddot{v_2}$, (12) becomes,

$$\ddot{v}_2 = \frac{1}{J_{ay}}[T_y + T_D + d_p - J_{ay}\dot{q}_k]. \tag{13}$$

Using (6), (11) and (13)

$$\begin{aligned} \ddot{v}_2 &= \frac{1}{J_{ay}}[0.128V_m - 0.068\dot{v}_2 - 1.014 \times 10^{-2}\ddot{v}_2 \\ &+ T_D + d_p - J_{ay}\dot{q}_k]. \end{aligned} \tag{14}$$

The inertia matrix parameters of the pitch frame described in [5] are used to get,

$$\begin{aligned} \ddot{v}_2 &= 12.26V_m - 6.51\dot{v}_2 + [95.81T_D + 95.81d_p \\ &- 0.029\dot{q}_k] \\ \ddot{v}_2 &= 12.26V_m - 6.51\dot{v}_2 + \xi \end{aligned} \tag{15}$$

where $\xi = [95.81T_D + 95.81d_p - 0.029\dot{q}_k]$.
Define $x_1 = v_2 \; and \; x_2 = \dot{v}_2$ to rewrite (15),

$$\dot{\mathbf{x}} = A\mathbf{x} + \mathbf{b}u + \mathbf{e}\xi. \tag{16}$$

Where
$A = \begin{bmatrix} 0 & 1 \\ 0 & -6.51 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 0 \\ 12.26 \end{bmatrix}$ and $\mathbf{e} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.
Eqn (16) can be rewritten as,

$$\dot{\mathbf{x}} = A\mathbf{x} + \mathbf{b}u + \mathbf{e}\xi_p + \mathbf{e}d_p' \tag{17}$$

where ξ_p is known disturbance, $\xi_p = 95.81T_D - 0.029\dot{q}_k$ and d_p' is unknown disturbance with known bounds, $d_p' = 95.81d_p$.

2.2.2 Yaw Channel Dynamics

The inertia matrix of yaw gimbal can be given as,

$$J_K = \begin{bmatrix} J_{kx} & d_{xy} & d_{xz} \\ d_{xy} & J_{ky} & d_{yz} \\ d_{xz} & d_{yz} & J_{kz} \end{bmatrix}$$

The moments of inertia are denoted by J and products of inertia by d. The angular momentum $\overline{H}$ of the total gimbal system, as yaw is outer gimbal, is the sum of the angular momentum of the pitch and yaw gimbals.i.e.

$$\begin{aligned} \bar{H} = \begin{bmatrix} H_x & H_y & H_z \end{bmatrix}^T &= J_K \begin{bmatrix} p_k & q_k & r_k \end{bmatrix}^T \\ &+ L_{AK}^T J_A \begin{bmatrix} p_a & q_a & r_a \end{bmatrix}^T \end{aligned}$$

where L_{AK} is the direction cosine matrix for the transformation from yaw frame K to pitch frame A.
Then the torque equation for a rotating yaw frame will be obtained as,

$$\bar{T} = \frac{d\bar{H}}{dt} + \bar{\omega} \times \bar{H}. \tag{18}$$

Where $\overline{\omega} = \begin{bmatrix} p_k & q_k & r_k \end{bmatrix}^T$. Similar to the pitch channel dynamics, the total external torque T_z will be given as,

$$\begin{aligned} T_z &= [d_{xz}\dot{p_k} + d_{yz}\dot{q_k} + J_{kz}\dot{r_k} \\ &- \dot{p_a}\sin v_2 J_{ax} - p_a\cos v_2 J_{ax}\dot{v_2} + \dot{p_a}\cos v_2 D_{xz} \\ &- p_a \sin v_2 D_{xz}\dot{v_2} - \dot{q_a}\sin v_2 D_{xz} - q_a\cos v_2 D_{xz}\dot{v_2} \\ &+ \dot{q_a}\cos v_2 D_{yz} - q_a \sin v_2 D_{yz}\dot{v_2} - \dot{r_a}\sin v_2 D_{xz} \\ &- r_a\cos v_2 D_{xz}\dot{v_2} + \dot{r_a}\cos v_2 J_{az} - r_a\sin v_2 J_{az}\dot{v_2}] \\ &+ p_k[d_{xy}p_k + J_{ky}q_k + d_{yz}r_k \\ &+ (D_{xy}p_a + J_{ay}q_a + D_{yz}r_a)] \\ &- q_k[J_{kx}p_k + d_{xy}q_k + d_{xz}r_k \\ &+ \cos v_2(J_{ax}p_a + D_{xy}q_a + D_{xz}r_a) \\ &+ \sin v_2(D_{xz}p_a + D_{yz}q_a + J_{az}r_a)] \end{aligned} \tag{19}$$

T_z is the actuating torque provided by the DC motor.

$$T_z = T_l \tag{20}$$

(19) can be rewritten as below,

$$T_z = J_k\dot{r}_k - T_d \tag{21}$$

T_d includes cross coupling effect and nonlinearities.
Further T_d includes,

$$\begin{aligned}
T_d &= T_{d1} + T_{d2} + T_{d3} \\
T_{d1} &= [J_{kx} + J_{ax}\cos^2 v_2 + J_{az}\sin^2 v_2 + D_{xz}\sin(2v_2) \\
&- (J_{ky} + J_{ay})]p_k q_k \\
T_{d2} &= -[d_{xz} + (J_{az} - J_{ax})\sin v_2 \cos v_2 \\
&+ D_{xz}\cos(2v_2)] \times (\dot{p}_k - q_k r_k) \\
&- (d_{yz} + D_{yz}\cos v_2 - D_{xy}\sin v_2)(\dot{q}_k + p_k r_k) \\
&- (d_{xy} + D_{xy}\cos v_2 + D_{yz}\sin v_2)(p_k^2 - q_k^2) \\
T_{d3} &= \ddot{v}_2(D_{xy}\sin v_2 - D_{yz}\cos v_2) \\
&+ \dot{v}_2[(J_{ax} - J_{az})(p_k\cos(2v_2) - r_k\sin(2v_2)) \\
&+ 2D_{xz}(p_k\sin(2v_2) + r_k\cos(2v_2)) \\
&+ (D_{yz}\sin v_2 + D_{xy}\cos v_2)(q_a + q_k) - J_{ay}p_k] \\
J_k &= J_{kz} + J_{ax}\sin^2 v_2 + J_{az}\cos^2 v_2 \\
&- D_{xz}\sin(2v_2) \qquad (22)
\end{aligned}$$

If d_y is a smooth matched bounded lumped external disturbance in the yaw channel,the dynamics becomes,

$$J_k\dot{r}_k = T_z + T_d + d_y. \qquad (23)$$

From equation (22) and (23),

$$[J_{kz} + J_{ax}\sin^2 v_2 + J_{az}\cos^2 v_2 - D_{xz}\sin(2v_2)]\dot{r}_k = T_z + T_d + d_y.$$

Defining $\Delta J_k = J_{ax}\sin^2 v_2 + J_{az}\cos^2 v_2 - D_{xz}\sin(2v_2)]\dot{r}_k$,

$$\begin{aligned}
[J_{kz} + \Delta J_k]\dot{r}_k &= T_z + T_d + d_y \\
\dot{r}_k &= \frac{1}{J_{kz}}[T_z + T_d + d_y - \Delta J_k\dot{r}_k]. \qquad (24)
\end{aligned}$$

Since $\dot{r}_k = \dot{r} + \ddot{v}_1$, (24) becomes,

$$\ddot{v}_1 = \frac{1}{J_{kz}}[T_z + T_d + d_y - \Delta J_k\dot{r}_k - J_{kz}\dot{r}]. \qquad (25)$$

Using (6), (20) and (25),

$$\begin{aligned}
\ddot{v}_1 &= \frac{1}{J_{kz}}[0.128V_m - 0.068\dot{v}_1 - 1.014\times 10^{-2}\ddot{v}_1 \\
&+ T_d + d_y - \Delta J_k\dot{r}_k - J_{kz}\dot{r}]. \qquad (26)
\end{aligned}$$

Substituting for inertia matrix parameters from [5],

$$\begin{aligned}
\ddot{v}_1 &= 2000[0.128V_m - 0.068\dot{v}_1 - 1.014\times 10^{-2}\ddot{v}_1 \\
&+ T_d + d_y - \Delta J_k\dot{r}_k - J_{kz}\dot{r}] \\
\ddot{v}_1 &= 12.03V_m - 6.39\dot{v}_1 + [93.98T_d + 93.98d_y \\
&- 93.98\Delta J_k\dot{r}_k - 0.04699\dot{r}] \\
\ddot{v}_1 &= 12.03V_m - 6.39\dot{v}_1 + \xi', \qquad (27)
\end{aligned}$$

where $\xi' = [93.98T_d + 93.98d_y - 93.98\Delta J_k\dot{r}_k - 0.04699\dot{r}]$.
Define $x_1 = v_1 \; and \; x_2 = \dot{v}_1$ to rewrite (27),

$$\dot{\mathbf{x}} = A\mathbf{x} + \mathbf{b}u + \mathbf{e}\xi'. \qquad (28)$$

Where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -6.39 \end{bmatrix}, \; \mathbf{b} = \begin{bmatrix} 0 \\ 12.03 \end{bmatrix} \text{ and } \mathbf{e} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad (29)$$

Eqn. (28) describes yaw channel dynamics and can be rewritten as,

$$\dot{\mathbf{x}} = A\mathbf{x} + \mathbf{b}u + \mathbf{e}\xi'_y + \mathbf{e}d'_y \qquad (30)$$

where ξ'_y is known disturbance, $\xi'_y = 93.98T_d - 93.98\Delta J_k\dot{r}_k - 0.04699\dot{r}$ and d'_y is unknown disturbance, $d'_y = 93.98d_y$.

It may be noted that disturbance T_D in pitch channel and T_d in yaw channel due to cross coupling effect can be computed exactly for nominal parameters and hence can be compensated. However further parameter variations needs to be taken care of. SMC can reject this and the externel disturbance as well.

3. DESIGN OF CONTROLLER

The sliding mode controller design involves two stages, i.e. design of sliding surface and control law synthesis. The sliding surface is designed by pole placement technique. Gao's power rate reaching law [11] is used for control law synthesis.

3.1 Design of Sliding Surface

consider a sliding surface s as below.

$$s = \mathbf{c}^T(\mathbf{x} - \mathbf{x}_d) \qquad (31)$$

where, $\mathbf{x}$ is actual state vector, $\mathbf{x}_d$ is desired state vector $\mathbf{c}^T$ is sliding surface matrix, $\mathbf{c}^T \in \Re^{m\times n}$ where m is number of inputs and n is number of states. Now $\mathbf{c}^T$ can be designed by converting the nominal system $\dot{\mathbf{x}} = A\mathbf{x} + \mathbf{B}u$ into regular form using $\mathbf{Z} \leftrightarrow T_r\mathbf{X}$ such that,

$$T_r\mathbf{B} = \begin{bmatrix} 0 & B_2 \end{bmatrix}^T$$

where, $B_2 \in \Re^{m\times m}$.

$$\begin{bmatrix} \dot{\mathbf{z}}_1 \\ \dot{\mathbf{z}}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} \dot{\mathbf{z}}_1 \\ \dot{\mathbf{z}}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b_2 \end{bmatrix}u \qquad (32)$$

where, $\mathbf{z}_1 \in \Re^{(n-m)\times 1}$ and $\mathbf{z}_2 \in \Re^{m\times 1}$.
The switching function matrix partitioned compatibly in Z co-ordinate as, $s = c_1\mathbf{z}_1 + c_2\mathbf{z}_2 = 0 \equiv M_s\mathbf{z}_1 + \mathbf{z}_2 = 0$, where $M_s = c_2^{-1}c_1$ and $M_s \in \Re^{1\times(n-m)}$. Using this, null space dynamics of (32) can be written as,

$$\dot{\mathbf{z}}_1 = A_{11}\mathbf{z}_1 + A_{12}\mathbf{z}_2 = (A_{11} - A_{12}M_s)\mathbf{z}_1 \qquad (33)$$

Here M_s is designed to ensure stable $(A_{11} - A_{12}M_s)$.

3.2 Design of SM Control Law

Differentiating equation (31) to get

$$\dot{s} = \mathbf{c}^T(\dot{\mathbf{x}} - \dot{\mathbf{x}}_d) \qquad (34)$$

Power rate reaching law [11] is used which is

$$\dot{s} = -k|s|^\alpha \text{sgn}(s) \qquad (35)$$

From (34) and (35)

$$\begin{aligned}
\mathbf{c}^T(\dot{\mathbf{x}} - \dot{\mathbf{x}}_d) &= -k|s|^\alpha \text{sgn}(s) \\
\mathbf{c}^T\dot{\mathbf{x}} &= -k|s|^\alpha \text{sgn}(s) + \mathbf{c}^T\dot{\mathbf{x}}_d, \qquad (36)
\end{aligned}$$

where k is switching gain. It is chosen to ensure the existence of sliding. The α is tuning parameter such that $0 < \alpha < 1$. By substituting plant dynamics in (36), a control can be synthesized.

3.2.1 SMC for Pitch channel

Neglecting the unknown disturbance term in equation (17),

$$\dot{\mathbf{x}} = A\mathbf{x} + \mathbf{b}u + \mathbf{e}\xi_p \tag{37}$$

Now from (36) and (37) considering k_p and α_p as tuning parameters,

$$\mathbf{c}^T(A\mathbf{x} + \mathbf{b}u + \mathbf{e}\xi_p - \dot{\mathbf{x}}_\mathbf{d}) = -k_p|s|^{\alpha_p}\mathrm{sgn}(s) \tag{38}$$

$$\begin{aligned} u &= (\mathbf{c}^T\mathbf{b})^{-1}[-k_p|s|^{\alpha_p}\mathrm{sgn}(s) + \mathbf{c}^T\dot{\mathbf{x}}_\mathbf{d} - \mathbf{c}^T A\mathbf{x} \\ &- \mathbf{c}^T\mathbf{e}\xi_p] \end{aligned} \tag{39}$$

This is the necessary control that compensates disturbance torque due to cross coupling.
To prove the existence of sliding: from (31) and (34),

$$s\dot{s} = s(\mathbf{c}^T(\dot{\mathbf{x}} - \dot{\mathbf{x}}_\mathbf{d})) \tag{40}$$

From (17) and (39),

$$\begin{aligned} s\dot{s} &= s(\mathbf{c}^T(A\mathbf{x} + \mathbf{b}u + \mathbf{e}\xi_p + \mathbf{e}d_p' - \dot{\mathbf{x}}_\mathbf{d})) \\ s\dot{s} &= s((\mathbf{c}^T A\mathbf{x} + \mathbf{c}^T\mathbf{b}((\mathbf{c}^T\mathbf{b})^{-1}(-k_p|s|^{\alpha_p}\mathrm{sgn}(s) + \mathbf{c}^T\dot{\mathbf{x}}_\mathbf{d} \\ &- \mathbf{c}^T A\mathbf{x} - \mathbf{c}^T\mathbf{e}\xi_p)) + \mathbf{c}^T\mathbf{e}\xi_p + \mathbf{c}^T\mathbf{e}d_p' - \mathbf{c}^T\dot{\mathbf{x}}_\mathbf{d})) \\ s\dot{s} &= -k_p|s|^{\alpha_p}\mathrm{sgn}(s) + \mathbf{c}^T\mathbf{e}d_p' \end{aligned}$$

To satisfy η reachability and to ensure sliding [10], $k_p > (\mathbf{c}^T\mathbf{e}d_p')_{\max}$

3.2.2 SMC for Yaw channel

Similar to the pitch channel, SM controller is developed for yaw channel. Neglecting the unknown disturbance term in equation (30),

$$\dot{\mathbf{x}} = A\mathbf{x} + \mathbf{b}u + \mathbf{e}\xi_y' \tag{41}$$

Now from (36) and (41) with tuning parameters k_y and α_y the control is,

$$\begin{aligned} u &= (\mathbf{c}^T\mathbf{b})^{-1}[-k_y|s|^{\alpha_y}\mathrm{sgn}(s) + \mathbf{c}^T\dot{\mathbf{x}}_\mathbf{d} - \mathbf{c}^T A\mathbf{x} \\ &- \mathbf{c}^T\mathbf{e}\xi_y'] \end{aligned} \tag{42}$$

This is the necessary control that compensates disturbance torque due to cross coupling and uncertainties.
As discussed in pitch control, existence of sliding is guaranteed for yaw dynamics if $k_y > (\mathbf{c}^T\mathbf{e}d_y')_{\max}$

4. SIMULATION RESULTS

If we apply a sine command of varying amplitude to the pitch gimbal and a cosine command with varying amplitude to yaw gimbal and the control ensures the gimbals to track their reference commands then the resultant motion of seeker will be a spiral.

4.1 For Pitch channel

In simulation a sine command of varying amplitude of pitch angle(v_2) was given. The simulation parameter are $k_p = 50$, $\alpha = 0.9$, initial conditions are $\begin{bmatrix} v_2 & \dot{v}_2 \end{bmatrix}^T = \begin{bmatrix} 0.5 & 0 \end{bmatrix}^T$, d_p is external disturbance $= 0.02 sin 5.18t$. The sliding surface matrix designed was $\mathbf{c}^T = [-5 \quad -1]$. Fig. 5-7 shows the simulated results. Fig. 5 shows evolution of v_2. Simulation performance shows that the actual output angle is tracking the desired reference angle properly. Fig. 6 shows the plot of control effort needed. Fig. 7 shows sliding surface.

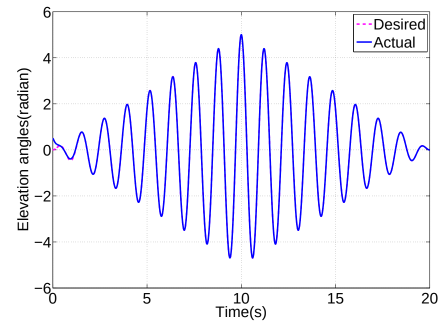


Fig. 5 Actual and desired angles for pitch gimbal

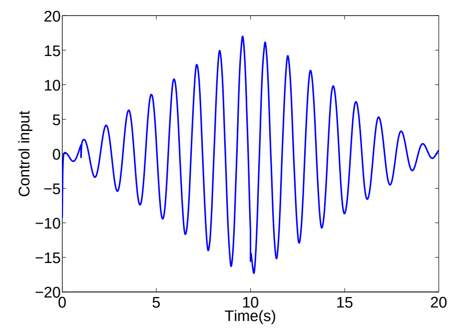


Fig. 6 Control input to the system

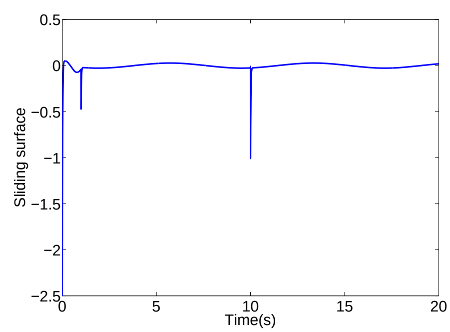


Fig. 7 Sliding surface

4.2 For Yaw channel

In simulation a cosine command of varying amplitude of yaw angle(v_1) was given. The simulation parameters for yaw channel are same as pitch channel, besides the initial conditions as $\begin{bmatrix} v_1 & \dot{v}_1 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$. Fig. 8-10 shows the simulated results. Fig. 8 shows evolution of v_1. Simulation performance shows that the yaw output angle follows the desired one nicely. Fig. 9 shows the plot of control effort needed. Fig. 10 shows that sliding begins very quickly.

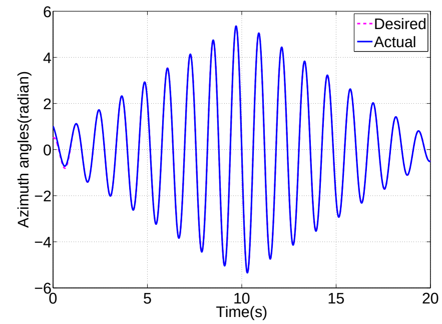


Fig. 8 Actual and desired angles for yaw gimbal

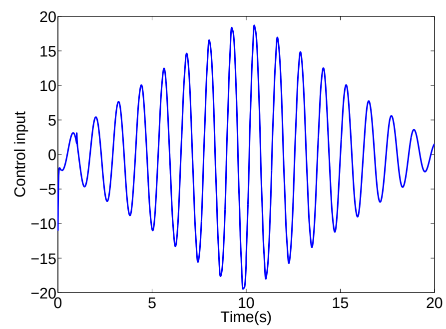


Fig. 9 Control input to the system

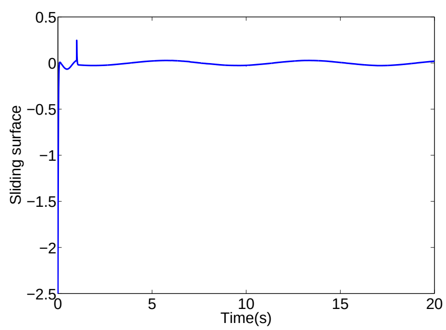


Fig. 10 Sliding surface

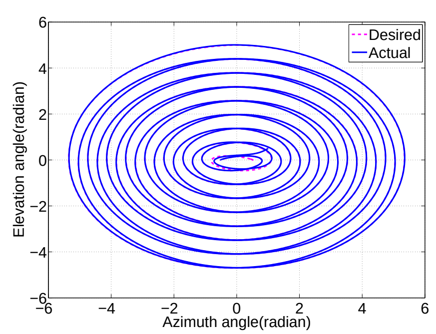


Fig. 11 Spiral motion

Fig. 11 illustrates the spiral motion of the seeker when both yaw and pitch controls are executed together.

5. CONCLUSION

In this paper a sliding mode control has been developed for pitch and yaw channel for the scan phase of seeker. The disturbance torque due to cross coupling effect has been computed analytically to compensate its effect. A sliding mode control ensures robust performance on the face of variation in cross coupling disturbance torque due to parameter variation and external disturbance. Simulation results show that sliding commences quickly yielding robust performance. The control is smooth.

REFERENCES

[1] W.E.Schmitendorf and Y.K. Kao and H.Y. Hwang, “Robust Tracking Controller for a Seeker Scan Loop,” *IEEE Transactions On Control Systems Technology*, Vol. 7, No. 2, 1999.

[2] Won-Sang Ra and Ick-Ho Whang, “A Robust Horizontal Los Rate Estimator for 2-Axes Gimballed Seeker,” *Proceedings of 41st IEEE conference on Decision and Control*, 2002.

[3] H.P. Lee and D.K. Schimidt, “Robust two-degree-of-freedom H_∞ control of a seeker scan loop system,” *IEE Proc.-Control Theory Appl.*, Vol. 149, No. 2, 2002.

[4] H.P. Lee, “Scan loop control design for a spin-stabilized seeker,” *IEE Proc.-Control Theory Appl.*, Vol. 145, No. 2, 1998.

[5] Jacques Waldmann, “Line-of-Sight Rate Estimation and Linearizing Control of an Imaging Seeker in a Tactical Missile Guided by Proportional Navigation,” *IEEE Transactions on Control Systems Technology*, Vol. 10, No. 4, 2002.

[6] Ming Xin and S.N. Balakrishnan and Ernest J. Ohlmeyer, “Integrated Guidance and Control of Missiles with θ-D Method,” *IEEE Transaction On Control Systems Technology*, Vol. 14, No. 6, 2006.

[7] Chun-Liang Lin and Yi-Hsing Hsiao, “Adaptive Feedforward Control for Disturbance Torque Rejection in Seeker Stabiling Loop,” *IEEE Transaction On Control Systems Technology*, Vol. 9, No. 1, 2001.

[8] B.J. Smith and W.J. Schrenk and W.B. Gass and Y.B. Shtessel, “Sliding mode control in a two-axis gimbal system,” *Aerospace Conference*, Vol. 5, 1999.

[9] Vadim Ivanovich Utkin and Juergen Gulder, “Sliding Mode Control in Electromechanical Systems,” *CRC Press*, 2nd edition.

[10] C Edward and S Spurgeon, “Sliding Mode Control: Theory and Applications,” *Taylor and Frank*, 1998.

[11] John Y. Hung and Weibing Gao and James C. Hung, “Variable Structure Control: A Survey,” *IEEE Transactions on Industrial Electronics*, Vol. 40,No. 1, 1993.

[12] Ashish Tewari, “Atmospheric and Space Flight Dynamics: Modeling and Simulation with MATLAB and Simulink,” $Birkh\ddot{a}user$, 2007.